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Searching for a Gap Acceptance Theory Basis for Linear Capacity Models

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Searching for a Gap Acceptance Theory Basis for Linear Capacity Models

CONTENTS

Reference:

AKÇELIK, R. (2022). *Searching for a Gap Acceptance Theory Basis for Linear Capacity Models*. Technical Note. Akcelik & Associates Pty Ltd, Melbourne, Australia.

Accompanying report:

AKÇELIK, R., SHIRKE, C., BESLEY, M., ESPADA, I. and BILLINGHURST, D. (2022). *A Comparative Analysis of Exponential and Linear Roundabout Capacity Models Using HCM Research Data*. Technical Note. Akcelik & Associates Pty Ltd, Melbourne, Australia.

ABSTRACT

This report presents an investigation to explore if a *linear capacity model* can be derived as a gap-acceptance capacity model assuming a uniform or linear arrival headway distribution of the opposing traffic stream. The uniform and linear arrival headway distributions are introduced and the gap-acceptance capacity models based on these headway distributions are presented. The derivations follow the *signal analogy* method used by the author in deriving the gap-acceptance capacity equations used in the SIDRA INTERSECTION software. These uniform and linear headway distributions are not realistic given the random nature of arrival headways including bunching considerations. It is shown that both headway distributions result in non-linear gap-acceptance capacity models. Although a form close to a linear model could be obtained by choosing low values of critical gap, these chosen critical gap values were not realistic when compared with observed values indicated by the HCM and Australian research data. The report also discusses application of a simplified *SIDRA geometry method* for estimating the parameters of the HCM (Siegloch) exponential roundabout capacity model. Results are promising when applied to the HCM roundabout capacity research data with favourable comparisons with the TRL linear empirical model. Alternative calibration methods for the TRL linear roundabout capacity model and HCM (Siegloch) exponential model are also discussed. It is shown that satisfactory results are obtained when the method was applied to the HCM model with a simplified version of the SIDRA geometry method added.

1 Introduction

This report presents an investigation to explore if a *linear capacity model* can be derived as a gap-acceptance capacity model assuming a *uniform* or *linear* arrival headway distribution of the opposing traffic stream. Best fit regression analyses based on statistical methods may provide reasonable empirical capacity models with linear or non-linear form. On the other hand, gap acceptance theory applied to drivers at roundabouts and sign-controlled (priority) intersections provides models with a causal basis that help modellers understand and calibrate the models they employ for their impact assessments and design decisions.

Recently a question was asked about whether all gap-acceptance capacity models are of the exponential form. A key assumption in deriving a gap-acceptance capacity model is the form of arrival headway distribution of the opposing traffic stream. For roundabout capacity models, the *circulating* stream is the *opposing (conflicting)* stream. Since most gap-acceptance capacity models use exponential forms of arrival headway distribution, the resulting capacity models have exponential form (Akçelik 1994, 2007, 2018).

The *TRL roundabout capacity model* was developed as an empirical linear model on the premise that gap acceptance modelling does not work for roundabouts (Kimber 1980, 1985, 1989). On this basis, regression methods were used to derive a linear roundabout capacity model using capacity data (hence the name *empirical* model). The investigation reported here was prepared particularly with this assertion in mind.

The TRL model applies the linear model as an *approach-based* model using parameters describing the roundabout geometry, namely inscribed diameter, entry radius, entry angle, entry lane width, approach half width and effective flare length. The search for a linear gap-acceptance model reported here considers *fixed y-intercept and slope parameters*. However, if a gap-acceptance basis could be found for a linear capacity model, this would mean that some implied *follow-up headway* and *critical gap (headway)* values could be determined for the model, and then the y-intercept and slope parameters could be related to geometry parameters by association with the TRL model. The model would then be similar to the SIDRA roundabout capacity model (Akçelik 2011, 2012, 2017a,b; 2018; Akçelik and Besley 2005; Akçelik, Chung, and Besley 1997; Akçelik and Troutbeck 1991) which was derived as a gap-acceptance capacity model using follow-up headway and critical gap parameters related to roundabout geometry parameters on a *lane-by-lane* modelling basis.

The SIDRA roundabout capacity model is based on a *bunched exponential* (M3) arrival headway distribution (Akçelik 1994, 2007, 2018) whereas the Highway Capacity Manual (HCM) roundabout capacity model is based on a simple negative exponential model (M1). The exponential arrival headway models are described in *Section 2*.

The Siegloch M1, Akçelik M1 and Traditional M1 exponential gap-acceptance capacity models are described in *Section 3*. These are based on the negative exponential headway distributions model (M1) but the capacity equations are derived using different methods. The capacity estimates from these models are shown to be very close.

This report uses the negative exponential form of arrival headway distribution as a basis for comparison of capacity models since the HCM (Siegloch M1) roundabout capacity model is based on this headway distribution, and the investigation reported here used the HCM roundabout research data (FHWA 2015).

The Linear capacity model is described in *Section 4*. There have been discussions about the linear vs non-linear (especially the exponential) forms of roundabout capacity model, comparing the TRL linear model vs the HCM exponential model (Lenters and Rudy 2010; Johnson and Lin, 2018). A detailed exploration of this issue using the HCM roundabout research data has been presented in a recent report (Akçelik, et al 2022).

The uniform and linear arrival headway distributions introduced in this report are described in *Section 5*. Gap-acceptance capacity models based on the uniform and linear headway distributions are presented in

Section 6. It should be noted that, in SIDRA INTERSECTION, estimates from roundabout and sign-controlled intersection capacity models incorporate the following effects that are not included in the basic capacity formulations given in this report:

(i) The follow-up headway (and the critical gap as a result) is adjusted by applying a flow-weighted ***Gap Acceptance Factor*** to allow for Movement Classes representing different types of vehicle groups in the opposed stream.

(ii) The opposing / circulating stream flow rate is adjusted by applying a flow-weighted ***Opposing Vehicle Factor*** for Movement Classes in the opposing / circulating stream,

(iii) A ***minimum capacity*** is applied at very high opposing / circulating flow rates.

(iv) For roundabouts, to help with modelling the effects of unbalanced flow conditions at high demand flows:

- o the unblocked time ratio is adjusted by an ***O-D factor*** to allow for the origin-destination and approach queuing pattern of the component streams of circulating flow, and
- o the follow-up headway of the entry stream is adjusted for the ***arrival flow to circulating flow ratio***.

In *Section 7*, a method is described for estimating the follow-up headway and critical gap parameters as a function of roundabout geometry parameters in order to determine the HCM (Siegloch M1) roundabout capacity model parameters. A simplified version of the SIDRA roundabout capacity model is used for this purpose (referred to as the *basic SIDRA geometry method*).

Calibration methods for the linear and exponential gap-acceptance capacity models are discussed in *Section 8*. The methods are applied to the TRL linear model and the HCM (Siegloch M1) model with the basic SIDRA geometry method applied.

2 Exponential Arrival Headway Distributions

Three different models of exponential arrival headway distribution, namely the negative exponential (M1), shifted negative exponential (M2) and bunched exponential (M3) models, have been discussed and used extensively in the literature as models of random arrivals. The bunched exponential distribution of arrival headways (M3) was proposed by Cowan (1975, 1984, 1987). The shifted negative exponential model (M2) is normally used for single-lane traffic only. The bunched exponential distribution (M3) offers improved accuracy in the prediction of small arrival headways. These arrival headway distributions and the gap-acceptance capacity models based on their use are discussed in Akçelik (2007) and Luttinen (1999, 2003). The following is a summary of these exponential headway distributions.

The cumulative distribution function, F(t), for the **bunched exponential** distribution of arrival headways, representing the *probability of a headway less than t seconds*, is:

$$F(t) = 1 - \varphi \exp(-\lambda (t - \Delta)) \quad \text{for } t \geq \Delta \qquad (2.1)$$
$$= 0 \quad \text{for } t < \Delta$$

where

Δ = average intrabunch (minimum) headway (seconds),

φ = proportion of free (unbunched) vehicles, and

λ = a model parameter calculated as:

$$\lambda = \varphi q_s / (1 - \Delta q_s) \quad \text{subject to } q_s \leq 0.98 / \Delta \qquad (2.2)$$

where q_s is the total arrival flow rate in all lanes of the opposing stream in passenger car units per second (pcu/s).

The probability density (frequency) function of arrival headways for the bunched exponential model, representing the *probability of a headway of t seconds*, is:

$$f(t) = \varphi \lambda \exp(-\lambda (t - \Delta)) \quad \text{for } t > \Delta \qquad (2.3)$$
$$= 1 - \varphi \quad \text{for } t = \Delta$$
$$= 0 \quad \text{for } t < \Delta$$

According to the bunched exponential model, the traffic stream consists of:

(i) *bunched vehicles* with all intrabunch headways equal to the minimum arrival headway, Δ (proportion of bunched vehicles = $1 - \varphi$), and

(ii) *free vehicles* with headways greater than the minimum arrival headway, Δ (thus, the proportion of free vehicles, φ, represents the unbunched vehicles with randomly distributed headways).

The M1 and M2 models can be derived as special cases of the M3 model through simplifying assumptions about the bunching characteristics of the arrival stream as shown below.

Negative exponential (M1) model:

$$\Delta = 0 \qquad (2.4)$$
$$\varphi = 1.0$$

Therefore:

$$\lambda = q_s \qquad (2.5)$$

Shifted negative exponential (M2) model:

$$\varphi = 1.0 \qquad (2.6)$$

Therefore:

$$\lambda = q_s / (1 - \Delta q_s) \quad \text{subject to } q_s \leq 0.98 / \Delta \qquad (2.7)$$

The maximum value of $\Delta\ q_s = 0.98$ is used for computational reasons.

Thus, models M1 and M2 assume no bunching ($\varphi = 1.0$) for all levels of arrival flows. On the other hand, model M3 can be used either with a known (measured) value of φ, or more generally, using a bunching model that estimates the value of φ as a function of the arrival flow rate.

The bunching models are discussed in Akçelik (2007). The bunching model with a delay parameter used in the SIDRA gap acceptance model is given by:

$$\varphi = (1 - \Delta\ q_s) / [1 - (1 - k_d)\ \Delta\ q_s] \qquad \textit{subject to } 1.0 \geq \varphi \geq 0.10 \tag{2.8}$$

where Δ and q_s, are as in *Equations (2.1) and (2.2)*, and k_d is the delay parameter.

According to *Equation (2.8)*, bunching increases (proportion unbunched, φ decreases) with increasing flow rate.

The graphs showing the cumulative and frequency distributions for M1, M2 and M3D exponential headway distribution functions are given in *Figure 1*. M3D model refers to M3 headway distribution using the proportion of unbunched vehicles estimated from *Equation (2.8)*.

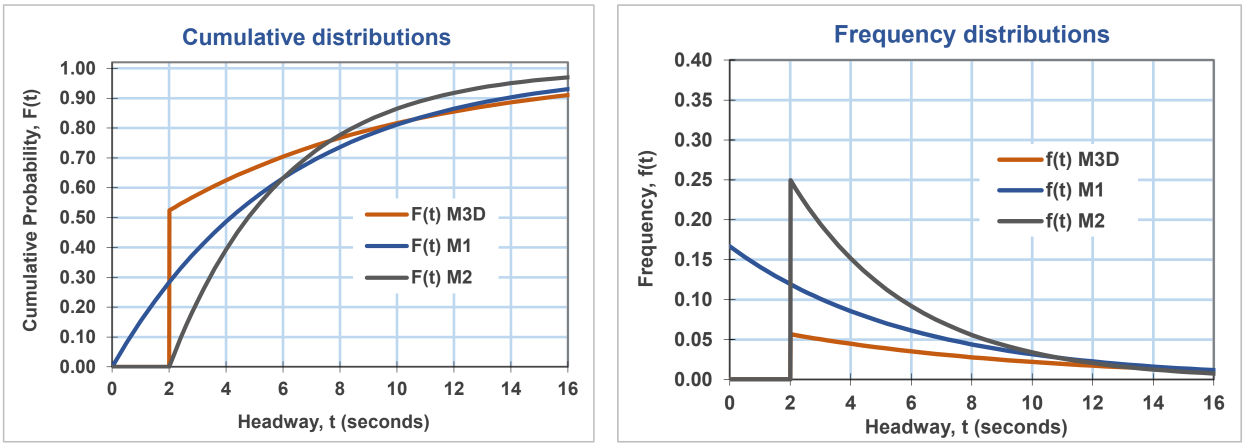


Figure 1 - Cumulative distributions and frequency distributions for M1, M2 and M3D (exponential) headway distribution functions

3 Exponential Gap-acceptance Capacity Models

Numerous exponential capacity models based on different methods of derivation and different exponential headway distributions (bunched exponential (M3D), simple negative exponential (M1) and shifted negative exponential (M2) described in *Section 2*) are given in (Akçelik 2007). In this report, only the following three capacity models based on the simple negative exponential (M1) headway distribution model will be considered for model comparison purposes.

The capacity models given here are expressed specifically for roundabouts, therefore capacity is expressed as a function of the *circulation flow* (hence the subscript c). However, discussions apply to all gap acceptance processes (minor road movements at unsignalised intersections controlled by stop and give-way signs, and filter / permitted turn and slip / bypass lane movements at signalised intersections) as well. The terms *opposing flow* and *conflicting flow* have been used for priority movements in gap-acceptance processes.

Siegloch M1 Model

The Siegloch (1973) capacity model, which is used in the German guidelines (Brilon 1988, Brilon and Grossman 1991), and forms the basis of the HCM roundabout capacity model (TRB 2016), assumes a negative exponential model of arrival headways (M1), and is given as:

$$Q_g = (3600 / t_f)\ exp\ (-t_o\, q_s) \quad (3.1)$$

$$t_o = t_c - 0.5\ t_f. \quad (3.2)$$

where t_o is the unused part of average accepted headway, t_f is the follow-up headway, t_c is the critical gap (headway) and q_s is the circulating (conflicting / opposing) flow in passenger car units per second (pcu/s).

The HCM roundabout capacity model uses parameters $A = 3600 / t_f$ and $B = t_o / 3600$:

$$Q_g = A\ exp\ (-B\, q_c) \quad (3.3)$$

where $q_c = 3600\ q_s$ is the circulating flow rate in passenger car units per hour (pcu/h).

From *Equations (3.1) to (3.3)*, it can be seen that the critical gap can be estimated from parameter B using:

$$t_c = 3600\ B + 0.5\ t_f \quad (3.4)$$

The slope of the exponential capacity curve given by *Equation (3.1)* is:

$$dQ_g/dq_c = -A\ B\ exp\ (-B\ q_c) = -B\ Q_g \quad (3.5)$$

For single-lane roundabouts, the HCM model parameters are $A = 1380$ ($t_f = 2.61$) and $B = 0.00102$ ($t_c = 4.98$).

Akçelik M1 Model

For the Akçelik M1 model derived using signal analogy method (Akçelik 1994, 2007, 2018), the simple negative exponential model of headway distribution (M1) is assumed using $\Delta_m = 0$, $\varphi_m = 1.0$ and $\lambda = q_s$ as given in *Equations (2.4) and (2.5)*:

$$Q_g = (3600 / t_f)\ (1 + 0.5\ t_f\, q_s)\ exp\ (-t_c\, q_s) \quad (3.6)$$

where parameters are as in *Equations (3.1) and (3.2)*.

The concept signal analogy concept used in deriving the gap-acceptance capacity models is shown in *Figure 2*.

Traditional M1 Model

The Traditional M1 model is also based on the simple negative exponential model of headway distribution (M1):

$$
\begin{aligned}
Q_g &= 3600\, q_s\, exp\, (-t_c\, q_s) / (1 - exp\, (-t_f\, q_s)) && \textit{for } q_s > 0 \\
&= (3600 / t_f) && \textit{for } q_s = 0
\end{aligned}
\tag{3.7}
$$

Refer to Tanner (1962, 1967) and Troutbeck (1989) for more general forms of this model.

This capacity model is used in the HCM (TRB 2016, Chapter 20) as the *potential capacity* for Two-Way Stop Control. It should be noted that HCM applies *impedance factors* that reduce the potential capacity for entry streams that give way to movements which themselves are subject to gap-acceptance. In determining the opposing flow rates, HCM also applies other factors to increase some opposing flow rates (the flow rates of opposed turns from the major road are doubled), therefore decreasing the potential capacity significantly.

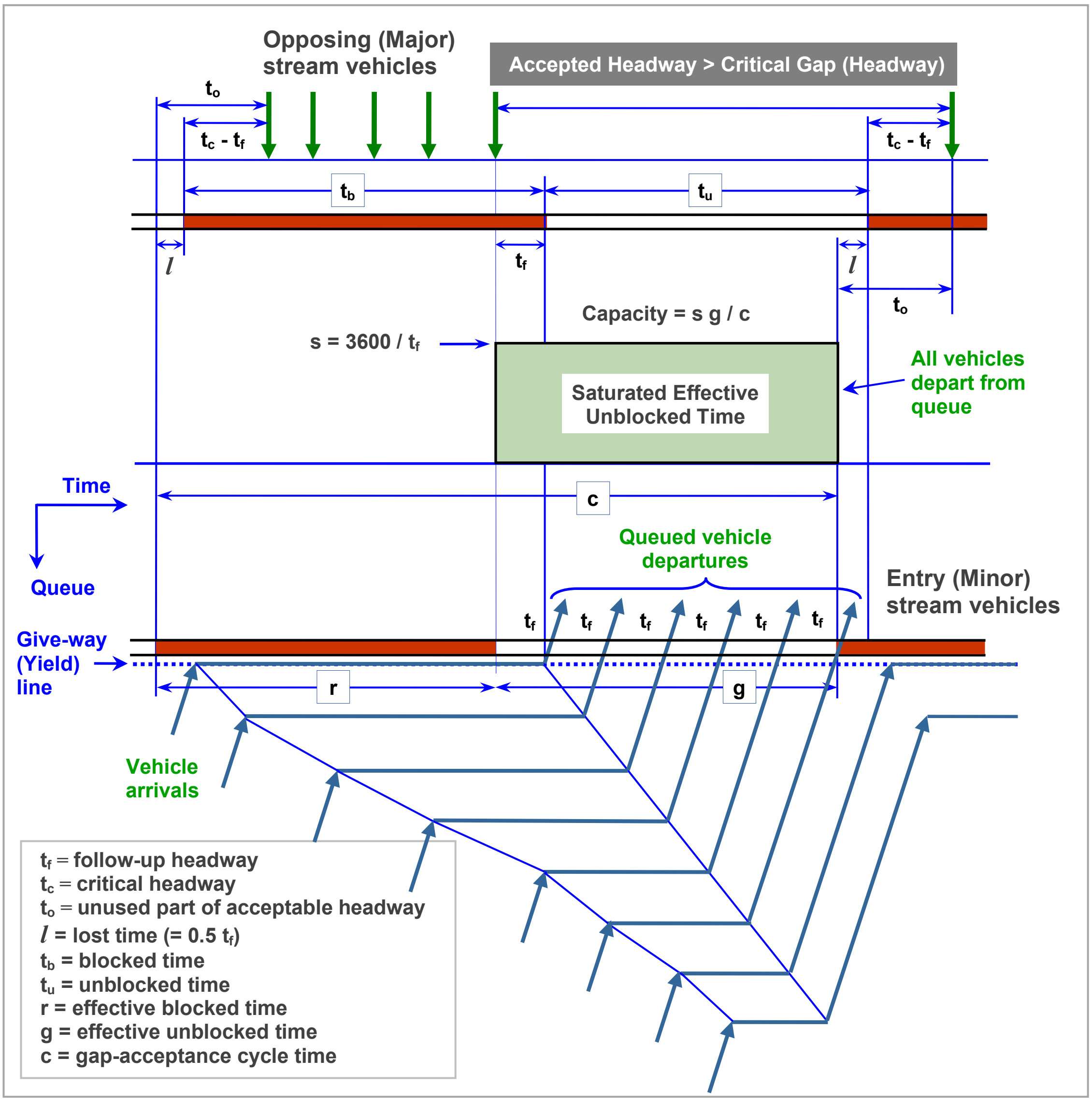
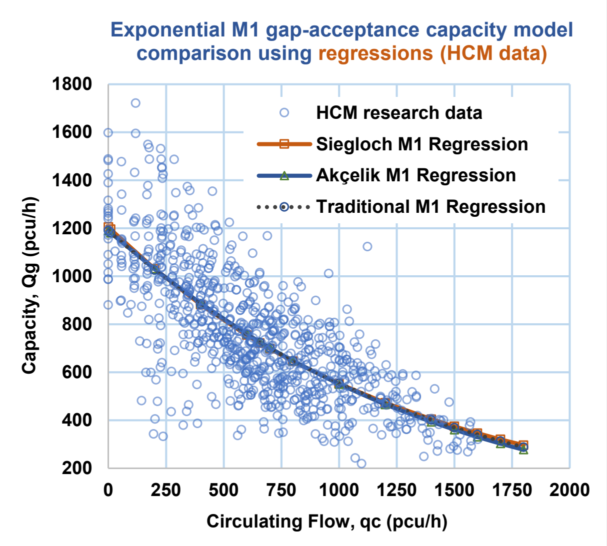


Figure 2 - Gap-acceptance capacity signal analogy concept



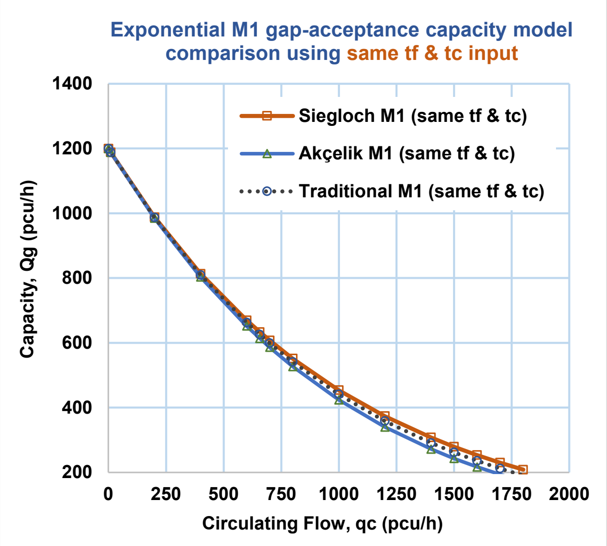


Figure 3 - Comparison of three exponential capacity models based on the simple negative exponential (M1) headway distribution

Comparison of Models

Three models given in this section based on the same exponential arrival headway (M1) distribution but produced from different model derivation methods give very close results. *Figure 3* shows comparison of these three models from best fit regression using the HCM roundabout research data with all sites included (FHWA 2015). Parameter values from regression are shown in *Figure 3* and given in *Table 1*. For the graphs using the same follow-up headway and critical gap input, $t_f = 3.0$ s and $t_c = 5.0$ s ($t_f / t_c = 0.60$) were used.

Table 1 - Comparison of three exponential models based on the simple negative exponential (M1) headway distribution

Regression type	t_f	t_c	t_f / t_c	RMSE	R^2
Akçelik M1	3.027	4.039	0.75	180.4	0.546
Traditional M1	3.025	4.163	0.73	180.3	0.546
Siegloch M1	2.988	4.302	0.69	180.2	0.547

4 Linear Capacity Model

The basic ***Linear roundabout capacity model*** form is:

$$Q_g = A + B\,q_c \tag{4.1}$$

where $q_c = 3600\ q_s$ is the circulating flow rate in pcu/h.

Parameters A (y-intercept) and B (slope) in *Equation (4.1)* have constant values. Parameter A can be expressed as:

$$A = 3600 / t_f \tag{4.2}$$

where t_f can be treated as an implied follow-up headway (seconds).

Figure 4 shows comparison of the Linear and the Siegloch M1 exponential capacity models using HCM roundabout capacity research data (FHWA 2015). The parameters for these best fit regression models are A= 1115 (t_f = 3.229 s) and B = -0.5570 for the Linear model (RMSE = 183.5) and A = 1205 and B = 0.00078 (t_f = 2.988 s, t_c = 4.302 s) for the Siegloch M1 exponential model (RMSE = 180.2). The critical gap, t_c is not determined for the Linear model.

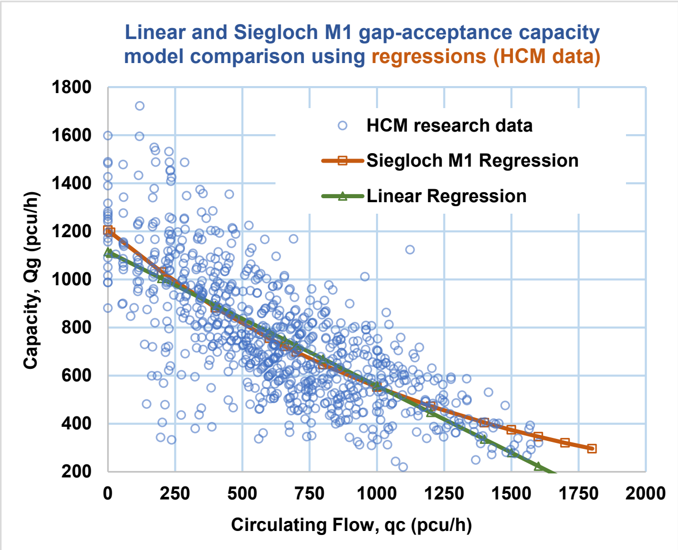


Figure 4 - Comparison of linear model and the Siegloch M1 exponential capacity models using HCM roundabout capacity research data

5 Uniform and Linear Arrival Headway Distributions

Two arrival headway distributions conceived by the author for analyses in search of a linear gap-acceptance capacity model are described in this section.

5.1 Uniform Arrival Headway Distribution

The cumulative and frequency distribution functions for the *uniform* arrival headway distribution model (M-U) are:

$$F(t) = f_u\, t \quad \textit{for } 0 \le t \le t_m \qquad (5.1)$$
$$= 1.0 \quad \textit{for } t > t_m$$

$$f(t) = f_u \quad \textit{for } 0 \le t \le t_m \qquad (5.2)$$
$$= 0 \quad \textit{for } t > t_m$$

where f_u is the probability of a headway of t seconds (constant) and t_m is the maximum headway (sec) where the cumulative probability of opposing stream headways is 1.0.

The values of parameters t_m and f_u depend on the opposing (conflicting / circulating) flow rate:

$$t_m = 2 / q_s = 2\, h_a \qquad (5.3)$$
$$f_u = 1 / t_m = 0.5\, q_s \qquad (5.4)$$
$$h_a = 1 / q_s \qquad (5.5)$$

where q_s is the total arrival flow rate in all lanes of the opposing stream (pcu/s) and h_a is the average headway (sec).

The cumulative and frequency distributions for the *uniform* (M-U) headway distribution function are given in *Figure 5*. The negative exponential (M1) distribution is included for comparison.

Uniform headway distribution means:

- The probability of small and large headways is the same for headways in the range for $0 \le t \le t_m$ where t_m is twice the average headway, and.
- the probability of a headway larger than twice the average headway ($t_m = 2\, h_a$) is zero.

For example, for circulating flow of $q_c = 900$ pcu/h ($q_s = 0.250$ pcu/s), the average headway is $h_a = 4.0$ s, therefore $t_m = 8.0$, and the probability of headways in the range 0 to 8.0 seconds is $f_u = 0.125$ and the probability of headways larger than 8.0 seconds is zero.

This distribution is clearly unrealistic and in contrast with the generally accepted exponential headway distributions described in *Section 2*.

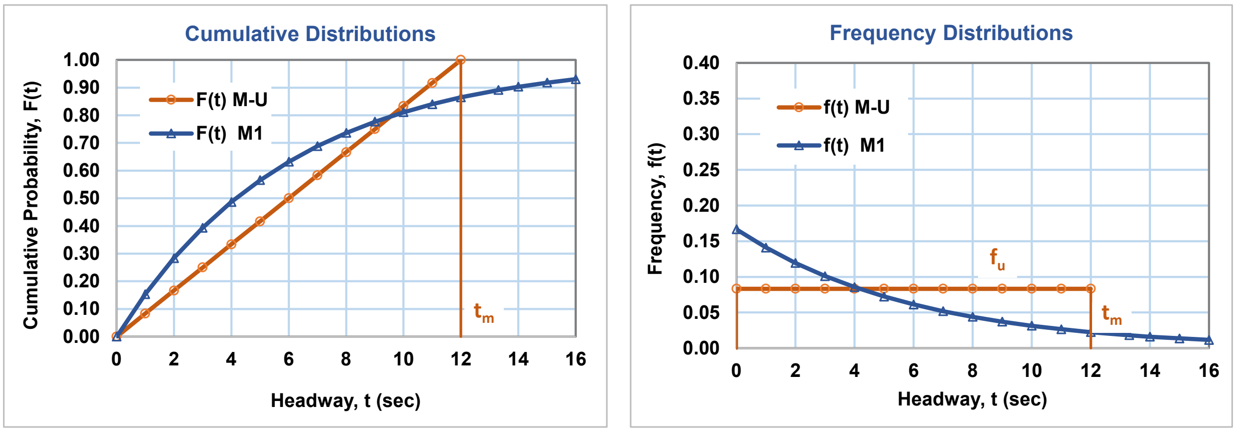


Figure 5 - Cumulative distributions and frequency distributions for uniform (M-U) headway distribution function (negative exponential (M1) distribution included for comparison)

5.2 Linear Arrival Headway Distribution

The cumulative and frequency distribution functions for the *linear* arrival headway distribution model (M-L) are:

$$F(t) = f_o\,t + b\,t^2 \qquad \textit{for } 0 \le t \le t_m \tag{5.6}$$
$$= 1.0 \qquad \textit{for } t > t_m$$

$$f(t) = f_o + b\,t \qquad \textit{for } 0 \le t \le t_m \tag{5.7}$$
$$= 0 \qquad \textit{for } t > t_m$$

where f_o is the probability of a headway of t = 0 seconds and t_m is the maximum headway (sec) where the cumulative probability of opposing stream headways is 1.0.

The values of parameters t_m and f_u depend on the opposing (conflicting / circulating) flow rate:

$$t_m = 3 / q_s = 3\,h_a \tag{5.8}$$
$$f_o = 2 / t_m = (2/3)\,q_s \tag{5.9}$$
$$b = f_o / t_m = 0.5\,f_o^2 \tag{5.10}$$
$$h_a = 1 / q_s \tag{5.11}$$

where q_s is the total arrival flow rate in all lanes of the opposing stream (pcu/s) and h_a is the average headway (sec).

The cumulative and frequency distributions for the *linear* (M-L) headway distribution function are given in *Figure 6*. The negative exponential (M1) distribution is included for comparison.

Compared with the uniform headway distribution, the linear distribution is more realistic, but still it is only an approximation to the more widely accepted exponential headway distributions.

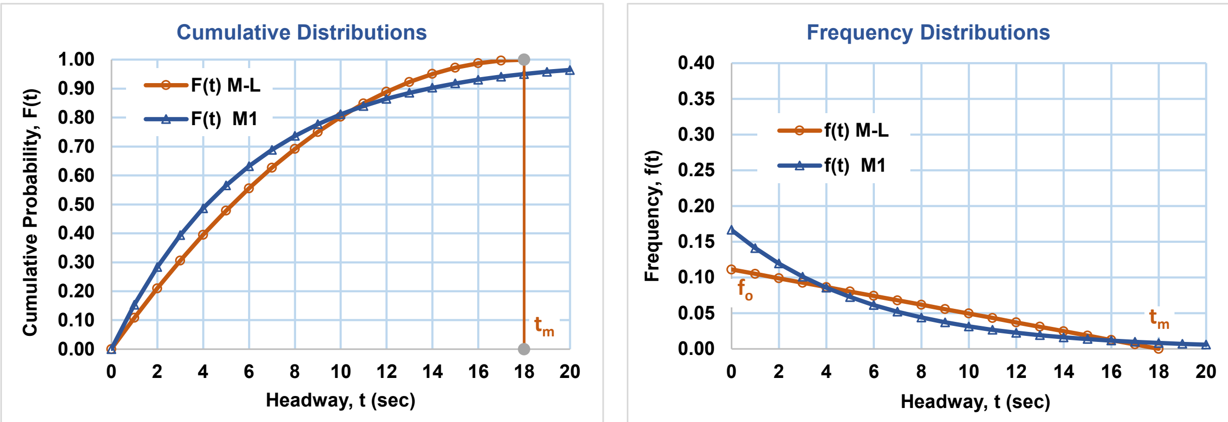


Figure 6 - Cumulative distributions and frequency distributions for linear (M-L) headway distribution function (negative exponential (M1) distribution included for comparison)

6 Gap-Acceptance Capacity Models for Uniform and Linear Headway Distributions

The derivation of the capacity equations for the uniform and linear headway distributions given in this section follow the "*signal analogy*" method used by the author in deriving the gap-acceptance capacity equations used in SIDRA (Akçelik 1994, 2007, 2018). Refer to *Figure 2* in *Section 3*.

Similarly to the capacity equation for signalised intersection, gap-acceptance capacity, Q_q can be expressed as:

$$Q_q = s\ u = (3600 / t_f)\ u \qquad (6.1)$$

where

s = 3600 / t_f = saturation flow rate (veh/h),

t_f = follow-up headway of the opposed stream (roundabout entry, minor road at sign-controlled intersections, or filter / permitted turns at signals) (seconds),

u = *unblocked time ratio* (ratio of the effective unblocked time to the average gap-acceptance cycle time) which is the time in the gap-acceptance cycle when vehicles depart from the queue,

g = effective unblocked time (seconds),

c = average gap-acceptance cycle time in the circulating or exiting stream (seconds).

The follow-up headway is the saturation headway, i.e. the minimum headway between vehicles that is achieved when they are departing from the queue. For example, t_f = 2.5 seconds implies a saturation flow rate of s = 1440 veh/h. This is the maximum capacity that can be achieved when the opposing flow is close to zero.

The capacity is reduced from this value with increased opposing flow rates resulting in decreased unblocked time ratio (u) as shown in *Figure 7*. The critical gap, t_c plays a key role in determining the slope of the capacity line. The capacity decreases more quickly if the critical gap is larger (drivers need larger acceptable gaps).

All gap-acceptance capacity models predict decreased capacity with increased circulating flow. This is due to the *blocked periods* that result when the opposed stream vehicles cannot find an acceptable gap in the circulating stream. *Unblocked periods* represent the times when queued or unqueued vehicles can enter the circulating road when a gap is available in the circulating flow (capacity is obtained when all opposed stream vehicles are queued). Blocked and unblocked periods are like *effective red and green times* at signals.

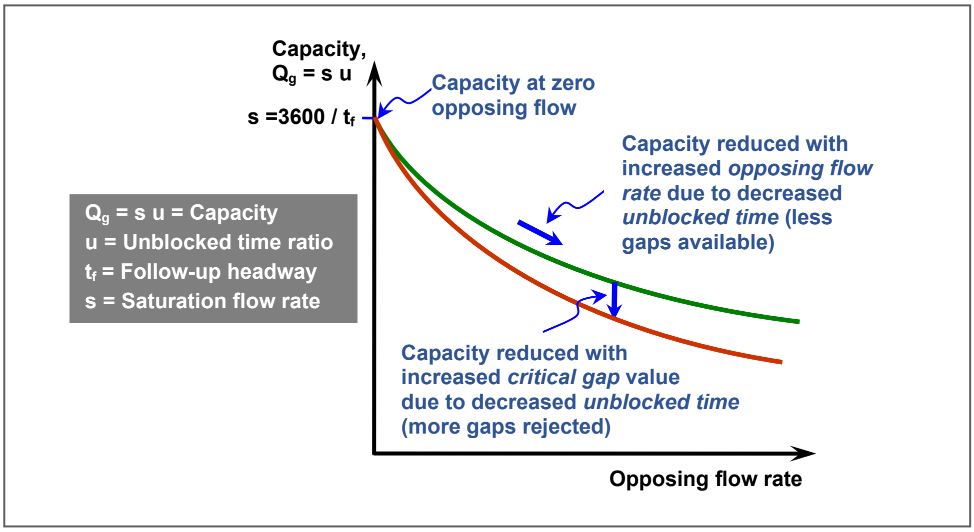


Figure 7 - Gap acceptance capacity

6.1 Gap-Acceptance Capacity Model for Uniform (M-U) Arrival Headway Distribution

The capacity model parameters for the uniform (M-U) headway distribution defined in *Section 4.1* are derived as follows.

Average unblocked time, t_u:

$$t_u = h_u - t_c = 0.5\ (t_m - t_c) \tag{6.2}$$

where $h_u = 0.5\ (t_m + t_c)$ is the average acceptable opposing stream headway (average for $t_c \le t \le t_m$ and t_c is the critical gap).

Average blocked time, t_b is found as an average considering blocks of k vehicles and as a function of the average blocking headway, $h_b = 0.5\ t_c$ (average for $0 \le t < t_c$). It is given by

$$\begin{aligned} t_b &= t_c + 0.5\ t_c\ F(t_c) / (1 - F(t_c)) \\ &= t_c\ (1 + 0.5\ t_c / (t_m - t_c)) \end{aligned} \tag{6.3}$$

where $F(t_c) = f_u\ t_c$ is the probability of a blocked headway (probability of $t < t_c$) and $(1 - F(t_c)) = 1 - f_u\ t_c$ is the probability of an unblocked headway (probability of $t \ge t_c$).

Effective unblocked time, g allowing for a lost time of $l = 0.5\ t_f$:

$$g = t_u + t_f - l = t_u + 0.5\ t_f = 0.5\ (t_m - t_c + t_f) \tag{6.4}$$

Effective blocked time, r:

$$r = t_b - t_f + l = t_b - 0.5\ t_f \tag{6.5}$$

Gap-acceptance cycle time, c as the sum of unblocked and blocked times, or the effective unblocked and effective blocked times:

$$\begin{aligned} c &= t_u + t_b = g + r \\ &= 0.5\ (t_m + t_c + 0.5\ t_c^2 / (t_m - t_c)) \end{aligned} \tag{6.6}$$

Unblocked time ratio, u:

$$u = g / c = (t_m - t_c + t_f) / (t_m + t_c + 0.5\ t_c^2 / (t_m - t_c)) \tag{6.7}$$

From *Equations (6.1) and (6.7)*, the gap-acceptance capacity is given by:

$$Q_g = s\ u = (3600 / t_f)\ (t_m - t_c + t_f) / (t_m + t_c + 0.5\ t_c^2 / (t_m - t_c)) \tag{6.8}$$

Putting $t_m = 2 / q_s$ in *Equation (6.8)*, the uniform headway (M-U) distribution can be seen not to produce a linear model of capacity as a function of the opposing / circulating flow rate. A form close to a linear model can be obtained by choosing a low value of critical gap in *Equation 6.8*.

Figure 8 compares the gap-acceptance capacity model based on the uniform headway (M-U) distribution with a basic linear model from best fit regression using the HCM roundabout capacity research data (FHWA 2015) as discussed in *Section 4*. The follow-up headway value chosen is 3.229 s (A = 1115) based on the linear regression model. The HCM research data indicates $t_c = 1.8\ t_f$ (or $t_f / t_c = 0.556$) as a reasonable relationship to estimate the critical gap, t_c from known follow-up headway, t_f (Akçelik, et al 2022). When the large value of $t_c = 5.812$ s found from this relationship is used in *Equation 6.8*, the non-linear nature of the M-U capacity model is seen clearly in *Figure 8*.

When $t_c = 4.302$ s found for the Siegloch M1 regression model is used, the M-U capacity curve is closer to the Linear regression model line. By varying the critical gap value, $t_c = 3.500$ s is found to give a very close result to the Linear regression model line. However, the resulting $t_c / t_f = 1.08$ is too low (or $t_f / t_c = 0.92$ is too high) compared with the values indicated by the HCM research data.

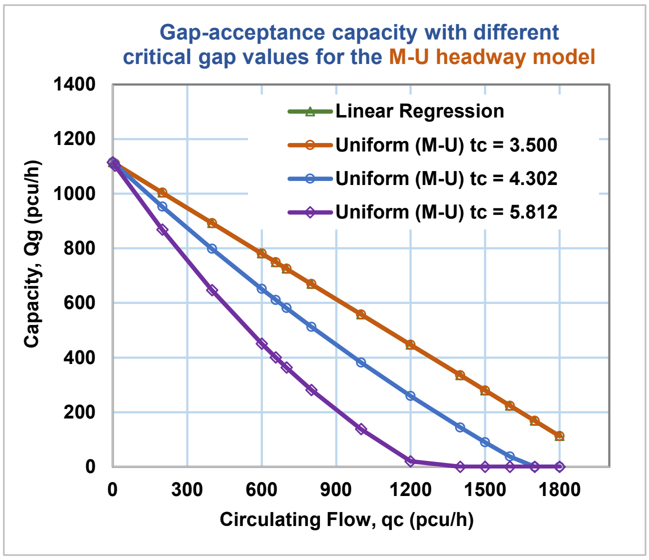


Figure 8 - Comparison of the gap-acceptance capacity model based on the uniform headway distribution (M-U) with a basic Linear model from regression using the HCM roundabout capacity research data using different critical gap, t_c values

6.2 Gap-Acceptance Capacity Model for Linear (M-L) Arrival Headway Distribution

The capacity model parameters for the uniform (M-L) headway distribution defined in *Section 4.2* are derived using the signal analogy method in a way similar to the M-U capacity model given in *Section 6.1*.

Average unblocked time, t_u:

$$t_u = h_u - t_c = - t_c + ((1/3)\ (t_m + t_c) - (2/3)\ t_c^2 / t_m) / (1 - t_c / t_m) \tag{6.9}$$

where h_u is the average acceptable opposing stream headway (average for $t_c \le t \le t_m$) and t_c is the critical gap.

Average blocked time, t_b is found as an average considering blocks of k vehicles and as a function of the average blocking headway, $h_b = t_c / 3$ (average for $0 \le t < t_c$). It is given by:

$$t_b = t_c + (1/3)\ t_c\ F(t_c) / (1 - F(t_c)) \tag{6.10}$$

where $F(t_c) = f_o\ t_c - 0.25\ f_o^2\ t_c^2$ is the probability of a blocked headway (probability of $t < t_c$) and $(1 - F(t_c))$ is the probability of an unblocked headway (probability of $t \ge t_c$).

Effective unblocked time, g allowing for a lost time of $l = 0.5\ t_f$:

$$g = t_u + t_f - l = t_u + 0.5\ t_f \tag{6.11}$$

Effective blocked time, r:

$$r = t_b - t_f + l = t_b - 0.5\ t_f \tag{6.12}$$

Gap-acceptance cycle time, c as the sum of unblocked and blocked times, or the effective unblocked and effective blocked times:

$$\begin{aligned} c &= t_u + t_b = g + r \\ &= ((1/3)\ (t_m + t_c) - (2/3)\ t_c^2 / t_m) / (1 - t_c / t_m) + (1/3)\ t_c\ F(t_c) / (1 - F(t_c)) \end{aligned} \tag{6.13}$$

Unblocked time ratio, u:

$$u = g / c = \tag{6.14}$$

From *Equations (6.1) and (6.14)*, the gap-acceptance capacity is given by:

$$Q_g = s\,u = = (3600 / t_f)\,(t_u + 0.5\,t_f) / [\,((1/3)\,(t_m + t_c) - (2/3)\,t_c^2 / t_m) / (1 - t_c / t_m) + (1/3)\,t_c\,F(t_c) / (1 - F(t_c))\,] \tag{6.15}$$

Putting $t_m = 3 / q_s$ in *Equation (6.15)*, the linear headway (M-L) distribution can be seen not to produce a linear model of capacity as a function of the opposing / circulating flow rate. A form close to a linear model can be obtained choosing a low value of critical gap in *Equation 6.15*.

Figure 9 compares the gap-acceptance capacity model based on the linear (M-L) headway distribution with a basic linear model from best fit regression using the HCM roundabout capacity research data (FHWA 2015) as discussed in *Section 4*. As in the case of the gap-acceptance capacity model based on the M-U headway distribution, the follow-up headway value chosen is 3.229 s (A = 1115) based on the linear regression model. The large value of t_c = 5.812 s used in *Equation 6.15* shows the non-linear nature of the M-L capacity model clearly in *Figure 9*.

When t_c = 4.302 s found for the Siegloch M1 regression model is used, the M-U capacity curve is closer to the linear regression model line. By varying the critical gap value, t_c = 3.950 s is found to give a close result to the linear regression model line. However, as in the case of the gap-acceptance capacity model based on the M-U headway distribution, the resulting t_c / t_f = 1.21 is low (or t_f / t_c = 0.83 is high) compared with the values indicated by the HCM research data.

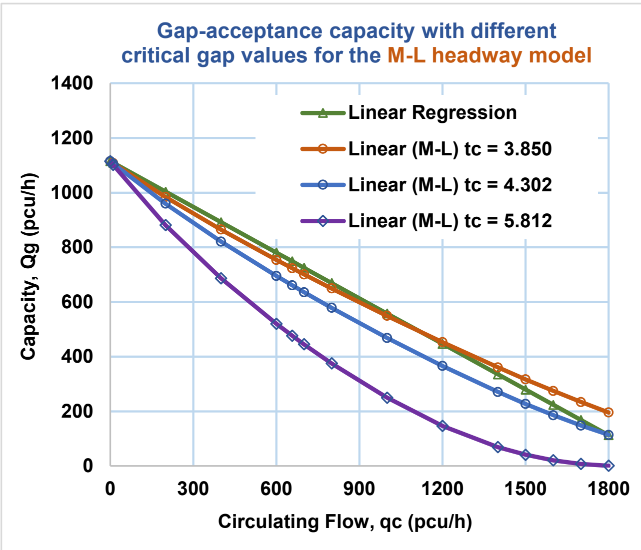


Figure 9 - Comparison of the gap-acceptance capacity model based on the linear headway distribution (M-L) with a basic Linear model from regression using the HCM roundabout capacity research data using different critical gap, t_c values

7 Adding Geometry Parameters to the HCM (Siegloch M1) Roundabout Capacity Model

The following method can be used to add geometry parameters to the HCM (Siegloch M1) roundabout capacity model. This is a simplified application of the SIDRA model for estimating the follow-up headway and critical gap parameters as a function of roundabout geometry. In the SIDRA model, the follow-up headway and critical gap parameters are also dependent on the opposing (circulating or exiting) flow rate. This is not included in the simplified model described here. The method is further simplified by not including the effect of the entry lane width in estimating the critical gap.

The aim of the method described here is to show the feasibility of a basic geometry model for use with the HCM exponential (Siegloch M1) capacity model for roundabouts. Therefore:

- the method is given for single-lane roundabouts only, and
- various important elements of the SIDRA model including adjustment of follow-up headway and critical gap parameters for vehicle movement class effects, or for the effect of the ratio of arrival flow to circulating flow, are not applied (the full list is included as listed in *Section 1*).

The method will be referred to as the "*basic SIDRA geometry method*". It has been tested for single-lane roundabouts using the HCM roundabout research data (FHWA 2015) as shown in this section.

Follow-up Headway Estimation

The *follow-up headway*, t_f (seconds) of the entry lane traffic can be estimated from:

$$t_f = f_e \, f_a \, f_r \, t_f' \qquad \textit{subject to } 1.0 \le t_f \le 5.0 \tag{7.1}$$

where

f_e = environment (calibration) factor,

f_a = entry angle adjustment factor,

f_r = entry radius adjustment factor, and

t_f' = unadjusted follow-up headway (seconds) when $f_e = 1.0$, $f_a = 1.0$ and $f_r = 1.0$.

The restriction to the range of follow-up headway values in *Equation (7.1)* is based on the Australian research data. The average follow-up headway values in the HCM single-lane roundabout research data are in the range 1.7 to 2.9 seconds (average 2.6 seconds).

Definitions of inscribed diameter, entry radius and entry angle measurements (as used in SIDRA INTERSECTION) are given in *Figure 10*.

The *environment (calibration) factor*, $f_e = 1.05$ is used as the default value in the HCM setup of SIDRA INTERSECTION. This can be modified for best fit to a given capacity dataset.

The entry angle adjustment factor, f_a is given by:

$$f_a = 0.94 + 0.000026 \, \phi_e^{1.6} \tag{7.2}$$

where ϕ_e is the entry angle (degrees).

The *entry radius adjustment factor*, f_r is given by:

<u>in US customary units</u>

$$f_r = 0.95 + 3.28 / r_e \tag{7.3}$$

<u>in metric units:</u>

$$f_r = 0.95 + 1 / r_e \tag{7.4}$$

where r_e is the entry radius (metres or feet),

The unadjusted follow-up headway is given below as a function of the inscribed diameter:

in US customary units

$$t_f' = 3.18 - 0.0061\ D_i + 7.8\text{x}10^{-6}\ D_i^2 \qquad \textit{subject to } 50 \le D_i \le 820 \qquad (7.5)$$

in metric units:

$$t_f' = 3.18 - 0.02\ D_i + 8.4\text{x}10^{-5}\ D_i^2 \qquad \textit{subject to } 15 \le D_i \le 250 \qquad (7.6)$$

where D_i is inscribed diameter (metres or feet).

The restriction to the range of inscribed diameter values in *Equation (7.6)* is based on the Australian research data. The average inscribed diameter values in the HCM single-lane roundabout research data are in the range 116 to 174 ft (35 to 53 m) with an average value of 141 ft (43 m).

Critical Gap (Headway) Estimation

As a simple method, the *critical gap (headway)*, t_c (seconds) of the entry lane traffic can be estimated from:

$$t_c = 1.8\ t_f \qquad \textit{subject to } 2.0 \le t_f \le 8.0 \qquad (7.7)$$

where t_f is the follow-up headway estimated from *Equation (7.1)* or measured in the field.

The factor of 1.8 in Equation (7.7) is close to 1.803 calculated as the average t_c / t_f value (weighted average using the number of critical gap data points) for the HCM research data. The values of t_c / t_f are in the range 1.57 to 2.53.

The restriction to the range of critical gap values in *Equation (7.7)* is based on the Australian research data. The average critical gap values in the HCM single-lane roundabout research data are in the range 3.3 to 6.5 seconds (weighted average 4.7 seconds).

The value of t_f / t_c corresponding to *Equation (7.7)* is 0.56. Australian research data limited to the inscribed diameter range shown in the HCM research data indicates average values of $t_c / t_f = 1.644$ ($t_f / t_c = 0.64$). The values of t_c / t_f for the Australian research data are in the range 1.18 to 2.32.

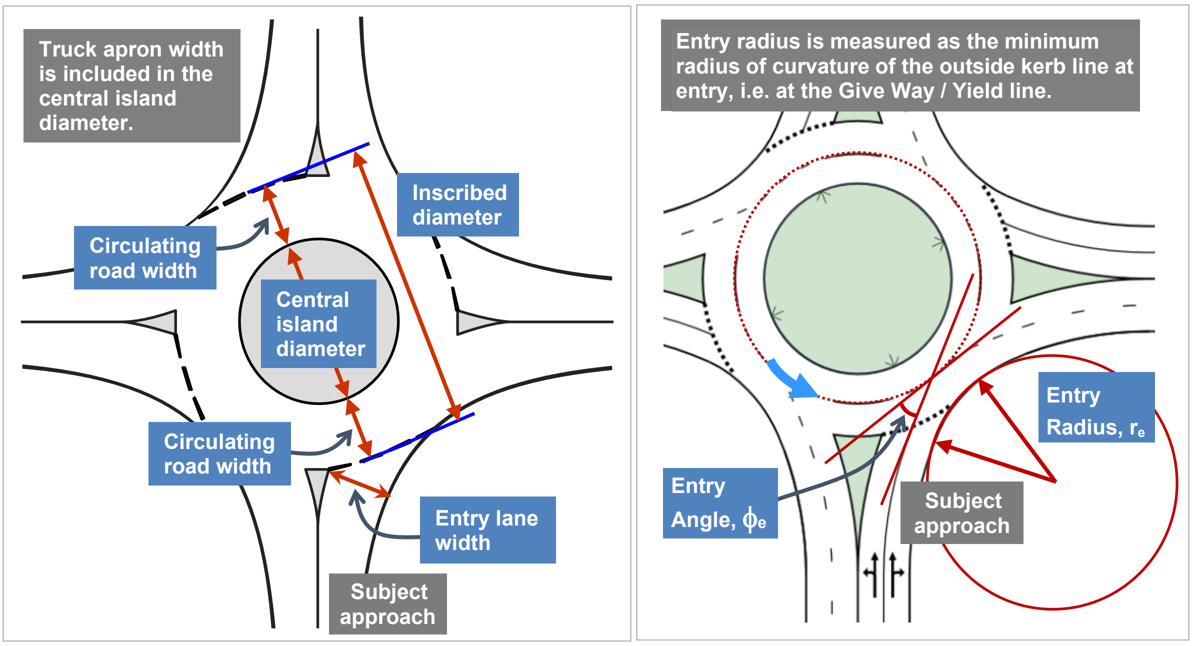


Figure 10 - Inscribed diameter, entry radius and entry angle measurements in SIDRA INTERSECTION

Example

The following example is given for the HCM roundabout capacity research data (FHWA2015) for Carmel sites. The roundabout geometry data used for estimating the HCM (Siegloch M1) capacity model parameters are given in *Table 2*. Data are as given in the paper by Johnson and Lin (2018). Parameters w_L (entry lane width), w_a (approach half width) and L_f (effective flare length) are for the TRL-Kimber linear model only.

Capacity model parameters estimated for Carmel sites are given in *Table 3* including RMSE results.

The capacity data and the lines showing capacity estimates with parameters determined using the basic SIDRA geometry method given in this section are given in *Figure 11*. All estimates are seen to be close to the best fit regression line in *Figure 11* (A = 1391, B = 0.00082) which has RMSE of 152.4. The estimate from the calibrated exponential model (f_e = 1.03, A = 1460, B = 0.00089) has an RMSE value of 153.7 which is within 1 percent of the RMSE value for the best fit regression model. The implied follow-up headway (2.466) is 3 percent larger than the measured follow-up headway (2.405), and the implied critical gap (4.438) is 18 percent larger than the measured critical gap (3.769).

Table 2 - Roundabout geometry data for Carmel sites

D_i	r_e	ϕ_e	w_L	w_a	L_f
ft	ft	degrees	ft	ft	ft
138	65	16	14	12	23
m	m		m	m	m
42.1	19.8		4.27	3.66	7.0

Table 3 - Capacity model parameter estimates for Carmel sites

Best fit regressions using R

Regression	A	B	t_f	t_c	t_f / t_c	RMSE
Exponential - Siegloch M1	1391	0.00082	2.588	4.246	0.610	152.4
Basic Linear	1260	-0.6322	2.857	-	-	152.0

HCM (Siegloch M1) Exponential model with SIDRA Basic Geometry method

Calibration	A	B	t_f	t_c	t_f / t_c	RMSE
	pcu/h	sec	sec	sec		
Default (f_e = 1.05)	1432	0.00091	2.514	4.524	0.556	154.8
Calibrated (f_e =1.03)	1460	0.00089	2.466	4.438	0.556	153.7

TRL-Kimber Linear model

Calibration	A	B	t_f	t_c	t_f / t_c	RMSE
	pcu/h		sec	sec	-	
Default	1314	-0.5745	2.740	-	-	181.7
Calibrated (B fixed)	1217	-0.5745	2.959	-	-	153.5

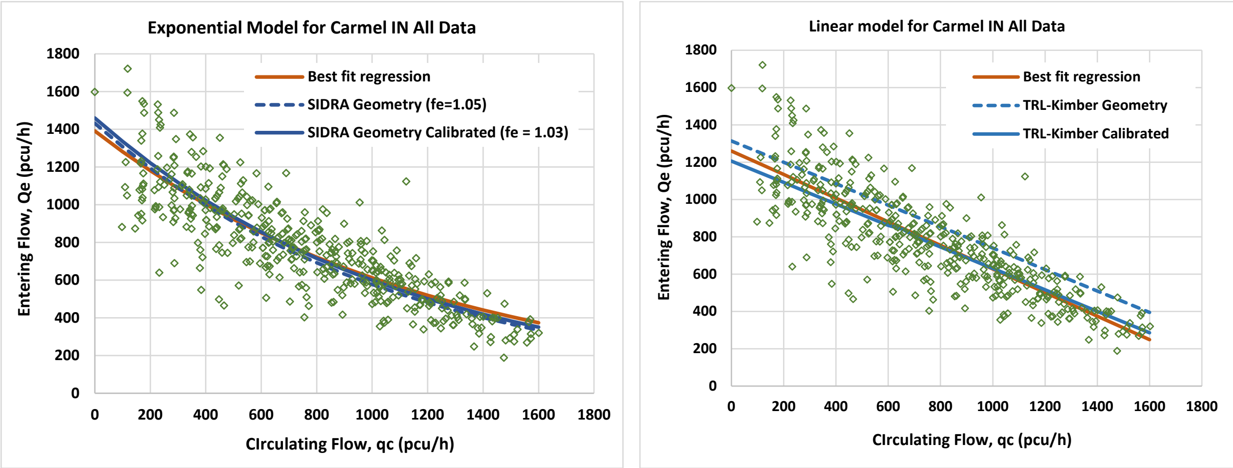


Figure 11 - Capacity data and estimates from regressions, HCM (Siegloch M1) Exponential model with SIDRA Basic Geometry method and TRL-Kimber Linear model

8 Model Calibration

The following calibration method recommended by (Kimber 1980) for the calibration of the TRL linear roundabout capacity model and its variations are considered in this section. Similar methods given here can be used for calibration of exponential gap-acceptance capacity models generally (*Figure 12*).

The application of the calibration method to the HCM (Siegloch M1) exponential model given here can be used for the original HCM model (*Section 3*) as well as the model with the basic SIDRA geometry method added (*Section 7*).

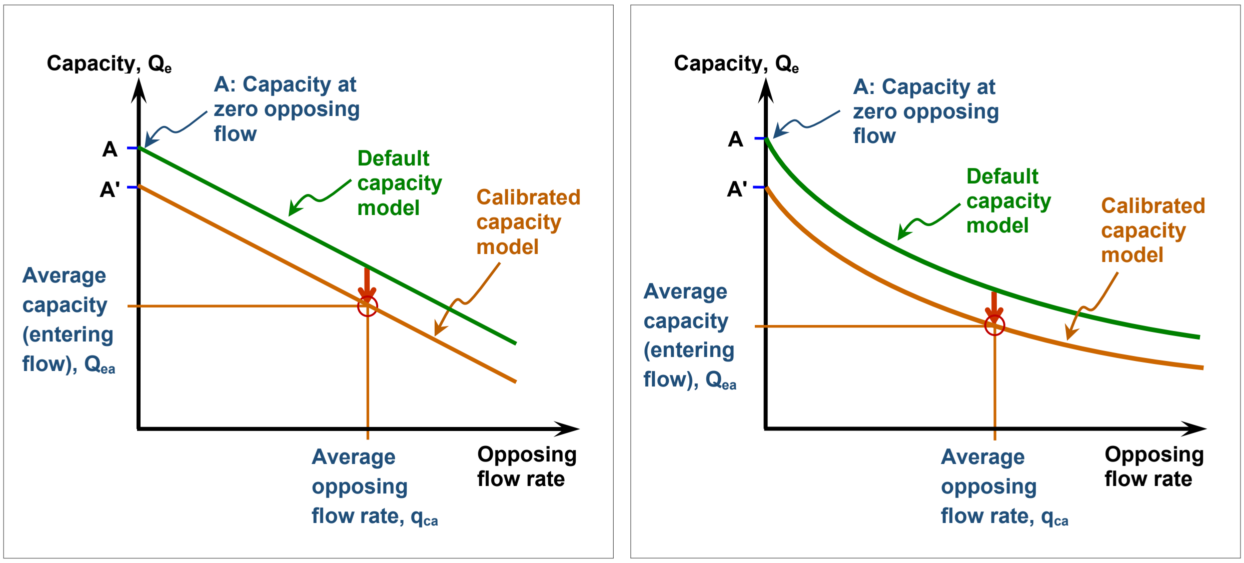


Figure 12 - Calibration of capacity models

The calibration method uses the capacity survey data (entry flow and corresponding opposing / circulating flow) collected during continuous queueing in the approach lane. While the TRL linear model calibration was specified per total entry flow for all lanes of the approach, the application here is for entry flow data per approach lane.

Using the capacity survey data, calculate the average entry (capacity) flow, Q_{ea} and the average opposing (conflicting) / circulating flow, q_{ca}. This is used as the key parameter for model calibration. For the HCM roundabout capacity data for the full dataset, Q_{ea} = 758 pcu/h and q_{ca} = 642 pcu/h.

The calibration methods given here are approximate methods given the non-linear character and large variability of the data used.

8.1 Calibrating TRL Linear Capacity Model

For the TRL linear capacity model which has the form $Q_e = A + B\ q_c$ (where $B < 0$) as discussed in *Section 4*, Kimber (1980) recommended the use of the slope, B, determined for the default model is used as given and the y-intercept (saturation flow), A is calculated from:

$$A = Q_{ea} + B\ q_{ca} \tag{8.1}$$

As an alternative method, the y-intercept (saturation flow), A, determined for the default model is used as given and the slope, B, is calculated from:

$$B = (Q_{ea} - A) / q_{ca} \tag{8.2}$$

A variation to the use of *Equation (8.2)* is to use measured follow-up headway to determine $A = 3600 / t_f$ and use this in the equation to determine B (rather than using A determined for the default model). This method is similar to regression with the y-intercept (A) anchored. It usually gives poor result for the linear model since the A parameter is not a good estimator of the follow-up headway for this model due to the linear nature of the model.

Example

In *Table 3*, the TRL linear model estimate for ***Carmel sites*** (HCM roundabout capacity research data) is seen to give A = 1314 (t_f = 2.740) and B = -0.5748 (RMSE = 181.5).

Using Q_{ea} = 783 pcu/h and q_{ca} = 754 pcu/h for this dataset and B = -0.5745 fixed in *Equation (8.1)*, A = 1217 (t_f = 2.959) is obtained (RMSE = 153.5).

Using the method in *Equation (8.2)* with the model estimate of A = 1314, the slope is determined as B = 0.7031 (RMSE = 154.3).

Using the method in *Equation (8.2)* with measured follow-up headway of t_f = 2.405 for this dataset, hence A = 3600 / 2.405 = 1497, the slope is calculated as B = -0.9454 (RMSE = 191.5). The RMSE value of 191.5 for this calibration is about 26 % larger than the value of RMSE for the best fit linear regression.

These results are shown in *Figure 13* including the HCM roundabout capacity data as well as the best fit linear regression model (A = 1260, B = -0.6322, RMSE =152.0) for ***Carmel sites*** used for the TRL linear capacity model calibration.

Figure 13 shows that the calibration methods using *Equations (8.1) and (8.2)* perform well for the TRL linear capacity model but the use of parameter A based on measured follow-up headway performs poorly for this model. The linear regression with the y-intercept A = 1497 anchored gave B = - 0.8845 with RMSE = 184.6 which is about 21 % larger than the value of RMSE for the best fit linear regression.

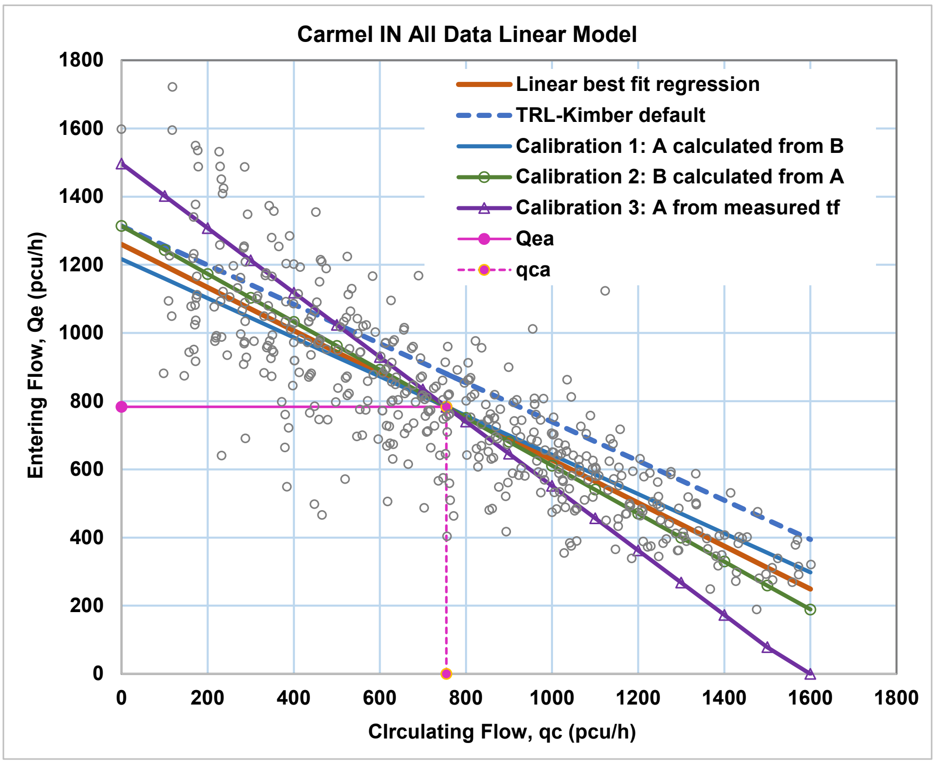


Figure 13 - Various methods of calibration of linear gap acceptance capacity models applied to the TRL linear model

8.2 Calibrating HCM (Siegloch M1) Exponential Capacity Model

For the HCM (Siegloch M1) exponential capacity model which has the form $Q_e = A \; exp\,(-B\, q_c)$ as discussed in *Section 3*, the y-intercept (saturation flow), A, determined for the default model can be used as given and the slope, B, calculated from:

$$B \quad = -ln\,(Q_{ea} / A) / q_{ca} \qquad (8.3)$$

This method can be applied to the use of A from the default (original) HCM model, the model with the basic SIDRA geometry model added, and with the measured follow-up headway value.

The application of the calibration method to the HCM (Siegloch M1) exponential model given here can be used for the original HCM model (*Section 3*) as well as the model with the basic SIDRA geometry method added (*Section 7*). The HCM model with the basic SIDRA geometry method can also be calibrated by modifying the environment (calibration) factor, f_e.

It is also possible to use the average value of $ln\,(Q_e / A)$ instead of using the average Q_{ea} in $ln\,(Q_{ea} / A)$ in *Equation (8.3)*.

Example

As seen in *Table 3*, using the HCM (Siegloch M1) exponential capacity model with the basic SIDRA geometry method added for ***Carmel sites***, A = 1432 (t_f = 2.514) and B = 0.000908 (t_c = 4.524) were obtained (RMSE = 154.8). This was based on the default value of f_e = 1.05.

Calibrating the model using f_e = 1.03 for the HCM model with the basic SIDRA geometry method, A = 1460 (t_f = 2.466) and B = 0.000890 (t_c = 4.438) were obtained (RMSE = 153.7).

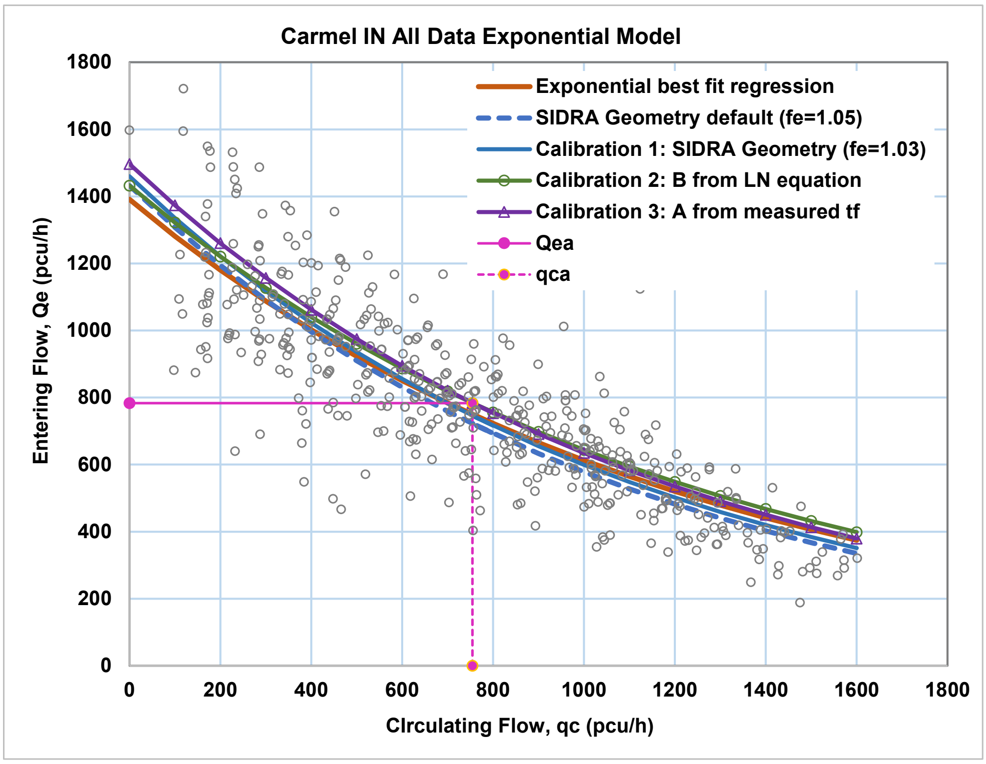


Figure 14 - Various methods of calibration of linear gap acceptance capacity models applied to the HCM (Siegloch M1) exponential model with basic SIDRA geometry method

Using Q_{ea} = 783 pcu/h and q_{ca} = 754 pcu/h for this dataset and A = 1432 (t_f = 2.514) fixed (as obtained using f_e = 1.05 with the basic SIDRA geometry method) in *Equation (8.3)*, B = 0.000800 (t_c = 4.136) was obtained (RMSE = 156.1).

With measured follow-up headway of t_f = 2.405 for this dataset, hence A = 3600 / 2.405 = 1497, used in *Equation (8.3)*, the calibrated model had B = 0.000858. The RMSE value of 158.9 for this calibration is about 4 % larger than the value of RMSE for the best fit exponential regression.

These results are shown in *Figure 14* including the HCM roundabout capacity data as well as the best fit regression model (A = 1391, B = 0.00082, RMSE =152.4). The exponential regression with the y-intercept A = 1497 anchored gave B = 0.00092 (RMSE = 155.5) which is close to the calibration method using the measured follow-up headway given above.

Figure 14 shows that the calibration methods using *Equation (8.3)* perform well for the HCM (Siegloch M1) exponential capacity model with the basic SIDRA geometry method. The use of parameter A based on measured follow-up headway (2.405) has an RMSE value which is 4% higher than the best fit regression). On the other hand, the calibration method for the linear model using the measured follow-up headway gives an RMSE value which is 26% higher than the best fit regression.

The results for alternative calibration methods for the linear and exponential models for Carmel sites are summarised in *Table 4*. Also see *Table 3*.

Table 4 - Results for alternative calibration methods for the linear and exponential methods for Carmel IN sites

Calibration of the TRL linear model			
Default TRL-Kimber model	Calibration 1	Calibration 2	Calibration 3
Default model using geometry parameters	A from *Equation (8.1)* using B from default model	B from *Equation (8.2)* using A from default model	B from *Equation (8.2)* using A from measured follow-up headway
A = 1314	A = 1217	A = 1314	A =1497
B = -0.5745	B = -0.5745	B = -0.7031	B = -0.9454
RMSE = 181.7	RMSE = 153.5	RMSE = 154.3	RMSE = 191.5
Calibration of the HCM (Siegloch M1) exponential model with the SIDRA basic geometry method			
Default HCM model with Basic SIDRA Geometry Method	Calibration 1	Calibration 2	Calibration 3
Default model using *Equations (7.1) to (7.8)* with f_e = 1.0	f_e in *Equation (7.1)* adjusted to 1.03 in SIDRA Basic Geometry Method	B from *Equation (8.3)* using A from SIDRA Basic Geometry Method with default f_e = 1.05	B from *Equation (8.3)* using A from measured follow-up headway
A = 1432	A= 1460	A = 1432	A = 1497
B = 0.000908	B =0.000890	B = 0.000800	B = 0.000858
RMSE = 154.8	RMSE = 153.7	RMSE =156.1	RMSE =158.9

9 Conclusions

This report has presented the results of a search for a *linear gap-acceptance capacity model* derived assuming a *uniform* or *linear* arrival headway distribution of the opposing traffic stream. These headway distributions are not realistic given the random nature of arrival headways including bunching considerations. However, the purpose of this exercise was to see if a linear gap-acceptance model is possible.

It was found that both uniform and linear headway distributions resulted in non-linear gap-acceptance capacity models. A form close to a linear model could be obtained by choosing low values of critical gap in these models. However, the chosen critical gap values were too low and not realistic when compared with observed values indicated by the HCM and Australian research data.

The report has also discussed application of a simplified *SIDRA geometry method* for estimating the parameters of the HCM (Siegloch M1) roundabout capacity model. Results are promising when applied to the HCM roundabout capacity research data with favourable comparisons with the TRL linear model.

The alternative calibration methods for the TRL linear roundabout capacity model and HCM (Siegloch M1) exponential model have been discussed. The calibration method given in this report for the HCM exponential model can be used for the original HCM model as well as the model with the basic SIDRA geometry method added. It is shown that satisfactory results were obtained when the method was applied to the HCM model with the basic SIDRA geometry method added.

The regression models and calibrated analytical models were found to imply larger values of follow-up headway compared with the measured values. The follow-up headways implied by the exponential model appear to be closer to the measured values compared with the linear model. Similarly, the increase in statistical errors when regressions or calibrations with the y-intercept (3600 / follow-up headway) anchored using the measured follow-up headway were small for the exponential model but large for the linear model. This subject is important in modelling roundabout capacities for specific cases of unbalanced flows at high demand levels. The critical gap values implied by exponential regression models and calibrated exponential analytical models also differ significantly from the measured ones. This is discussed in more detail in the accompanying report (Akcelik, et al 2022). Research is recommended into causes of these discrepancies including the survey methods used for follow-up headway and critical gap.

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REFERENCES

AKÇELIK, R. (1994). Gap acceptance modelling by traffic signal analogy. *Traffic Eng. and Control*, 35 (9), pp 498-506.

AKÇELIK, R. (2007). A Review of Gap-Acceptance Capacity Models. *29th Conference of Australian Institutes of Transport Research (CAITR)*, University of South Australia, Adelaide, Australia.

AKÇELIK, R. (2011). Some common and differing aspects of alternative models for roundabout capacity and performance estimation. *TRB International Roundabout Conference*, Carmel, Indiana, USA.

AKÇELIK, R. (2012). Alternative models for roundabout capacity. *Traffic Engineering and Control*. July 2012.

AKÇELIK, R. (2017a). An assessment of the Highway Capacity Manual Edition 6 roundabout capacity model. *5th International Roundabout Conferenc*e, Transportation Research Board, Green Bay, Wisconsin, USA.

AKÇELIK, R. (2017b). Roundabout Model Comparison Table. Akcelik and Associates Pty Ltd, Melbourne, Australia.

AKÇELIK, R. (2018). Gap Acceptance Cycles for Modelling Roundabout Capacity and Performance. In: *Roundabouts as Safe and Modern Solutions in Transport Networks and Systems* (Eds E. Macioszek, R. Akçelik and G. Sierpiński). Proceedings of the *15th Scientific and Technical Conference, Transport Systems Theory and Practice.* Department of Transport Systems and Traffic Engineering, Faculty of Transport, Silesian University of Technology, Katowice, Poland, pp 89-98.

AKÇELIK, R. and BESLEY, M. (2005). Differences between the AUSTROADS Roundabout Guide and aaSIDRA roundabout analysis methods. *Road & Transport Research* 14(1), pp 44-64.

AKÇELIK, R., CHUNG, E. and BESLEY, M. (1997). Analysis of Roundabout Performance by Modelling Approach Flow Interactions. In: Kyte, M. (Ed.), *Proceedings of the Third International Symposium on Intersections Without Traffic Signals, July 1997, Portland, Oregon, USA*, University of Idaho, Moscow, Idaho, USA, pp 15-25.

AKÇELIK, R., SHIRKE, C., BESLEY, M., ESPADA, I. and BILLINGHURST, D. (2022). *A comparative analysis of exponential and linear roundabout capacity models using HCM research data*. Technical Note. Akcelik & Associates Pty Ltd, Melbourne, Australia.

AKÇELIK, R. and TROUTBECK, R. (1991). Implementation of the Australian Roundabout Analysis Method in SIDRA. In: U. Brannolte (Ed.), *Highway Capacity and Level of Service – Proc. of the International Symposium on Highway Capacity, Karlsruhe, July 1991*. A.A. Balkema, Rotterdam, pp 17-34.

CAMPBELL, J.R., OLSSON, S.M and STERNKE, C.R. (2021). Using vehicle tracking software to validate roundabout capacity models. ITE Journal 91 (12), pp 43-49.

COWAN, R.J. (1975). Useful headway models. *Transportation Research* 9 (6), pp 371-375.

COWAN, R.J. (1984). Adam's formula revised. *Traffic Engineering and Control*, 25 (5), pp. 272-274.

COWAN, R.J. (1987). An extension of Tanner's results on uncontrolled intersections. *Queueing Systems* Vol.1, pp. 249-263.

FHWA (2015). *Assessment of Roundabout Capacity Models for the Highway Capacity Manual.* Accelerating Roundabout Implementation in the United States - Volume II of VII. Publication No. FHWA-SA-15-070. US Department of Transportation, Federal Highway Administration, McLean, Virginia, USA.

JOHNSON, M.T. and LIN, T-L. (2018). Impact of Geometric factors on the capacity of Single-Lane Roundabouts. *Transportation Research Record* 2672 (34), pp 10-19.

KIMBER, R.M. (1980). *The Traffic Capacity of Roundabouts*. TRRL Laboratory Report 942. Transport and Road Research Laboratory, Crowthorne, Berkshire, UK.

KIMBER, R.M. (1985). Roundabouts (Letter to the Editor). *Australian Road Research,* 15 (4), pp 295-297.

KIMBER, R.M. (1989). Gap-acceptance and empiricism in capacity prediction. *Transportation Science* 23 (2), pp 100-111.

LENTERS, M. and RUDY, C. (2010). HCM roundabout capacity methods and alternative capacity models. *ITE Journal*, 80 (7), pp 22-27.

LUTTINEN, R.T. (1999). Properties of Cowan's M3 headway distribution. *Transportation Research Record 1678*, pp 189-196.

LUTTINEN, R.T. (2003). Capacity of Unsignalised Intersections. *TL Research Report No. 3.* TL Consulting Engineers, Lahti, Finland.

TANNER, J.C. (1962). A theoretical analysis of delays at an uncontrolled intersection. *Biometrika*, 49(1&2), pp 163-170.

TANNER, J.C. (1967). The capacity of an uncontrolled intersection. *Biometrika*, 54(3&4), pp 163-170.

TRB (2016). *Highway Capacity Manual Edition 6*. Transportation Research Board, National Research Council, Washington, DC, USA.

TROUTBECK, R.J. (1989). *Evaluating the Performance of a Roundabout***.** Special Report SR 45. ARRB Transport Research Ltd, Vermont South, Australia.