

## REPRINT

# Traffic performance models for unsignalised intersections and fixed-time signals

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### NOTE:

This paper is related to the intersection analysis methodology used in the SIDRA INTERSECTION software. Since the publication of this paper, many related aspects of the traffic model have been further developed in later versions of SIDRA INTERSECTION. Though some aspects of this paper may be outdated, this reprint is provided as a record of important aspects of the SIDRA INTERSECTION software, and in order to promote software assessment and further research.

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# **PREPRINT**

## **Traffic performance models for unsignalised intersections and fixed-time signals**

by

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**Paper for 2nd International Symposium on Highway Capacity,  
9-13 August 1994, Sydney, Australia**

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# **Traffic performance models for unsignalised intersections and fixed-time signals**

**Paper for 2nd International Symposium on Highway Capacity,  
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## **ABSTRACT**

New analytical models are presented for predicting various performance statistics (delay, queue length, proportion queued, queue move-up rate and stop rate) for traffic in approach lanes controlled by give-way and stop signs and fixed-time signals. The models are also applicable to roundabouts. An integrated modelling framework is employed for consistency among different statistics and among models for different intersection types. The models have the traditional two-term form used for fixed-time signals, with additional calibration factors introduced for each term of each model. The additional factors help to allow for the effects of variations in arrival flow rates and cycle capacities. The models for unsignalised intersections were developed by converting the block and unblock periods in traditional gap acceptance modelling to effective red and green periods by analogy to traffic signal operations. This enabled the modelling of the average back of queue, proportion queued and queue move-up rates in a manner consistent with models for signalised intersections. Equations to predict the 90th, 95th and 98th percentile queue lengths are also presented. Data generated by a modified version of the microscopic simulation model MODEL C were used for model calibration. Models were derived using a bunched exponential model of arrival headway distributions for all traffic streams.

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## 1. INTRODUCTION

This paper presents new analytical models of traffic performance (delay, queue length, proportion queued, queue move-up rate and stop rate) for approach lanes controlled by give-way (yield) and stop signs and fixed-time signals. The models are also applicable to roundabouts. The performance models are based on the theoretical framework previously developed for modelling delay, queue length and stop rate at fixed-time signals in an integrated manner (Akçelik 1980, 1981, 1988, 1990a). This framework is employed for consistency in modelling different statistics and modelling different intersection types.

The modelling framework presented here makes use of the work of many researchers who contributed to the modelling of signalised intersection delays previously, but is unique in predicting queue length, proportion queued, queue move-up rate and stop rate in a way integrated with the modelling of delay. Overflow queue formulation is central to the modelling of delay, queue length and queue move-up rate. This method provides a convenient link between steady-state and time-dependent formulations (Akçelik 1980), thus allowing for easy model calibration using field or simulation data. The models presented here also differ from most traditional models in the use of capacity per cycle as an additional parameter in performance prediction.

The traditional two-term form is used with additional calibration factors introduced for each term of each model. The additional factors help to allow for the effects of variations in arrival flow rates and cycle capacities. They also help to achieve improved accuracy in predicting relative values of a given statistic for low and high degrees of saturation. Although results for fixed-time (pretimed) isolated signals are given in this paper, the model structure allows for the development of similar models for vehicle-actuated signals and closely-spaced intersections (Akçelik 1994a,c; Akçelik and Roupail 1994).

The models for unsignalised intersections represent a new development to fill the gap in modelling queue length, proportion queued and queue move-up rates in the context of gap acceptance modelling. The traditional gap-acceptance and queueing theory models do not give sufficient information for intersection design purposes since they predict the average cycle-based queue length rather than the average back of queue, and models for predicting queue move-up and stop rates do not exist other than recent work by Troutbeck (1993).

The commonly-used average cycle-based queue length incorporates all queue states including zero queues. The back of queue is a more useful statistic since it is relevant to the design of appropriate queueing space (e.g. for short lane design). The back of queue is also used for the prediction of such statistics as the saturated portion of the green period

and for modelling short lane capacities. In addition to the average values of the back of queue, this paper presents equations to predict the 90th, 95th and 98th percentile queue lengths.

The models for unsignalised intersections were derived by extending the traditional gap acceptance modelling by treating *block* and *unblock* periods in a priority (major) stream as *red* and *green* periods in a way similar to the modelling of signal-controlled traffic streams (Akçelik 1994b). This enabled the modelling of the average back of queue, proportion queued and queue move-up rate for the entry (minor) stream in a manner consistent with models for traffic signals. This presents a methodological advantage in that the same conceptual framework is employed in models for different types of intersection.

The performance models presented in this paper were developed using the bunched exponential model of arrival headway distribution which is more realistic than the commonly used simple exponential and shifted exponential models (Akçelik and Chung 1994). The models given here are based on the use of the bunched exponential model for capacity and performance prediction for all types of intersection. In modelling capacity of entry streams at unsignalised intersections, the headway distribution of total traffic demand in all lanes of the major stream(s) is adopted with different values of minimum headway and bunching parameters for single-lane and multi-lane cases. For performance modelling, a lane-by-lane method is adopted, and therefore, arrival headway distribution in a single lane of the approach road is considered.

The calibration of performance models was carried out using data generated by the microscopic simulation model MODEL C (Chung, Young and Akçelik 1992a,b). MODEL C was modified to incorporate the calibrated arrival headway distribution model and generate data required for the calibration of the new performance models. The models for unsignalised intersections were derived by simulating a single uninterrupted opposing (major) stream. As such, they are applicable to all basic gap acceptance situations including roundabouts. Of course, different sets of gap acceptance parameters are used for roundabout and sign-control cases.

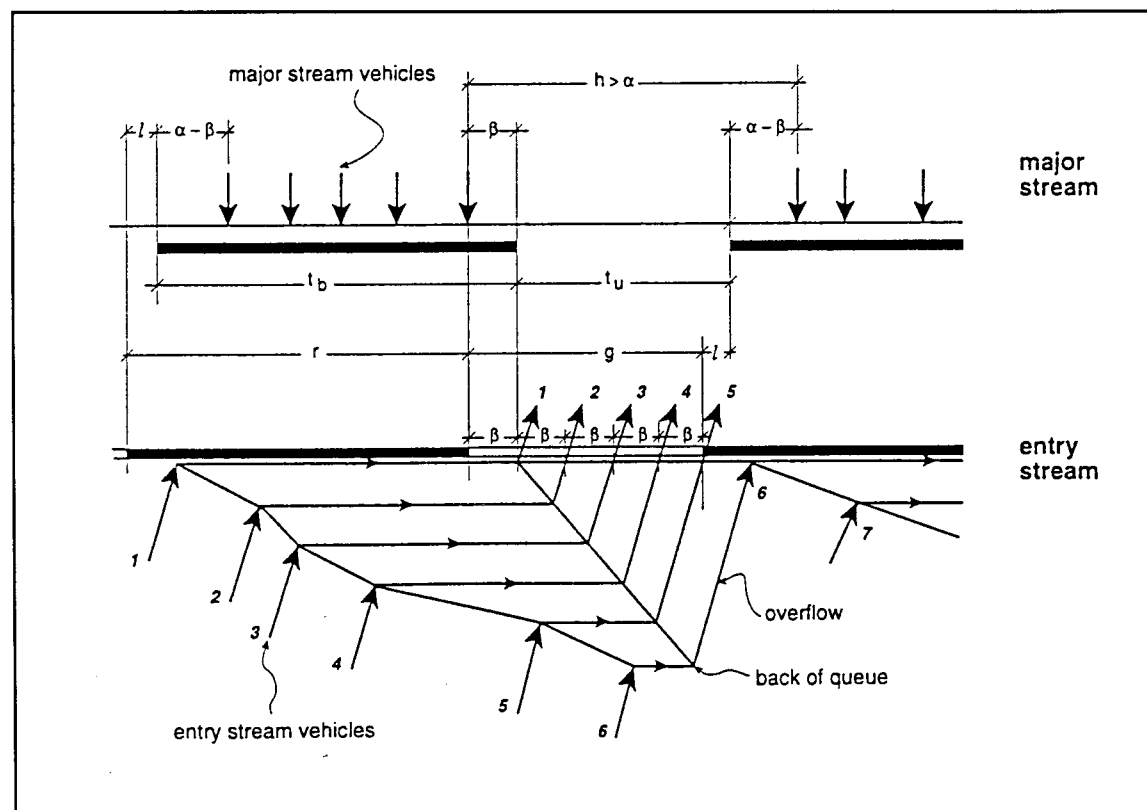
Originally a roundabout simulation model, MODEL C was extended to simulate fixed-time signal control conditions in a simple way making use of the analogy between gap acceptance and signal operations. The models for fixed-time signal operations represent a refinement of the current models to achieve improvements related to a more flexible model structure, the use of the bunched exponential headway distribution model, the need for performance models for lane-by-lane application (earlier models were calibrated for lane groups), and the need for information on the 90th, 95th and 98th percentile queue lengths for design purposes. The comparison of results from MODEL C with known models for traffic signal operations helped to validate various simulation algorithms.

Before presenting the new performance formulae, the traffic signal analogy for gap acceptance processes is explained.

## 2. UNSIGNALISED INTERSECTION ANALYSIS BY SIGNAL OPERATIONS ANALOGY

A method for treating the traditional gap acceptance modelling used for unsignalised intersections by analogy to traffic signal operations was conceived by Akçelik (1991) and is discussed in more detail in Akçelik (1994b). The underlying assumptions are shown in *Figure 1* where an entry (minor) stream gives way to an uninterrupted major (priority) stream. The method derives *equivalent average red, green and cycle times* ( $r$ ,  $g$ ,  $c$ ) for use in capacity and performance models considering average durations of *block* and *unblock* periods ( $t_b$ ,  $t_u$ ) in the major (priority) stream as used in traditional gap acceptance modelling (Tanner 1962, 1967; Troutbeck 1986, 1988, 1989, 1990, 1991a, 1993; Akçelik and Troutbeck 1991).

The basic formulae for unsignalised intersection modelling, including the capacity and minimum delay formulae, are expressed by *Equations (1) to (8)*. All capacity and delay calculations are carried out for individual lanes of entry (minor) movements, but traffic in all lanes of the major (conflicting) movement is treated together as one stream. When there are several conflicting (higher priority) streams, they are combined together and treated as one stream.



**Fig. 1 – Signal operations analogy for gap acceptance modelling**

$$t_b = \frac{e^{\lambda(\alpha-\Delta_m)}}{\varphi_m q_m} - \frac{1}{\lambda} \quad (1a)$$

$$t_u = \frac{1}{\lambda} \quad (1b)$$

$$c = t_b + t_u = r + g = \frac{e^{\lambda(\alpha-\Delta_m)}}{\varphi_m q_m} \quad (2a)$$

$$g = t_u + \beta - l = \frac{1}{\lambda} + \beta - l \quad (2b)$$

$$r = c - g = t_b - \beta + l = \frac{e^{\lambda(\alpha-\Delta_m)}}{\varphi_m q_m} - \frac{1}{\lambda} - \beta + l \quad (2c)$$

$$l = 0.5 \beta \quad (3)$$

$$u = g / c = \left( \frac{1}{\lambda} + \beta - l \right) \varphi_m q_m e^{-\lambda(\alpha-\Delta_m)} \quad (4a)$$

$$= (1 - \Delta_m q_m + 0.5 \beta \varphi_m q_m) e^{-\lambda(\alpha-\Delta_m)}$$

$$y = q_e / s = \beta q_e \quad (4b)$$

$$sg = g / \beta \quad (4c)$$

$$Q = \max(Q_g, Q_m) \quad (5)$$

$$Q_g = \frac{sg}{c} = \frac{3600 u}{\beta} = \quad (5a)$$

$$= \frac{3600 \varphi_m q_m}{\beta} \left( \frac{1}{\lambda} + \beta - l \right) e^{-\lambda(\alpha-\Delta_m)}$$

$$= \frac{3600}{\beta} (1 - \Delta_m q_m + 0.5 \beta \varphi_m q_m) e^{-\lambda(\alpha-\Delta_m)}$$

$$Q_m = \min(q_e, 60 n_m) \quad (5b)$$

$$d_m = \frac{e^{\lambda(\alpha-\Delta_m)}}{\varphi_m q_m} - \alpha - \frac{1}{\lambda} + \frac{\lambda \Delta_m^2 - 2\Delta_m + 2\Delta_m \varphi_m}{2(\lambda \Delta_m + \varphi_m)} \quad (6)$$

$$\lambda = \varphi_m q_m / (1 - \Delta_m q_m) \quad (7)$$

*subject to  $q_m \leq 0.98 / \Delta_m$*

$$\varphi = e^{-b \Delta q} \quad (8)$$

where

- $t_b, t_u$  = average durations of the block and unblock periods in the major traffic stream (sec);
- $c$  = equivalent average cycle time corresponding to the block and unblock periods in the major traffic stream ( $c = r + g$ ) (sec);
- $r, g$  = equivalent average red and green times corresponding to the block and unblock periods in the major traffic stream (sec);
- $l$  = equivalent lost time that corresponds to the unused portion of the unblock period (sec);
- $u, y$  = equivalent green time ratio (green time/cycle time) and flow ratio (arrival flow/saturation flow) for the entry stream;
- $sg$  = equivalent capacity per cycle for the entry stream, i.e. the maximum number of vehicles that can discharge during the average unblock period (veh), where  $s$  is in veh/s;
- $s$  = saturation flow ( $s = 3600/\beta$ ) (veh/h);
- $\alpha, \beta$  = mean critical gap and follow-up (saturation) headway for the entry stream (sec);
- $Q$  = capacity of the entry stream (veh/h);
- $Q_g$  = capacity estimate using the gap-acceptance method (veh/h);
- $Q_m$  = minimum capacity (veh/h);
- $n_m$  = minimum number of entry stream vehicles that can depart under heavy major stream flow conditions (veh/min);
- $d_m$  = minimum delay experienced by the entry stream vehicles (sec) (see Cowan 1984, 1987; Troutbeck 1986, 1989, 1991a, 1993);
- $\lambda$  = a parameter in the exponential arrival headway distribution model;
- $\phi$  = proportion of free (unbunched) vehicles in the traffic stream ( $\phi_m$  for the major stream,  $\phi_e$  for the entry stream);
- $\Delta$  = minimum arrival (intra-bunch) headway in the traffic stream (sec) ( $\Delta_m$  for the major stream,  $\Delta_e$  for the entry stream);
- $b$  = a bunching factor in the formula for estimating proportion of free (unbunched) vehicles in the traffic stream;
- $q_e$  = arrival flow of the entry lane (veh/h);
- $q_m$  = total arrival flow of the major stream (veh/s or veh/h; expressed in pcu/s or pcu/h if adjusted for heavy vehicle effects using the passenger car equivalents method – see the *Concluding Remarks*).

When there are several conflicting (higher priority) streams, the total major stream flow ( $q_m$ ) is calculated as the sum of all conflicting stream flows and parameters  $\Delta_m, \phi_m$  are determined accordingly.



*Equations (1) to (6)* should be used for  $q_m > 0$  (for  $q_m = 0$ ,  $r = 0$ ,  $g = c$ ,  $u = 1.0$ ,  $Q_g = 3600/\beta$ ,  $d_m = 0$ , and the gap-acceptance based delay, queue length, etc. will all be zero.

*Equations (1) to (8)* are based on a bunched exponential distribution model of arrival headways, known as Model M3 (Cowan 1975, Troutbeck 1986, 1989, 1991a). A detailed discussion of this model and the results of its calibration using real-life data for single-lane traffic streams and simulation data for multi-lane streams are given in Akçelik and Chung (1994). The bunched exponential distribution is relatively new and its use is less common than the simple negative exponential (Model M1) and shifted negative exponential distributions (Model M2) which are used in the traffic analysis literature as models of *random arrivals*. This paper adopts the more realistic M3 model for all analyses to replace the M1 and M2 models.

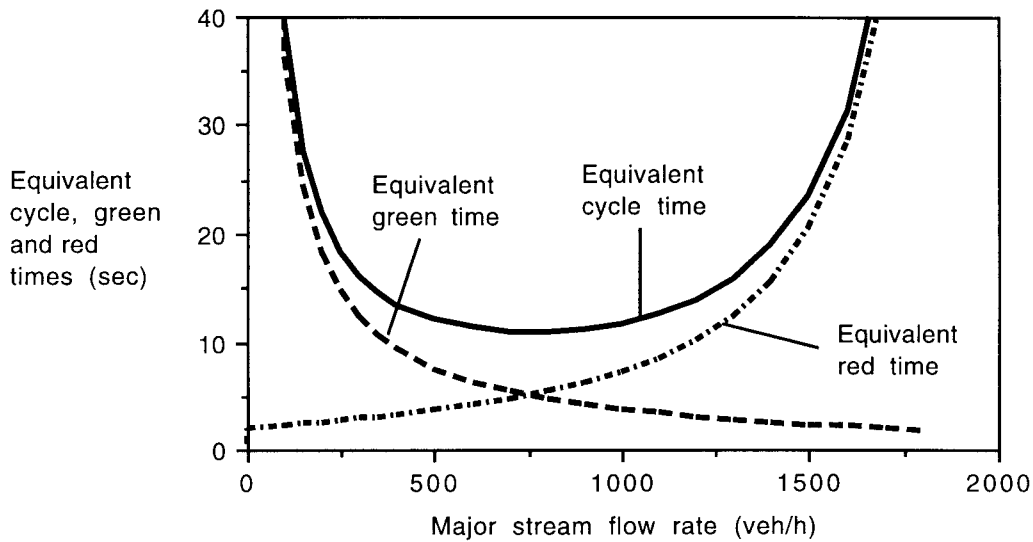
Important parameters which describe Model M3 are  $\Delta$  and  $\phi$ . The M3 model with  $\phi$  estimated from *Equation (8)* will be referred to as Model M3A. In addition to the use of an estimate of  $\phi$  from *Equation (8)*, Model M3 can be used with a specified (measured) value of  $\phi$ . The parameters for the M3A model calibrated for uninterrupted flow conditions, and for roundabout circulating streams are summarised in *Table 1*. Note that Models M1 and M2 can be derived as special cases of the M3 model through simplifying assumptions about the bunching characteristics of the arrival stream: both models M1 and M2 assume no bunching for all levels of arrival flows ( $\phi = 1$ ), and Model M1 also assumes  $\Delta = 0$ , therefore  $\lambda$  equals the total flow. Also note that the shifted negative exponential model (M2) is normally used for single-lane traffic only.

An example of equivalent red, green and cycle times is given in *Figure 2* for the case of a simple gap-acceptance situation with a single-lane major stream ( $\Delta_m = 1.5$  from *Table 1*) with  $\alpha = 4$  s,  $\beta = 2$  s.

**Table 1**

**Summary of parameter values for the bunched exponential arrival headway distribution model M3A**

| Total number of lanes | Uninterrupted traffic streams |     |               | Roundabout circulating streams |     |              |
|-----------------------|-------------------------------|-----|---------------|--------------------------------|-----|--------------|
|                       | $\Delta$                      | $b$ | $\phi$        | $\Delta$                       | $b$ | $\phi$       |
| 1                     | 1.5                           | 0.6 | $e^{-0.9 q}$  | 2                              | 25  | $e^{-5.0 q}$ |
| 2                     | 0.5                           | 0.5 | $e^{-0.25 q}$ | 1                              | 25  | $e^{-2.5 q}$ |
| > 2                   | 0.5                           | 0.8 | $e^{-0.4 q}$  | Same as the 2-lane case        |     |              |



**Fig. 2 – The equivalent red, green and cycle times as a function of the major stream flow rate for a simple gap acceptance example**

### 3. PERFORMANCE MODELS

The traditional two-term model structure for delays at traffic signals is adopted for unsignalised intersection modelling through the use of traffic signal analogy for gap acceptance processes discussed in the previous section and by introducing a separate calibration factor for each term of each performance statistic (Akçelik 1994c). In the traditional delay models, the first terms represent a *uniform-flow* model only, and any variational effects are included in the second term (the *overflow* term). In the new models, the first term calibration factors help to predict the effect of variations in queue clearance times under low to medium flow conditions (when there are no overflow queues), and any additional delays, etc. due to overflow queues are included in the second (overflow) terms.

Overflow queues and the associated delays and stops (queue move-ups) result from insufficient cycle capacity due to (i) *temporary* cycle oversaturation due to the random variations in arrival flows (and in capacities in the case of gap acceptance processes), and (ii) *permanent* oversaturation when the average flow rate exceeds the overall capacity during the specified flow period.

Separate calibration of the first and second terms of each performance model helps to obtain better predictions of the proportion queued, saturated portion of green period, and the number of major stops and queue move-ups (multiple stops in the queue). Separate information

about major stops and queue move-ups is useful for more accurate prediction of fuel consumption, pollutant emissions, operating cost and similar statistics.

This paper uses the term *proportion queued* rather than the term *proportion stopped*. This helps to distinguish between *geometric stops* (associated with delays in negotiating the intersection with no queueing effects) and the *stops due to traffic control* (red signal, or time until an acceptable gap occurs in the major stream). The latter corresponds to the delays predicted by the performance models given in this paper, and is better expressed by the term *queueing*. For example, all vehicles have to stop at a stop sign (geometric stops), but only a proportion of these stops are due to the gap acceptance process (queueing stops).

Similarly, a distinction between *queueing delay* and *stopped delay* is observed. The former includes the stopped delay as well as the delays associated with queue move-ups. However, the queueing delay can be so small that it corresponds to a slow-down only. The delays given in this paper are overall average delays including deceleration and acceleration delays for major stops, but excluding any geometric delays. The HCM method (TRB 1985) uses the average stopped (or queueing) delay calculated as  $d/1.3$  where  $d$  is the average overall delay (it is not clear if the factor 1.3 is meant to apply to stopped delay or queueing delay). However, the ratio of the stopped (or queueing) delay to the overall delay is expected to decrease with increased queueing delay. This ratio would also depend on approach and exit speeds since deceleration and acceleration delays depend on the initial and final speeds involved in a stop-start cycle.

The basic two-term models for average delay, average back of queue and effective stop rate can be expressed as the following interrelated set of equations:

$$d = d_1 + d_2 = f_{d1} d_u + f_{d2} d_o = f_{d1} d_u + f_{d2} \frac{N_o}{Q} \quad (9)$$

$$N_b = N_{b1} + N_{b2} = f_{b1} N_{bu} + f_{b2} N_o \quad (10)$$

$$h = h_1 + h_2 = f_{h1} h_u + f_{h2} h_o = f_{h1} h_u + f_{h2} \frac{N_o}{q c} \quad (11)$$

where

$d$  = average delay in seconds per vehicle,

$N_b$  = average back of queue (vehicles),

$h$  = effective stop rate,

$d_u, N_{bu}, h_u$  = *uniform flow* components,

$d_o, N_o, h_o$  = *overflow* components,

$f_{d1}, f_{b1}, f_{h1}$  = first-term calibration factors,

$f_{d2}, f_{b2}, f_{h2}$  = second-term calibration factors,

$N_o$  = average overflow queue,

- $Q$  = approach lane capacity in vehicles per *second* (see *Equations 5 and 17*), and
- $qc$  = average number of arrivals per cycle ( $q$  = average arrival flow rate in vehicles per *second* and average cycle time in seconds).

The effective stop rate is obtained by expressing the major stops (first term) and the queue move-ups (second term) in *equivalent stop values* (ESVs) using adjustment factors  $e_{ms}$  and  $e_{qm}$ , respectively:

$$h = e_{ms} p_q + e_{qm} h_{qm} \quad (11a)$$

where

$e_{ms}$  = ESV factor for major stops (corresponding to  $h_1$  as the effective stop rate for major stops:  $h_1 = f_{h1} h_u = e_{ms} p_q$ ),

$e_{qm}$  = ESV factor for queue move-ups (corresponding to  $h_2$  as the effective stop rate for queue move-ups:  $h_2 = f_{h2} h_o = e_{qm} h_{qm}$ ),

$p_q$  = proportion queued (corresponding to major stops) given by

$$p_q = f_{pq} h_u \quad \text{subject to } p_q \leq 1.0 \quad (12)$$

$h_{qm}$  = queue move-up rate (corresponding to multiple stops in queue before clearing the approach lane) given by

$$h_{qm} = f_{qm} h_o = f_{qm} (N_o / qc) \quad (13)$$

In the above models delay, queue length, stop rate and queue move-up rate statistics represent average values for all vehicles queued and unqueued. The relationships between effective stop rate, proportion queued and queue move-up rate will be further discussed later in this section.

### Non-Overflow Queue Terms

The first terms of *Equations (9) to (11)*, namely  $d_1$ ,  $N_{b1}$  and  $h_1$ , represent the performance of traffic in the approach lane under low to medium flow conditions since the overflow term is zero or negligible in those conditions. The corresponding calibration factors  $f_{d1}$ ,  $f_{b1}$ ,  $f_{h1}$  (also  $f_{pq}$  in *Equation 12*) are used to allow for the effect of variations from uniform-flow conditions. These variations depend on (i) arrival characteristics: uniform or platooned arrivals considering isolated and closely-spaced/coordinated intersections; and (ii) service characteristics: constant green and red times for fixed-time signals, or variable green and red times for actuated signals (Akçelik 1994a,c) and unsignalised intersections (by signal operations analogy as discussed in *Section 2*). The calibration factors given in this paper apply to isolated intersection conditions (i.e. without any platooned arrivals or signal coordination effects).

The *uniform-flow* components ( $d_u$ ,  $N_{bu}$ ,  $h_u$ ) are derived assuming that the number of vehicles which arrive during each signal cycle is fixed and equivalent to the average arrivals per cycle ( $qc$ ), and that arrivals are

distributed *uniformly* throughout the signal cycle at the average flow rate ( $q$ ). These are expressed as follows:

$$\begin{aligned} d_u &= \frac{0.5 r (1 - u)}{1 - y} & \text{for } x \leq 1.0 \\ &= 0.5 r & \text{for } x > 1.0 \end{aligned} \quad (14)$$

$$\begin{aligned} N_{bu} &= \frac{q r}{1 - y} & \text{for } x \leq 1.0 \\ &= q c & \text{for } x > 1.0 \end{aligned} \quad (15)$$

$$\begin{aligned} h_u &= \frac{1 - u}{1 - y} & \text{for } x \leq 1.0 \\ &= 1.0 & \text{for } x > 1.0 \end{aligned} \quad (16)$$

where

$r, g, c$  = effective red, green and cycle time in seconds (for unsignalised intersections, these are estimated using the equations given in Section 2),

$u$  = green time ratio (ratio of effective green time to cycle time):

$$u = g/c \quad (17a)$$

$y$  = flow ratio (ratio of arrival flow rate to saturation flow rate):

$$y = q/s \quad (17b)$$

$x$  = degree of saturation (ratio of arrival flow rate to capacity):

$$x = q/Q = qc/sg \quad (17c)$$

$Q$  = capacity under the specified flow conditions in vehicles per hour or per second:

$$Q = sg/c \quad (17d)$$

$q$  = arrival (demand) flow rate during the specified flow period in vehicles per hour (or per second),

$qc$  = average number of arrivals per cycle (vehicles),

$s$  = saturation (queue discharge) flow rate in vehicles per hour or per second (for unsignalised intersections,  $s = 3600/\beta$  is used where  $\beta$  is the follow-up headway in seconds and  $s$  is in vehicles per hour), and

$sg$  = capacity per cycle (vehicles),

For oversaturated conditions ( $x > 1$ ), the values of  $f_{d1}$ ,  $f_{b1}$  and  $f_{h1}$  at  $x=1$  ( $f_{d1(x=1)}$ ,  $f_{b1(x=1)}$  and  $f_{h1(x=1)}$ ) are used so that the first terms of  $d_1$ ,  $N_{b1}$ ,  $h_1$  models are constant for  $x > 1$ .

For minimum flow conditions, the queue length is approximately zero, and the minimum delay ( $d_m$ ) is given by Equation (14) with  $y = 0$ , therefore:

$$d_m = 0.5 r (1 - u) \quad (18a)$$

Similarly, the minimum proportion queued ( $p_{qm}$ ) is obtained by putting  $y = 0$  in *Equation (16)*:

$$p_{qm} = (1 - u) \quad (18b)$$

*Equation (14)* is commonly used as the first term of most traditional delay models for isolated fixed-time signals (Webster 1958; Webster and Cobbe 1966; Miller 1968; Akçelik 1981; Teply 1984; TRB 1985). For signalised intersections, the traffic movements which receive *two green periods per cycle* (e.g. protected and permissive turns), more complex forms of *Equations (14) to (16)* are needed. Most existing delay models apply *Equation (14)* by combining the two green periods as a single period with an equivalent capacity. However, this method is not sufficiently accurate, particularly in the case of non-consecutive green periods. SIDRA (Akçelik 1990a, Akçelik and Besley 1992) uses an extended model to estimate  $d_u$ ,  $N_{bu}$  and  $h_u$  for the case of two green periods, allowing for different conditions of residual queues between the two green periods. The details of this model are yet to be published.

For closely-spaced signalised intersections (see Akçelik and Roupail 1994), platooned arrivals cause a variation from uniform flow conditions because arrivals occur at different rates during different intervals of the signal cycle (regularly in the case of coordinated signalised intersections). The US Highway Capacity Manual (TRB 1985) uses *progression factors* as a simple method of modelling the effects of platooned arrivals. Further research is being carried out on this subject.

### Overflow Queue Terms

From *Equations (9) to (11)*, the second terms of the delay, queue length and stop rate models are seen to be commonly expressed as a function the average overflow queue ( $N_o$ ). The following *steady-state* expression for average overflow queue was developed by Akçelik (1980, 1988, 1990a,b) by generalising an expression he originally used as a simple approximation to Miller's (1968) expression for delay at signalised intersections:

$$N_o = \frac{k_o (x - x_o)}{1 - x} \quad \text{for } x > x_o \quad (19)$$

$$= 0 \quad \text{otherwise}$$

where  $k_o$  (used as  $k$  in previous publications) is a parameter that determines the steepness of the overflow queue function, and  $x_o$  is the degree of saturation below which the average overflow queue is approximately zero.

*Equation (19)* can be considered to form the basis of various delay equations used in the literature for fixed-time traffic signal operations as discussed previously (Akçelik 1988, 1990a,b). This formula has recently been adopted by Tarko and Roupail (1994) as "a distribution-free model for estimating random queues in signalised networks".

*Equation (19)* as a steady-state expression is based on the assumption that arrival demand conditions last indefinitely. It is valid only for degrees of saturation up to about 0.95. A *time-dependent* function corresponding to *Equation (19)* can be derived using the well-known coordinate transformation technique (Akçelik 1980). This function is based on the assumption that arrival demand conditions last for a finite period of time ( $T_f$ ). Derived on this basis, the time-dependent function can be used for oversaturated conditions. Using the time-dependent form of *Equation (19)* in *Equations (9), (10) and (13)*, the following time-dependent functions for the overflow terms of average delay and back of queue, as well as the queue move-up rate are obtained:

$$\begin{aligned} d_2 &= 900T_f \left[ \left( z + \sqrt{z^2 + \frac{8k_d(x-x_0)}{QT_f}} \right) \right] && \text{for } x > x_0 \\ &= 0 && \text{otherwise} \end{aligned} \quad (20)$$

$$\begin{aligned} N_{b2} &= 0.25 QT_f \left[ \left( z + \sqrt{z^2 + \frac{8k_b(x-x_0)}{QT_f}} \right) \right] && \text{for } x > x_0 \\ &= 0 && \text{otherwise} \end{aligned} \quad (21)$$

$$\begin{aligned} h_{qm} &= \frac{0.25 QT_f}{qc} \left[ \left( z + \sqrt{z^2 + \frac{8k_{qm}(x-x_0)}{QT_f}} \right) \right] && \text{for } x > x_0 \\ &= 0 && \text{otherwise} \end{aligned} \quad (22)$$

where  $z = x - 1$ ,  $T_f$  = flow (analysis) period in hours,  $Q$  = capacity in veh/h,  $QT_f$  = throughput (maximum number of vehicles that can be discharged during the flow period),  $qc$  = number of arrivals per cycle ( $q$  is the arrival flow rate in veh/s,  $c$  in seconds), and the second-term parameters are defined as:  $k_d = f_{d2} k_0$ ,  $k_b = f_{b2} k_0$  and  $k_{qm} = f_{qm} k_0$  where  $f_{d2}$ ,  $f_{b2}$  and  $f_{qm}$  are as used in *Equations (9), (10) and (13)*.

The duration of the flow period affects the estimates of performance statistics significantly. Larger delays, queue lengths, and stop rates will result from longer flow periods for a given demand level.  $T_f = 0.25$  h is built into the HCM delay formula (TRB 1985) whereas the models given here allow  $T_f$  to be variable.

### Calibration Method

Using the simulation data generated by ModelC, model parameters  $f_{d1}$ ,  $f_{b1}$ ,  $e_{ms}$ ,  $f_{pq}$ ,  $x_0$ ,  $k_0$ ,  $f_{d2}$ ,  $f_{b2}$  and  $f_{qm}$  were determined using the following calibration method:

- (a) Calibrate the steady-state expression for the average overflow queue  $N_0$  (*Equation 19*) by determining  $k_0$  and  $x_0$  parameters using overflow queue data for undersaturated flow conditions ( $x \leq 0.95$ ). Determine  $x_0$  as the degree of saturation below which  $N_0$  is negligible ( $N_0 \leq 0.05$  vehicles).

- (b) Determine the first term calibration factors ( $f_{d1}$ ,  $f_{b1}$ ,  $e_{ms}$ ,  $f_{pq}$ ) using data representing the cases where  $N_0$  is negligible ( $x < x_0$  or  $N_0 \leq 0.05$ ) and therefore the second term is zero.
- (c) Determine the second term calibration factors for delay and queue length ( $f_{d2}$  and  $f_{b2}$  in *Equations 9 and 10*) using the overflow data calculated as the total value of delay or queue length less the value of the calibrated first term.
- (d) Determine the calibration factor for queue move-up rate ( $f_{qm}$ ) using the move-up rate data ( $h_{qm}$ ) and  $h_0 = N_0 / q_c$  calculated from *Equation (13)*.
- (e) Calculate composite parameters  $f_{h1} = e_{ms} f_{pq}$  for the first term of the effective stop rate formula (*Equation 11*), and  $k_d = f_{d2} k_0$ ,  $k_b = f_{b2} k_0$  and  $k_{qm} = f_{qm} k_0$  for the time dependent functions for second terms of delay, queue length and queue move-up rate (*Equations 20 to 22*).

### Calibration Results

Calibration results are given in *Equations (23) to (26)* for unsignalised intersections and *Equations (27) to (30)* for fixed-time signals. Graphs showing simulated vs estimated average overflow queue length ( $N_0$ ), average delay ( $d$ ), average back of queue ( $N_b$ ), proportion queued ( $p_q$ ) and queue move-up rate ( $h_{qm}$ ) for unsignalised and signalised intersections are shown in *Figures 3 to 7*. The comparisons are based on steady-state model predictions.

For a simple gap-acceptance case with a single-lane major stream ( $\Delta_m = 1.5$ ,  $\alpha = 4$  s,  $\beta = 2$  s), the average delay ( $d$ ) and proportion queued ( $p_q$ ) as a function of the entry lane degree of saturation ( $x$ ) are shown in *Figures 8 and 9* for major stream flow rates of  $q_m = 360, 720$  and  $1080$  veh/h (to represent low, medium and high flow levels). Both the major stream and the entry stream are considered to consist of cars only. *Figures 10 and 11* show the average back of queue ( $N_b$ ) and queue move-up rate ( $h_{qm}$ ) as a function of the major stream flow rate ( $q_m$ ) for entry flow rates of  $q_e = 300, 600$  and  $900$  veh/h. The time-dependent forms of the delay, average back of queue and queue move-up rate models are used for graphs given in *Figures 8, 10 and 11* with flow period duration  $T_f = 0.5$  h.

### Unsignalised intersections

#### First-term parameters

$$f_{d1} = \frac{2 d_m (1 + 0.3 y^{0.20})}{r (1 - u)} \quad \text{subject to } f_{d1} > 1.0 \quad (23a)$$

$$f_{b1} = 1.2 \varphi_e^{0.8} \quad \text{subject to } f_{b1} > 1.0 \quad (23b)$$

$$f_{pq} = 0.75 \varphi_e (sg)^{0.40} \quad \text{subject to } f_{pq} > 1.0 \quad (24a)$$

$$e_{ms} = 1.65 (sg)^{-0.40} y^{0.10} \quad \text{subject to } e_{ms} \leq 1.0 \quad (24b)$$



*Second-term parameters*

$$x_o = 0.14 (sg)^{0.55} \quad \text{subject to } x_o \leq 0.95 \quad (25a)$$

$$k_o = 0.3 \varphi_e (sg)^{1.10} (d_m Q) \quad (25b)$$

$$k_d = 0.17 \varphi_e (sg)^{1.40} y^{-0.40} (d_m Q) \quad (26a)$$

$$k_b = 0.45 \varphi_e (sg)^{1.70} y^{0.40} (d_m Q) \quad (26b)$$

$$k_{qm} = 1.1 \varphi_e (sg)^{1.10} y^{0.50} (d_m Q) \quad (26c)$$

where  $\varphi_e$  is the proportion of unbunched traffic in the entry lane,  $sg$  is the capacity per cycle (vehicles),  $y$  is the flow ratio,  $d_m$  is the minimum delay (in seconds),  $Q$  is the entry lane capacity (in vehicles per second), and  $r, u$  are the effective red time and green time ratio (see Section 2).

*Fixed-time signals**First-term parameters*

$$f_{d1} = 1 + 0.1 \varphi_e (sg)^{0.25} y^{0.10} \quad (27a)$$

$$f_{b1} = 1 + 0.1 \varphi_e (sg)^{0.10} y \quad (27b)$$

$$f_{pq} = 1 + 0.004 \varphi_e (sg)^{1.25} y^{0.25} \quad (28a)$$

$$e_{ms} = 1.04 (sg)^{-0.07} y^{0.03} \quad \text{subject to } e_{ms} \leq 1.0 \quad (28b)$$

*Second-term parameters*

$$x_o = 0.4 (sg)^{0.20} \quad \text{subject to } x_o \leq 0.95 \quad (29a)$$

$$k_o = 0.55 \quad (29b)$$

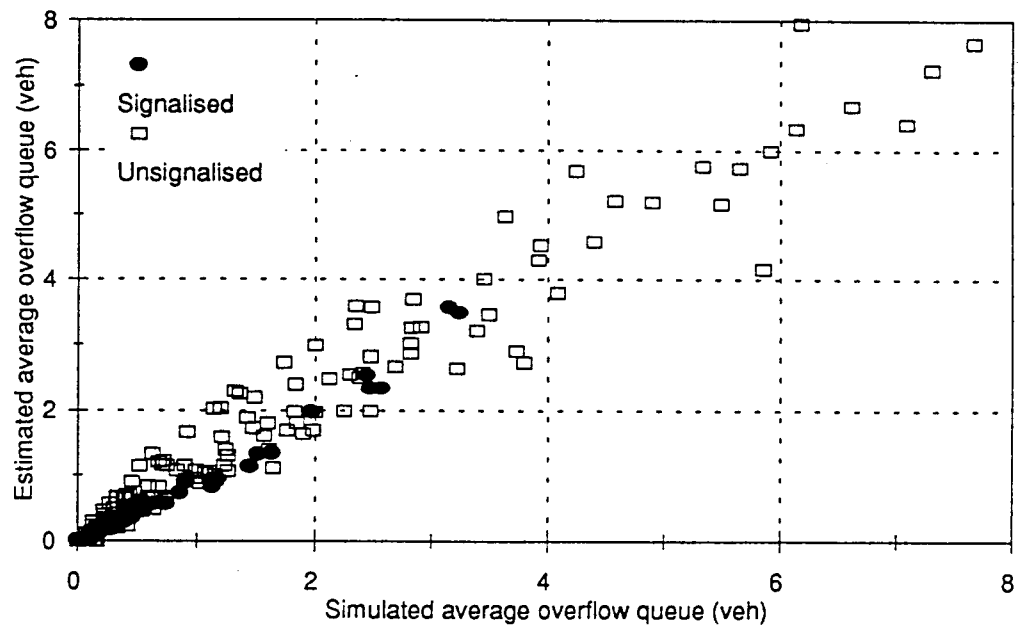
$$k_d = 0.55 \quad (30a)$$

$$k_b = 0.55 \quad (30b)$$

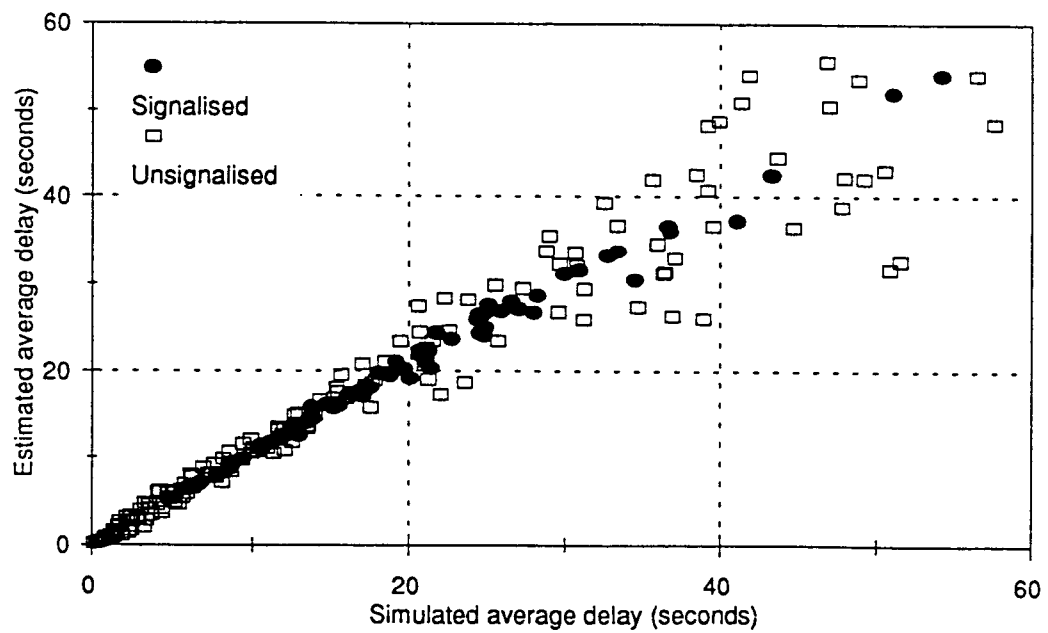
$$k_{qm} = 0.55 + 0.22 y^{0.30} \quad (30c)$$

where  $\varphi_e$  is the proportion of unbunched traffic in the entry lane,  $sg$  is the capacity per cycle (veh), and  $y$  is the flow ratio.

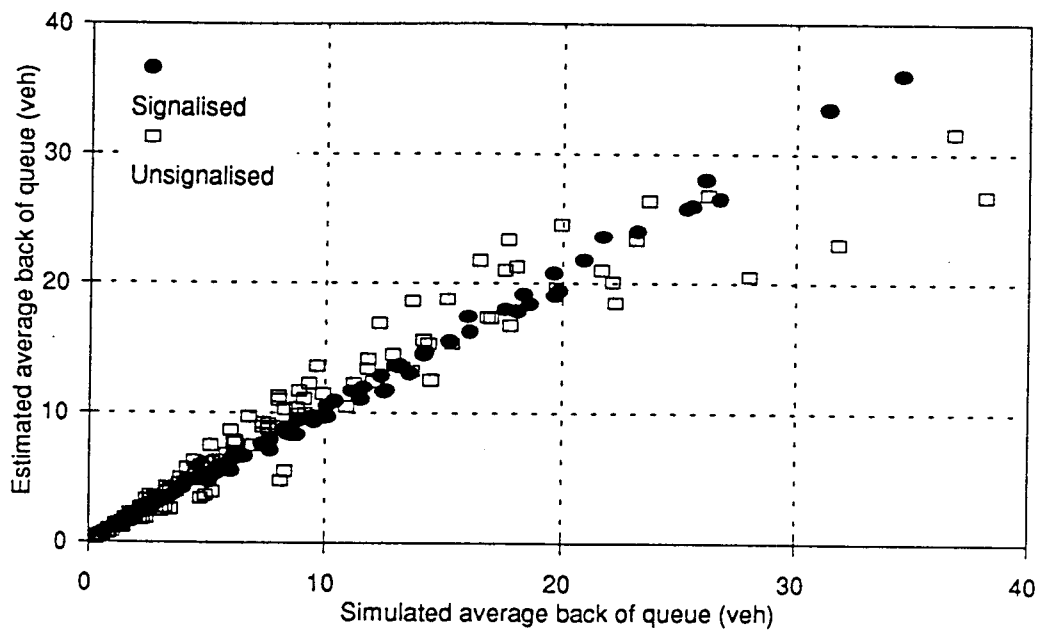
It is seen that capacity per cycle, flow ratio and proportion of unbunched traffic in the entry lane are the parameters that appear as variational factors for both low to medium flow conditions (in the first term) and high flow conditions (in the second term). Capacity per cycle was used in earlier Australian models for overflow queue at fixed-time signals (Miller 1968; Akçelik 1980, 1981, 1988, 1990a,b). For unsignalised intersections,  $d_m Q$  appears as an important factor as in previous models (Troutbeck 1989; Akçelik and Troutbeck 1991). The form of the first-term delay factor ( $f_{d1}$ ) for unsignalised intersections is a result of the use of minimum delay parameter in the formulation of delay in line with current models (Troutbeck 1989; Akçelik and Troutbeck 1991).



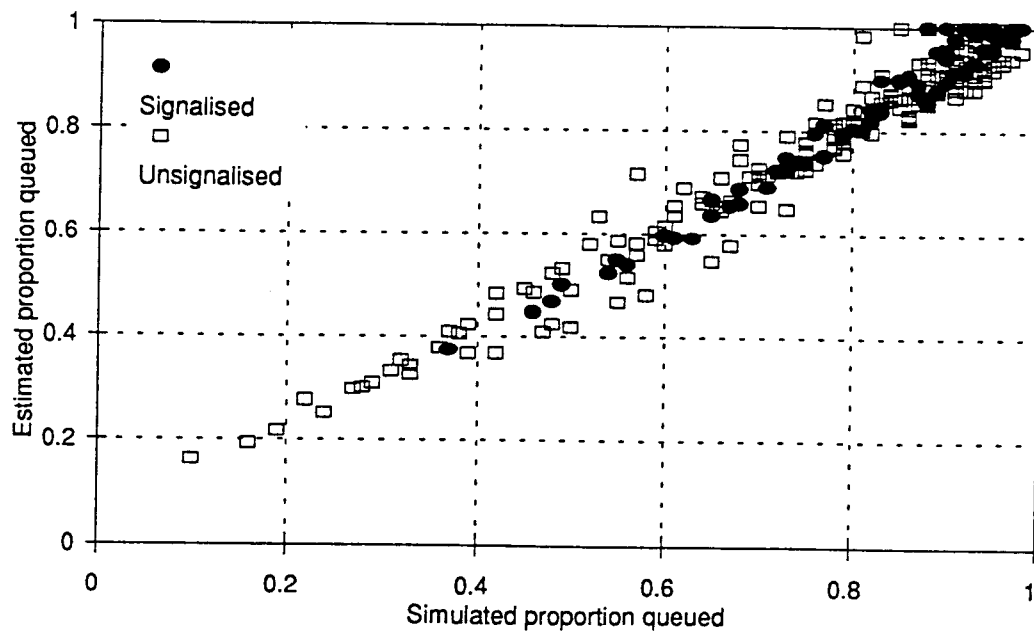
**Fig. 3 – Comparison of simulated and estimated values of *average overflow queue* for signalised and unsignalised intersections**



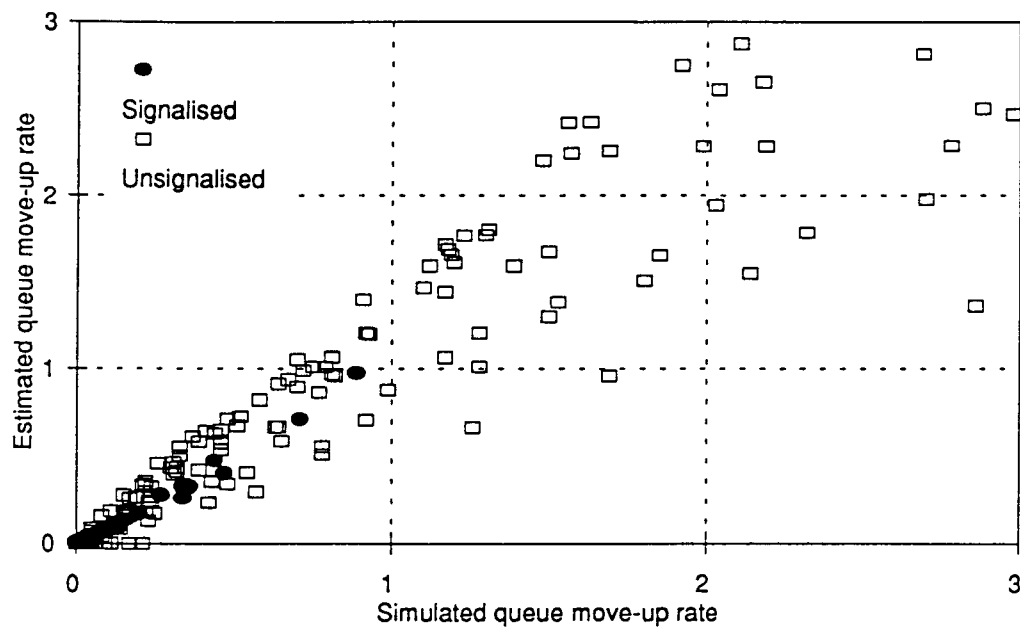
**Fig. 4 – Comparison of simulated and estimated values of *average delay* for signalised and unsignalised intersections**



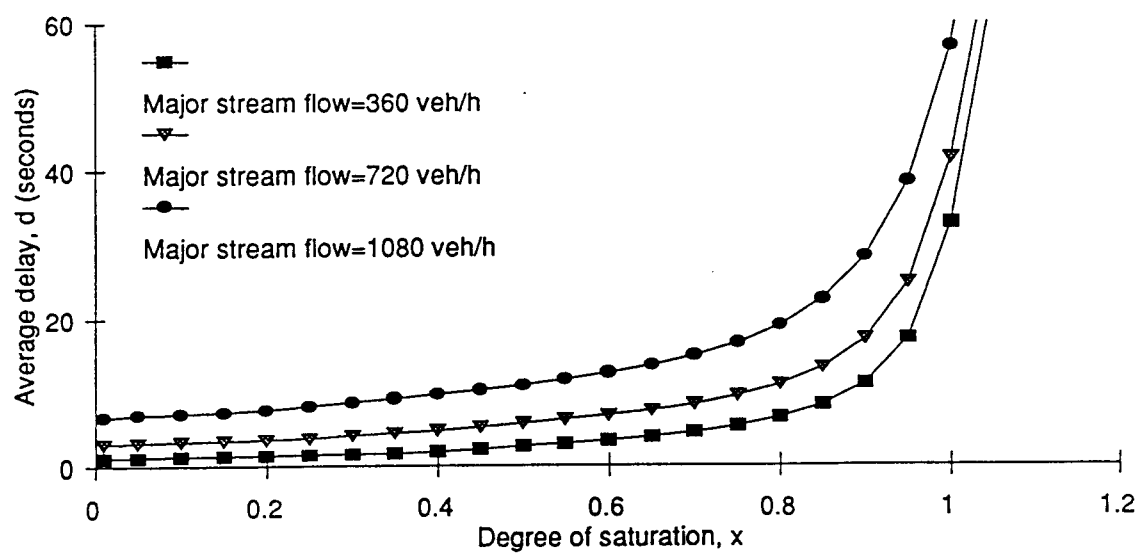
**Fig. 5 – Comparison of simulated and estimated values of *average back of queue* for signalised and unsignalised intersections**



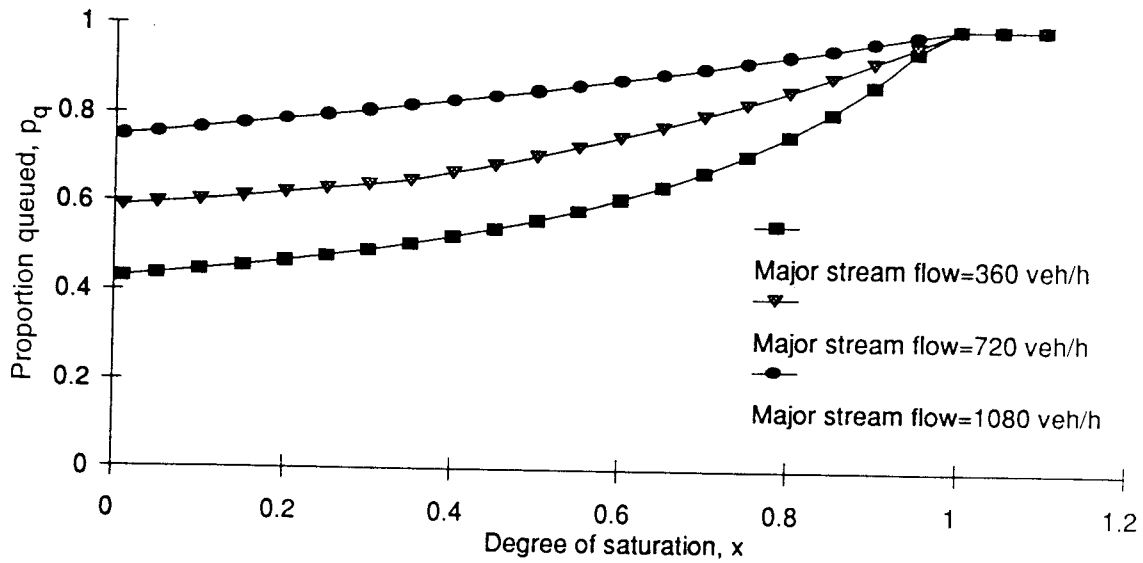
**Fig. 6 – Comparison of simulated and estimated values of *proportion queued* for signalised and unsignalised intersections**



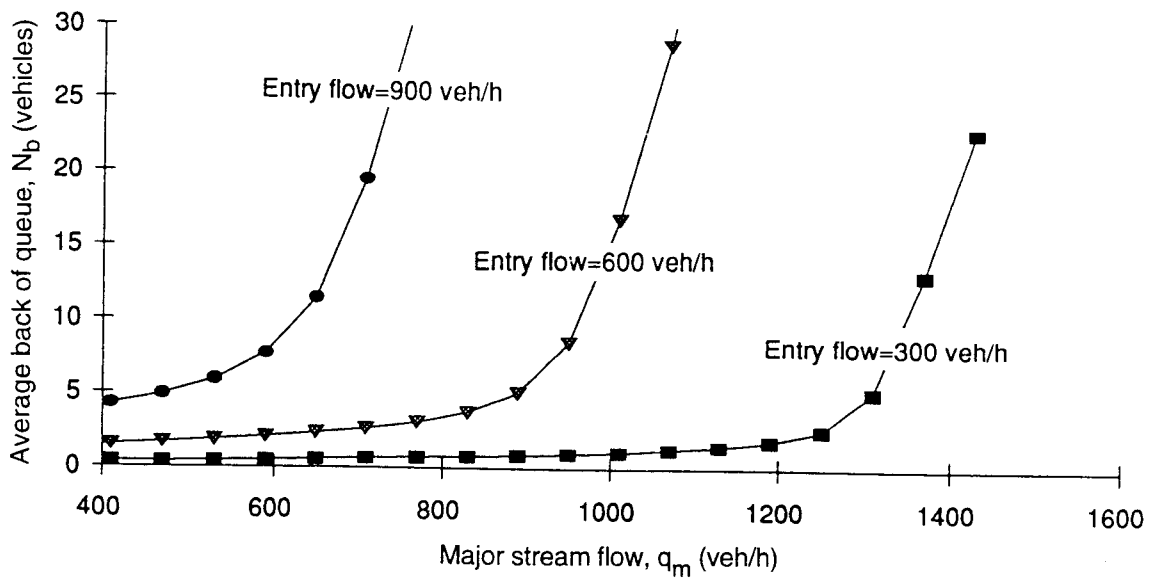
**Fig. 7 – Comparison of simulated and estimated values of *queue move-up rate* for signalised and unsignalised intersections**



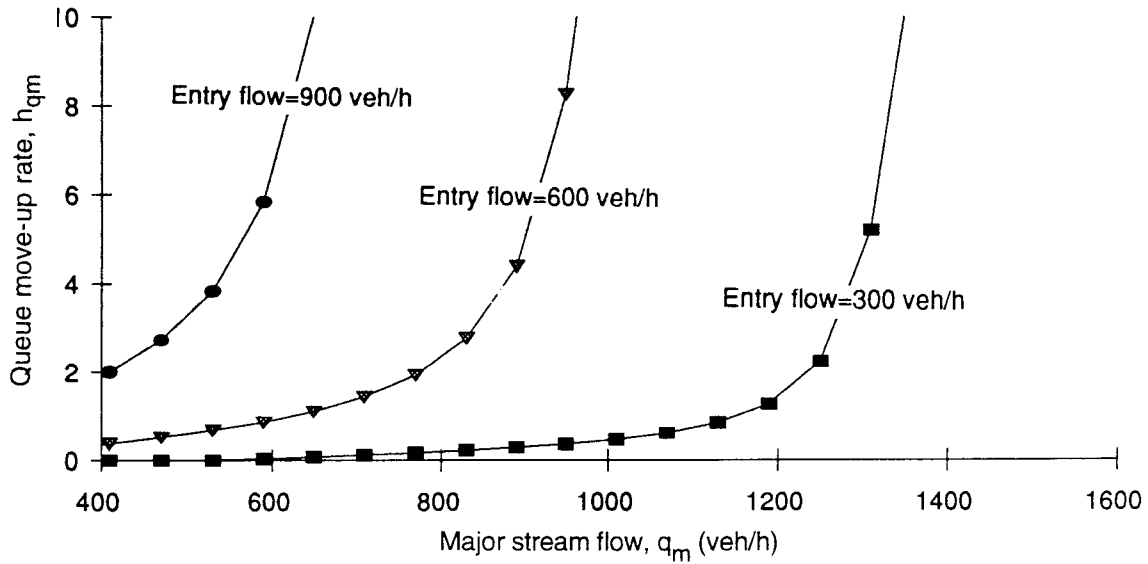
**Fig. 8 – *Average delay* as a function of the entry lane degree of saturation for three levels of major stream arrival flow rate**



**Fig. 9 – Proportion queued as a function of the entry lane degree of saturation for three levels of major stream arrival flow rate**



**Fig. 10 – Average back of queue as a function of the major stream flow rate for various entry flow rates**



**Fig. 11 – Queue move-up rate as a function of the major stream flow rate for various entry flow rates**

### Queue Length

The average back of queue ( $N_b$ ) represents the maximum extent of queue in an average cycle. The first term ( $N_{b1}$ ) is useful for the prediction of such statistics as the saturated portion of the green period and for modelling short lane capacities. The prediction of the back of queue is required for the design of appropriate queueing space (e.g. for short lane design). The traditional gap-acceptance and queueing theory models do not give sufficient information for intersection design purposes since they predict the cycle-average queue ( $N_c$ ). This is the average queue length considering all instances during the cycle including the zero-queue states. The commonly-used formula to calculate the cycle-average queue ( $N_c$ ) is:

$$N_c = d q_e \quad (31)$$

where  $d$  is the average delay from Equation (9) and  $q_e$  is the average arrival flow rate for the entry stream. Thus, the cycle-average queue is equivalent to the *total delay*, or *delay rate* (strictly speaking, this relationship applies to undersaturated conditions,  $x < 1$ , only).

The 90th, 95th and 98th percentile values of the back of queue ( $N_{bp\%}$ ) and the cycle-average queue ( $N_{cp\%}$ ) can be expressed as a function of the average value ( $N_b$  or  $N_c$ ):

$$N_{bp\%} = f_{bp\%} N_b \quad (32a)$$

$$N_{cp\%} = f_{cp\%} N_c \quad (32b)$$

where  $f_{bp\%}$  and  $f_{cp\%}$  are the factors for  $p$ th percentile queue. The calibration results for percentile queue length prediction are given below.

*Unsignalised intersections*

$$f_{b90\%} = 1.9 + 0.7 e^{-N_b/8} \quad (33a)$$

$$f_{b95\%} = 2.5 + 0.7 e^{-N_b/8} \quad (33b)$$

$$f_{b98\%} = 3.0 + 0.7 e^{-N_b/8} \quad (33c)$$

$$f_{c90\%} = 2.0 + 0.6 e^{-N_c/8} \quad (34a)$$

$$f_{c95\%} = 2.5 + 0.7 e^{-N_c/8} \quad (34b)$$

$$f_{c98\%} = 3.2 + 1.0 e^{-N_c/2} \quad (34c)$$

*Fixed-time signals*

$$f_{b90\%} = 1.3 + 0.5 e^{-N_b/13} \quad (35a)$$

$$f_{b95\%} = 1.4 + 0.9 e^{-N_b/12} \quad (35b)$$

$$f_{b98\%} = 1.5 + 1.3 e^{-N_b/11} \quad (35c)$$

$$f_{c90\%} = 1.7 + 1.3 e^{-N_c/3} \quad (36a)$$

$$f_{c95\%} = 2.1 + 2.4 e^{-N_c/2} \quad (36b)$$

$$f_{c98\%} = 2.3 + 4.0 e^{-N_c/2} \quad (36c)$$

Graphs showing simulated vs estimated 90th, 95th and 98th percentile values of back of queue ( $N_{b90\%}$ ,  $N_{b95\%}$ ,  $N_{b98\%}$ ) for unsignalised and signalised intersections are shown in *Figures 12 to 14*.

For a simple gap-acceptance case with a single-lane major stream ( $\Delta_m = 1.5$ ,  $\alpha = 4$  s,  $\beta = 2$  s), the average, 90th, 95th and 98th percentile back of queue values as a function of the entry lane degree of saturation ( $x$ ) for major stream flow rate of  $q_m = 720$  veh/h are shown in *Figure 15*. These graphs are based on the time-dependent form of the model for the average back of queue (flow period duration  $T_f = 0.5$  h).

**Effective Stop Rate**

The first terms of the equations for the average back of queue and effective stop rate (*Equations 10 and 11*) correspond to *major stops*, whereas the second terms of these equations relate to *queue move-ups*. The model for the proportion queued (*Equation 12*) is related to major stops only. The major stops include some *partial stops*, i.e. slow-downs corresponding to small delays at the back of the queue, or slow-downs corresponding to small delays generally. For proportion queued, each major stop is counted as one stop irrespective of the corresponding delay value (i.e. even if it is a very small delay). The effects of *partial stops* can be taken into account in calculating effective stop rates for major stops by expressing the first term of the effective stop rate model ( $h_1$ ) in terms of *equivalent stop values* (*ESV*). In the simulation model, this was achieved by converting each slow-down to an equivalent full stop value (less than one). As an

approximate method, the ratio of deceleration-acceleration delays for a partial stop and full stop was used for deriving an ESV factor for major stops ( $e_{ms}$ ). Thus, the calibration factor  $f_{h1}$  allows for partial stop effects as well as any variational effects (see *Equations 11 and 11a*):

$$f_{h1} = e_{ms} f_{pq} \quad \text{subject to } f_{h1} > 1.0 \quad (37a)$$

The second term of the effective stop rate model corresponds to queue move-ups, or multiple stops in queue. The queue move-up rate,  $h_{qm}$  (average number of queue move-ups per vehicle) is given by *Equation (13)* where the calibration factor is  $f_{qm}$ . To convert the queue move-up rate to an effective stop rate, an ESV factor for queue move-ups ( $e_{qm}$ ) is used, and  $h_2$  is calculated from:

$$h_2 = e_{qm} h_{qm} \quad (37b)$$

This method is appropriate considering both steady-state (*Equation 11a*) and time-dependent (*Equation 22*) formulations.

To calculate  $e_{qm}$ , the ratio of acceleration-deceleration delays for a queue move-up manoeuvre (accelerate to a queue move-up speed  $v_{qm}$  and decelerate to zero speed) and a full acceleration-deceleration cycle to the approach cruise speed (accelerate to  $v_{ac}$  and decelerate to zero speed) can be used. Although the use of full acceleration and deceleration distance and time models are preferred for this purpose (as used in SIDRA), the following approximate formula is useful for quick calculations:

$$e_{qm} = \frac{0.33 v_{qm} - 0.02 v_{qm}^{1.5}}{0.33 v_{ac} - 0.02 v_{ac}^{1.5}} \quad (38a)$$

The speed involved in queue move-up manoeuvres ( $v_{qm}$ ) is less than the approach cruise speed ( $v_{ac}$ ), especially in the case of unsignalised intersections. The queue move-up speed can be estimated from:

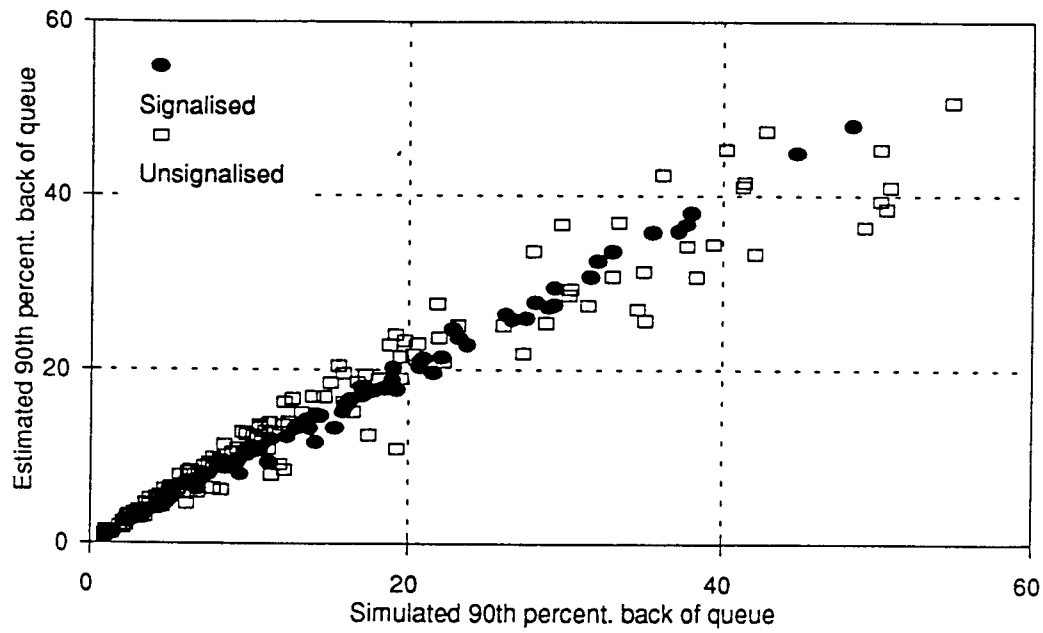
$$v_{qm} = 3.88 (L_j \text{ sg})^{0.5} \quad \text{subject to } v_{qm} \leq v_{ac} \quad (38b)$$

where  $L_j$  is the average queue space per vehicle (m/veh) and  $sg$  is the cycle capacity (veh). This formula is based on the assumption that, when  $sg$  vehicles depart in a saturated cycle (queue move-ups are relevant to saturated cycles), all following vehicles in the queue move up (accelerate to  $v_{qm}$  and decelerate to zero speed) within a distance equivalent to  $(L_j \text{ sg})$ . For example, for the case when  $L_j = 6.6$  m/veh and  $v_{ac} = 60$  km/h,  $e_{qm} = 0.31 (sg)^{0.5} - 0.06 (sg)^{0.75}$  (subject to  $e_{qm} \leq 1.0$ ) is obtained.

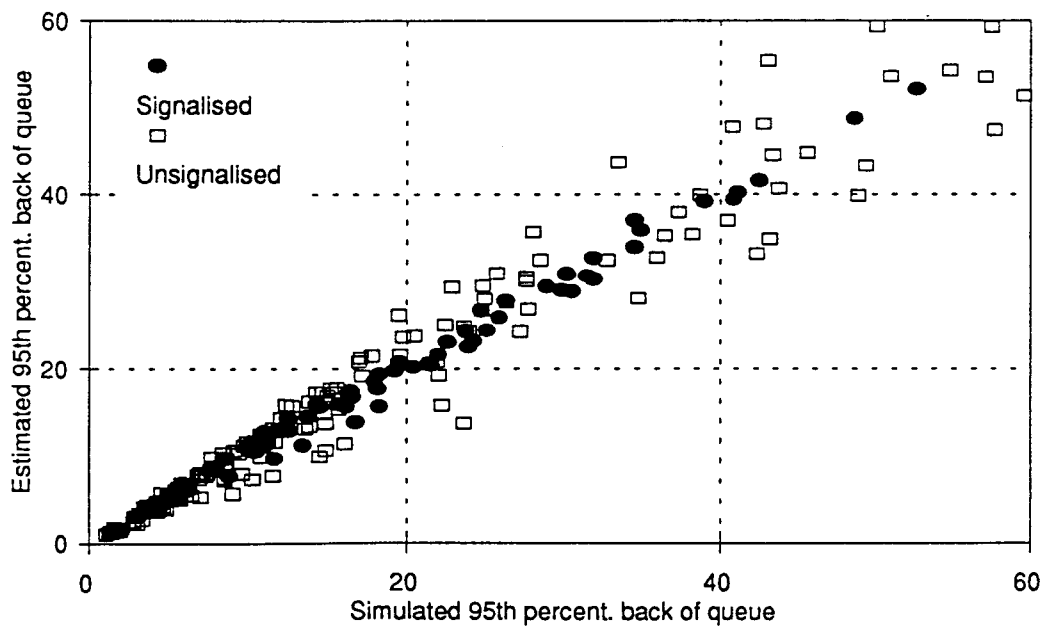
Note that the effective stop rate in ESVs (*Equation 11*) calculated explicitly as described above is an improvement over the earlier method for fixed-time signals which used a constant factor of  $e_{ms} = e_{qm} = 0.9$  (Akçelik 1981). Explicit calculation is particularly needed for unsignalised intersection operations where  $sg$ , therefore  $v_{qm}$  values are low and the queue-move-up rates ( $h_{qm}$ ) are high.

Graphs showing simulated vs estimated effective stop rates for major stops ( $h_1$ ) for unsignalised and signalised intersections are shown in *Figure 16*.

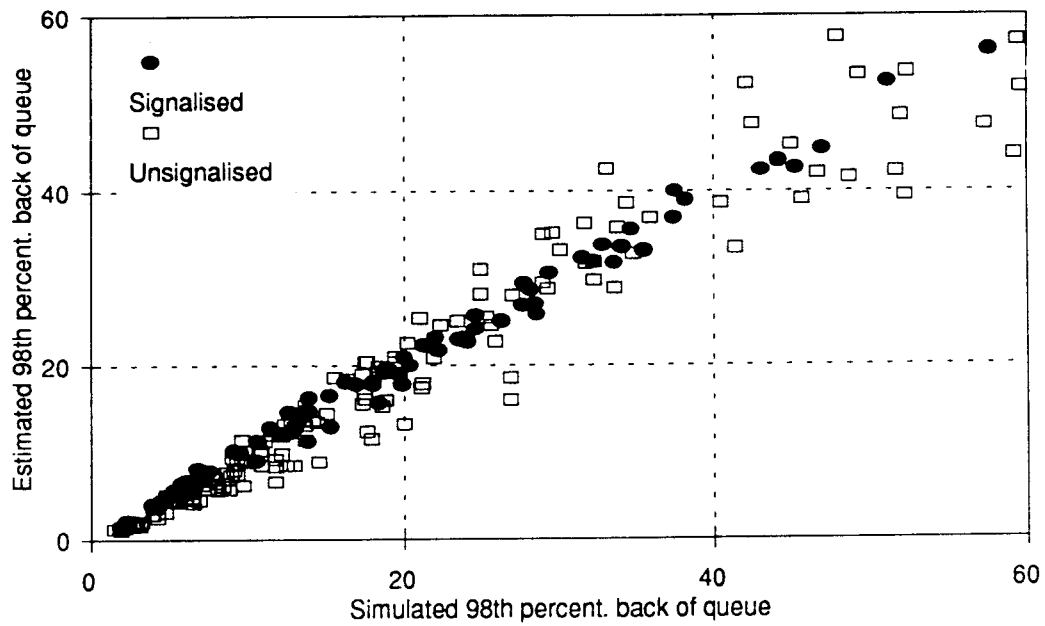




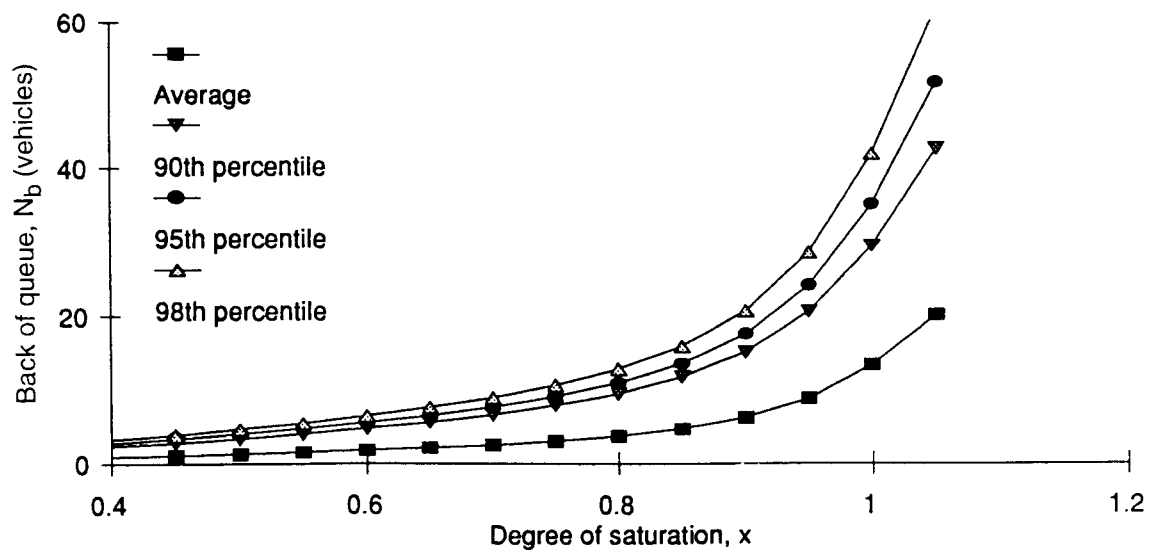
**Fig. 12 – Comparison of simulated and estimated values of 90th percentile back of queue for signalised and unsignalised intersections**



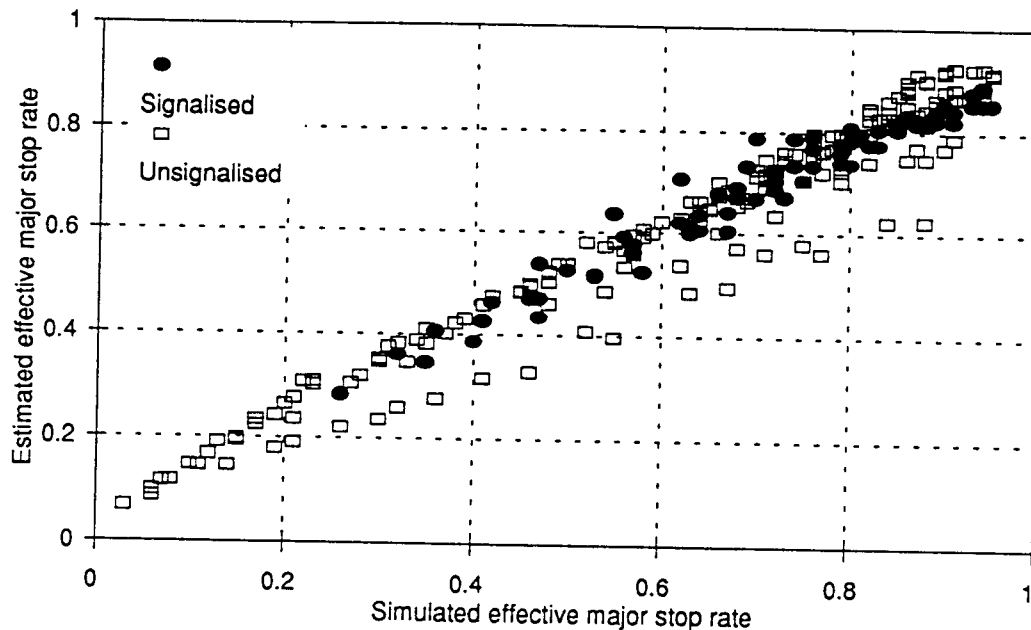
**Fig. 13 – Comparison of simulated and estimated values of 95th percentile back of queue for signalised and unsignalised intersections**



**Fig. 14 – Comparison of simulated and estimated values of 98th percentile back of queue for signalised and unsignalised intersections**



**Fig. 15 – The average, 90th, 95th and 98th percentile back of queue functions for major stream flow rate of 720 veh/h**



**Fig. 16 – Comparison of simulated and estimated values of *effective stop rate* for major stops for signalised and unsignalised intersections**

#### 4. CONCLUDING REMARKS

The analytical models presented in this paper provide a consistent modelling framework for the comparison of different types of intersections. The modelling of unsignalised intersections by analogy to traffic signal operations enabled the modelling of the average back of queue and effective stop rate (including major stops and queue move-ups) in a manner consistent with models for signalised intersections. The models have been structured in a form appropriate for developing performance models for vehicle-actuated signals.

Equations to predict the 90th, 95th and 98th percentile queue lengths will provide valuable information to practitioners for the design of queueing spaces. Effective stop rates predicted in equivalent stop values (ESVs) can be used in simple methods for estimating fuel consumption, pollutant emissions, operating cost and similar statistics (e.g. using excess fuel consumption rate per major stop). Separate prediction of major stops and queue move-up rates is useful for more accurate estimation of such statistics (e.g. using the four-mode elemental model in SIDRA).

The use of capacity per cycle as a parameter in the formulation of overflow terms provides sensitivity to the relative characteristics of major and minor movements. This may have an important impact on signal timing optimisation when there are marked differences between major and minor movement characteristics. Through the use of the bunched exponential model of arrival headways for all traffic streams, the performance models now take into account the effect of bunching in approach (entry) flows as well as major (opposing or circulating) flows.

The effects of heavy vehicles in the major stream and the entry stream can be taken into account either by adjusting gap acceptance parameters or using passenger car equivalents (Troutbeck 1991b). The use of passenger car equivalents to convert major stream arrival flow rates and entry stream capacities as used in the SIDRA software package (Akçelik 1990a; Akçelik and Besley 1992) is described in Akçelik (1991). Further research is recommended on the effects of heavy vehicles on arrival headway distributions and gap acceptance processes.

The capacity model for unsignalised intersections given in this paper differs from those published previously. Various forms of the new model based on different assumptions about arrival headway distributions were compared with more traditional models (Akçelik 1994b). It was found that (i) there is little difference between various models for low major stream flows, (ii) the differences among models which use the same arrival headway distribution are negligible, and (iii) the impact of the assumption about the arrival headway distribution is significant at high major stream flow levels.

The capacity model given in this paper for unsignalised intersections is relevant to a basic gap-acceptance situation where an entry (minor) stream gives way to a single uninterrupted opposing (major) stream. Further considerations apply to the prediction of the capacities of entry streams at sign-controlled intersections and roundabouts.

The German and US Highway Capacity Manual models adjust the basic gap-acceptance capacity using *impedance factors* to allow for interactions among various conflicting movements subject to several levels of priority (TRB 1985; Brilon 1988; Brilon and Grossman 1991). A critical examination of this method is currently being undertaken.

Traditionally, roundabouts are analysed as a series of T-junctions, i.e. as a basic gap-acceptance process where an entry stream gives way to a circulating stream. The only dependence among traffic streams entering from various approaches is modelled through the contributions of entry flows to circulating stream flow rates. This method has been found to overestimate capacities especially under heavy circulating flow conditions. Work is in progress to develop models to adjust basic gap-acceptance capacities at roundabouts to allow for the effects of arrival (origin-destination) patterns and the amount of queueing of approach (entry) streams.

It is possible to use the models presented in this paper on a *lane-by-lane* or *lane group* basis by choosing appropriate parameters relevant to the application. The lane-by-lane method is preferred due to better accuracy levels that can be achieved, especially in the prediction of queue lengths.

The new arrival headway distribution, capacity and performance models were being incorporated into the SIDRA software package at the time of the writing of this paper.

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