THE IMPLICATIONS OF VOLATILITY IN DAY-TO-DAY TRAVEL FLOW AND ROAD CAPACITY ON TRAFFIC NETWORK DESIGN PROJECTS

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ABSTRACT:

This work addresses the traffic network design problem when day-to-day uncertainties in travel demand and link capacity are taken into account. Specifically, this work proposes a network design formulation that uses a strategic behavior approach, where total demand and link capacity are treated as random variables and a strategic user equilibrium results in fixed equilibrium link proportions. The bilevel model is formulated, system performance metrics derived, and then a solution method is developed based on a tailored genetic algorithm. Results under varying levels of volatility reflect possible suboptimal project selection when using a deterministic modeling approach.

1. INTRODUCTION

One of the primary purposes of transport planning models is the ranking and evaluation of infrastructure design projects. Planning tools, such as traffic assignment models based on the Wardropian equilibrium principle, can capture the effect that improvements in the network have on route choice in vehicle travelers. However, traditional models do not account for the inherent uncertainty in these methods, leading to an important question for researchers and practitioners alike: how do optimal project designs change in the face of non-deterministic network parameters?

Uncertainties in network modeling are well-established phenomena in traffic settings. However traditional equilibrium-based network design approaches are primarily deterministic and therefore make a single prediction that is usually interpreted as an average, rather than any specific manifestation of network conditions. While there are important reasons to use such traditional models (model stability, uniqueness, tractability), this approach will almost certainly misrepresent real network conditions, particularly in networks that deviate significantly from the average.

To complicate the matter, network assignment models are often used to evaluate the effects of changes in the network, such as infrastructure design. It follows that deviant model behavior, particularly that resulting from uncertain demand and capacity, is important to capture due to its unpredictable impact on design projects.

This paper focuses on the network design problem (NDP) wherein the planner seeks the optimal links to which to add capacity to improve a stated network performance measure. Specifically, this work applies a novel strategic-based assignment approach (*1*), in which total travel demand in treated as a random variable. Additionally, we consider the day-to-day volatility in link capacity, due to factors such as weather conditions or driving behavior, by incorporating the model extension (*2*) and treating link capacity as a random variable. The contribution of this work is as follows:

- We propose a novel formulation for the NDP that integrates the strategic user equilibrium (StrUE) model to capture user behavior in the face of day-to-day variation in demand and the strategic user equilibrium with capacity (StrUEC) model to represent the day-to-day variation in link capacity;
- Results examine the impact of uncertain modelling parameters on design project selection and evaluation; we highlight the differences in project selection when accounting for no uncertainty, day-to-day demand uncertainty, and day-to-day capacity uncertainty.

2. BACKGROUND

This work focuses on incorporating two sources of uncertainty into the network design problem. While accounting for different sources of uncertainty makes the NDP still more complex, it is essential that researchers develop approaches to quantify how those uncertainties impact infrastructure projects. Network design is an active field in the research and as such, only selected relevant works are discussed here; see Yang and Bell (*3*) for an overview and historical developments, Chen et al (*4*) for a review of uncertainty in the NDP specifically, and Wismans et al (*5*) for an in depth review of NDP applications using a dynamic approach.

Most generally, network design is conceptually simple: the problem of finding the optimal location(s) to enhance a network given a limited "budget." In this work, such enhancements are generally vehicle capacity improvements that can have a variety of interpretations, from the discrete additions (e.g., lanes, roads) to projects that may have a more continuous nature (e.g., optimized signal timing plans, widening of shoulders, elimination of parking, etc). The NDP is traditionally formulated as a bi-level

mathematical programming problem, where the upper level represents the "planner's" perspective that measures the impact in the network due to the change, and the lower level represents the users' reaction to those changes (*3*). Due to the nonconvex cost function resulting from the addition of capacity, the NDP can't be solved by traditional optimization techniques and heuristic methods are necessary. A few previous examples of bilevel network design formulations include multi-objective signal timing (*6*), accounting for long term demand uncertainty (*7*), total travel time reliability with stochastic route choice (*8*), optimal toll pricing strategies (*9*), examining the impact of environmental justice considerations (*10*), and minimizing emissions (*11* and *12*).

Uncertainty in transport network modeling is usually viewed as arising from demand, capacity, and user behaviour. Previous research has looked at the strategic behavior from users in terms of hyperpaths that are formed due to the possibility of being unable to enter capacitated links (*13*). A recent work by Xie and Liu provides a background of the stochastic user equilibrium approach and introduces a new approach based on behavioral inertia, a reflection of how likely a traveler is to deviate from a chosen route *(14)*. The current work also employs strategic user behavior in the sense that people will choose a route based on a range of possible network conditions they may encounter during travel, but the underlying modelling approach is based on the strategic user equilibrium (StrUE) introduced by Dixit et al (*1*). StrUE finds equilibrium flow proportions based on expected path costs, and is detailed in Section 3.1. The output from the strategic assignment approach is link volumes that will vary from day-to-day, thus accounting for short-term demand uncertainty that users face making day-to-day route choice decisions. Other approaches accounting for the strategic based behaviour include an extension to capacity uncertainty (*2*), the design problem of setting optimal tolls (*15*), and dynamic optimal routing for discrete design scenarios (*16*).

This work extends the strategic assignment model to form the subproblem for a network design scenario focused on link capacity additions. Previous work has examined the impact of short-term demand uncertainty and link capacity uncertainty, but rarely in combination. This work proposes a novel approach to address this gap.

3. MODEL FORMULATION

This work captures users' reactions to day-to-day demand fluctuation using the strategic assignment model described in Section 3.1. However, the "capacity" of links as employed by most static traffic assignment approaches is another non-deterministic quantity that users consider when selecting a route and should be included in the evaluation design projects. Therefore, Section 3.2 describes the strategic behavior approach that accounts for the variability in link capacity. Finally, Section 3.3 formulates the bilevel network design model incorporating strategic route choice assignment.

3.1 Strategic user equilibrium (StrUE) model

The core assumption of strategic behavior route choice model is that users have knowledge of day-to-day demand distribution, although on any given day they do not know what conditions they will encounter during travel. Therefore, they employ a simple strategy: they choose the *expected* shortest cost path and they do not deviate from this path regardless of experienced conditions.

The modelling implications of this behavior is a mathematical equilibrium based a fixed assignment pattern of link proportions where the expected cost for the proportion of total travelers on each used route between an origin and destination is equal. However, the day-to-day demand is a random variable. Thus, while link proportions remain fixed, total demand changes, resulting in

flows that vary according to a set of demand realizations, reflecting disequilibrium similar to what is observed in traffic networks.

The strategic approach has two main advantages: it accounts for demand uncertainty, which adds realism as compared to deterministic models, and due to the relative simplicity of its implementation, the StrUE approach can be applied to practically sized problems. Additionally, the strategic approach quantifies the variance in link travel time. This variance can be interpreted as a measure of reliability. In a network design setting, the planner can consider link variance and variance in total system travel time as part of the decision making process.

Consider a directed graph $G = (V, A)$ where V is the set of nodes (vertices) and A is the set of links (edges), where an individual link is indexed by a. Let $r \in R$ index an origin and $s \in S$ index one destination from the set of destinations. Let W be the set of origin-destination pairs connecting origins r with destinations s, where q_{rs} indicates the proportion of total demand between origin r and destination s. The total demand is a random variable T with associated probability distribution $g(T)$. The travel cost on a link is $t_a(p, T)$, which is a function of the proportion of the total flow on the link $p_a : \sum_{a \in A} p_a = 1$, and T. Furthermore let K_{rs} be the set of paths connecting origin r and destination s, and let f_k^{rs} represent the proportion of the total travel demand on that path. Finally, let $\delta_{a,k}^{rs}$ be the incidence matrix that is equal to 1 if link a is on path k between origin r and destination s and 0 otherwise. The StrUE model as previously introduced may then be written as:

StrUE Model:

$$minmize \ \ z(p, T) = \int_0^\infty \sum_{a \in A} \int_0^{p_a} t_a(p, T) \, g(T) dp dT \tag{1}$$

subject to

$$\sum_{k \in K_{rs}} f_k^{rs} = q^{rs} \qquad\qquad \forall r \in R, \forall s \in S \tag{2}$$

$$f_k^{rs} \geq 0 \qquad\qquad \forall k \in K_{rs}, \forall r \in R, \forall s \in S \tag{3}$$

$$p_a = \sum_{r \in R} \sum_{s \in S} \sum_{k \in K_{rs}} f_k^{rs} \delta_{a,k}^{rs} \qquad\qquad \forall a \in A \tag{4}$$

The StrUE model formulation provides a straightforward framework that can be applied to practically sized problems by modifying well established solution methods. As discussed in Duell et al *(1)*, to ensure uniqueness of link flows, for each origin-destination, path flow proportion is assumed to be equal under all demand scenarios. Therefore, the equilibrium flow on each path will vary in a proportional manner when the total origin-destination demand varies. The system performance metrics in the strategic approach can be found through analytical derivations or simulation-based sampling methods, and will be detailed in the next section.

3.2 Strategic user equilibrium with capacity uncertainty (StrUEC) model

In both the deterministic approach and the StrUE model, capacity serves as a model input and assumed to be a fixed value. Capacity is inherently a static representation of a dynamic concept. It is often intended to be a proxy to capture the effects of congestion, where the travel time increases as the ratio of flow to capacity on a link increases. In spite of this, the capacity of a road will fluctuate due to factors such as

driving behavior and adverse weather conditions, phenomena that is captured in this work by the concept of "day-to-day" capacity. Drivers may consider this fluctuating capacity when making route selections; therefore, it is important to consider its impact on network design project rankings.

The StrUEC model, introduced by Wen et al *(2)*, accounts for the day-to-day volatility in capacity and vehicle users reaction to knowledge of that volatility. In the StrUEC approach, we assume that the inverse capacity $C_{inv,a}$ on each link is a random variable with a known probability distribution $h_a(C_{inv})$. Vehicle users have knowledge of this capacity distribution and choose the expected least cost route, where the expected cost is based on the probability distribution of both the demand and the capacity distribution on each link, assumed to be independent from one another. Wen et al *(2)* show the uniqueness of the model assignment solution.

Continuing the notation previously introduced, the StrUEC model seeks the set of link flow proportions that satisfy the mathematical program in Equation 5. The difference between the objective function for the StrUE model in Equation 1 and the StrUEC model in Equation 5 is that the expected cost is a function of flow proportion, total demand, and link capacity.

StrUEC Model:

$$minimize\ z_C(p,T) = \int_0^\infty \int_0^\infty \sum_{a \in A} \int_0^{p_a} t_a(p,T,C_{inv})\, g(T) h_a(C_{inv}) dp dC_{inv,a} dT \qquad (5)$$

subject to

Constraints [2] – [4]

In traditional network design, the capacity of a link is increased in vehicles per hour, which will lower total system travel time and make the link more attract to drivers. However in network design with uncertain capacity, the *expected* capacity of the link, which is one of the parameters of the distribution of the link capacity $h_a(C_{inv})$, is increased because capacity is not a deterministic quantity. There might be other possibilities where the coefficient of variation or the variance of the capacity is decreased (for example policy targeting illegal parking practices). For simplicity, the most straightforward interpretation is utilized in this work. Alternate possibilities will be the topic of future research.

3.3 Network design formulation
The network design problem with uncertainty is formulated as a bilevel nonlinear mathematical programming problem. The upper level seeks to minimize the planning objective accounting for volatility in the network, for example, expected total system travel time or standard deviation of travel time, both of which are a function of proportion of flow on each link and capacity changes in the transportation network. The lower level represents drivers' reactions to changes in the road network, modelled by the StrUE or StrUEC approaches presented in Section 3.1 and 3.2.

This work focuses on ranking and evaluating design projects in a traffic network, although principles similar to those discussed here would apply to other NDP applications. Let S be a predetermined set of possible network design scenarios indexed by s, each of which is defined by the amount of capacity or expected capacity p to add to some number of links such that the total cost of improving the links is below the budget B in order to minimize objective $w \in \Omega$. δ_{ij}^s is a binary decision variable equal to 1 if link (i,j) is an optimal location to add capacity in project scenario s. Note that links which are not available to be improved by the amount $p \in P_S$ will be constrained such that $\delta_{ij}^s = 0$.

Additionally, the binary constraint on δ^s could be relaxed, in which case this would be a continuous network design formulation. The binary approach was utilized here because it puts realistic bounds on the solution space, which is already quite large.

      The upper level problem represents the "planner's" perspective, who seeks the optimal links to which to add capacity for each design scenario in order to minimize an objective $s_{B,p}^w$. The upper level decision variables also impact the lower level problem, which is the strategic traffic equilibrium approach that accounts for different sources of network uncertainty. For each design scenario $s_{B,p}^w$, the formulation to minimize the objective w follows:

$$minimize\ w \tag{6}$$

subject to

$$\sum_{a \in A} \alpha_a \delta_a^s \leq B_s \tag{7}$$

$$\delta_a^s \in \{0,1\} \quad \forall a \in A \tag{8}$$

subject to

$$StrUE\ or\ StrUEC \tag{9}$$

The benefit of this formulation is that the lower level StrUE model can be treated independently from the design problem, making it conducive to heuristic methods such as genetic algorithms.

The defining point of this formulation is which objective function should be utilized for Equation (6). There are two basic system objectives that are of interest in this work: expected total system travel time E and the standard deviation of total system travel time STD. Due to the assumptions in the strategic assignment model, there are two approaches to solving for these objective measures. The first is to use the analytical equations that are derived based on a travel cost function and the distribution of the random variables. The expression for $\diamond E$ is given in Equation 10.

$$\diamond E = \int_0^\infty \sum_{a \in A} p_a t_a(p, T, \delta^s) g(T) dT \tag{10}$$

The analytical expression for the total system travel time where link capacity is a random variable given in Equation 11.

$$\diamond E_C = \int_0^\infty \int_0^\infty \sum_{a \in A} p_a t_a(p, T, C_{inv}, \delta^s) h_a(C_{inv}) g(T) dT \tag{11}$$

In this work, the $\diamond$ symbol indicates that a quantity is analytically derived. Alternatively, a quantity can be found using a simulation procedure that involves sampling from the distribution of the random variable, which is denoted as $\odot$. The simulation based expected total system travel time and standard deviation are $\odot E$ and $\odot S$. A procedure to find these values is outlined in *(1)*.

      The variance of total system travel time is denoted as $\diamond V$, and it is the expected value of the square of the total system travel time minus the square of the expected value of total system travel time.

Note that expectation is denoted $Ex(X)$ in this work. Equations 12 and 13 contain the $\diamond V$ for StrUE and StrUEC models respectively. The work uses the standard deviation of total travel time $\diamond STD = \sqrt{\diamond V}$ in order to be more directly comparable to $\diamond E$.

$$\diamond V = Ex\left(\left(\sum_{a\in A} p_a t_a(p,T,\delta^s)\right)^2\right) - \left(Ex\left(\sum_{a\in A} p_a t_a(p,T,\delta^s)\right)\right)^2 \tag{12}$$

$$\diamond V_C = Ex\left(\left(\sum_{a\in A} p_a t_a(p,T,C_{inv},\delta^s)h_a(C_{inv})\right)^2\right) - \left(Ex\left(\sum_{a\in A} p_a t_a(p,T,C_{inv},\delta^s)h_a(C_{inv})\right)\right)^2 \tag{13}$$

The next section outlines the assumptions to provide tractable forms of equations (10) – (13), as well as the solution method for the bilevel program proposed in equations (6) – (9).

4. SOLVING THE MODEL
This section details the assumptions and methodology to solve the bilevel network design model presented in Section 3.3. The upper level of the model is solved using a heuristic based on natural evolution known as a genetic algorithm. The strategic assignment submodel presented in Sections 3.1 and 3.2 is solved using a straightforward approach based on the well-known Frank Wolfe method. Additionally, the strategic assignment approach both with and without capacity uncertainty require a number of assumptions in order to provide a tractable form of the model to solve analytically. All solution methods and assumptions are detailed Sections 4.1 – 4.3.

4.1 Strategic assignment model
In order to solve the strategic assignment model, this approach assumes a lognormal distribution for the total travel demand $T \sim LN(E_{str}, CV_{str})$, where E_{Str} is the total expected demand, the CV_{str} is the coefficient of variation of total trips, and that the OD demand follows fixed, specified proportions. Travelers make their route choices based on knowledge of the distribution and the resulting expected travel costs. In order to solve the StrUE and StrUEC models that are the subproblem of this work, we assume that travel cost varies with flow according to a variation of the BPR function, where flow is a function of link proportion p_a and the random variable for total demand T:

$$t_a(p,T) = t_a^f \left(1 + \alpha \left(\frac{p_a T}{c_a + \delta_a^s n_s}\right)^\beta\right) \tag{14}$$

Where t_a^f is the free flow travel time on link a, c_a is the capacity, α and β are BPR shaping parameters that are commonly assumed to be 0.15 and 4, respectively. For simplicity, this work assumes that the α and β parameters in the BPR function are the same on every link. The flow proportion on each link p_a is an output from solving the StrUE model.

The expected total system travel time is an important metric for planners, especially for the ranking of design projects. For simplicity, unless stated otherwise, we assume the link capacity c_a includes the additional projects and exclude $\delta_a^s n_s$ from the travel cost function. Using Equations 10 and 14, the $\diamond E$ can be calculated as:

$$\diamond E = \int_0^\infty \sum_{a\in A} p_a t_a(p,T)g(T)dT = \sum_{a\in A}\left(t_a^f p_a M_1 + \left(\frac{\alpha t_a^f}{c_a^\beta}\right)p_a^{\beta+1} M_{\beta+1}\right) \tag{15}$$

Where M is the analytical moment of the lognormal demand distribution that is found as:

$$M_\beta = e^{\beta\mu + \frac{1}{2}\beta^2\sigma^2} \tag{16}$$

In order to aid with the presentation of system performance metrics, consider the two parts of total system travel time as that resulting from sum of the free flow travel time on each link F and that resulting from the sum of the delays on each link, D.

$$F = \sum_{a \in A} t_a^f p_a \tag{17}$$

$$D = \sum_{a \in A} \left(\frac{\alpha t_a^f}{c_a^\beta}\right) p_a^{\beta+1} \tag{18}$$

Using this notation, the expected total system travel time can then be written as:

$$\diamond E_{str} = FM_1 + DM_{\beta+1} \tag{19}$$

The standard deviation is more complex because we need to find the expected value with respect to the total demand T, of the sum of link travel times squared. However, assuming that T is not link dependent (implying β is the same on all links), then the total trips T can be factored out and the standard deviation calculated by summing each different quantity on each link, and then computing the $\diamond STD_{str} = \sqrt{\diamond V}$.

$$\diamond V = F^2 M_2 + D^2 M_{2\beta+2} + 2FDM_{\beta+2} - \left(FM_1 + DM_{\beta+1}\right)^2 \tag{20}$$

While $\diamond STD_{str}$ is somewhat nonstandard, it can still be calculated relatively easily using a single pass through the array of links. Next we consider the differences in calculating strategic model performance metrics when including link capacity as random variables.

4.2 StrUEC model assumptions

Additionally, this work considers the model in which capacity is also a random variable that users consider for when making a route choice decision and that may have a corresponding impact on network design projects. In order to capture the variation in day-to-day capacity, we assume that capacity follows a gamma distribution. The inverse of capacity therefore follows an inverse gamma distribution $C \sim Inv\_\Gamma(k, \frac{1}{\theta})$, where k and θ are the distribution shaping and scaling parameters respectively and specific to link a. Assume that the expected capacity on a link is C_a, and the standard deviation is C_a^{std}. Furthermore, we assume that capacity distributions of each link are independent from one another and independent from the demand.

As input to the StrUEC model let the coefficient of variation on link a be $cov_a = \left(\frac{C_a^{std}}{C_a}\right)$. We calculate the link distribution parameters as:

$$k_a = \frac{C_a}{\theta_a} = \frac{1}{cov_a^2} \tag{21}$$

$$\theta_a = \frac{C_a}{k_a} = C_a \times cov_a^2 \tag{22}$$

Again, the system performance metrics of interest are the analytical total system travel time $\diamond E_C$ and analytical standard deviation of total system travel time $\diamond S_C$, where the subnote C indicates metrics from the StrUEC model. The $\diamond E$ can be found by combining Equations 11 and 14.

$$\diamond E_C = \int_0^\infty \int_0^\infty \sum_{a \in A} p_a t_a(p, T, C_{inv}, \delta^s) h_a(C_{inv}) g(T) dT$$

$$= \sum_{a \in A} \left(t_a^f p_a M_1 L_{a,1} + \alpha t_a^f p_a^{\beta+1} M_{\beta+1} L_{a,\beta} \right) \tag{23}$$

Where $L_{a,\beta}$ is the βth moment of the inverse gamma link capacity distribution that is computed as:

$$L_{a,\beta} = \frac{1/\theta^\beta}{\prod_{i=1}^\beta (k - i)} \tag{24}$$

Note that this definition does place constraints on the feasible values of cov_a.

In order to aid with the presentation of metrics, consider the two parts of the travel cost function, where $F_C = F$. To calculate D_C, the link capacity distribution stays inside the summation because it is link specific. However, it remains independent from the capacity distribution on any other link and therefore the expectation becomes the moment to the power of β.

$$D_C = \sum_{a \in A} \alpha t_a^f p_a^{\beta+1} L_{a,\beta} \tag{25}$$

Using notation meant to aid in computation, the analytical total system travel time for the StrUEC model may be calculated as:

$$\diamond E_C = FM_1 + D_C M_{\beta+1} \tag{26}$$

The derivation of standard deviation from Equation 13 is more algebraically demanding. We need to square D_C before deriving the expectation, so the link specific capacity random variables will be multiplied. For clarity, we factor out the "constant" segment of each link quantity attributed to the delay as D_a:

$$D_a = t_a^f \alpha p_a^{\beta+1} \tag{27}$$

Finding the expectation of the square of a summation of each link is less straightforward. In the demand case, we assumed the value of β is not link specific. Therefore T factored out. However in the case of Equation 13, when the summation of the travel time on each link is squared, the capacity random variable on each link must be multiplied by the capacity random variable on every other link, after which the expected value is calculated, shown in Equation 28.

$$D_{C,2} = \int_0^\infty \int_0^\infty \left(\sum_{a \in A} D_a T^\beta C_a^\beta \right)^2 g(T) h_a(C_{inv}) dC_a dT \tag{28}$$

Using the property that for independent, real value variables, $E(XY) = E(X)E(Y)$ and ordering the links in a "list", it is still relatively simple to calculate $D_{C,2}$. Using manipulation to arrange the equation in a form that is computable, the squared part of the system "delay" can be found as:

$$D_{C,2} = M_{2\beta+2} \sum_{a \in A} \left(D_a L_{a,2\beta} + 2 \sum_{b=1}^{b<a} D_b D_a L_{b,\beta} L_{a,\beta} \right) \tag{29}$$

Therefore the $\diamond STD_C$ is calculated as $\sqrt{\diamond V_C}$, with the variance $\diamond V_C$ shown in Equation 30.

$$\diamond V_C = F^2 M_2 + D_{C,2} M_{2\beta+2} + 2FD_C M_{\beta+2} - \left(FM_1 + D_C M_{\beta+1}\right)^2 \tag{30}$$

4.2 Solving the design problem

The network design problem as formulated in Section 3.3 cannot be solved to a guaranteed global optimal value using standard optimization techniques because of the non-convex cost function (Equation 14). Therefore heuristic solution methods are necessary. This work applied a genetic algorithm, an optimization technique inspired by principles of natural evolution. GAs provide a flexible, rigorous framework to solve challenging optimization problems, and are a relatively common research method to solve the bi-level traffic network design problem. Karoonsoontawong and Waller (*17*) showed that in terms of heuristic approaches to solve the continuous NDP, GAs perform better than simulated annealing or random search algorithms. A GA will correctly identify local extrema, but as is the case with all heuristics, the solution is not guaranteed to be the globally optimal value. In this approach, steps were taken to ensure that the GA had converged on the best solution.

A GA locates an optimal solution by searching for promising regions in which there are a high proportion of "good" solutions. It begins with a randomly generated initial population of individuals that represent potential solutions (called chromosomes). Over "time", the population evolves according to a natural selection process, in which the best individuals are selected and combined using a crossover technique to form new populations of individuals.

This work utilized a single-objective binary-coded variation of the nondominant sorting genetic algorithm II (NSGA-II) by Deb (*18*). NSGA-II is a well-known algorithm that has proven to be the best GA tool for solving multi-objective optimization problems, and utilizes several techniques that provide superior performance. See (*19*), (*10*), (*11*) for other examples that utilize NSGA II for various applications of the traffic network design problem.

An important aspect of using a GA to solve any optimization problem is how the problem variables are represented. As in previous applications, this work uses a "binary" approach in order to limit the feasible solution space. Each "chromosome" is specified to have as many bits as there links in the network. Then a "0" represents the decision not to add capacity to a link and a "1" means to add capacity (where the amount of capacity to add is a model input). In general, GAs perform better without constraints. However, a constraint was unavoidable in this application due to the fact that we consider the cost of adding capacity to a link to be related to the length of the link. We avoided the use of a penalty function by initializing each population (set of GA chromosomes) to be feasible. A GA relies on crossover and selection procedures to explore the solution space. However, for binary approaches where the solution contains many more "0"s than "1"s, there is a much higher probability that crossover or selection will result in infeasible solutions. This issue was addressed by running the GA for more generations to give it more time to explore the solution space. A crossover probability of 0.9 and a mutation probability of 0.001 were used in this work.

In order to evaluate a design scenario, this work employs two performance metrics to measure the relative impact of each design scenario. For the performance metrics, $\Delta(\cdot,\cdot)$ is used to indicate the percentage difference between two quantities. The decrease in expected total system travel time is the percentage difference between the E_0, the system travel time in the base case with no design changes, and E_s, the expected system travel time that results from design scenario s, with the same principle applying to the case of $\diamond STD$.

$$\Delta E_s = \Delta(\diamond E_s, \diamond E_0) = 1 - \frac{\diamond E_s}{\diamond E_0} \qquad (31)$$

$$\Delta STD_s = \Delta(\diamond STD_s, \diamond STD_0) = 1 - \frac{\diamond STD_s}{\diamond STD_0} \qquad (32)$$

5. RESULTS AND DISCUSSION

This section demonstrates the NDP model accounting for strategic user behavior and discusses the implications for planning for uncertainty in transport networks. The GA is used to solve a variety of design scenarios. Results are presented for a small network in order to demonstrate the model and then on a slightly larger network where more rerouting effects can be captured. Three modelling approaches are compared: StrUE, StrUEC, and a deterministic UE approach.

In the design scenarios, there are three sets of input parameters that can be changed: the total budget, the cost of building on a link, the amount to be added to the link (in the binary relaxation). The user inputs regarding the demand are: the expected value of total trips, and the coefficient of variation of the demand distribution. Networks with a higher degree of fluctuation in the realized demand will have a higher CV_{str}. The link capacity follows an inverse gamma distribution, where for each link, the primary input is the expected value of the capacity for each link and the coefficient of variation for each link.

In the strategic assignment network design application, the planner seeks to rank and compare different design scenarios, indexed by $s_{B,p}^w \in S$. In this experiment, the objective is $w{:}\diamond E$, and we focus on the case where $p = 1,500 \ vph$. Lacking the appropriate data, we assume that the cost to add capacity to each link is equivalent to the length of that link. Essentially, this captures the fact that the cost to add capacity to all links is not equivalent, but links that are longer will cost more to enhance their vehicle capacity. If the cost is \$100K/km, then a budget of 10 is a proxy for \$1M budget.

5.1 Small network demonstration

The first demonstration utilizes a test case based on the Nguyen-Dupuis network that is popular for small transport demonstrations. The network data can be found in Appendix I. There are two origins (1 and 4) and two destinations (2 and 3) with a strategic demand parameter $g(T{:}6,240, CV_{str})$. Note that the original demand resulted in a highly congested network and therefore a deflated demand was utilized in this work.

Figure 1 presents results for the StrUE network (where $cov_a = 0, \forall a$) for a specific design scenario $s_{25,1500}^E$, meaning that a total of 25 "length units" of 1,500 vph capacity were added to the network. The horizontal axis shows the value of CV_{str} as it varies between 0 and 0.6. Figure 1(a) shows the value (absolute not relative) of $\diamond E_s$ and $\diamond STD_s$ (in minutes). Figure 1(b) shows the performance metrics ΔE_s and ΔSTD_s for the same cases of CV_{str}.
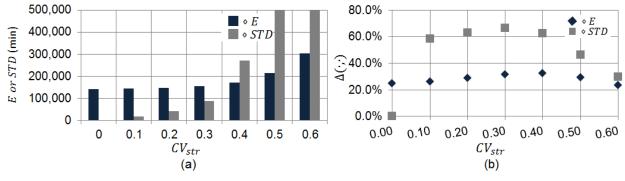
FIGURE 1   Results for Nguyen Dupuis network and StrUE subproblem

Figure 1 suggests that for a small network and low levels of volatility, a design scenario will receive similar evaluations of performance. However, once the CV_{str} reaches a certain point, the $\diamond STD$ becomes much larger and the reduction in STD is less. It is also empirically observed that in most cases, but not all, the GA identifies the same optimal set of projects.

Figure 2 presents the same demonstration where the demand is treated as a deterministic quantity. It is assumed that all links in the network have the same level of volatility. The horizontal axis of Figure 2 shows the values of cov_a as it varies between $0 - 0.4$ in increments of 0.05.
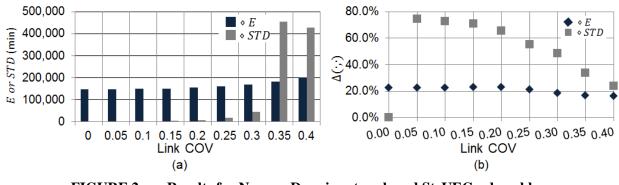

FIGURE 2   Results for Nguyen Dupuis network and StrUEC subproblem

Increasing levels of link volatility did not have an immense impact on project evaluation. This is likely due to the fact that the links were all treated as uniform, i.e., identical C_a and cov_a. Therefore the design projects affect $\diamond STD$ more than $\diamond E$. In networks where certain links have higher levels of volatility, this might not be the case.

Next we examine the case where there is volatility in both the demand and the capacity. For the results in Figure 3, $CV_{str} = 0.3$ and $cov_a = 0.3, \forall a$. In this experiment, we examine the impact of different budgets, which are shown on the horizontal axis. The vertical axis shows the system performance metric, where the blue columns represent $\diamond E$ and the grey columns represent the results for $\diamond STD$. The red crosses in Figure 3 represent the predicted performance of the design scenario in the deterministic case (where $CV_{str} = 0$ and $cov_a = 0, \forall a$).
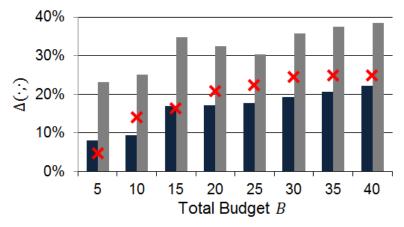
FIGURE 3   **Results for Nguyen Dupuis where $CV_{str} = 0.3$ and $cov_a = 0.3 \forall a$**

In many cases, the project selection is different when network uncertainty is accounted for. Of course, project evaluation is also different. Network design projects can impact the network by either lowering the travel time on routes and thereby lowering the total system travel time, according to the travel cost function, or by causing people to change routes, which will have unintuitive and unpredictable impacts on system performance metrics. For the simple case of the Nguyen Dupuis network, a deterministic approach appears to overestimate the impact of design projects.

5.2 Medium network demonstration

While the network used in Section 5.1 is useful to isolate individual behaviors, it is too small to capture any significant effects of route choice. Therefore, this work presents results from a second experiment on the well-known Sioux Falls network, the data for which was obtained from Bar-Gera *(20)*. Sioux Falls consists of 24 nodes, 76 links and 24 zones. The strategic demand parameter is $g(T: 360,000, CV_{str})$.

Figure 4 presents the results for a design scenario $s_{B,1500}^E$ for the when the budget varies from 20, 40, 60, or 80. The horizontal axis indicates the total budget, while the vertical axis indicates the performance metric, which is the reduction in travel time or standard deviation due to the design scenario. Figure 4(a) presents the result for the StrUE model where $CV = 0.3$ (and $cov_a = 0, \forall a$) and Figure 4(b) presents the results for StrUEC, where cov_a is based on normalizing the capacity to a range of 0.0-0.2.
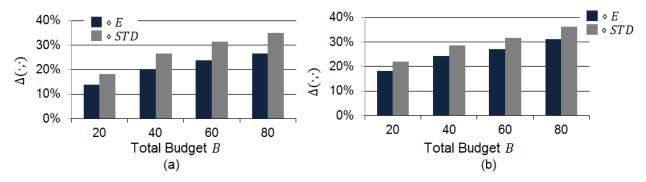

FIGURE 4   **Results for the Sioux Falls network with (a) StrUE subproblem and (b) StrUEC subproblem**

The GA found similar solutions, although the reduction in $\diamond E$ and $\diamond STD$ was greater for the results of the StrUEC model where there was also more volatility. In nearly all cases, the GA identifies a

different set of links for capacity addition for the deterministic versus the stochastic models. Additionally, the level of volatility (as captured by increasing the coefficient of variation of the probability distributions) affects the selection of optimal links. In most cases, increasing the budget resulted in links being added to the optimal set for capacity addition. More research is needed to determine the relationship between the volatility on individual links and network performance metrics.

6. Conclusion and future directions

The network design problem has a solid foundation in the literature, but remains a challenging topic among researchers. The problem becomes more complex when uncertainty in real-world parameters is included in the modelling procedure. However, model predictions may be impacted by the extreme behaviors caused by driver reaction to uncertainty, which could change project ranking and evaluation. Additionally, the performance of infrastructure design scenarios will almost certainly be forecasted incorrectly and it is not intuitive whether they will overestimate or underestimate project performance.

This work proposed a network design model that uses the strategic assignment approach to capture the reaction of vehicle travelers to day-to-day demand uncertainty. Additionally, an extension of the strategic approach where day-to-day link capacity is also a random variable is compared. The model is solved using a tailored genetic algorithm. Results show that at low levels of volatility, project rankings may not be as significantly impacted; however, project evaluations will change. As the volatility in the network increases, not accounting for uncertainties in modelling parameters means that suboptimal projects could be selected. Future work will explore more in-depth implications of link capacity uncertainty on the network design problem and incorporating reliability into the strategic route choice decisions of users.

APPENDIX

Nguyen Dupius network data:

From Node	To Node	Length	FF Speed (mph)
1	5	6	50
1	12	8	50
4	5	3	50
4	9	6	50
5	6	2	50
5	9	4	50
6	7	3	50
6	10	10	50
7	8	4	50
7	11	12	50
8	2	4	50
9	10	3	50
9	13	12	50
10	11	4	50
11	2	3	50
11	3	3	50

12	6	4	50
12	8	12	50
13	3	7	50

REFERENCES
(*1*) Dixit, V., L. M. Gardner, and S. T. Waller. Strategic User Equilibrium Assignment Under Trip Variability. in *Transportation Research Board 92nd Annual Meeting*. 2013.
(*2*) Wen, T., L. Gardner, V. Dixit, M. Duell, and S. T. Waller. A Strategic User Equilibrium Model Incorporating Both Demand and Capacity Uncertainty. in *Transportation Research Board 93rd Annual Meeting*. 2014.
(*3*) Yang, H. and M. G. H. Bell, Models and algorithms for road network design: a review and some new developments. *Transport Reviews*. Vol. 18, No. 3, 1998, pp. 257-278.
(*4*) Chen, A., et al., Transport Network Design Problem under Uncertainty: A Review and New Developments. *Transport Reviews*. Vol. 31, No. 6, 2011, pp. 743-768.
(*5*) Wismans, L., E. Van Berkum, and M. Bliemer, Modelling externalities using dynamic traffic assignment models: A review. *Transport Reviews*. Vol. 31, No. 4, 2011, pp. 521-545.
(*6*) Sun, D., R. F. Benekohal, and S. T. Waller. Multiobjective traffic signal timing optimization using non-dominated sorting genetic algorithm. in *Intelligent Vehicles Symposium, 2003. Proceedings. IEEE*. 2003. IEEE.
(*7*) Ukkusuri, S. V., T. V. Mathew, and S. T. Waller, Robust transportation network design under demand uncertainty. *Computer‐Aided Civil and Infrastructure Engineering*. Vol. 22, No. 1, 2006, pp. 6-18.
(*8*) Sumalee, A., D. P. Watling, and S. Nakayama, Reliable network design problem: case with uncertain demand and total travel time reliability. *Transportation Research Record: Journal of the Transportation Research Board*. Vol. 1964, No. 1, 2006, pp. 81-90.
(*9*) Gardner, L. M., A. Unnikrishnan, and S. T. Waller, Robust pricing of transportation networks under uncertain demand. *Transportation Research Record: Journal of the Transportation Research Board*. Vol. 2085, No. -1, 2008, pp. 21-30.
(*10*) Duthie, J. and S. T. Waller, Incorporating environmental justice measures into equilibrium-based network design. *Transportation Research Record: Journal of the Transportation Research Board*. Vol. 2089, No. -1, 2008, pp. 58-65.
(*11*) Sharma, S. and T. V. Mathew, Multiobjective network design for emission and travel-time trade-off for a sustainable large urban transportation network. *Environment and Planning-Part B*. Vol. 38, No. 3, 2011, pp. 520.
(*12*) Ferguson, E. M., J. Duthie, and S. Travis Waller, Comparing Delay Minimization and Emissions Minimization in the Network Design Problem. *Computer‐Aided Civil and Infrastructure Engineering*. Vol. No. 2012,
(*13*) Marcotte, P., S. Nguyen, and A. Schoeb, A strategic flow model of traffic assignment in static capacitated networks. *Operations Research*. Vol. 52, No. 2, 2004, pp. 191-212.
(*14*) Xie, C., & Liu, Z. (2014). On the stochastic network equilibrium with heterogeneous choice inertia. *Transportation Research Part B: Methodological*, Vol. 66, 2014, pp. 90-109.
(*15*) Duell, M., L. Gardner, V. Dixit, and S. T. Waller. Evaluation of a strategic road pricing scheme accounting for day-to-day and long term demand uncertainty. in *Transportation Research Board 93rd Annual Meeting*. 2014.
(*16*) Waller, S. T., D. Fajardo, M. Duell, and V. Dixit, Linear Programming Formulation for Strategic Dynamic Traffic Assignment. *Networks and Spatial Economics*. Vol. No. 2013, pp. 1-17.
(*17*) Karoonsoontawong, A. and S. T. Waller, Dynamic continuous network design problem: linear bilevel programming and metaheuristic approaches. *Transportation Research Record: Journal of the Transportation Research Board*. Vol. 1964, No. -1, 2006, pp. 104-117.

(*18*) Deb, K., A. Pratap, S. Agarwal, and T. Meyarivan, A fast and elitist multiobjective genetic algorithm: NSGA-II. *Evolutionary Computation, IEEE Transactions on*. Vol. 6, No. 2, 2002, pp. 182-197.

(*19*) Gardner, L. M., A. Unnikrishnan, and S. T. Waller, Solution Methods for Robust Pricing of Transportation Networks under Uncertain Demand. *Transportation Research Part C: Emerging Technologies*. Vol. 18, No. 5, 2010, pp. 656-667.

(20) Bar-Gera, H. *Transportation Test Problems*. 2012 [cited 2013 February 29]; Available from: http://www.bgu.ac.il/~bargera/tntp/.