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REPRINT

Comments on the Application of Queueing Theory to Delays at Signals

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NOTE:

This technical note is related to the intersection analysis methodology used in the SIDRA INTERSECTION software. Since the publication of this technical note, many related aspects of the traffic model have been further developed in later versions of SIDRA INTERSECTION. Though some aspects of this technical note may be outdated, this reprint is provided as a record of important aspects of the SIDRA INTERSECTION software, and in order to promote software assessment and further research.

Comments On the Application of Queueing Theory to Delays at Signals

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ABSTRACT

This technical note presents comments on the results of the application of queueing theory to delays at signalised intersections given in a paper by Blunden and Vandebona (1989). The relationship between the Australian and other delay models used around the world is explained. Importance of a continuous delay model that applies to both undersaturated and oversaturated conditions is emphasised. The simulation results given by Blunden and Vandebona are analysed. It is concluded that the simulation model used in deriving the delay results needs to be specified in detail to facilitate comparison with analytical models.

INTRODUCTION

This technical note presents comments on the results of the application of queueing theory to delays at signalised intersections given in a paper by Blunden and Vandebona (1989).

Firstly, as a historical note on the development of delay models, the relationship between the Australian work (Akçelik 1980, 1981) and the U.S. Highway Capacity Manual (Transportation Research Board 1985) should be clarified. Blunden and Vandebona state that

> "The many attempts to solve (the intersection delay problem) range from the rationally based approximations of Webster, Miller, Newell, Blunden to the ingenious curve fitting effort presented in the new Highway Capacity Manual." (p. 763)
>
> "... when one considers the curve fitting ingenuity used in obtaining the new Highway Capacity Manual formula and that produced by the Australian Road Research Board for local application." (p.766).

These statements underestimate the role and significance of the Australian work. The historical sequence of events were clearly explained by one of the principal authors of the Highway Capacity Manual (Roess 1987, p. 15):

> "The basic algorithm form was taken from the *Australian Road Capacity Guide*, developed primarily by Akçelik (ARR No. 123). JHK & Associates revised the formula to better fit delay data from U.S. intersections. Carroll Messer introduced a further modification that had a simpler form, yielded similar results to the Australian and JHK equations, and appeared to have the potential to better predict cases in which V/C ratio was marginally higher than 1.00."

A generalised delay model which embraces the Highway Capacity Manual (HCM), ARR No.123 and Canadian (Teply 1984) delay equations was presented in a paper by the author (Akçelik 1988; also see Akçelik 1990a,b). The paper explained the development of time-dependent delay formulation. In summary, the time-dependent delay models are derived by converting a steady-state delay function (as the random, or overflow, term of the delay equation) which is applicable to undersaturated conditions only, to an asymptotic time-dependent function, which becomes applicable to oversaturated conditions also (see *Figs 1* and *2*).

The asymptotic curve is an important characteristic of time-dependent delay formulation and was originally developed by researchers at the U.K. Transport and Road Research Laboratory for the TRANSYT program. In fact, the Canadian delay formula is the same as the formula given by Robertson (1979).

A general form of the steady-state delay function is

$$d_{os} = \frac{k\,(x - x_o)}{Q\,(1 - x)} \quad \text{for } x > x_o \qquad (1)$$

$$= \text{zero} \quad \text{for } x \le x_o$$

where d_{os} is the average overflow delay in seconds per vehicle, Q is the capacity in vehicles per second, x is the degree of saturation (arrival flow/capacity ratio), x_o is a limiting value of x below which the overflow (random) delay is zero ($d_{os} = 0$), and k is a model parameter (k and x_o are the calibration coefficients in the model).

The Canadian formula (Teply 1984, Robertson 1979) corresponds to Eqn (1) with k = 0.5 and $x_o = 0$, which is the random delay term of the well-known Webster (1958) formula. The Australian formula (Akçelik 1981) corresponds to k = 1.5 with a variable value of x_o, which is an approximation to Miller's (1968) delay formula. Unlike the Webster formula, Miller's original equation was based on the formulation of overflow queues. This was extended to oversaturated conditions by the author using the TRRL time-dependent delay method and introducing a simplification to the original Miller formula (Akçelik 1980). The reader is also referred to a paper by Hurdle (1984), which discusses the relationship between the steady-state and time-dependent delay equations.

Akçelik (1988) pointed out that the HCM formula produced a curve that did not have the fundamental characteristic of the time-dependent delay formulation (assuming constant demand). For x above 1.0, it diverged from the deterministic delay line and predicted very large delay values as indicated by the shaded area in *Fig. 1*. See a recent paper by Messer (1990) on this issue.

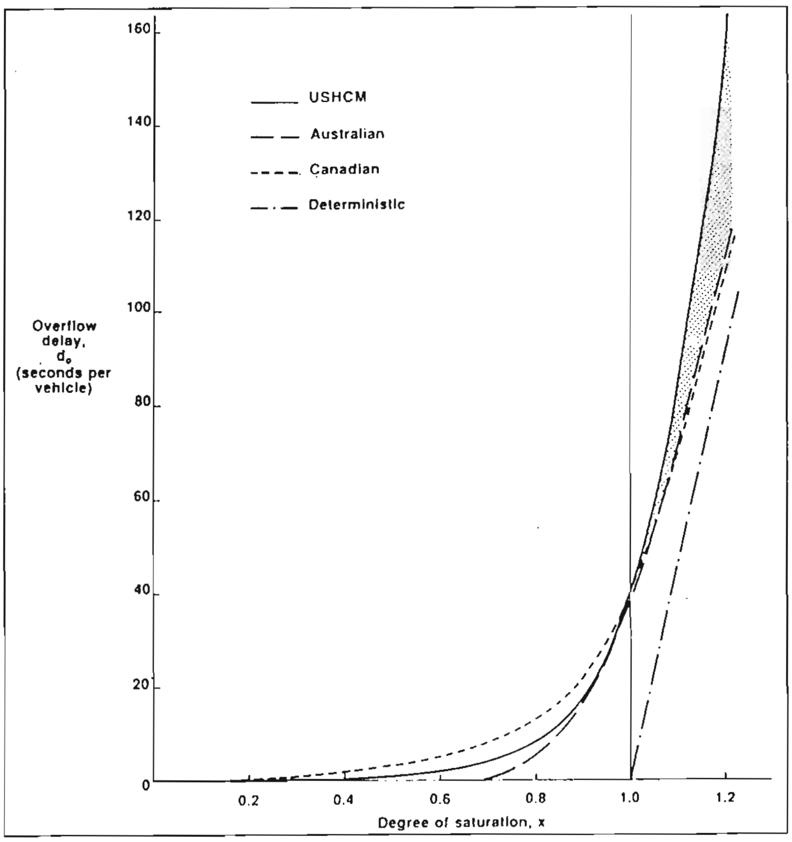

Fig. 1 – Overflow delays predicted by the Highway Capacity Manual (USHCM), Australian, and Canadian formulae (c = 90 s, g= 30 s, Q = 500 veh/h, and T = 0.25 h)

Thus, it is difficult to agree with Blunden and Vandebona that the time-dependent arrivals issue presents an "*intractable problem*" or that "*there is much confusion*" on the problem of the overloaded intersection. The solutions to this problem exist as explained above and have been in use in practice for more than two decades.

The simple model considered by Blunden and Vandebona for x > 1 (DBAR equation in p.766) is equivalent to the deterministic delay equation

$$d_d = 0.5\,r + 0.5\,T_f(x - 1) \qquad (2)$$

where the first term is the fixed uniform delay value at capacity (r = effective red time), and the second term is the oversaturation delay which depends on the flow period, T_f (d_d, r and T_f are in seconds). The basis of this equation can be found in Akçelik (1980, Eqn 5) and Akçelik (1988, Eqn 6). This is illustrated in Fig. 2 from Akçelik (1980).

Overloading for an extended, but finite period is possible in practice, and time-dependent delay formulation deals with this satisfactorily. In fact, it is the steady-state theory which assumes that arrival flow rates are sustained indefinitely!

Importantly, the time-dependent delay formulation implies that delays for undersaturated conditions (x < 1) also depend on the flow period. This effect is more pronounced as flows approach capacity (x approaches 1.0).

The deterministic delay formula (Eqn 2) is useful in providing a lower bound for delay when x is very high, but this equation has problems when x is just above 1.0. As discussed in Akçelik (1980), the equation predicts zero random (overflow) delay at x = 1.0, which is incorrect (the random effects are largest at capacity). In practice, most signalised intersections operate around capacity (at peak and near-peak flow conditions) and therefore the prediction of delays for x near 1.0 is an important consideration in intersection analysis.

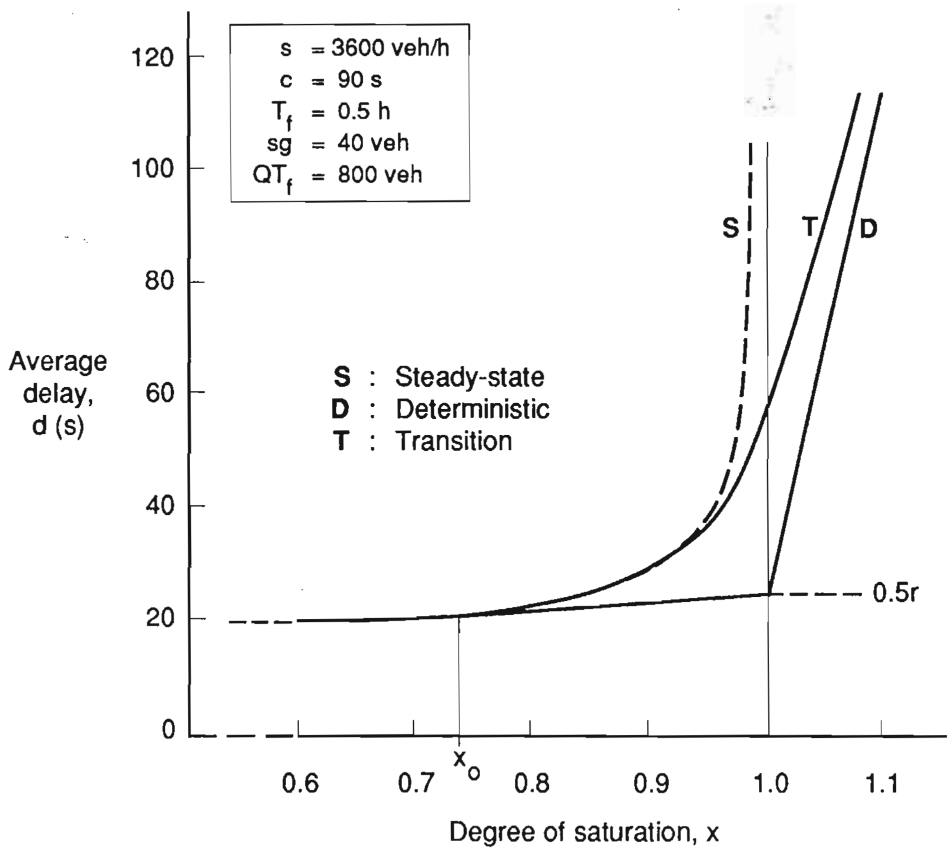

Fig. 2 – Average delay as a function of the degree of saturation

For this reason, a continuous delay model that applies to both undersaturated and oversaturated conditions is needed. The generalised time-dependent model discussed above satisfies this condition and it is at the same time consistent with the "*rationally-based*" classical models of Webster (1958), Miller (1963, 1968), and others.

ANALYSIS OF RESULTS

The results obtained using the SIDRA program (Akcelik 1986, 1987, 1990a, b) for the Highway Capacity Manual and ARR No.123 models are given in Table I for the Blunden-Vandebona example (Table 4, p.766). It should be noted that the HCM model gives *stopped delay* which is not directly comparable with the results from other models (it needs to be multiplied by a factor of 1.3 before comparing). The generalised model used in SIDRA provides HCM delay values for T_f other than 15 minutes.

The results for x = 1.0 and 1.02 in Table I show that the deterministic equation (DBAR as defined in p. 766 of Blunden and Vandebona, which is equivalent to Eqn 2 of this note) grossly underestimates delays for these conditions. It would be useful to obtain results from simulation for these conditions.

It is seen in Table I that the delays from simulation are less than those given by the deterministic equation. Theoretically, this should not be possible. There could be various reasons for this result:

(a) The equation for SBAR in p.764 as the mean service time (3600/Q) implies a capacity of Q = 796 veh/h (3600/Q = 4.5231 s) whereas Q = sg/c = 1500 x 0.5 = 750 veh/h. Thus, the degrees of saturation in simulation are effectively about 6% less than those used for analytical equations (e.g. x = 1.1 for DBAR and HCM but x = 1.036 for simulation). The results from SIDRA using capacities of Q = 750 and 796 veh/h are given in Table II (note that the SBAR equation in p.764 has unmatching brackets and TBAR in the equation for DBAR should be SBAR).

(b) It is not clear if simulations have been executed for the arrival period only (T_f) or for the arrival period as well as the period when the vehicles that are in queue at the end of the T_f period are cleared. Equation 2 applies to the latter case. The deterministic equation for delay during the T_f period only is:

$$d_d^* = 0.5\, r + 0.5\, T_f\, (x - 1) / x \qquad (3)$$

where d_d^*, r and T_f are in seconds (as in Eqn 2). The results from this equation are shown in Table II as DBAR(*). It is seen that there are simulation results that are still less than the deterministic delay value, especially for x = 1.1 (using Q = 750 veh/h). Using Eqn (2) with Q = 796 veh/h as explained in (a) gives deterministic delays lower than the simulation results, but in some cases these are too low, especially for x = 1.036 (corresponding to x = 1.1) case.

(c) Simulation could be predicting the stopped delay as in the HCM formula, rather than the overall delay (including major deceleration and acceleration delays) as in the ARR No.123 formula. However, applying a factor of 1.3 still leaves some values less than the deterministic delay value.

Of course, a combination of the above (and other?) factors could be at work. It is therefore necessary to specify the simulation method adopted in more detail. Without this specification, it is difficult to reach any conclusion from the results in Tables I and II. However, the ARR 123 model seems to give close results to the simulation estimates using Q = 796 veh/h. Note that SIDRA results in Tables I and II assume a single lane approach.

The results from SIDRA for the undersaturated case are given in Table III. A reasonable match between simulation and ARR 123 results are observed in this case also (although simulation estimates are higher for low degrees of saturation). The results for the P–K formula (DBAR formula in p.764) confirm that Q = 796 veh/h has been used:

$$d = \frac{3600}{Q} \left(1 + \frac{x}{2\,(1-x)} \right) \qquad (4)$$

For example, for x = 0.1, d = (3600/796) (1 + 0.1/1.8) = 4.77 s. Note that the delays from the P–K formula are smaller than the uniform delay results for x values of 0.7 and less. The results from the HCM formula are smaller than other models because of the stopped delay factor of 1.3.

CONCLUSION AND SUGGESTIONS FOR FURTHER WORK

In principle, we should be careful not to over-promote the *simulation* approach. Basically, simulation models are approximate models which are as prone to misspecification as analytical models. Reliance on simulation models is not healthy when simulation details are not known (people tend to believe in simulation results simply because *it is simulation*!). Therefore, simulation models need to be specified clearly. Furthermore, simulation may not always provide a good benchmark. In the case of oversaturation delays, the deterministic equation appears to provide a better basis for comparisons.

The general form of the time-dependent delay model described by Akçelik (1988, 1990a, b) can be easily calibrated by calibrating the basic steady state model (Eqn 1) for undersaturated conditions. For example, the simulation results can be used to find k and x_o (but setting $x_o = 0$, and finding k only could be satisfactory). An interesting development could be to relate k in Eqn (1) and the Erlang factor (K + 1) / 2K as in the general delay formula proposed by Blunden and Vandebona (p.763). This would simply mean replacing the first term of the P–K formula by the uniform term of the traditional signalised intersection delay model. In this, a wide range of arrival and departure distributions should be covered as in Pretty and Blunden (1964).

The reader is also referred to the Appendix of a recent paper by the author (Akçelik 1990b) for further discussion on delay models in response to a paper by Burrow (1989).

NOTATIONS

Blunden-Vandebona notation

s	=	start-up (capacity) rate
g	=	effective time green ratio
Y	=	the ratio of mean arrival rate to total capacity rate
T	=	arrival flow period
TBAR	=	mean service time

Notation in this note

s	=	saturation flow (veh/h)
u	=	g/c = effective green time ratio
g	=	effective green time (seconds)
c	=	cycle time (seconds)
x	=	arrival flow/capacity (q/Q, where q = arrival flow rate, and Q = sg/c = capacity)
T_f	=	arrival flow period
3600/Q	=	3600c/sg (not used in signalised intersection analysis)

Table I

Comparison of Delays from SIDRA* and the Delays in Table 4 of Blunden and Vandebona: the Oversaturated Case

Time, T_f (mins)	Degree of Saturation (x)	*1.1*		*1.2*		*1.3*		***1.02***	***1.0***
	SIM'L'N	53.0		79.3		125.6		***?***	***?***
15	DBAR	60.0	***60.0***	105.0	***105.0***	150.0	***150.0***	***24.0***	***15.0***
	HCM†	64.5	***70.8***	121.5	***125.2***	198.2	***199.7***	***41.9***	***36.8***
	ARR 123		***80.3***		***120.6***		***163.3***	***52.5***	***46.6***
	SIM'L'N	92.5		169.4		250.1		***?***	***?***
30	DBAR	105.0	***105.0***	195.0	***195.0***	285.0	***285.0***	***33.0***	***15.0***
	HCM†	—	***115.2***	—	***225.8***	—	***375.6***	***57.0***	***47.3***
	ARR 123		***128.4***		***211.8***		***298.9***	***71.1***	***59.8***
	SIM'L'N	133.2		277.9		470.9		***?***	***?***
60	DBAR	195.0	***195.0***	375.0	***375.0***	555.0	***555.0***	***51.0***	***15.0***
	HCM†	—	***200.8***	—	***425.7***	—	***726.8***	***81.0***	***62.1***
	ARR 123		***220.8***		***392.5***		***569.2***	***100.7***	***78.3***
	SIM'L'N	245.2		561.9		875.7		***?***	***?***
120	DBAR	375.0	***375.0***	735.0	***735.0***	1095.0	***1095.0***	***87.0***	***15.0***
	HCM†	—	***369.6***	—	***824.7***	—	***1428.9***	***120.8***	***83.0***
	ARR 123		***402.4***		***752.9***		***1109.4***	***150.1***	***104.5***
	Canada		***399.7***		***749.1***		***1105.3***	***151.5***	***108.0***

* Delays from SIDRA are in bold italic (for capacity, Q = 750 veh/h). DBAR = 0.5 r + 0.5 T_f(x – 1).

† HCM model results are stopped delay (factor of 1/1.3). Blunden and Vandebona gave results for 15-min flow period only.

Table II

Comparison of Delays with Capacity Values of Q = 750 veh/h and 796 veh/h
(Refer to Table 4 of Bluden and Vandebona): the Oversaturated Case*

Time, T_f (mins)	*Capacity and Degree of Saturation*	***Q = 750*** *x = 1.1*	***Q = 796*** ***x = 1.036***	***Q = 750*** *x = 1.2*	***Q = 796*** ***x = 1.131***	***Q = 750*** *x = 1.3*	***Q = 796*** ***x = 1.225***
	SIM'L'N	53.0		79.3		125.6	
15	***DBAR(*)***	***55.9***		***90.0***		***118.8***	
	DBAR	60.0	***31.4***	105.0	***73.8***	150.0	***116.2***
	HCM†	64.5	***46.0***	121.5	***84.6***	198.2	***141.1***
	ARR 123		***56.6***		***91.3***		***130.3***
	SIM'L'N	92.5		169.4		250.1	
30	***DBAR(*)***	***96.8***		***165.0***		***222.7***	
	DBAR	105.0	***47.8***	195.0	***132.6***	285.0	***217.4***
	HCM†	—	***65.4***	—	***144.0***	—	***258.6***
	ARR 123		***80.1***		***152.1***		***232.4***
	SIM'L'N	133.2		277.9		470.9	
60	***DBAR(*)***	**178.7**		***315.0***		***430.4***	
	DBAR	195.0	***80.6***	375.0	***250.2***	555.0	***419.8***
	HCM†	—	***98.8***	—	***260.7***	—	***492.5***
	ARR 123		***120.4***		***271.1***		***435.3***
	SIM'L'N	245.2		561.9		875.7	
120	***DBAR(*)***	**342.3**		***615.0***		***845.8***	
	DBAR	375.0	***146.2***	735.0	***485.4***	1095.0	***824.5***
	HCM†	—	***159.0***	—	***492.6***	—	***959.8***
	ARR 123		***193.4***		***507.1***		***840.3***
	Canada		***193.4***		***504.2***		***836.7***

* Delays from SIDRA are in bold italic. DBAR = $0.5\,r + 0.5\,T_f\,(x - 1)$. DBAR(*) = $0.5\,r + 0.5T_f\,(x - 1)/x$.

† HCM model results are stopped delay (factor of 1/1.3). Blunden and Vandebona gave results for 15-min flow period only.

Table III

Comparison of Delays from SIDRA* and the Delays in
Table 3 of Blunden and Vandebona: the Undersaturated Case

Average Flow	*Y (≡ x)*	*Sim'l'd Delay*	*Std.Dev.*	*P-K Formula*	*HCM†*		*Webster*	***ARR 123***	***Canada***	***Deterministic (Uniform Delay)***
75	0.1	9.86	2.36	4.77	6.00	***6.1***	7.28	***7.9***	***8.2***	***7.9***
150	0.2	10.28	2.39	5.09	6.34	***6.4***	8.11	***8.3***	***8.9***	***8.3***
225	0.3	10.47	2.32	5.49	6.74	***6.9***	8.89	***8.8***	***9.8***	***8.8***
300	0.4	11.28	2.48	6.01	7.22	***7.4***	9.86	***9.4***	***11.0***	***9.4***
375	0.5	12.05	2.67	6.78	7.83	***8.1***	11.21	***10.0***	***12.4***	***10.0***
450	0.6	13.30	2.92	7.92	8.64	***9.2***	12.91	***10.7***	***14.2***	***10.7***
525	0.7	14.63	3.23	9.87	9.80	***10.9***	15.39	***11.8***	***16.9***	***11.5***
600	0.8	17.85	4.76	13.57	11.75	***13.9***	19.91	***16.3***	***21.2***	***12.5***
675	0.9	24.84	9.15	24.88	15.97	***20.4***	31.51	***25.5***	***29.6***	***13.6***
750	***1.0***	—	—	—	—	***36.8***	—	***46.6***	***47.9***	***15.0***

* Delays from SIDRA are in bold italic (for capacity, Q = 750 veh/h). T_f = 15 minutes used.

† HCM model results are stopped delay (factor of 1/1.3).

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