

REPRINT

A hierarchy of vehicle fuel consumption models

R. AKÇELIK, C. BAYLEY, D. P. BOWYER, D.C. BIGGS

REFERENCE:

R. Akçelik, C. Bayley, D.P Bowyer, D.C. Biggs (1983). A hierarchy of vehicle fuel consumption models. *Traffic Engineering and Control*, Vol. 24, No. 1, pp 491-495, October 1983.

NOTE:

This paper is related to the intersection analysis methodology used in the SIDRA INTERSECTION software. Since the publication of this paper, many related aspects of the traffic model have been further developed in later versions of SIDRA INTERSECTION. Though some aspects of this paper may be outdated, this reprint is provided as a record of important aspects of the SIDRA INTERSECTION software, and in order to promote software assessment and further research.

A hierarchy of vehicle fuel consumption models

by **R. Akçelik**, *Principal Research Scientist, Australian Road Research Board*
C. Bayley, *Senior Partner, COMMED Associates*
D. P. Bowyer, *Principal Research Scientist, Australian Road Research Board*
and **D. C. Biggs**, *Experimental Officer, Australian Road Research Board*

INTRODUCTION

This paper presents a brief review of the existing vehicle fuel consumption models in the light of recent developments in this area. Emphasis will be on models relating to road-based passenger vehicles and on Australian developments¹. Various levels of models are identified with regards to model simplicity and accuracy. Relationships of the models to each other and to fuel consumption data are considered in an effort to demonstrate the fundamental principles employed in the development and use of various fuel consumption models.

A model is a concise way of providing information about the system it represents. Models are of particular value for testing in advance new designs and policies when real-life testing is not practical (i.e. costly or unsafe). As a general principle, the model should be kept as simple as possible while providing an adequate level of accuracy for the purposes for which it is needed. It is therefore inevitable that a model which is best suited for a particular area of use (i.e. the range of conditions to which the model is to be applied) may be either unduly complicated or of insufficient accuracy for another area of use.

The following are the main factors which will be considered in discussing fuel consumption models:

- (a) area of use;
- (b) required model response and sensitivities (i.e. ability to predict total fuel consumption as well as the individual contributions of system components); and
- (c) availability of, or effort required to collect, suitable data.

There is continuing, active interest in a wide range of techniques for reducing vehicle fuel consumption. These techniques can be conveniently classified into the following four primary areas:

- (a) automotive engineering;
- (b) traffic management;
- (c) transport management; and
- (d) transport infrastructure and urban form.

A discussion on information requirements from energy studies in these areas of interest is given in a recent report by the authors², which sets the scene for classifying existing fuel consumption models and specifying desirable model features. Considering the four areas of interest and the three main factors stated above, a hierarchy of models can be identified, which range from detailed models for automotive engineering purposes (Level 0 Models) to coarser models for traffic, transport and urban form analyses (Level I to III Models). This model hierarchy is described below. The questions of model response and sensitivities and input data availability are discussed in subsequent sections. The model hierarchy is illustrated and the important points of the discussion are summarised in Fig 1 and Table I. As indicated in the Figure and Table, the levels in the model hierarchy can be associated with levels of traffic/transport systems, ranging from micro to macro scale.

THE MODELS

Level 0 models

In this category, detailed models of single vehicle systems are considered. These models are useful as aids to analyses for optimum vehicle design. Detailed vehicle/engine mapping models which use input parameters describing engine and transmission characteristics, body and tyre characteristics, gear-change behaviour and other driver performance characteristics are such models^{3,4}. Detailed travel information specified as a speed-time trace of the individual vehicle is required for such models. Although automotive engineering usually relies on drive cycle methods as the main source of this information, real-life measurements of joint speed-acceleration probability distributions or microscopic traffic simulation models can be used to generate this information.

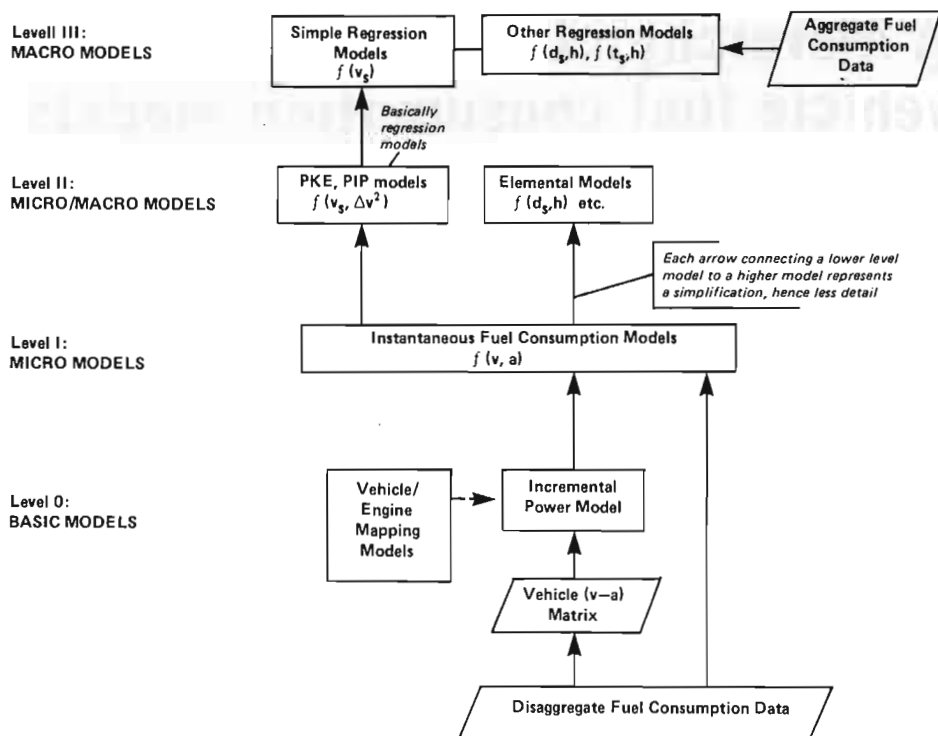


Fig 1. Hierarchy of fuel consumption models (for symbols, see Table 1).

A simpler model which retains relation to vehicle design parameters is the 'incremental power model'⁵. This model expresses the instantaneous (second-by-second) fuel consumption as a linear function of the instantaneous power requirement of the vehicle. Idling fuel consumption rate is accounted for as a constant which depends on the engine capacity. The model was developed using fuel consumption data obtained from chassis dynamometer tests and stored in vehicle (speed-acceleration) matrices. In principle, the model is applicable to on-road conditions since the power can be expressed as an explicit function of on-road speed and acceleration, vehicle mass, aerodynamic drag and rolling resistance (also road gradient as an external factor).

The incremental power model provides the link between detailed models for use in the automotive engineering area and the Level I to III models for use in the traffic/transport area. The Level I to III models discussed in detail below use traffic related variables only; road gradient is considered to be allowed as an additional term in each model.

Level I models

Models that are in the form of an instantaneous fuel consumption function are considered in this category^{6,7}. These models also require continuous speed-time traces of individual vehicles (or instantaneous speed and acceleration values) as input data, but detailed vehicle design and driver performance characteristics are not explicitly allowed for. These data can be generated by a microscopic traffic simulation model such as NETSIM⁸, MULTSIM⁹, SCATSIM¹⁰, or can be

Table 1. Fuel consumption models for traffic/transport system analysis

Model	Basic variables	Input data required	Input data source	Model output	Main area and method of use
Simple regression models (Level III: MACRO MODELS)	\bar{t}_s, \bar{h} , etc	t_s, h, x_s , etc	Real-life survey Manual counts and stopwatch	Traffic model Analytical model (e.g. Davidson's travel-time function)	Fuel consumption per trip
	\bar{t}_s = average travel time per unit distance (t_s/x_s)		In-car traffic data collection methods are suitable		—Suitable for transport analysis and other macro-level applications where overall fuel consumption estimates per trip are sufficient —Simple to use, but not sensitive to incremental effects of traffic variables
PKE & PIP models (Level II: MICRO/MACRO MODELS)	$v_s, \Delta v^2$ (or $v_r, d_s, \Delta v^2$)	$t_s, (v_i, v_f), x_s$ (or t_r, d_s , etc.)	Instrumented car (e.g. tachographs)	Microscopic model (e.g. NETSIM, MULTSIM)	Fuel consumption per link or per trip
	v_s = average travel speed v_r = average running speed t_s = average travel time t_r = average running time $v^2 = v_f^2 - v_i^2$ where v_i, v_f are initial and final speeds in a positive acceleration		In-car traffic data collection methods are used		—Model is macro-level, but input requirements are micro-level: simple formulae suitable for manual calculation are used, but data collection is relatively costly —Suitable only when an overall fuel consumption estimate (per link or per trip) is needed, but not for analysing the incremental effects of stops and delays, hence not suitable for traffic control/management studies, in particular if based on free regression method (PKE model with v_r, d_s based on two-step regression has the sensitivity to changes in delay)
Elemental models (Level II: MICRO/MACRO MODELS)	\bar{d}_s, \bar{h}, v_c	d_s, h, x_s, t_c	Manual counts and stopwatch	Macroscopic model (e.g. TRANSYT) or analytical model (e.g. ARR No. 123)	Fuel consumption per link or per trip
	\bar{d}_s = idling time \bar{h} = no. of stop/starts x_s = section distance t_c = average uninterrupted travel time $\bar{d}_s = d_s/x_s$ $\bar{h} = h/x_s$ $v_c = t_c/x_s$		Road-side or in-car traffic data collection methods can be used		—Suitable for manual calculations using macroscopic data on travel delay (or time) and number of stop/starts. Survey or computing costs are low for small-scale studies —Suitable for predicting the incremental effects of delays and number of stop/starts due to traffic control devices, hence useful for the design of traffic control/management schemes —Requires an appropriate definition of a 'stop' for simple counts; data can also be extracted from speed-time, $v(t)$, information
Instantaneous fuel consumption models (Level I: MICRO MODELS)	v, a	$v(t)$	Instrumented car (e.g. ARRB TTDAS)	Microscopic model (e.g. NETSIM, MULTSIM)	Instantaneous (second-by-second) fuel consumption
	v = instantaneous speed a = instantaneous acceleration $v(t)$ = speed-time trace		In-car traffic data collection methods are used		—Provides a basis for deriving simpler models —Requires a computer program to calculate instantaneous acceleration values and to carry out the method of summations to predict fuel consumption per mode, per link or per trip using $v(t)$ data —Suitable for detailed traffic control/management studies, but expensive in terms of data collection and computation

Level I models also provide a basis for deriving simpler, more aggregate Level II models which are discussed below.

These are micro/macro models useful for traffic and transport management purposes. These models are derived from Level I models. Depending on the assumptions involved in the derivation process and depending on the form of the Level I function, these models can take various forms, namely various PKE-average speed models¹⁵⁻¹⁷, the PIP-average speed model¹², and elemental models^{6,13,14,18,19}. PKE (Positive Kinetic Energy), PIP (Positive Inertial Power) and 'number of effective stops' in the elemental model are the variables which attempt to account for speed-smoothness of traffic. The relation between the PKE-average speed model and the elemental model is shown in Reference 20. The PIP variable is directly related to PKE, hence all these models are closely inter-related. However, there are significant differences in the methods used to derive and calibrate these models, hence they have different abilities with respect to output quality and model sensitivity (see the following section).

(a) Elemental models typically require information on items such as idling time, undelayed travel (cruise) time and speed, number of stop/starts and travel distance. (b) PKE and PIP models need total travel time, minimum and maximum speeds during each positive acceleration, and

Data produced by macroscopic traffic simulation models such as TRANSYT²¹ and SATURN²² or by analytical models²³ are suitable for elemental model applications. For real-life data, simple manual survey methods can be used. An adequate definition of a 'stop/start manoeuvre' is necessary for this purpose²⁴.

To generate data for the PKE or PIP models, it is necessary to use microscopic traffic simulation models as for Level I models, because minimum and maximum speeds for each positive acceleration necessitates information of speed-time traces of individual vehicles. However, if such information is available, Level I fuel consumption models can be used for better accuracy and model sensitivity, but at the expense of increased computing cost. This also applies to real-life traffic data, although data can be obtained using instrumented cars with less sophisticated equipment such as tachographs.

In summary, the PKE and PIP models are macro-level models, but their input data requirements are micro-level. They are simple functions suitable for manual calculations, but data collection is relatively costly. Data collection for elemental modelling should be less costly, but a commonly-used model form requires an

appropriate definition of a stop-start manoeuvre. An important feature of the PKE and PIP models is that, unlike elemental models, they are limited in predicting the incremental effects of delays and number of stop/starts as discussed further in the following section. On the other hand, these models are able to reflect the effects of changes in speed fluctuations in 'cruise' conditions better than existing elemental models.

Level III models

Simple regression models that use total travel time and distance (hence average travel speed) as input data^{25,26} are considered in this category. Because these models do not have a variable describing speed-smoothness explicitly, they are insensitive to small changes in traffic conditions in this respect. The models are simple to use and appear to be suitable for macro transport or urban form analyses where total fuel consumption estimates per vehicle trip are sufficient. Data can be collected using simple manual survey methods. Adequate estimates of average trip time (or speed) can be obtained using simple analytical models such as Davidson's function²⁷.

More complicated regression models have been considered in the literature by adding more variables to Level III models, such as the number of stops, number of gear-changes, number of brake applications, etc.²⁸. Additional terms increase prediction accuracy, but all regression models suffer from inadequate sensitivity to small changes in traffic conditions due to the problem of multi-collinearity in regression. This is discussed in detail below.

MODEL RESPONSE AND SENSITIVITIES

To be of use for design and optimisation purposes a model must have the ability not only to predict total fuel consumption, but also to predict the contribution of individual system components accurately. This question is best discussed by comparing the 'elemental' and 'regression' modelling approaches.

Although discussed under Level II models above, the principles of 'elemental modelling' can apply to all model levels. An *elemental* model and a *regression* model may be confused because they may appear to have exactly the same form, but the definitions of the model coefficients and the way they are derived are very different. Hence, this is a question of both model structure and calibration.

Regression models are *descriptive* but not necessarily *causal* relations. The coefficients of a regression model are determined jointly on the basis of best statistical fit. Hence, regression models can provide good overall prediction of the statistic concerned (fuel consumption), but they are not suited for analyses in terms of changes in the values of individual predictor variables (speed, idling time, number of stop/starts, etc.). This is due to the problem of multi-collinearity, i.e.

unreliability of estimated regression parameters caused by correlations among predictor (independent) variables. In fact, fuel consumption modelling is found to pose an extreme case of multi-collinearity since relevant traffic variables are highly correlated. As a result of this problem, the individual coefficient values cannot be determined accurately and uniquely by regression analysis.

The problem of multi-collinearity is best demonstrated by the fact that as new variables are added to the model, coefficient values of the other predictor variables will change and they may take negative or positive values. It is possible that the sign of a coefficient does not correspond to the physical behaviour of the relevant variable (e.g. a negative rolling resistance coefficient). Furthermore, a variable may be found statistically not significant where it is expected to be physically significant. The coefficient values and the levels of significance will also depend on the particular data set. It is likely that different results will be obtained for fuel consumption data sets representing different ranges of speed and acceleration rate.

The elemental modelling approach overcomes the above problems because it is structured according to the assumption that individual predictor variables are *independent*. Hence coefficient values must be determined separately either by direct measurement (e.g. idling fuel consumption rate) or by independent estimation (e.g. cruise fuel consumption rate separate from idle and stop/start rates). Thus, proper use of regression *techniques* are made in the elemental modelling approach. Examples of this can be found in Reference 19 for the Level II elemental model, and in Reference 6 for a Level I model (instantaneous fuel consumption function). The incremental power model⁵ can also be viewed as an elemental model in that it separates the idling fuel consumption and the fuel consumption due to positive power requirements, and furthermore the individual power terms (drag, inertia and grade) are determined separately. This leads to a Level I model which has high sensitivity to a small changes in traffic conditions. This does not apply to a Level I model based on free regression (e.g. see Reference 29).

Similarly, the PKE and PIP models will suffer from inadequate sensitivity to small changes in traffic conditions if based on free regression. Using a method such as 'two-step regression'¹⁷, these models can be calibrated better, but the problem will still exist to a certain extent due to the model structure. Specifically, sensitivity to cruise speed and idling time changes may become satisfactory but sensitivity to the number of stop/starts will not be satisfactory (see Reference 14 for detailed discussions on this matter).

The use of Level III models is limited to overall fuel consumption prediction, which is the primary information required at the macro travel system level.

In summary, the considerations of elemental *vs* regression modelling approach apply to all model levels and should be the primary concern in the development and choice of fuel consumption models.

INPUT DATA AVAILABILITY

The type and quality of the input data available is also an important consideration in model development and use in a particular area. While basic (Level 0) models for automotive engineering require detailed data on vehicle design and driver performance characteristics (e.g. gear-change behaviour), this type of data is not generally available to the traffic and transport analyst (Level I to III models). Detailed data for individual vehicles need to be aggregated to represent the on-road vehicle fleet characteristics, hence models need to be calibrated for this purpose. This can be done in a number of ways. A common method is to determine the model coefficients for a 'composite' vehicle^{30,31} as an average of the coefficients for individual vehicle types weighted according to the proportion of these vehicle types in the on-road traffic stream (traffic composition). An alternative, and perhaps more useful, approach is to determine model coefficients for a selected 'representative' vehicle. For more detailed analyses, several representative or average vehicle types can be employed such as a 'representative car' and a 'representative heavy vehicle'. The method adopted in this respect determines the usefulness of the fuel consumption model as a 'single vehicle' or a 'traffic stream' model.

The form of data is important in model derivation and calibration. For example, disaggregate data in the form of continuous (second-by-second) fuel consumption record, or fuel consumption values per mode (idle, constant-speed cruise, acceleration and deceleration), are suitable for deriving Level I models. On the other hand, fuel consumption data per link or per trip are only useful for calibrating Level III models.

From the user's point of view, the model choice at a particular level in the model hierarchy will depend on the type of data (simulation or real-life) available as well as the model sensitivity required, as discussed above. For example, for Level II model usage, PKE or PIP model would be appropriate if instantaneous speed-time trace data are available and incremental prediction ability is not the primary concern. On the other hand, if traffic data such as idle time, number of stop-starts and average cruise speed are available and/or incremental prediction ability is of prime importance, then an elemental model would be appropriate.

CONCLUSION

It is concluded that different levels of models are appropriate for different areas of use. However, in a particular area of use, it is preferable to use a single model which is applicable to a wide (ideally full) range of operating characteristics, i.e. low to high speeds and acceleration rates. It is therefore important that the models are calibrated with the widest possible data range as determined by real-life traffic operating conditions.

Table I and Fig 1 provide a summary of the above discussion. Table I is recommended as a guide for the choice of a model which is appropriate for a particular application. Whatever level of model detail is decided upon, special attention should be paid to the model sensitivity question and the limitation of free regression models should be kept in mind.

Considerations of on-road *versus* dynamometer data, and of aggregate *versus* disaggregate data, are also important in this context. Most existing models are derived and calibrated using dynamometer data. Future research should place more emphasis on the use of on-road data.

ACKNOWLEDGMENTS

The authors thank the Executive Director of ARRB, Dr M. G. Lay, for permission to publish this paper. The work reported in this paper was undertaken through a project supported by the National Energy Research, Development and Demonstration Council (NERDDC) of Australia. The views expressed are those of the authors, and not necessarily those of ARRB or NERDDC.

REFERENCES

- ¹SAE-A/ARRB (SOCIETY OF AUTOMOTIVE ENGINEERS—AUSTRALASIA AND AUSTRALIAN ROAD RESEARCH BOARD). 2nd Conf. 'Traffic Energy and Emissions', Melbourne, May 1982.
- ²BOWYER, D. P., R. AKÇELIK, C. BAYLEY and D. C. BIGGS. An audit of energy savings from traffic management: Stage 1 report. Internal Report AIR 390-1, Australian Road Research Board, 1982.
- ³MILKINS, E. E. and B. ALIMORADIAN. Sensitivity of car fuel consumption to vehicle design factors. Paper No. 1, SAE-A/ARRB 2nd Conf. 'Traffic Energy and Emissions', Melbourne, 1982.
- ⁴GIBSON, T. J. and R. W. BILGER. Standardised transient engine maps for Australian vehicles and their use in fuel economy modelling. Paper No. 4, SAE-A/ARRB 2nd Conf. 'Traffic Energy and Emissions', Melbourne, 1982.
- ⁵POST, K., J. TOMLIN, D. PITT, N. CARRUTHERS, A. MAUNDER, T. GIBSON, J. H. KENT and R. W. BILGER. Fuel economy and emissions research annual report for 1980-81. Charles Kolling Research Laboratory Technical Note ER-36. Department of Mechanical Engineering, University of Sydney, 1981.
- ⁶AKÇELIK, R. Derivation and calibration of fuel consumption models. Internal Report AIR 367-3, Australian Road Research Board, 1982.
- ⁷RICHARDSON, A. J. The use of vehicle maps in the derivation of stop-start fuel consumption rates. Paper No. 11, SAE-A/ARRB 2nd Conf. 'Traffic Energy and Emissions', Melbourne, 1982.
- ⁸LIEBERMAN, E., R. D. WORRALL, D. WICKS and J. WOO. NETSIM Model (5 vols). Report No. FHWA-RD-77-41 to 77-45. U.S. Federal Highway Administration, Washington, DC, 1979.
- ⁹GIPPS, P. G. and B. G. WILSON. MULTSIM: A computer package for simulating multi-lane traffic flows. Proc., 4th Biennial Conf., Simulation Soc. Aust., 1980.
- ¹⁰NEGUS, B. J. and K. J. FEHON. Fuel usage evaluation of linked signal systems. Paper No. 29, SAE-A/ARRB 2nd Conf. 'Traffic energy and Emissions', Melbourne, 1982.
- ¹¹RICHARDS, B. E. The ARRB Travel Time Data Acquisition System. Internal Report AIR 807-11, Australian Road Research Board, 1980.
- ¹²KENT, J. H., K. POST and J. TOMLIN. Fuel consumption and emission modelling in traffic links. Paper No. 10, SAE-A/ARRB 2nd Conf. 'Traffic Energy and Emissions', Melbourne, 1982.
- ¹³AKÇELIK, R. Formulae for predicting fuel consumption of cars. *Traff. Engng. Control*, 24(3), March 1983, 115-118.
- ¹⁴AKÇELIK, R. (Ed.) Progress in fuel consumption modelling for urban traffic management. Research Report ARR No. 124, Australian Road Research Board, 1983.
- ¹⁵WATSON, H. C. Sensivity of fuel consumption and emissions to driving patterns and vehicle design. SAE-A/ARRB 1st Conf., 'Can traffic management reduce vehicle fuel consumption and emissions?', Melbourne, 1980.
- ¹⁶WATSON, H. C., E. E. MILKINS and G. A. MARSHALL. A simplified method for quantifying fuel consumption of vehicles in urban traffic. *SAE-Aust.* 40(1), 1980, 6-13.
- ¹⁷WATSON, H. C., E. MILKINS, M. PRESTON, P. BEARDSLEY and C. CHITTLEBOROUGH. Further application of the PKE-average speed fuel consumption and emissions model. Paper No. 9, SAE-A/ARRB 2nd Conf. 'Traffic Energy and Emissions', Melbourne, 1982.
- ¹⁸AKÇELIK, R. Fuel efficiency and other objectives in traffic system management. *Traff. Engng Control*, 22(2), February 1981, 54-65.
- ¹⁹AKÇELIK, R. and C. BAYLEY. Derivation of fuel consumption models. Paper No. 8, SAE-A/ARRB 2nd Conf. 'Traffic Energy and Emissions', Melbourne, 1982.
- ²⁰AKÇELIK, R., A. J. RICHARDSON and H. C. WATSON. Relation between two fuel consumption models. Paper No. 7, SAE-A/ARRB 2nd Conf. 'Traffic Energy and Emissions', Melbourne, 1982.
- ²¹VINCENT, R. A., A. I. MITCHELL and D. I. ROBERTSON. User Guide to TRANSYT Version 8. TRRL Report LR888, Transport and Road Research Laboratory, Crowthorne, 1980.
- ²²FERREIRA, L. J. A., M. D. HALL and D. VAN VLIET. SATURN—a user's guide. Working Paper No. 146. Inst. of Transport Studies, Univ. of Leeds, 1982.
- ²³AKÇELIK, R. Traffic signals—capacity and timing analysis. Research Report ARR No. 123, Australian Road Research Board, 1981.
- ²⁴LUK, J. Y. K., A. G. SIMS and P. R. LOWRIE. Fuel consumption in area traffic control schemes. Paper No. 28, SAE-A/ARRB 2nd Conf. 'Traffic Energy and Emissions', Melbourne, 1982.
- ²⁵EVANS, L. and R. HERMAN. A simplified approach to calculations of fuel consumption in urban traffic systems. *Traff. Engng Control*, 17(8/9), August/September 1976, 352-354.
- ²⁶EVERALL, P. F. The effect of road and traffic conditions on fuel consumption. RRL Report LR226, Road Research Laboratory, Crowthorne, 1968.
- ²⁷AKÇELIK, R. A new look at Davidson's travel time function. *Traff. Engng Control*, 19(10), October 1978, 459-463.
- ²⁸PELENSKY, E. Cost of urban car travel. Special Report SR No. 5, Australian Road Research Board, 1970.
- ²⁹GIPPS, P. G. The effect of trams and buses on the fuel consumption of other vehicles. Paper No. 25, SAE-A/ARRB 2nd Conf. 'Traffic Energy and Emissions', Melbourne, 1982.
- ³⁰CLAFFEY, P. J. Running costs of motor vehicles as affected by road design and traffic. NCHRP Report 111, Highway Research Board, Washington, DC, 1971.
- ³¹ROBERTSON, D. I., C. F. LUCAS and R. T. BAKER. Co-ordinating traffic signals to reduce fuel consumption. TRRL Report LR 934, Transport and Road Research Laboratory, Crowthorne, 1980.

The address of Dr Akçelik and of Messrs Bowyer and Biggs: Australian Road Research Board, P.O. Box 156 (Bag 4), Nunawading 3131, Victoria, Australia.