REPRINT

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REFERENCE:

NOTE:

This paper is related to the intersection analysis methodology used in the SIDRA INTERSECTION software. Since the publication of this paper, many related aspects of the traffic model have been further developed in later versions of SIDRA INTERSECTION. Though some aspects of this paper may be outdated, this reprint is provided as a record of important aspects of the SIDRA INTERSECTION software, and in order to promote software assessment and further research.

ESTIMATION OF DELAYS AT TRAFFIC SIGNALS FOR VARIABLE DEMAND CONDITIONS

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Abstract — This paper proposes a delay model for signalized intersections that is suitable for variable demand conditions. The model is applicable to the entire range of expected operations, including highly oversaturated conditions with initial queues at the start of the analysis period. The proposed model clarifies several issues related to the determination of the peak flow period, as well as the periods immediately preceding and following the peak. Separate formulas are provided for estimating delay in each of the designated flow periods as well as in the total flow period. Formulas are also provided to estimate the duration of the oversaturation period where applicable. The strength of the model lies in the use of simple rules for determining flow rates within and outside the peak, using the peak flow factor, a generalization of the well-known peak hour factor parameter. Simple rules are also provided for the identification of the location and duration of the peak flow period from observations of the demand profile. Such information is considered vital from an intersection design and evaluation viewpoint. Application of the model to a variety of operating conditions indicates that the estimated delay for vehicles arriving in the peak flow period is an acceptable predictor of the average delay incurred during the total flow period, even when oversaturation persists beyond the total flow period. On the other hand, the use of the average degree of saturation with no consideration of peaking can lead to significant underestimation of delay, particularly when operating at or near capacity conditions. These findings were confirmed by comparing the model results with other models found in the literature. The significant contribution of this work is not simply in the development of improved delay estimates, but, more important, in providing an integrated framework for an estimation process that incorporates (a) the peaking characteristics in the demand flow pattern, (b) the designation of flow-specific periods within the total flow period in accordance with the observed peaking and (c) the estimation of performance parameters associated within each flow period and in combination with other periods. A revised delay formula for the U.S. Highway Capacity Manual (HCM) is proposed. The revised formula has no constraints on the peak flow period degree of saturation, unlike the current HCM formula. It is also recommended that a simple formula for estimating the duration of oversaturation be used in conjunction with the revised delay formula.

NOTATION

Symbol	Definition
T	: Duration of the total flow period
T_i	: Duration of the nonpeak flow period at the start of the total flow period
T_f	: Duration of the nonpeak flow period at the end of the total flow period
T_p	: Duration of the peak flow period in the total flow period
T_o, T'_o T''_o	: Duration of the oversaturation period
T_{pp}	: Duration of the postpeak oversaturation period
T_e, T'_e	: Duration of the posttotal flow oversaturation period (included in T_{pp})
q	: Arrival (demand) flow rate
q_p	: Average flow rate in the peak flow period (during T_p)
q_n	: Average flow rate in the nonpeak flow period (during $T_i + T_f$)
q_a	: Average flow rate during the total flow period (during T)
q_l	: Low flow rate before and after the total flow period
q_h	: Highest flow rate during the total flow period
z	: Peaking parameter, $z = (q_h - q_l)/q_a$
α	: The ratio of nonpeak and peak flow rates, $\alpha = q_n/q_p$
α'	: The ratio of low (posttotal flow period) and peak flow rates, $\alpha' = q_l/q_p$
c	: Capacity

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c_p : Peak period capacity throughout the oversaturation period, which applies to the peak flow period and the postpeak period until the oversaturation queues clear

c_n : Nonpeak period capacity outside the oversaturation period

x : Degree of saturation (volume/capacity [v/c] ratio), $x = q/c$

x_p : Peak period degree of saturation, $x_p = q_p/c_p$

αx_p : Degree of saturation during the oversaturation period following the peak flow period (before the end of the total flow period), $\alpha x_p = q_n/c_p$

$\alpha' x_p$: Degree of saturation during the oversaturation period following the total flow period, $\alpha' x_p = q_t/c_p$

x_n : Nonpeak period degree of saturation outside the oversaturation period, $x_n = q_n/c_n$

PFF   : Peak flow factor: the ratio of average flow rates in the total and peak flow periods, PFF $= q_a/q_p$

PHF   : Peak hour factor: special case of PFF in which the total flow period T is 1 h, PHF $= q_a/q_p$

PTF   : Peak time factor: the ratio of durations of the peak and total flow periods, PTF $= T_p/T$

d : Average delay per vehicle during a specified flow period (path-trace definition of delay is used in the delay model given in this paper; hence the delay estimate includes any delay experienced by these vehicles after the specified flow period)

d_p : Average delay to vehicles arriving in the peak period

d_n : Average delay to vehicles arriving in the nonpeak flow period

d_{pp} : Average delay to vehicles arriving in the postpeak oversaturation period

d_{pT} : Average delay to vehicles arriving in the posttotal flow period while oversaturation lasts

d_a : Average delay to vehicles arriving in the total flow period, estimated as an average of d_p, d_{pp} and d_n

d'_a : Average delay to vehicles arriving in the total flow period, estimated by using the average degree of saturation (x_a) and T in the general delay equation

d''_a : Average delay to vehicles arriving in the total flow period and the posttotal flow period while oversaturation lasts (average of d_a and d_{pT})

Units: Flow and capacity in vehicles per hour, flow periods in hours, average delay in seconds per vehicle.

1. INTRODUCTION

This paper proposes a delay model for signalized intersections as a simple application to the variable demand case (Kimber and Hollis, 1979; Burrow, 1989; Brilon and Wu, 1990). The extra oversaturation delay experienced by vehicles arriving after an oversaturated peak flow period is the key aspect of this application. The delay formula is consistent with models published previously (see Akçelik 1981, 1988, 1990a,b; Messer, 1990; McShane and Roess, 1990), but its use is clarified in terms of application to the peak flow period and the total flow period. The variable demand specification is given in terms of peak flow factor (PFF), which is a generalization of the well-known peak hour factor (PHF) parameter. This process approximates the variable demand pattern by using a step function that divides the total flow period into peak and nonpeak flow periods, each with a constant average flow rate.

Different delay definitions are considered, namely the average delays to vehicles arriving during (a) the peak flow period, (b) the postpeak flow period when the peak flow period is oversaturated, (c) the nonpeak flow period, and (d) the total flow period. The period following the total flow period (posttotal flow period), which may occur in highly oversaturated conditions, is also considered. In all cases, the path-trace method of delay measurement is assumed. For more detailed discussion on delay definitions and measurement methods, see Rouphail and Akçelik (1991).

The general form of the delay formula is presented, and the parameter values corresponding to the Australian (Akçelik 1981, 1988, 1990a,b) and the U.S. Highway Capacity Manual (HCM) delay equations (Transportation Research Board, 1985) are given. The proposed model resolves the question of differences between these two models (Akçelik, 1988; Messer, 1990). With its explicit formulation of the peak and postpeak period delays, the new formula can be used for degrees of saturation above 1.2 (a restriction stated in the HCM). Prediction of oversaturation delays without any limitation is desirable from a congestion-management viewpoint. Continuity in the application of the delay equation is also preferred from an analytical viewpoint.

Selection of the location and duration of the peak flow period (and the resulting peak and nonpeak flow rates), according to the peaking pattern exhibited in the total flow period, is discussed. Equations for triangular and parabolic demand patterns are provided. Comparisons of the cumulative demand patterns from the triangular and parabolic functions with the step function are given.

Graphs are given to demonstrate the use of the model with different delay definitions and with different peaking levels as represented by the PFF.

2. THE GENERAL DELAY MODEL

The average delay per vehicle, d, to vehicles arriving in a specified flow period at traffic signals can be expressed as the sum of two delay terms:

$$d = d_1 + d_2 \tag{1}$$

where d_1 = non-random delay term (delay due to signal cycle effects calculated assuming nonrandom arrivals at the average flow rate) and d_2 = overflow delay term, including effects of random arrivals as well as any oversaturation delays experienced by vehicles arriving during the specified flow period. The formulas for the two components of delay are given in Sections 2.1 and 2.2. An example illustrating the components of delay is given in Fig. 1.

The delay to vehicles arriving in a specified flow period includes delays incurred by those vehicles arriving after the termination of the flow period when such period is oversaturated. This delay definition corresponds to the path-trace method of measurement and is also consistent with the queue-sampling method (e.g. as used in the HCM) when there is no oversaturation or if the oversaturation queues are cleared within the period of delay measurement. The path-trace method of measurement is appropriate for level of service assessment and traffic assignment purposes. See Rouphail and Akçelik (1991) for detailed discussion of these issues.

It would also be possible to modify the queue-sampling method to measure delays corresponding to the path-trace method. For this purpose, the last vehicle in the queue at the end of the specified flow period would be tagged, and the queue sampling process would continue until that vehicle cleared. This count would be limited to the subject vehicle and the others in the queue preceding it.

2.1. The nonrandom delay component

Nonrandom delay is estimated by assuming that the number of vehicles that arrive during each signal cycle is fixed and equivalent to the average flow (demand) rate per cycle. Different expressions are used for the nonrandom delay term, according to the arrival characteristics (uniform or platooned) and the signal characteristics (one or two green periods). The uniform delay formula, which is valid for the case of a single green period with arrivals at a constant rate throughout the signal cycle, is:

$$d_1 = \frac{0.5C(1 - u)^2}{1 - ux} \quad \text{for } x \le 1.0$$
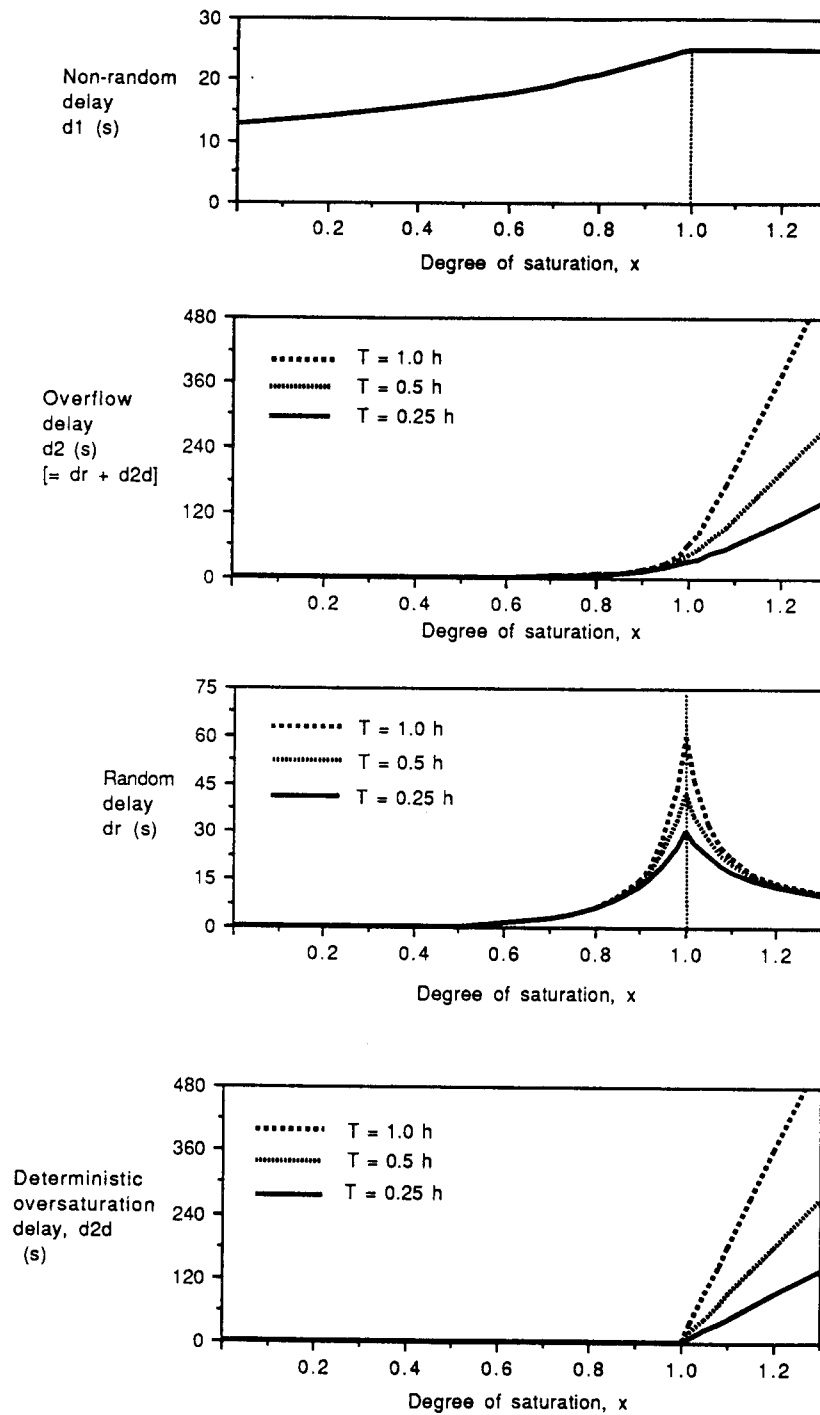
$$= 0.5(C - g) \quad \text{for } x > 1.0 \tag{2}$$

Fig. 1. The components of the general delay formula.

where c = cycle time in seconds, $u = g/C$ (ratio of effective green time, g, to cycle time, C), and x = degree of saturation (v/c ratio) given by

$$x = q/c \tag{3}$$

where q = arrival (demand) flow rate during the specified flow period in vehicles per hour or per second and c = capacity under the specified flow conditions in vehicles per hour or per second given by

$$c = sg/C \tag{4}$$

where s is the saturation flow rate in vehicles per hour or per second and g/C is the ratio of effective green time to cycle time (sg is the capacity per cycle in vehicles).

In the example shown in Fig. 1, $g = 50$ s, $C = 100$ s, and $s = 1800$ vehicles per hour are assumed, hence $u = 0.5$ and $c = 900$ vehicles per hour. Alternative formulas for the case of two green periods (as used in SIDRA, Akçelik, 1990b) and for platooned arrivals (Rouphail, 1988; Olszewski, 1988) can be used instead of eqn (2).

2.2 The overflow delay component

The overflow delay term represents the additional delay experienced by vehicles arriving during a specified flow period, which results from temporary oversaturation due to the random nature of arrivals (hence the expression "random delays") and due to persistent oversaturation when the average flow rate exceeds the capacity (which could include oversaturation delays experienced by these vehicles after the specified flow period). As discussed in Akçelik (1981, 1988, 1990a,b), the following simple formula provides a satisfactory general expression for overflow delay:

$$d_2 = 900T \left[(x - 1) + \sqrt{(x - 1)^2 + \frac{8k(x - x_o)}{cT}} \right]$$
for $x > x_o$ (zero otherwise) $\tag{5}$

where T = duration of the flow period in hours, x = degree of saturation in the specified flow period, x_o = the degree of saturation below which the overflow delay is zero ($x_o \leq 1.0$), k = a delay parameter, and c = capacity in vehicles per hour.

The steady-state delay expression which is the basis of the time-dependent expression given by eqn (5) is

$$d_s = \frac{k(x - x_o)}{c(1 - x)} \quad \text{for } x < 1.0 \tag{5a}$$

Parameters k and x_o, which determine the shape of the overflow delay curve, can be derived by calibrating the steady-state expression using data for undersaturated conditions (for x up to about 0.95).

In the HCM delay equation, $k = 0.5$ and $x_o = 0$ are used (together with an x^2 factor as discussed in Section 4). In the alternative HCM formula given by Akçelik (1988), $k = 1.0$ and $x_o = 0.5$. This model is used with $T = 0.25$ h in the example depicted in Fig. 1.

In the Australian formula (Akçelik, 1981), $k = 1.5$ and a variable x_o parameter are used, with x_o given by

$$x_o = 0.67 + sg/600 \tag{6}$$

where sg = capacity per cycle in vehicles (s = saturation flow rate in vehicles per second and g = effective green time in seconds).

The following formula, based on recent simulation work by the authors, provides an alternative form using a fixed value of $x_o = 0.5$ and a variable k parameter given by

$$k = 1.22\,sg^{-0.22} \tag{7}$$

Equation (7) gives k values in the range 1.0 to 0.5 for sg values in the range 3 to 60 vehicles per cycle.

The overflow delay term consists of two components (see Fig. 1): a random delay component (d_r), which applies to all x values, and a deterministic oversaturation delay component (d_{2d}), which applies at $x \geq 1$. Thus

$$d_2 = d_{2d} + d_r \tag{8}$$

For *undersaturated* conditions ($x \leq 1.0$), $d_{2d} = 0$, and the random delay is

$$d_r = \frac{900T \left[(x - 1) + \sqrt{(x - 1)^2 + \frac{8k(x - x_o)}{cT}} \right]}{\text{for } x_o < x \leq 1.0}$$

$$= 0 \quad \text{for } x \leq x_o \tag{9}$$

For *oversaturated* conditions ($x > 1.0$), the deterministic component of the delay experienced by vehicles arriving during the specified flow period (including delays experienced after the specified flow period) can be expressed as

$$d_{2d} = 1800 (x - 1) T \tag{10}$$

where T is in hours. Therefore, the random component of the overflow delay for oversaturated conditions is given by

$$d_r = d_2 - d_{2d} = 900T \left[- (x - 1) + \sqrt{(x - 1)^2 + \frac{8k(x - x_o)}{cT}} \right]$$
$$\text{for } x > 1.0 \tag{11}$$

As seen in Fig. 1, the random delay is negligible for low to medium degrees of saturation. It increases sharply as flows approach capacity and reaches its maximum value at capacity. As demand exceeds capacity, the oversaturation delays become more dominant, and the random delay effects are reduced gradually. It is also seen from Fig. 1 that the overflow delay and its deterministic oversaturation delay component are highly dependent on the duration of the flow period for high degrees of saturation ($x > 1$). The dependence of the random delay on the duration of the flow period is high at capacity ($x = 1$) but negligible for medium and very high degrees of saturation ($x < 0.9$ and $x > 1.1$).

3. DELAY DEFINITIONS

The general delay model given by eqns (1), (2) and (5) can be used for different delay definitions, as described in this section. The method described here allows for the effects of different forms of peaking in the total flow period.

Alternative definitions of delay require the use of different flow rates, capacities and degrees of saturation in the general model and involve the use of different averaging processes. The assumptions regarding the flow and capacity patterns, the corresponding degrees of saturation and a deterministic oversaturation queueing model that contributes to the various delay components in the general model are illustrated in Figs. 2 and 3. They are discussed below.

The model considers the total flow period (of duration T) to be divided into

1. A nonpeak period of duration T_i before the peak period and T_f after the peak period (or after the postpeak oversaturation period if applicable)
2. A peak period of duration T_p
3. A postpeak oversaturation period of duration T_{pp} ($T_{pp} = 0$ when the peak flow period is undersaturated). Note that in Fig. 2 the postpeak oversaturation period terminates in T, whereas in Fig. 3 it extends beyond T.

The parameters describing the characteristics of these three periods are explained in the following section.

(a) *Demand pattern*

(b) *Capacity pattern*

Applicable x:

(c) *Queueing pattern*
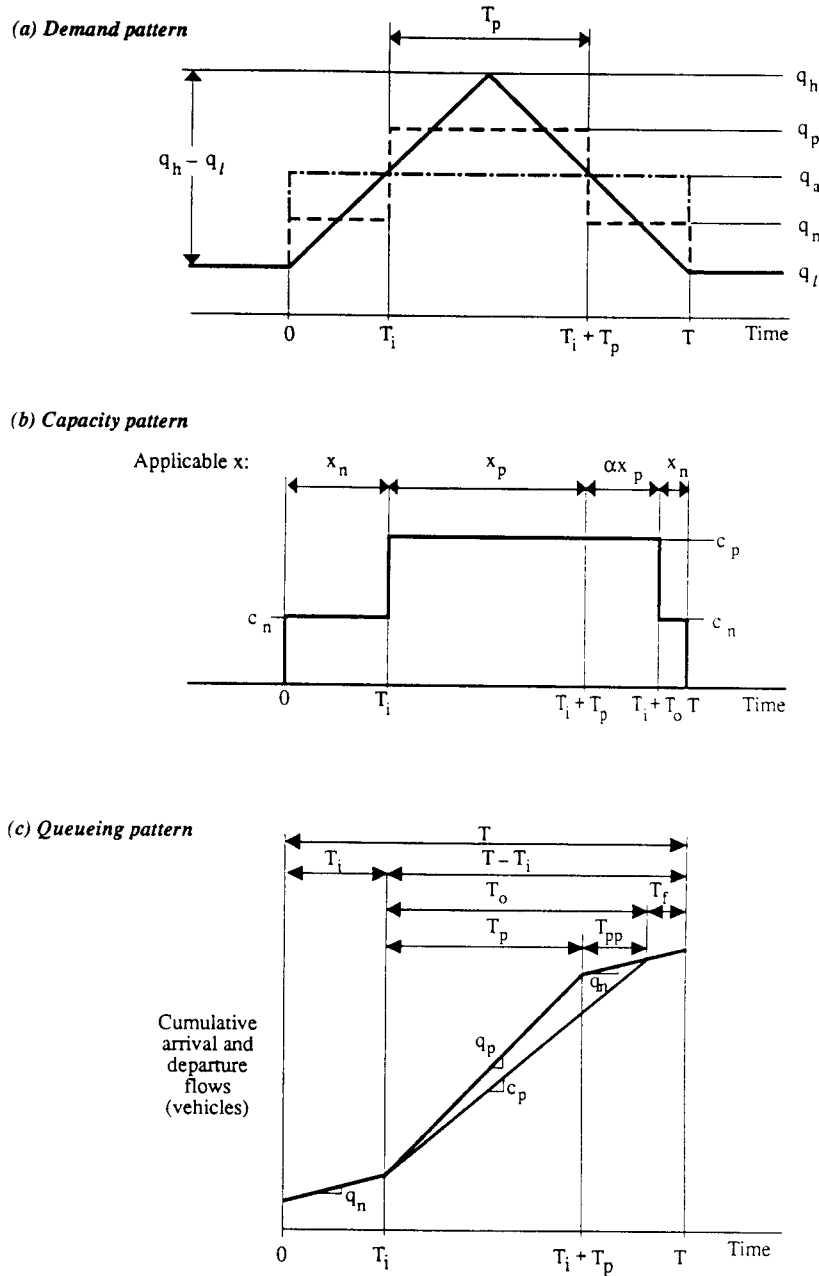
Cumulative
arrival and
departure
flows
(vehicles)

Fig. 2. Demand, capacity and deterministic oversaturation queueing patterns.
Case a: oversaturation ends *within* the total flow period.

3.1. Parameters

The degree of saturation in the peak flow period is

$$x_p = q_p/c_p \tag{12}$$

where q_p and c_p are the arrival flow rate and capacity during the peak flow period. If the peak flow period is oversaturated ($x_p > 1$), the degree of saturation for the postpeak oversaturation period, that is, the period from the end of peak flow period to the time the oversaturation queues clear, is

$$\alpha x_p = q_n/c_p \tag{13}$$

(a) Demand pattern

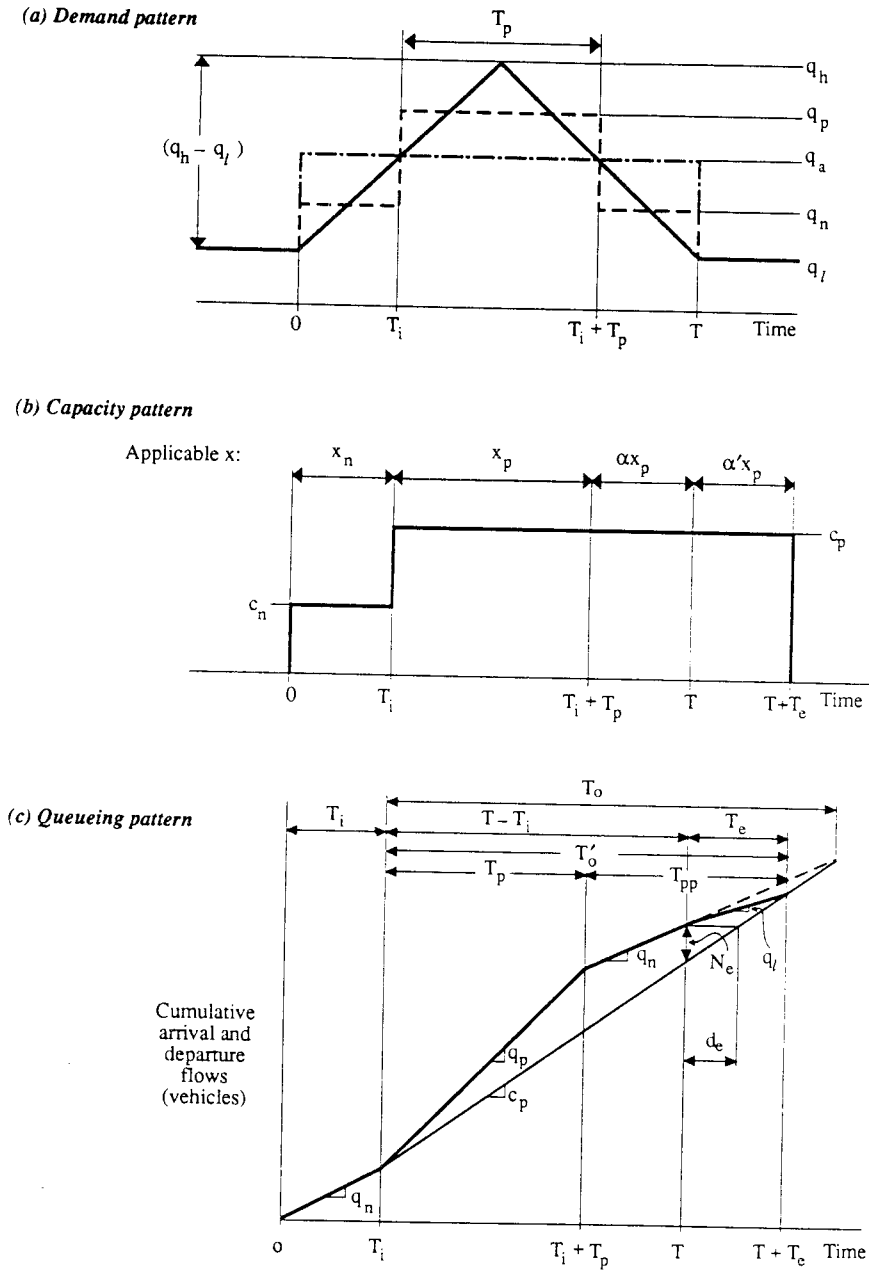

(b) Capacity pattern

(c) Queueing pattern

Fig. 3. Demand, capacity and deterministic oversaturation queueing patterns.
Case b: oversaturation ends *after* the total flow period.

where q_n is the nonpeak flow rate, c_p is the peak period capacity and the peaking parameter α is the ratio of nonpeak and peak flow rates $(0 < \alpha \leq 1.0)$:

$$\alpha = q_n/q_p \tag{14}$$

Equation (13) assumes that the peak period capacity, c_p, applies as long as oversaturation queues persist because this situation represents heavy demand conditions (e.g. leading to maximum green times). The peaking parameter α (eqn 14) can be expressed as

$$\alpha = \frac{PFF - PTF}{1 - PTF} \tag{15}$$

where *PFF* is defined as the ratio of the average flow rate during the total flow period to the average flow rate in the peak flow period (q_a, q_p):

$$PFF = q_a/q_p \tag{16}$$

and *PTF* is the ratio of the duration of peak flow period to the duration of the total flow period (T_p, T):

$$PTF = T_p/T \tag{17}$$

Note that $0 < T_p \leq T$ must be satisfied for a meaningful peaking definition. Detailed derivation of eqn (15) is given in Rouphail and Akçelik (1991).

A special case of PFF is the well known PHF, which corresponds to the case where $T = 1$ h. In the HCM, $T_p = 0.25$ h and $T = 1$ h are used, yielding $PTF = 0.25$ and $0.25 \leq PHF \leq 1.0$. In this case, the peaking parameter is

$$\alpha = \frac{PHF - 0.25}{0.75} \tag{18}$$

In the nonpeak flow period (outside the oversaturated portion of the total flow period), the degree of saturation is

$$x_n = q_n/c_n \tag{19}$$

where q_n and c_n are the arrival flow rate and capacity during the nonpeak flow period. This applies before the peak period (for duration T_i) and after the peak period, or postpeak oversaturation period if applicable (for duration T_f). The two periods are considered together as one nonpeak period. The accuracy of the resulting delay estimates are acceptable, considering the low to medium degrees of saturation assumed by the model for the nonpeak flow period.

The delay model given in the main text of this paper is valid so long as

$$x_n < 1.0 \tag{20}$$

so that there are no oversaturation queues before the start of the peak flow period. Thus, the basic assumption of the delay model, which is zero queue at the start of the peak flow period, is ensured in terms of deterministic oversaturation queues. Any transitional effects of random queues formed during the nonpeak period before the peak are therefore ignored. It is also assumed that any oversaturation queues developed during the peak flow period are eventually cleared after the peak flow period. Delay models for the case when the nonpeak period is oversaturated ($x_n > 1.0$) are discussed in the Appendix.

The oversaturation period, that is, the time from the start of the peak flow period until the time the oversaturation queue clears, is given by

$$
\begin{aligned}
T_o &= \frac{(1 - \alpha)x_p}{1 - \alpha x_p} T_p & \text{if } x_p > 1.0 \\
&= 0 & \text{if } x_p \leq 1.0
\end{aligned}
\tag{21}
$$

This expression assumes that the nonpeak flow rate (q_n) continues after the total flow period until the oversaturation queues are cleared. $T_o = 0$ represents the case when the peak period is undersaturated.

The condition for the oversaturation queues to clear sometime after the peak flow period is

$$\alpha x_p < 1.0 \tag{22}$$

Eqn (22) is equivalent to

$$x_p < 1.0/\alpha \quad \text{or} \quad q_n < c_p \tag{22a}$$

Substituting α from eqn (18) gives

$$x_p < \frac{0.75}{PHF - 0.25} \tag{23}$$

For example, when $PHF = 0.9$, x_p must be less than 1.15 for the oversaturation queues to clear sometime after the peak flow period.

There are two cases in relation to when the oversaturation period ends. The duration of the nonpeak period after the peak flow period (T_f) and the postpeak oversaturation period (T_{pp}) depends on these two cases as explained below.

Case (a): $T_o \leq T - T_i$. This is the case when the oversaturation queues clear within the total flow period (Fig. 2). In this case, the duration of the postpeak oversaturation period is given by

$$T_{pp} = T_o - T_p = \frac{x_p - 1}{1 - \alpha x_p} T_p \quad \text{if } x_p > 1.0$$
$$= 0 \quad \text{if } x_p \leq 1.0 \tag{24}$$

and the duration of the nonpeak period after the peak flow period is given by

$$T_f = T - T_i - T_p - T_{pp} \tag{25}$$

where T_{pp} is calculated from eqn (24).

Case (b): $T_o > T - T_i$. This is the case when the oversaturation queues clear after the total flow period (Fig. 3). In this case, the duration of the nonpeak period after the peak flow period is given by

$$T_f = 0 \tag{25a}$$

Note that T_o overestimates the oversaturation time unless the flow rate beyond the total flow period (T) is identical to the nonpeak period flow rate, q_n. Assuming the flow rate after the total flow period is $q_l < q_n$, hence $(q_l < c_p)$, then the corrected oversaturation period T_o' can be calculated from

$$T_o' = T - T_i + T_e \tag{26}$$

where T_e is the time to clear the oversaturation queues after the total flow period (see Fig. 3). T_e (in hours) can be expressed in terms of d_e (in seconds), which is the time it takes for the last vehicle in the queue at the end of the total flow period to clear (in accordance with the deterministic model shown in Fig. 3). Thus

$$T_e = \frac{d_e}{3600(1 - \alpha' x_p)} \tag{27}$$

where

$$d_e = 3600[(1 - \alpha)x_p T_p - (1 - \alpha x_p)(T - T_i)] \tag{27a}$$

and

$$\alpha' = q_l/q_p \tag{27b}$$

These expressions are derived by assuming that the peak period capacity (c_p) applies

after the total flow period, due to continued oversaturation. The duration of the postpeak oversaturation period is given by

$$T_{pp} = T'_o - T_p = T - T_i - T_p + T_e \qquad (28)$$

where T_e is calculated from eqn (27).

3.2. Average delay experienced by vehicles arriving during the peak flow period

If the objective is to obtain the average delay experienced by vehicles arriving during the peak flow period (T_p), then the general formula [eqns (1), (2) and (5)] is directly applicable by using the parameters T_p, c_p and x_p for the peak flow period. Thus, the average delay during the peak flow period is given by

$$d_p = 0.5(C - g) + 900T_p \left[(x_p - 1) + \sqrt{(x_p - 1)^2 + \frac{8k(x_p - x_o)}{c_p T_p}} \right] \text{ for } x_p > 1.0$$

$$= \frac{0.5C(1 - u)^2}{1 - ux_p} + 900T_p \left[(x_p - 1) + \sqrt{(x_p - 1)^2 + \frac{8k(x_p - x_o)}{c_p T_p}} \right]$$
$$\text{for } x_o < x_p \leq 1.0$$

$$= \frac{0.5C(1 - u)^2}{1 - ux_p} \quad \text{for } x_p \leq x_o \qquad (29)$$

This average delay applies to $q_p T_p$ vehicles that arrive during the peak flow period. Thus, the corresponding total delay is $d_p q_p T_p$ (vehicle seconds).

This model is equivalent to the delay equations described by Akçelik (1981, 1988, 1990a,b). Note that the model is applied to the peak flow period, not to the total flow period, unlike the assumption made by Burrow (1989) and Brilon and Wu (1990) in interpreting Akçelik's equations (i.e. the so-called low definition does not apply). See Section 3.5 for further discussion on this issue.

3.3. Average delay experienced by vehicles arriving during the postpeak oversaturation period

The average delay experienced by vehicles arriving during the postpeak oversaturation period $(T - T_i - T_f - T_p)$, which applies when $x_p > 1.0$, can be estimated from

$$d_{pp} = d_p + d_3 \qquad (30)$$

where d_p is the peak flow period delay given by eqn (29), and the additional delay term (d_3), which applies to the case when the oversaturation queues clear after the total flow period (Fig. 3), is expressed as

$$d_3 = 1800[(1 - \alpha)x_p T_p - (1 - \alpha x_p)(T - T_i)] \quad \text{if } T_o > T - T_i$$
$$= 0 \quad \text{if } T_o \leq T - T_i \qquad (31)$$

Note that $d_3 = 0.5 \, d_e$, where d_e is the time it takes for the last vehicle in the queue at the end of the total flow period to clear, as given by eqn (27a) (see Fig. 3). The delay given by eqn (30) applies to $q_n(T - T_i - T_f - T_p)$ vehicles.

For the postpeak oversaturation period, although the degree of saturation $\alpha x_p < 1.0$, the uniform delay is calculated as it is for oversaturated conditions $d_1 = 0.5 (C - g)$ because oversaturation persists. The signal timing parameters C and u are the same as the peak period values. The average deterministic oversaturation delay for the postpeak period is equal to the corresponding value for the peak flow period [d_{2d}, given by eqn (10)] when $T_o \leq T - T_i$, as depicted in Fig. 2. Assuming that the random delay is the

same as that in the peak flow period, the average postpeak oversaturation delay becomes equal to the peak period delay, $d_{pp} = d_p$. Thus, the average delay for the oversaturation period (T_o) is found to be d_p. Note that $d_{pp} = 0$ if $x_p \leq 1$.

When $T_o > T - T_i$, the additional delay term, d_3, applies and the average postpeak oversaturation delay is larger than the average peak period delay, $d_{pp} > d_p$. This is due to the fact that delays experienced by vehicles arriving after the total flow period are excluded and that the early part of the postpeak oversaturation period exhibits higher delays (closer to the maximum value of $2d_{2d}$). In this case, the overflow queue (in vehicles) at the end of the total flow period is given by

$$N_e = d_3 c_p/1800 = d_e c_p/3600 \tag{32}$$

where d_3 (in seconds) is as calculated from eqn (31), d_e (in seconds) from eqn (27a) and c_p (in vehicles per hour) is the peak period capacity.

In the case when $T_o > T - T_i$, the time to clear the oversaturation queues after the total flow period, T_e, can be calculated from eqn (27) if the flow rate after the total flow period is lower than the nonpeak period flow rate ($q_l < q_n$). If $q_l = q_n$, then $T_e = T_o - (T - T_i)$.

The average delay experienced by vehicles arriving in the posttotal flow period, T_e, which applies when $x_p > 1.0$ and $T_o > T - T_i$, can be estimated from

$$d_{pT} = d_p + d_3' \tag{33}$$

where d_p is the peak flow period delay given by eqn (29) and the additional delay term (d_3'), which applies to the case when the oversaturation queues clear after the total flow period, is expressed as

$$
\begin{aligned}
d_3' &= -1800(1 - \alpha x_p)(T - T_i - T_p) && \text{if } T_o > T - T_i \\
&= 0 && \text{if } T_o \leq T - T_i
\end{aligned}
\tag{34}
$$

Note that $d_3' = 0.5 d_e - 1800(x_p - 1)T_p$, where d_e is the delay experienced by the last vehicle in the queue at the end of the total flow period, as given by eqn (27a). The delay given by eqn (33) applies to $q_l T_e$ vehicles.

3.4. Average delay experienced by vehicles arriving during the nonpeak period

Average delay during the nonpeak period is experienced by vehicles arriving outside the oversaturation period, that is, during ($T_i + T_f$). Thus, it applies to $q_n(T_i + T_f)$ vehicles [T_f may be zero, as given by eqn (25a)]. To estimate this delay, the general formula [Eqns (1), (2) and (5)] should be used with the parameters c_n, x_n and $T_n = T_i + T_f$. It is assumed that the nonpeak period is undersaturated ($x_n < 1.0$), and therefore

$$d_n = \frac{0.5C(1 - u)^2}{1 - ux_n} + 900T_n \left[(x_n - 1) + \sqrt{(x_n - 1)^2 + \frac{8k(x_n - x_o)}{c_n T_n}} \right]$$

$$\text{for } x_o < x_n < 1.0$$

$$= \frac{0.5C(1 - u)^2}{1 - ux_n} \qquad \text{for } x_n \leq x_o \tag{35}$$

Note that the timing parameters C and u for the nonpeak period may have different values from those for the peak period (e.g. with actuated signals). Similarly, the nonpeak period capacity may be different from the peak period capacity (due to different signal timings or lower levels of opposing flows). The case when the nonpeak period is oversaturated ($x_n > 1$) is discussed in the Appendix.

3.5. Average delay experienced by vehicles arriving during the total flow period

The average delay experienced by vehicles arriving during the total flow period (T) can be calculated as the average of peak, postpeak and nonpeak period delays:

$$d_a = \frac{d_p q_p T_p + d_{pp} q_n (T - T_i - T_f - T_p) + d_n q_n (T_i + T_f)}{q_a T} \tag{36}$$

where q_a is the average flow rate in the total flow period. Note that the delay experienced by N_e vehicles [Eqn (32)] that depart after the total flow period is included in the delay given by this model.

It is interesting to compare the average delay given by eqn (36) with the average delay, d_a', estimated for the total flow period (T), using an average degree of saturation $x_a = q_a/c_a$, where c_a is the capacity for the total flow period (which may be different from the peak and nonpeak period capacities). Thus, the general delay formula [eqns (1), (2) and (5)] can be used with parameters T, c_a and x_a to estimate d_a' for the total flow period:

$$d_a' = 0.5(C - g) + 900T \left[(x_a - 1) + \sqrt{(x_a - 1)^2 + \frac{8k(x_a - x_o)}{c_a T}} \right]$$

$$\text{for } x_a > 1.0$$

$$= \frac{0.5C(1 - u)^2}{1 - u x_a} + 900T \left[(x_a - 1) + \sqrt{(x_a - 1)^2 + \frac{8k(x_a - x_o)}{c_a T}} \right]$$

$$\text{for } x_o < x_a \leq 1.0$$

$$= \frac{0.5C(1 - u)^2}{1 - u x_a} \qquad \text{for } x_a \leq x_o \tag{37}$$

This delay corresponds to the so-called low definition of delay, which has been discussed by Burrow (1989) and Brilon and Wu (1990). Simply due to the nature of the averaging process, which ignores the peaking of the demand pattern, d_a' will substantially underestimate the average delay compared to d_a for high degrees of saturation. However, the point made in this paper is that the peak period delay (d_p) rather than d_a' should be used if a single period analysis is to be carried out (the interpretations of Akçelik's equations by Burrow (1989) and Brilon and Wu (1990) miss this point).

3.6. Average delay experienced by vehicles arriving during the total flow period and the posttotal flow period while oversaturation lasts

In the case when $T_o > T - T_i$, as depicted in Fig. 3, the delays caused by oversaturation during the total flow period (T) for the vehicles that arrive after the total flow period are not included in d_a. A different average delay (d_a'') can be calculated to include the delay experienced by vehicles arriving in the posttotal flow period while oversaturation lasts, d_{pT} [given by eqn (33)]. This is given by

$$d_a'' = \frac{d_a q_a T + d_{pT} q_l T_e}{q_a T + q_l T_e} \tag{38}$$

where q_a is the average flow rate for the total flow period, d_a is given by eqn (36), T_e is the duration of delay period while oversaturation queues last after the total flow period and q_l is the flow rate during that period. It should be noted that if a separate analysis of the period after T is required, then the period T_e should be excluded from calculations as it is already accounted for in d_a''.

4. A PROPOSED REVISED DELAY MODEL FOR THE HCM

The HCM delay equation is basically a peak period delay formula with a built-in peak flow period duration of $T_p = 0.25$ h. The HCM method implies a total flow period of $T = 1$ h through the definition of the PHF [eqn (25)]. Furthermore, the HCM formula gives the average stopped delay calculated from $d_s = d/1.3$, where d is the average overall delay. The HCM delay formula is equivalent to $d_s = (d_1 + x^2 d_2)/1.3$ with $k = 0.5$ and $x_o = 0$. The x^2 factor is somewhat related to the highest delay observed during the oversaturation period (see Rouphail and Akçelik, 1991).

It is shown in this paper that the peak period delay is representative of postpeak oversaturation as well and therefore is adequate for level of service assessment purposes, which is the main focus of the HCM method. Therefore, the peak period delay formula given in this paper [eqn (29)] can be adopted for the HCM by using the delay parameters previously recommended by Akçelik (1988) ($k = 1.0$ and $x_o = 0.5$) which give delay values very close to the HCM formula for undersaturated conditions ($x \leq 1.0$):

$$d = 0.385(C - g) + 173 \left[(x - 1) + \sqrt{(x - 1)^2 + \frac{32(x - 0.5)}{c_p}} \right]$$

$$\text{for } x > 1.0$$

$$= \frac{0.385C(1 - u)^2}{1 - ux} + 173 \left[(x - 1) + \sqrt{(x - 1)^2 + \frac{32(x - 0.5)}{c_p}} \right]$$

$$\text{for } 0.5 < x \leq 1.0$$

$$= \frac{0.385C(1 - u)^2}{1 - ux} \qquad \text{for } x \leq 0.5 \tag{39}$$

where x is the peak period degree of saturation (x_p in other sections of this paper).

Consideration should be given to the use of a variable, or a fixed but longer, peak flow period duration T_p (a 30-min peak period appears to be more appropriate, as indicated by the analysis in the following section). For level of service assessment purposes, the estimation of the peak period delay along with an estimate of the duration of the oversaturation period is sufficient [T_o, T_o' or T_o'' from eqns (21), (26) or (59), respectively]. For example, eqn (21) expressed in terms of PHF (for $T_p = 0.25$ h) is

$$T_o = \frac{(1 - PHF)x_p}{3 - (4PHF - 1)x_p} \tag{39a}$$

which is valid for $1 < x_p < 3/(4PHF - 1)$, where the upper limit is in accordance with eqn (23).

5. DETERMINING THE PEAK FLOW PERIOD

The application of the general delay model with various delay definitions is based on the use of a step function to describe the peaking in the total flow period. Thus, a key aspect of the delay estimation process is the choice of the location and duration of the peak flow period (T_i and T_p) within a given total flow period of duration T (e.g. 1 h). All flow parameters used in the delay model depend on this choice. A simple yet effective method for this purpose is to use points where the average flow rate intersects the demand flow line (see Figs. 2 and 3).

Triangular, parabolic, trapezoidal and other functions have been considered in the literature as variable demand functions to approximate real-life demand patterns (May

and Keller, 1967; Burrow, 1987; Chodur and Tracz, 1984; Hansson and Bergh, 1988; Brilon and Wu, 1990). In this paper, symmetrical triangular (Figs. 2 and 3) and parabolic (Fig. 4) functions are considered. These functions are characterized by a low flow rate (q_l) that occurs at the start and end of the total flow period (at times $t = 0$ and $t = T$) and a high flow rate (q_h) that occurs in the middle of the total flow period (at time $t = 0.5\ T$).

For the symmetrical triangular and parabolic functions, the peaking of the demand pattern is represented by a parameter (z) that relates the low and high flow rates (q_l, q_h) to the average flow rate (q_a):

$$z = \frac{q_h - q_l}{q_a} \tag{40}$$

The parameter values for the step functions that approximate the symmetrical triangular and parabolic functions are listed below (similar equations can be provided for the non-symmetrical case).

For the triangular function:

$$q_l = (1 - 0.5z)q_a \tag{41}$$

$$q_h = (1 + 0.5z)q_a \tag{42}$$

$$q_p = (1 + 0.25z)q_a \tag{43}$$

$$q_n = (1 - 0.25z)q_a \tag{44}$$

$$T_i = 0.25T \tag{45}$$

$$T_p = 0.50T \tag{46}$$

$$PTF = T/T_p = 0.50 \tag{47}$$

$$PFF = q_a/q_p = \frac{1}{1 + 0.25z} \tag{48}$$

$$\alpha = q_n/q_p = \frac{1 - 0.25z}{1 + 0.25z} \tag{49}$$

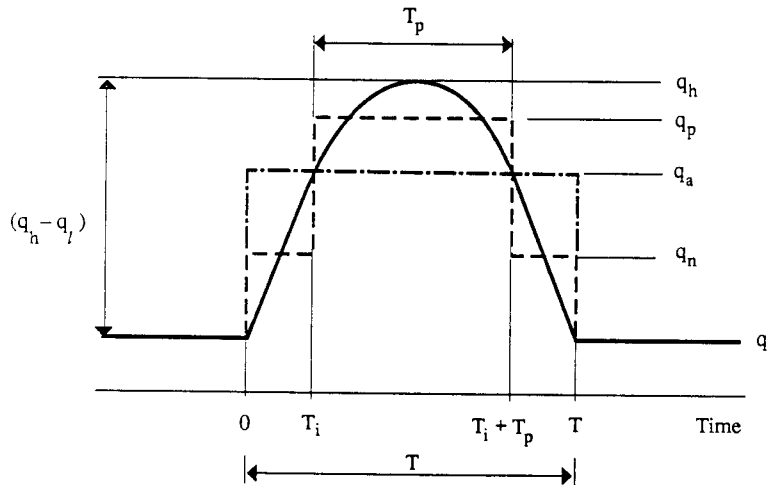For the parabolic function:

$$q_l = (1 - 2z/3)q_a \tag{50}$$

Fig. 4. The parabolic demand function and approximation by the step function.

$$q_h = (1 + z/3)q_a \tag{51}$$

$$q_p = (1 + 2z/9)q_a \tag{52}$$

$$q_n = (1 - 0.303z)q_a \tag{53}$$

$$T_i = 0.2115T \tag{54}$$

$$T_p = 0.577T \tag{55}$$

$$PTF = T/T_p = 0.577 \tag{56}$$

$$PFF = q_a/q_p = \frac{1}{1 + 2z/9} \tag{57}$$

$$\alpha = q_n/q_p = \frac{1 - 0.303z}{1 + 2z/9} \tag{58}$$

A comparison of cumulative patterns derived from the triangular and step functions for two levels of peaking corresponding to $PFF = 0.7$ (high peaking) and $PFF = 0.9$ (low peaking) is shown in Fig. 5. It is evident that the simple step function provides a very good fit to the more complex patterns under both peaking conditions. Similar results were obtained when comparing with the parabolic pattern. Thus, the step function is adequate for representing peaking in the total flow period, and the simple delay model described in this paper provides a satisfactory variable demand model for intersection design and evaluation purposes.

It is noted that the triangular and parabolic demand functions considered here produce initial and peak flow period durations (T_i, T_p) that are proportional to the duration of the total flow period (T). Because high peaking situations could be represented by small T and PFF values, T_i and T_p would be low as well. The opposite occurs for low peaking situations (high T, PFF, T_i and T_p values). When $T = 1$ h, a peak flow period of $T_p = 30$ min appears to be a good general-purpose figure (PHF parameter describes the level of peaking in this case).

6. DISCUSSION

In this section, a summary of the results obtained by approximating the triangular demand pattern with the step function is provided. The results are depicted in graphical form in Figs. 6 to 8. In this application, $C = 100\ s$, $g = 50\ s$ $(u = 0.5)$, $c_n = c_p = 900$ vehicles per hour, $s = 1800$ vehicles per hour ($sg = 25$ vehicles per cycle), $k = 0.6$, $x_o = 0.5$, $T = 1$ h, $T_i = 0.25$ h and $T_p = 0.50$ h [the latter two derived from eqns (45) and (46)]. Both the average demand rate (q_a) and the PHF were varied to estimate delays for varying average degrees of saturation (x_a) in the total flow period (T) and peaking conditions characterized by PHF. Thus, this process involved selecting values for q_a, then applying eqns (41) to (49) to derive all the necessary parameters for delay computation. Of particular interest here are comparisons of average delays in the peak and total flow periods and a study of the effect of PHF on the final delay estimates (for a given x_a).

In Fig. 6, three delay curves are shown, namely, the average delay (d_p) experienced by vehicles arriving in the peak period (T_p) from eqn (29); the average delay (d_a) experienced by vehicles arriving in the total flow period (T) from eqn (36); and, finally, the average delay (d'_a) experienced by vehicles arriving in (T), ignoring the peaking effects calculated from eqn (37). The peaking in this example is characterized by $PHF = 0.85$ (high peaking).

It is evident from Fig. 6 that, for undersaturated conditions up to about $x_a = 0.80$, all three definitions yield virtually identical delays. However, as x_a approaches 0.85 (which is equal to the PHF value), x_p approaches 1.0 and some divergence occurs. Beyond this region, the graph depicting d'_a consistently underestimates the average delay in T. This is because the da'_a model will not consider any deterministic oversaturation effects that
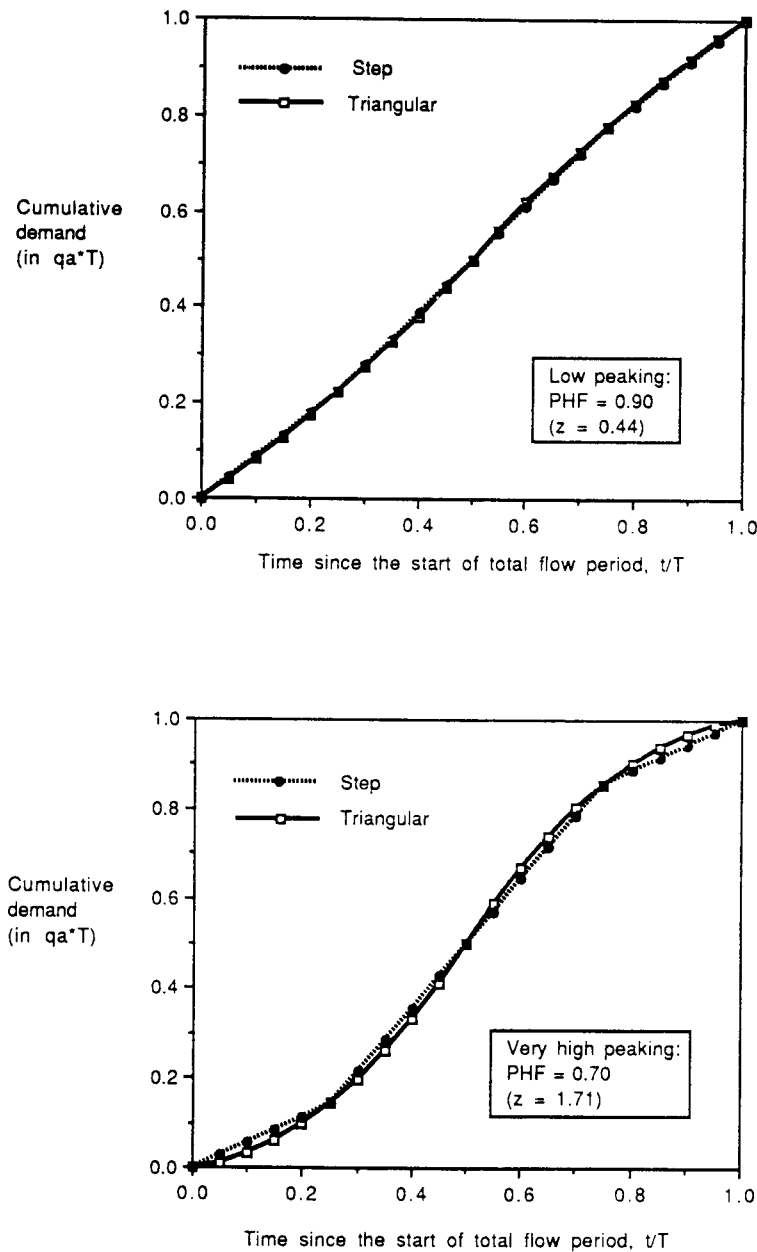
Fig. 5. Comparison of cumulative arrivals for the triangular and step function demand patterns (for peak hour factors [PHFs] of 0.7 and 0.9).

might have occurred during the peak and postpeak flow periods. This underestimation is critical when the total flow period is used as the design period because the design (practical) degree of saturation is in the range 0.85 to 1.00. As x_a exceeds 1.0, the difference is gradually reduced because the deterministic oversaturation components are partly accounted for.

It is also interesting to observe from Fig. 6 the rather small differences in the average delay to vehicles arriving in the peak (d_p) and the delay to all vehicles arriving in $T(d_a)$. Considering the high peaking situation implied by a PHF value of 0.85, it is apparent that d_p may be an acceptable estimate of d_a. Very small differences between d_a and d_a'' [eqn (38), considering oversaturation delays after the total flow period] were found for this and other examples.

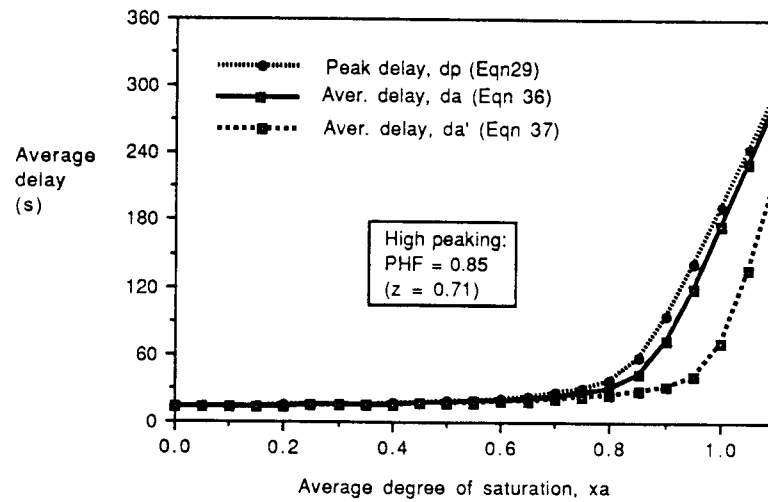The effect of the level of peaking on delays in the total and peak flow periods is

Fig. 6. The average delay for the peak flow period (d_p); the total flow period, using the average of peak, postpeak and nonpeak delays (d_a); and the total flow period using the average degree of saturation (d'_a).
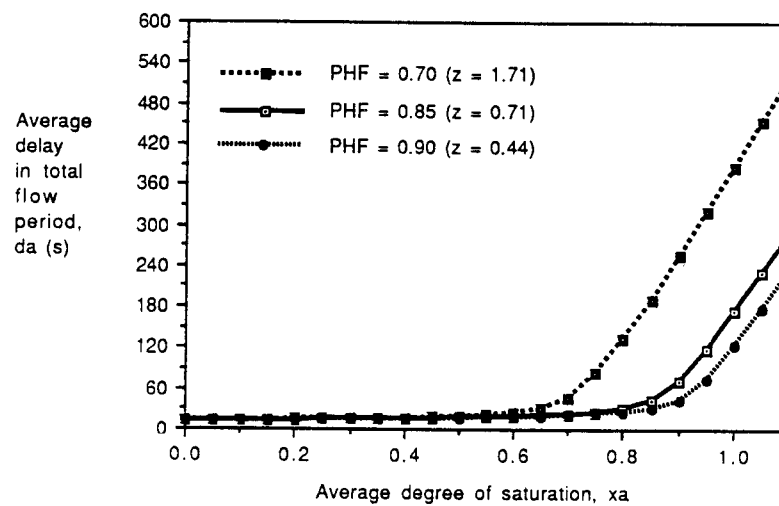

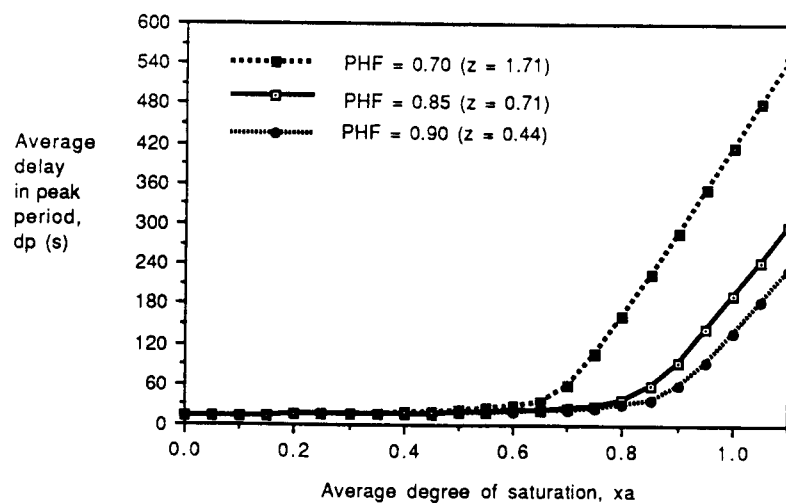Fig. 7. The effect of peak hour factor (PHF) on average delay for the *total* flow period.

Fig. 8. The effect of peak hour factor (PHF) on average delay for the *peak* flow period.

depicted in Figs. 7 and 8. In Fig. 7, the average delay to vehicles arriving in the total flow period (d_a) is compared at the PHF values of 0.70, 0.85, and 0.90. It is obvious that PHF is an important determinant of the average delay, with lower PHF values (higher peaking) yielding higher peak period delays. An almost identical pattern emerges when comparing the peak period delay (d_p) with the delays to vehicles arriving in the total flow period T (d_a), as shown in Fig. 8. This observation also confirms the previously stated finding that d_p appears to be a good estimate of d_a. Similar conclusions were drawn from the delay results obtained by using the parabolic demand function.

In Fig. 9, a comparison of the delay estimates using the method presented in this paper with the delay model proposed in a recent paper by Brilon and Wu (1990) is shown. The latter estimates the average delay to all vehicles arriving in the total flow period equivalent to d_a in this paper. The example in Fig. 9 is based on Brilon and Wu's eqn (13), which uses a parabolic demand pattern with $z = 0.4$ (equivalent to $PHF = 0.92$) and $T = 1$ h. The corresponding step function has $T_i = 0.21$ h and $T_p = 0.58$ h [eqns (54) and (55)]. Other parameters used in this example are $C = 100$ s, $g = 50$ s $(u = 0.5)$, $c_n = c_p = 900$ vehicles per hour, $s = 1800$ vehicles per hour $(sg = 25$ vehicles per cycle), $k = 1.5$, $x_o = 0.71$. The values of the delay parameters k and x_o are chosen to match the Brilon-Wu model and are the same as the original Akçelik (1981) model. For $x_a > 1$, the first term of the Brilon-Wu model is replaced by 0.5 $(C - g)$ to make the first terms of the delay equations identical.

It is seen from Fig. 9 that, compared to the average delay from eqn (36), the average delay estimated by the Brilon-Wu model somewhat overestimates delays for degrees of saturation above $x_a = 0.8$ (maximum difference around $x_a = 0.95$). This difference could be partly attributed to the transitional effects of random queues between the non-peak, peak and postpeak periods.

It is interesting to note that the average delay in the peak period [d_p from eqn (29)] is identical to the average delay in T predicted by the Brilon-Wu model (in Fig. 9, it is difficult to distinguish the two graphs). In fact, an analysis of Brilon-Wu's eqns (12) and (13) indicates that they approximate d_a by d_p for $\alpha x_p < 1.0$ and by d_a' [equivalent to eqn (37)] for $\alpha x_p > 1.0$ (note that the Brilon-Wu model is limited to $c_n = c_p$ and a symmetrical parabolic demand pattern).

Brilon-Wu's results confirm the observation from other examples in this paper that d_p may be an acceptable estimate of d_a. This simplifies the calculations considerably, as a single period analysis is sufficient.

The average delay (d_a') experienced by vehicles arriving in (T), ignoring the peaking
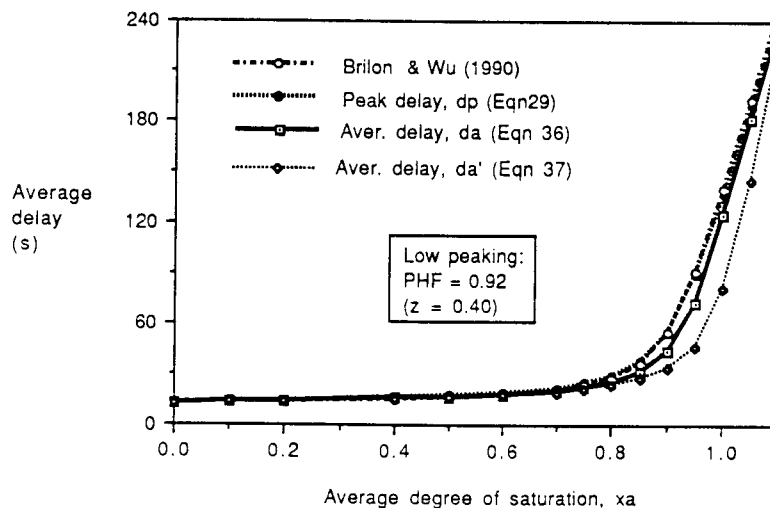

Fig. 9. Comparison of the average delay for the total flow period, estimated by using the method given in this paper and Brilon and Wu (1990) for parabolic demand function, with $z = 0.4$ and $T = 1$ h (peak hour factor [PHF] = 0.92 for step function).

effects as calculated from eqn (37), is also shown in Fig. 9. It is seen that d_a' underestimates delays substantially. Thus, Brilon and Wu's finding regarding the so-called low-definition is confirmed. However, it should be pointed out that their understanding that Akçelik's (1981) general delay equation applies to the total flow period, neglecting any peaking is somewhat misplaced. If the general equation is applied to the peak period [as represented by eqn (29)], this gives a good representation of the delay in the total flow period, including the postpeak oversaturation effects. This is valid for the Akçelik (1981) and HCM delay models.

7. CONCLUSION

This paper has presented a general delay model that uses the PFF concept as a simple variable demand model. Application of the general delay model using different delay definitions (the average delays for vehicles arriving in the peak, nonpeak and postpeak oversaturation and total flow periods) has been explained. For undersaturated conditions represented by low to medium v/c ratios, the differences between different delay definitions are small because the effect of time dependence and randomness of demand flows is negligible and no persistent oversaturation occurs.

However, as flows approach capacity (undersaturated but high v/c ratios near capacity) and exceed capacity (v/c ratio > 1), the selection of the duration of the flow period, delay definition and delay measurement method affects delay estimates significantly. The model given in this paper satisfies these requirements using a simple variable-demand model. Graphs showing the delay estimates using different delay definitions and levels of peaking demonstrate this point.

The estimation of the average delay for the total flow period appears to be simple, as it requires repeated use of the general model. In reality, the need to derive separate timing and capacity parameters for component delay periods (nonpeak and peak) makes this application rather complex. However, the method could be easily and efficiently applied in computer software such as SIDRA and HCS. Note that the method is much simpler and efficient in data requirements and computing time than a variable-demand method that would divide the total flow period into many subintervals (5 or 10 min). Furthermore, the delay model given in this paper is applicable to nonsymmetrical as well as symmetrical demand patterns.

The results of the work reported in this paper confirm that single-period analysis is adequate, provided the peak flow period is determined with due attention to peaking in the total flow period. The use of PFF (or PHF) parameter appears to be sufficient for this purpose. Thus, the delay estimates for the peak flow period can be used for the purpose of level of service assessment, and a simple application of the general delay formula is adequate in this case. For the purpose of total system performance analysis (including estimation of operating cost, fuel consumption and pollutant emission), the average delay for the total flow period (and beyond) is more appropriate, requiring more complex analysis. Nevertheless, the average delay to vehicles arriving in the peak flow period appears to be a reasonable estimate for the corresponding average delay in the total flow period. This needs to be confirmed for nonsymmetrical demand patterns.

A revised formula for the HCM is given that estimates the average delay for a peak flow period of 15 min and a total flow period of 1 h as in the original HCM formula. This paper clarifies the issue of postpeak delay implications of the delay model. The use of the peak period delay estimates, along with an estimate of the oversaturation period, is considered sufficient for level of service purposes.

The step function, which uses the PFF (or PHF) concept presented in this paper, is a simplification of the real-life variable demand patterns. Nevertheless, the concept builds on flow data that are gathered routinely as part of intersection operational analysis studies. It is important that some prior investigation is needed to select sensible duration for the peak and total flow periods taking into account the level of peaking. As a general guide, a 30-min peak flow period appears to be an appropriate choice when the total flow

period is 1 h. Generally, shorter peak flow periods are required in higher peaking cases (low PHF or PFF values, also lower T values).

The practical difficulty of measuring the true demand profile, which requires measuring arrival flows at the end of the queue, should also be recognized. Volume counts at the stopline cannot identify oversaturation because the stopline flows can never exceed the capacity. On the other hand, the stopline method would count the excess demand in subsequent intervals. This would indicate less peaking (a higher PHF or PFF value) than the real demand profile and would mean the use of a longer total flow period. Stopline volume counts supplemented by queue counts (Berry, 1987) could be used to estimate the true demand for oversaturated conditions.

REFERENCES

Akçelik R. (1981) Traffic signals — capacity and timing analysis. Research Report ARR 123, Australian Road Research Board, Melbourne, Australia.

Akçelik R. (1988) The highway capacity manual delay formula for signalized intersections. *ITE J.*, **58**(3), 23-27.

Akçelik R. (1990a) SIDRA for the highway capacity manual. Compendium of technical papers, 60th Annual Meeting of the Institute of Transportation Engineers, Orlando, FL, pp. 210-219.

Akçelik R. (1990b) Calibrating SIDRA. Research Report ARR 180, Australian Road Research Board, Melbourne, Australia.

Berry D. S. (1987) Volume counting for computing delays at signalized intersections. *ITE J.*, **57**(3), 21-23.

Brilon W. and Wu. N. (1990) Delays at fixed-time traffic signals under time-dependent traffic conditions. *Traf. Eng. Cont.*, **31**(12), 623-631.

Burrow I. J. (1987) OSCADY: A computer program to model capacities, queues and delays at isolated traffic signal junctions. Research Report 105, Transport and Road Research Laboratory, Crowthorne, U.K.

Burrow I. J. (1989) A note on traffic delay formulas. *ITE J.*, **59**(10), 29-32.

Chodur J. and Tracz M. (1984) Study and numerical modelling of non-stationary traffic flow demands at signalized intersections. In *Proceedings of the Ninth International Symposium on Transportation and Traffic Theory*, VNU Science Press, The Netherlands, Utrecht, pp. 133-154.

Hansson A. and Bergh T. (1988) A new Swedish capacity manual/Capcal 2. *Proc. 14th ARRB Conf.*, **14**(2), 38-47.

Kimber R. M. and Hollis E. M. (1979) Traffic queues and delays at road junctions. Laboratory Report 909, Transport and Road Research Laboratory, Crowthorne, U.K.

May A. D. and Keller H. M. (1967) A deterministic queueing model. *Trans. Res.*, **1**(2), 117-128.

McShane W. R. and Roess R. P. (1990) *Traffic Engineering*. Prentice-Hall, Englewood Cliffs, NJ.

Messer C. (1990) The 1985 highway capacity manual delay equation. Compendium of technical papers, 60th Annual Meeting of the Institute of Transportation Engineers, Orlando, FL, pp. 205-209.

Olszewski P. S. (1988) Efficiency of arterial signal coordination. *Proc. 14th ARRB Conf.*, **14**(2), 249-258.

Rouphail N. M. (1988) Delay models for mixed platoon and secondary flows. *J. Trans. Eng.*, **114**(2), 131-152.

Rouphail N. M. (1991) Cycle-by-cycle analysis of congested flow at signalized intersections. *ITE J.*, **61**(3), 33-36.

Rouphail N. M. and Akçelik R. (1991) Oversaturation delay estimates with consideration of peaking. Working Paper WD TE91/003, Australian Road Research Board, Melbourne, Australia.

Transportation Research Board (1985) Highway capacity manual. Special Report 209, Washington, DC.

APPENDIX

Delay estimation when the nonpeak period is oversaturated

In the main text of this paper, a fundamental assumption in the derivation of delay models is that the nonpeak flow period is undersaturated ($x_n \leq 1.0$). The extension of the model to the case when the nonpeak flow period is oversaturated ($x_n > 1.0$) is given here. This provides a very general model that allows the application of the step function to the peak flow period by dividing it into further peak and nonpeak periods.

The oversaturation queueing model that applies here is shown in Fig. 10. In this case, the peak period capacity (c_p) is assumed to apply to the nonpeak period as well. The duration of the nonpeak period is $T_n = T_i$ because $T_f = 0$, that is, oversaturation always continues beyond the total flow period. A new definition of the oversaturation period is required, as the initial period is also saturated. Thus, the oversaturation period can be defined as

$$T_o'' = T + T_e' \tag{59}$$

where T_e' is the time to clear the oversaturation queues after the total flow period assuming the period after the total flow period is undersaturated ($q_l < c_p$ or $\alpha'x_p < 1.0$ where $\alpha' = q_l/q_p$). T_e' (in hours) can be expressed as
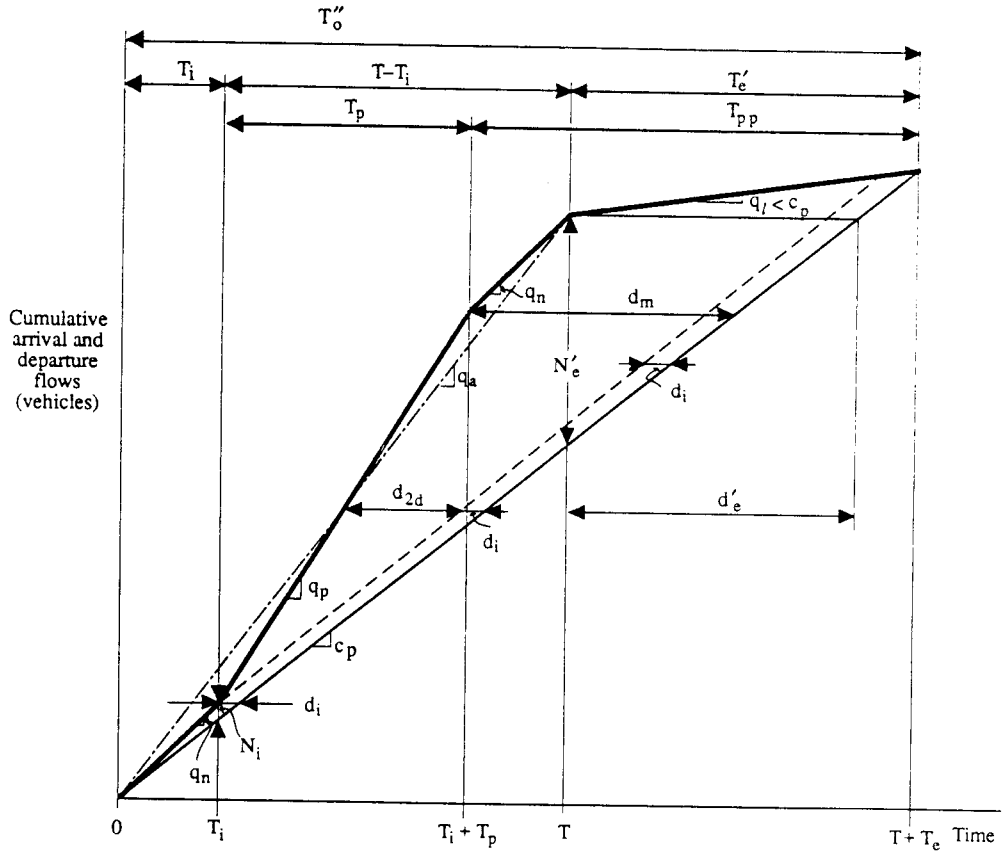
Fig. 10. Oversaturation queueing model with initial queue (nonpeak period oversaturated).

$$T'_e = \frac{d'_e}{3600(1 - \alpha'x_p)} \tag{60}$$

$$d'_e = d_i + 3600[(1 - \alpha)x_pT_p - (1 - \alpha x_p)(T - T_i)] \tag{61}$$

$$d_i = 3600N_ic_p = 3600(\alpha x_p - 1)T_i \tag{62}$$

where d'_e (in seconds) is the time it takes for the last vehicle in the queue at the end of the total flow period to clear and d_i (in seconds) is the time it takes for the last vehicle in the queue at the end of the initial (nonpeak) period to clear (in accordance with the deterministic model shown in Fig. 10); N_i is the length of the oversaturation queue at the end of the initial (nonpeak) period ($N_i = d_ic_p/3600$). Also note that the queue length at the end of the total flow period is $N'_e = d'_ec_p/3600 = (d_e + d_i)c_p/3600$, where d_e is given by eqn (27a).

The duration of the postpeak oversaturation period is given by

$$T_{pp} = T''_o - T_i - T_p = T - T_i - T_p + T'_e \tag{63}$$

where T'_e is calculated from eqn (60).

Thus, the delay equations in the main text of this paper can be modified to allow for the effect of the initial queue N_i at the start of the peak flow period and the corresponding delay d_i [from eqn (62)]:

1. The peak period delay

$$d'_p = d_p + d_i \tag{64}$$

where d_p is calculated from eqn (29) for the case when $x_p > 1.0$.
2. Postpeak period delay

$$d'_{pp} = d_{pp} + d_i \tag{65}$$

where d_{pp} is the nonzero value calculated from eqn (30).
3. Posttotal flow period delay

$$d'_{pT} = d_{pT} - 0.5 d_i \tag{66}$$

where d_{pT} is calculated from eqn (33).
4. Nonpeak period delay (for $x_n > 1.0$ and $T_n = T_i$)

$$d_n = 0.5(C - g) + 900T_i \left[(x_n - 1) + \sqrt{(x_n - 1)^2 + \frac{8k(x_n - x_o)}{c_pT_i}} \right] \tag{67}$$

5. For the average delay experienced by vehicles arriving during the total flow period (d_a), apply eqn (36) with $T_f = 0$ and use the delays from eqns (64), (65) and (67).

6. For the average delay experienced by vehicles arriving during the total flow period and the vehicles arriving in the posttotal flow period while oversaturation lasts (d_a''), apply eqn (38) and use d_a as calculated in item 5 above, d_{pT}' from eqn (66) instead of d_{pT} and T_e' from eqn (60) instead of T_e.

For symmetrical demand patterns (as for the symmetrical triangular and parabolic patterns described in Section 5), the average delay in the peak period equals the average delay in the total period, calculated as the average of peak, nonpeak and postpeak delays (in deterministic queueing terms, as shown in Fig. 10). This is also equal to the average delay in the total flow period calculated by using x_a and T in the general delay formula. However, this relationship is valid for symmetrical demand patterns only. Generally, full delay estimates (d_p', d_a, d_a') differ from each other due to the differences in the random delay components of delays for the nonpeak and peak flow periods.