

REPRINT

Fundamental Traffic Variables in Adaptive Control and the SCATS DS Parameter

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NOTE:

This paper is related to the intersection analysis methodology used in the SIDRA INTERSECTION software. Since the publication of this paper, many related aspects of the traffic model have been further developed in later versions of SIDRA INTERSECTION. Though some aspects of this paper may be outdated, this reprint is provided as a record of important aspects of the SIDRA INTERSECTION software, and in order to promote software assessment and further research.

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**Fundamental Traffic Variables in Adaptive Control and
the SCATS DS Parameter**

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ABSTRACT

This paper discusses the fundamental traffic variables relevant to adaptive traffic control, monitoring and incident detection. The variables considered include headway, gap, occupancy and space times, vehicle spacing, gap length and speed. Speed - flow, spacing - speed, headway - speed, occupancy time - speed and space time - speed functions as well as the relationship between space time and speed as a function of the detection zone length are given. These relationships are based on queue discharge models calibrated using data obtained through surveys in Melbourne. The application of the knowledge about fundamental traffic variables to the estimation of the SCATS DS parameter is presented. The discussion is relevant to the meaning of the basic DS parameter rather than the role of this parameter in actual SCATS operation which is outside the scope of this paper. The analysis and an example demonstrate the soundness of the SCATS DS parameter which is one of the most important parameters in SCATS control.

INTRODUCTION

1. This paper presents the definitions of the fundamental traffic variables relevant to adaptive traffic control, monitoring and incident detection. The variables considered include headway, gap, occupancy and space times, vehicle spacing, gap length and speed. Speed - flow, spacing - speed, headway - speed, occupancy time - speed and space time - speed functions as well as the relationship between space time and speed as a function of the detection zone length are given. These relationships are based on queue discharge models calibrated using data obtained through surveys in Melbourne.
2. The application of the knowledge about fundamental traffic variables to the estimation of the SCATS DS parameter is presented. Results of the related research on vehicle-actuated control can be found in recent publications (Akçelik 1995a,b; Akçelik and Chung 1995).
3. The relationships between fundamental traffic variables spacing, headway, speed, density, flow and travel time for uninterrupted and interrupted traffic flows conditions were discussed and the difference between two types of speed - flow relationships used in traditional traffic flow theory and in travel time prediction for congested traffic conditions was explained in a related publication (Akçelik 1996a).

QUEUE DISCHARGE MODEL

4. The established method in traffic signal analysis and control is to use a constant saturation flow during queue discharge, and then a departure flow rate that equals the arrival rate after the queue has been cleared (Akçelik 1981, TRB 1994, Webster and Cobbe 1966). An improved queue discharge model is described here for use in simulation and for analytical purposes to achieve more realistic estimates of flow rates and speeds (and therefore headways, occupancy and space times) for vehicles departing from the queue. This model is based on the following queue discharge flow rate and speed relationships (see *Figure 1*) developed using real-life data from recent paired intersection work (Akçelik and Besley 1996):

$$q_s = q_n (1 - e^{-m_q t}) \quad (1)$$

$$v_s = v_n (1 - e^{-m_v t}) \quad (2)$$

$$v_s = v_n \left[1 - \left(1 - \frac{q_s}{q_n} \right)^{m_v/m_q} \right] \quad (3)$$

where

q_s = queue discharge flow rate at time t (veh/h),

q_n = maximum queue discharge flow rate (veh/h),

m_q = a parameter in the queue discharge flow rate model,

v_s = queue discharge speed at time t (km/h),

v_n = maximum queue discharge speed (km/h), and

m_v = a parameter in the queue discharge speed model,

t = time since the start of the displayed green period (seconds).

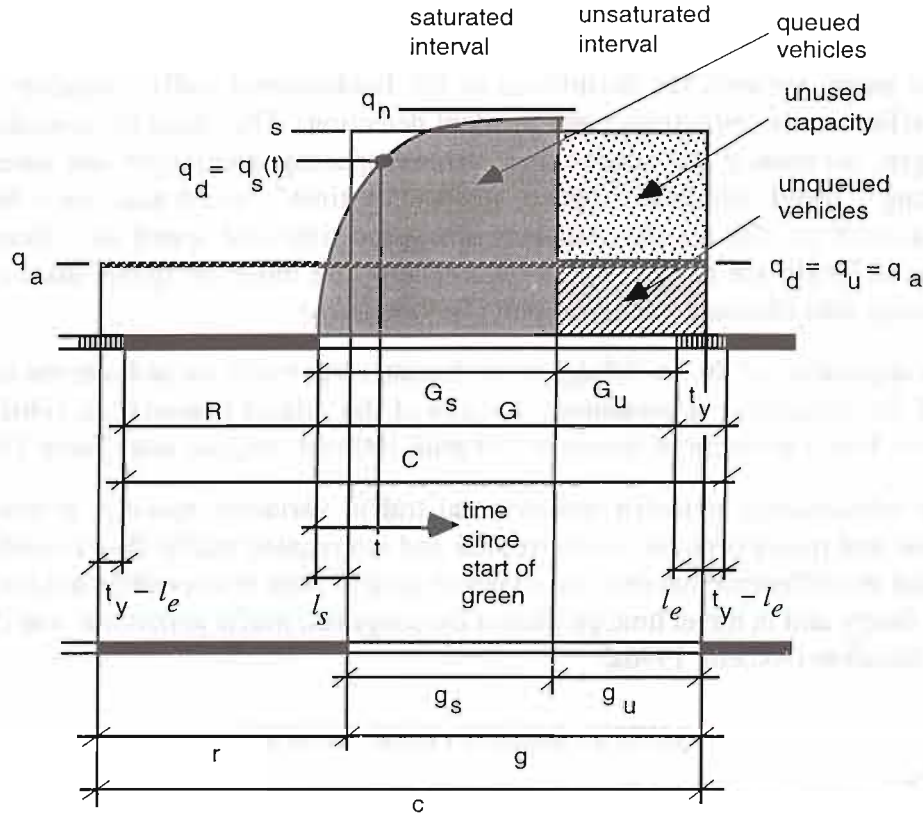


Fig. 1 – Departures during the saturated and unsaturated portions of the green period with the exponential queue discharge model

5. In *Equations (1) to (3)*, q_n and v_n are the parameters for the actual traffic mix including heavy vehicles. The corresponding parameters q_m and v_m can be used for a traffic stream consisting of passenger car units (light vehicles) only. In *Figure 1*, q_a and q_d represent the arrival and departure flow rates, respectively. The departure flow rate is $q_d = q_s(t)$ during the *saturated* part of the green period, and $q_d = q_u = q_a$ during the *unsaturated* part of the green period.

6. Applying the boundary condition that speed is zero ($v_s = 0$) when the vehicle spacing during queue discharge ($L_{hs} = v_s/q_s$) equals the jam spacing ($L_{hs} = L_{hj}$), the parameters m_v and m_q are related through:

$$m_v = m_q \frac{L_{hj}}{L_{hn}} \quad (4)$$

where L_{hj} is the average jam spacing (m/veh), which is the sum of vehicle length and average space length for vehicles in a stationary queue ($L_{hj} = L_v + L_{sj}$), and L_{hn} is the average spacing (m/veh) at the maximum queue discharge flow rate ($L_{hn} = v_n/q_n$).

7. The parameters for the queue discharge flow rate and speed models for seven sites in Melbourne (each representing a traffic lane) are given in *Table 1*. The parameter values were derived using an iterative non-linear regression method with $L_{hj} = 7.0$ m specified (Akçelik and Besley 1996). The value of $L_{hj} = 7.0$ m is based on limited surveys at Site 7. The parameters in Table 1 are for traffic streams with cars only (hence q_m , v_m , L_{hm} rather than q_n , v_n , L_{hn}). The speed limit that can be used as a free-flow speed (v_{fm}) is also given for each site.

Table 1

Parameters for the queue discharge flow rate and speed models for six sites in Melbourne

Site	1	2	3	4	5	6	7
q_m (veh/h)	2368	1796	2127	2030	1657	2042	2038
m_q	0.352	0.552	0.390	0.295	0.759	0.466	0.362
v_m (km/h)	56.1	31.1	46.1	48.2	27.0	34.5	52.2
m_v	0.104	0.223	0.126	0.087	0.326	0.193	0.099
m_v/m_q	0.30	0.40	0.32	0.29	0.43	0.41	0.27
L_{hm} (m/veh)	23.7	17.3	21.7	23.7	16.3	16.9	25.6
<i>Free-flow speed as the speed limit:</i>							
v_{fm} (km/h)	80	60	80	60	60	80	80
v_n/v_f	0.70	0.52	0.58	0.80	0.45	0.43	0.65

Sites (intersections):

- 1: Ferntree Gully Road and Stud Road
- 2: Kooyong Road and Dandenong Road
- 3: South Eastern Arterial and Burke Road
- 4: Canterbury Road and Middleborough Road
- 5: Pedestrian Crossing on Canterbury Road, East Camberwell
- 6: Boronia Road and Wantirna Road
- 7: Ferntree Gully Road and Scoresby Road

8. *Figure 1* shows the departure flow rates during the saturated and unsaturated parts of the green period using the exponential queue discharge flow rate model. Relationships between displayed and effective red, green and cycle times are shown in *Figure 1*, and the formulae are given below. Any intervals with no conflicting phase demand are ignored (Akçelik 1995a).

$$g = G - l_s + l_e - l_b \quad (5)$$

$$r = R + l_s + t_y - l_e + l_b \quad (6)$$

$$C = R + G + t_y \quad (7)$$

$$c = r + g = C \quad (8)$$

$$G_s = g_s + l_s \quad (9)$$

$$G_u = g_u - l_e = G - G_s - l_b \quad (10)$$

$$g = g_s + g_u = G_s + G_u - l_s + l_e = G - l_s + l_e - l_b \quad (11)$$

where r, g, c are the effective red, green and cycle times; R, G, C are the displayed red, green and cycle times; g_s, g_u are the durations of the saturated and unsaturated intervals of the effective green period; G_s, G_u are the corresponding displayed green values; t_y is the yellow time; l_s, l_e are the start loss and end gain times; and l_b is the blocked green time (e.g. due to pedestrians or opposing vehicle traffic), i.e. that part of the green period during which no vehicles can depart due to downstream queue interference, opposing vehicles, pedestrians, and so on (not shown in *Figure 1*).

SATURATION FLOW, START LOSS AND END GAIN

9. The average saturation flow (s), start loss (l_s) and end gain (l_e) can be derived from the exponential queue discharge model (*Equation 1*) using the principles of the method described in Appendix E of the report ARR 123 (Akçelik 1981).

10. The saturation flow rate, s , can be determined as the average queue discharge flow rate during an interval that starts at time t_i and ends at time G_s (at the end of the saturated part of the green period):

$$s = \frac{\int_{t_i}^{G_s} q_s dt}{G_s - t_i} \quad (12)$$

$$= q_n \left[1 - \frac{e^{-m_q t_i} - e^{-m_q G_s}}{m_q (G_s - t_i)} \right]$$

11. The method described in ARR 123 (Akçelik 1981) uses an initial interval of $t_i = 10$ s. Different methods to calculate the saturation flow rate is discussed in Akçelik and Besley (1996). In real-life surveys, G_s is used for determining the saturation flow as specified in ARR 123. For general analysis purposes, $G_s = G_{\max}$ (i.e. a saturation flow rate based on long saturated green periods) can be used:

$$s = q_n \left[1 - \frac{e^{-m_q t_i} - e^{-m_q G_{\max}}}{m_q (G_{\max} - t_i)} \right] \quad (13)$$

12. Using parameter q_m instead of q_n , $s = s_m$ (pcu/h) is obtained for the case of a traffic stream with passenger car units (light vehicles) only. The maximum queue discharge flow rates for the actual traffic mix and passenger cars only (q_n, q_m), and the saturation flow rates for the actual traffic mix and for passenger cars only (s, s_m) are related through:

$$q_n = q_m / f_c \quad (14)$$

$$s = s_m / f_c \quad (15)$$

where f_c is the traffic composition factor given by:

$$f_c = 1 + (f_{HV} - 1) p_{HV} \quad (16)$$

where f_{HV} is a heavy vehicle factor for saturation flow purposes (typically $f_{HV} = 1.5$ to 2.0 veh/pcu), and p_{HV} is the proportion of heavy vehicles in the traffic stream.

13. The start loss (l_s) can be calculated from:

$$l_s = t_i - \frac{3600 n_{vj}}{s} \quad (17)$$

where n_{vi} is the total number of vehicles that depart during the initial interval t_i :

$$\begin{aligned} n_{vi} &= \int_0^{t_i} (q_s / 3600) dt = \\ &= \frac{q_n}{3600} \left[t_i - \frac{1 - e^{-m_q t_i}}{m_q} \right] \end{aligned} \quad (18)$$

14. While the ARR 123 method measures the saturation flow as the average queue departure flow rate during the saturated part of the green period *excluding the vehicles departing during the first 10 seconds*, the US Highway Capacity Manual method (TRB 1994, Chapter 9, Appendix IV) measures the saturation flow as average queue departure flow rate during the saturated part of green period *excluding the first 4 vehicles*. The two methods give very close results (Akçelik and Besley 1996).

15. The end gain (l_e) can be calculated from:

$$l_e = \frac{3600 n_e}{s} \quad (19)$$

where n_e is the total number of vehicles that depart after the end of the green period, i.e. during the terminating intergreen period (yellow and all-red), which is used as an input parameter.

16. The use of a saturation flow rate based on zero start loss ($l_s = 0$) is also feasible. This can be obtained from Equation (13) by putting $t_i = 0$ ($s = s_{os}$):

$$s_{os} = q_n \left[1 - \frac{1 - e^{-m_q G_s}}{m_q G_s} \right] \quad (20)$$

AVERAGE SATURATION SPEED

17. The average saturation speed (v_{sa}) for the saturated part of the green interval can be calculated from:

$$v_{sa} = v_n \left[1 - \frac{1 - e^{-m_v G_s}}{m_v G_s} \right] \quad (21)$$

where G_s is the duration of the saturated part of the displayed green period (see Figure 1).

18. For speed adjustments to allow for heavy vehicle effects, the conditions during queue clearance and after the queue has cleared should be treated separately. Heavy vehicle speed factors f_{vsHV} for queue discharge flow conditions (e.g. $f_{vsHV} = 0.70$) and f_{vuHV} for uninterrupted flow conditions (e.g. $f_{vuHV} = 0.90$) can be defined as the corresponding ratios of heavy vehicle speed to light vehicle speed.

19. The maximum queue discharge speed for the actual traffic mix can be calculated as:

$$v_n = v_m [1 + (f_{vsHV} - 1) PHV] \quad (22)$$

where v_m is the maximum queue discharge speed for a traffic stream consisting of passenger car units only (km/h). The queue discharge speed calculated from Equation (2) using v_n from Equation (22) will also represent the actual traffic mix.

AVERAGE SPEED FOR THE UNSATURATED PART OF THE GREEN

20. Models developed for uninterrupted flow conditions can be used for calculating an average speed for the unsaturated part of the green period, i.e. after queue clearance. A general discussion of various speed-flow relationships for uninterrupted and interrupted flow conditions is given in Akçelik (1996a), and alternative speed-flow models for uninterrupted traffic are discussed in Akçelik and Besley (1996). The following model based on a time-dependent travel time function can be used by limiting its range of application to conditions of demand flow below the maximum flow, $q_u \leq q_n$ for the purpose of this analysis. Uninterrupted travel speed, v_u (in km/h), for a continuous movement is given by

$$v_u = \frac{v_f}{1 + 0.25 v_f T_p \left[z + \left(z^2 + \frac{m_c x}{q_n T_p} \right)^{0.5} \right]} \quad (23)$$

where

v_u = uninterrupted travel speed in km/h,

v_f = free-flow travel speed in km/h,

T_p = peak flow (analysis) period in hours,

q_n = uninterrupted stream capacity in vehicles per hour (= maximum flow rate, q_n used in the queue discharge flow rate model),

z = $x - 1$,

x = q_u/q_n (q_u is the average uninterrupted demand flow rate), and

m_c = cruise delay model parameter.

21. For detailed discussions on this model and the corresponding travel time model, including a method for the estimation of parameter m_c , the reader is referred to Akçelik (1996a) and Akçelik and Besley (1996).

22. The uninterrupted flow speeds for the actual traffic mix can be calculated as:

$$v_f = v_{fm} [1 + (f_{vuHV} - 1) PHV] \quad (24a)$$

$$v_u = v_{um} [1 + (f_{vuHV} - 1) PHV] \quad (24b)$$

where v_{fm} and v_{um} are the uninterrupted flow speeds for a traffic stream consisting of passenger car units only (km/h or m/s), and f_{vuHV} is the heavy vehicle speed factor for uninterrupted flow conditions.

23. Figure 2 shows the speed as a function of the ratio of flow to maximum flow (q/q_n) for queue discharge ($q = q_s$, $v = v_s$) and uninterrupted flow ($q = q_u$, $v = v_u$) regimes together for the case of Site 7 in Table 1 (with cars only). The parameter values for this example are $v_f = 67.8$ km/h (calibrated value), $v_n/v_f = 0.77$, $T_p = 0.25$ h, and $m_c = 2.53$. The corresponding vehicle spacing - speed ($L_h = v_s / q_s$) relationship is shown in Figure 3.

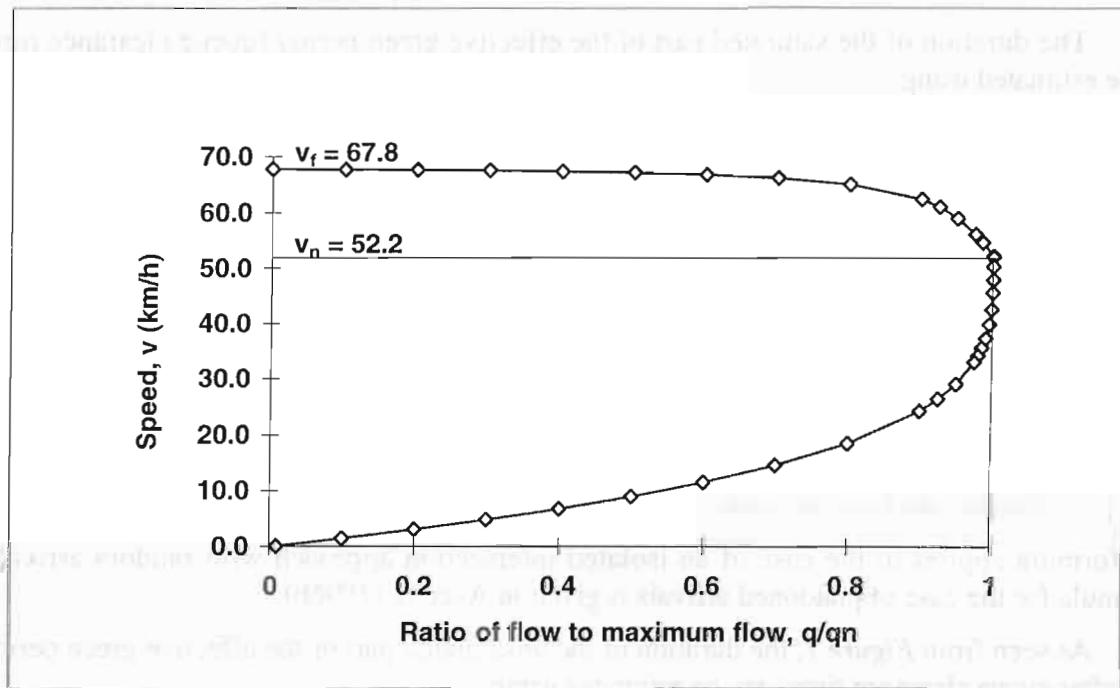


Fig. 2 – Speed as a function of the ratio of flow to maximum flow (q/q_n) for queue discharge ($q = q_s$, $v = v_s$) and uninterrupted flow ($q = q_u$, $v = v_u$) cases

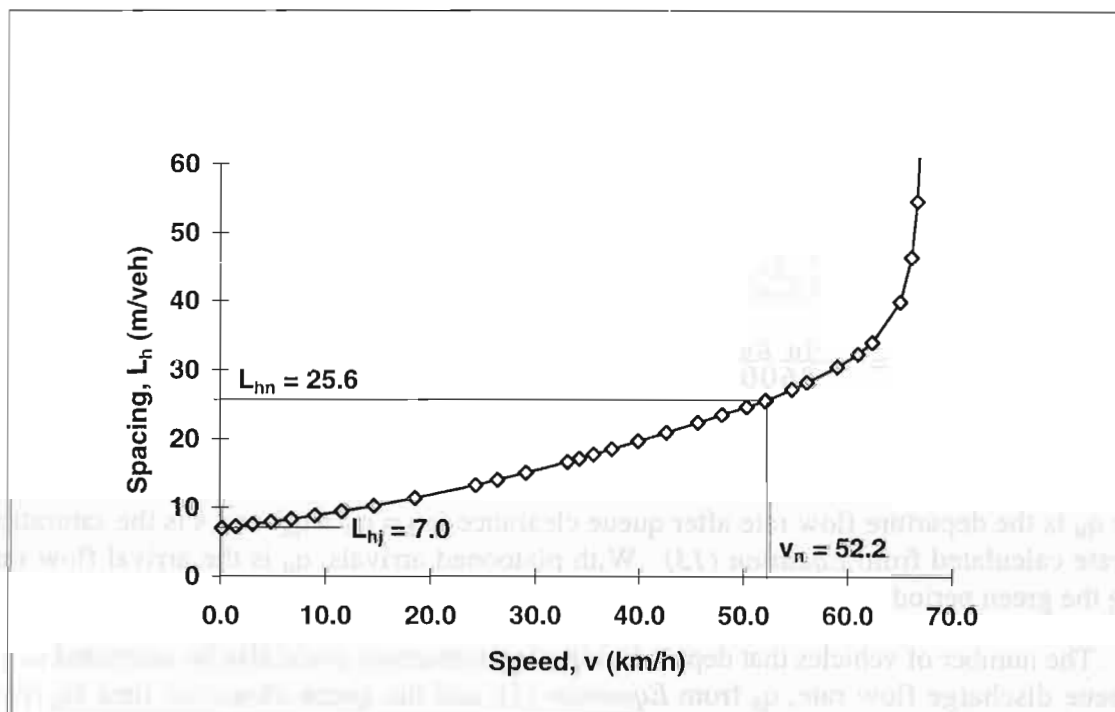


Fig. 3 – Vehicle spacing (m/veh) as a function of speed corresponding to Figure 2 ($L_h = v/q$)

QUEUE CLEARANCE TIME

24. The duration of the saturated part of the effective green period (queue clearance time) can be estimated using:

$$g_s = f_q \frac{(n_r / s) + y r}{1 - y} \quad (25)$$

where

f_q = a calibration factor that allows for cycle-by-cycle variations in the value of the queue clearance time,

n_r = residual queue, i.e. the number of vehicles in the queue at the start of the effective red period,

y = flow ratio, i.e. the ratio of the arrival flow rate to saturation flow rate, $y = q_a / s$, where s is the saturation flow rate calculated using *Equation (13)*,

r = effective red time (seconds).

This formula applies to the case of an isolated intersection approach with random arrivals. A formula for the case of platooned arrivals is given in Akçelik (1996b).

25. As seen from *Figure 1*, the duration of the unsaturated part of the effective green period (time after queue clearance time) can be estimated using:

$$g_u = g - g_s \quad (26)$$

The durations of the saturated and unsaturated parts of the displayed green period (G_s , G_u) can be obtained using *Equations (9) and (10)*.

DEPARTURES DURING SATURATED AND UNSATURATED PARTS OF THE GREEN PERIOD

26. Using the durations of the saturated and unsaturated intervals of the green period, the number of vehicles that depart during these intervals (n_{vs} and n_{vu} , respectively), and the total number of departures during the green period (n_{vg}) can be calculated from:

$$n_{vs} = \frac{s g_s}{3600} \quad (27)$$

$$n_{vu} = \frac{q_u g_u}{3600} \quad (28)$$

$$n_{vg} = n_{vs} + n_{vu} = \frac{s g_s + q_u g_u}{3600} \quad (29)$$

where q_u is the departure flow rate after queue clearance ($q_d = q_u = q_a$) and s is the saturation flow rate calculated from *Equation (13)*. With platooned arrivals, q_u is the arrival flow rate during the green period.

27. The number of vehicles that depart during queue clearance could also be estimated using the queue discharge flow rate, q_s from *Equation (1)*, and the queue clearance time G_s from *Equations (9) and (25)*:

$$n_{vs} = \int_0^{G_s} q_s dt = \quad (30)$$

$$= \frac{q_n}{3600} \left[G_s - \frac{1 - e^{-m_q G_s}}{m_q} \right]$$

The estimates of n_{vs} values from *Equations (27) and (30)* would differ due to the method used to derive the average saturation flow. *Equation (27)* will be used as an approximate solution, rather than using a more complicated iterative method to solve *Equations (30), (25) and (12)*.

OCCUPANCY AND SPACE TIME

28. The basic relationships among traffic variables in adaptive control are shown in *Figures 4 to 6*. *Figure 4* was developed from *Figure 7.1* of AUSTROADS (1993). Refer to Akçelik (1995a) for detailed discussions.

29. The occupancy time (t_o) and space time (t_s) are given by:

$$t_o = \frac{3.6 (L_v + L_p)}{v} \quad \text{subject to } t_o \leq h \quad (31)$$

$$t_s = h - t_o \quad \text{subject to } t_s \geq 0 \quad (32)$$

where

L_v = average vehicle length (m/veh),

L_p = detection zone length (m), typically 4.5 m in Australia,

v = vehicle speed (km/h), and

h = headway (seconds).

30. The condition $t_o \leq h$ is to allow for the case of *overlapping occupancy times* at slow speeds. This occurs when the following vehicle enters the detection zone before the leading vehicle leaves the zone. When $3.6 (L_v + L_p) / v > h$ is found, $t_o = h$ is set, therefore, $t_s = 0$ is obtained. In this case, the departing vehicle count (= number of spaces) will underestimate the actual number departing since no space will appear between the two vehicles. During the blocked green interval (duration l_b), no vehicles can depart but there is a vehicle present in the detection zone, therefore $t_o = l_b$ should be set for this interval, and this should be added to the total occupancy time (T_o) during the green period.

31. The vehicle length (L_v) should represent the actual traffic composition. Where the traffic stream is represented as a mixture of light vehicles (LVs) and heavy vehicles (LVs), the average vehicle length can be calculated as:

$$L_v = (1 - p_{HV}) L_{vm} + p_{HV} L_{vHV} \quad (33)$$

where

p_{HV} = proportion of heavy vehicles in the traffic stream,

L_{vm} = average vehicle length for light vehicles (passenger car units) (m/LV or m/pcu),

L_{vHV} = average vehicle length for heavy vehicles (m/HV).

32. *Figures 7 and 8* show the headway, gap time, occupancy time and space time as a function of speed for Site 7 in *Table 1* corresponding to the speed-flow relationship shown in *Figure 2* (detection zone length, $L_p = 4.5$ m; cars only with average vehicle spacing in queue, $L_{hj} = 7.0$ m/veh).

33. Figure 9 shows the relationship between space time and speed as a function of the detection zone length (L_p) using the speed-flow relationship shown in Figure 2 (cars only with average vehicle spacing in queue, $L_{hj} = 7.0$ m/veh). For the examples of the related speed-density and flow-density relationships, see Akçelik and Besley (1996).

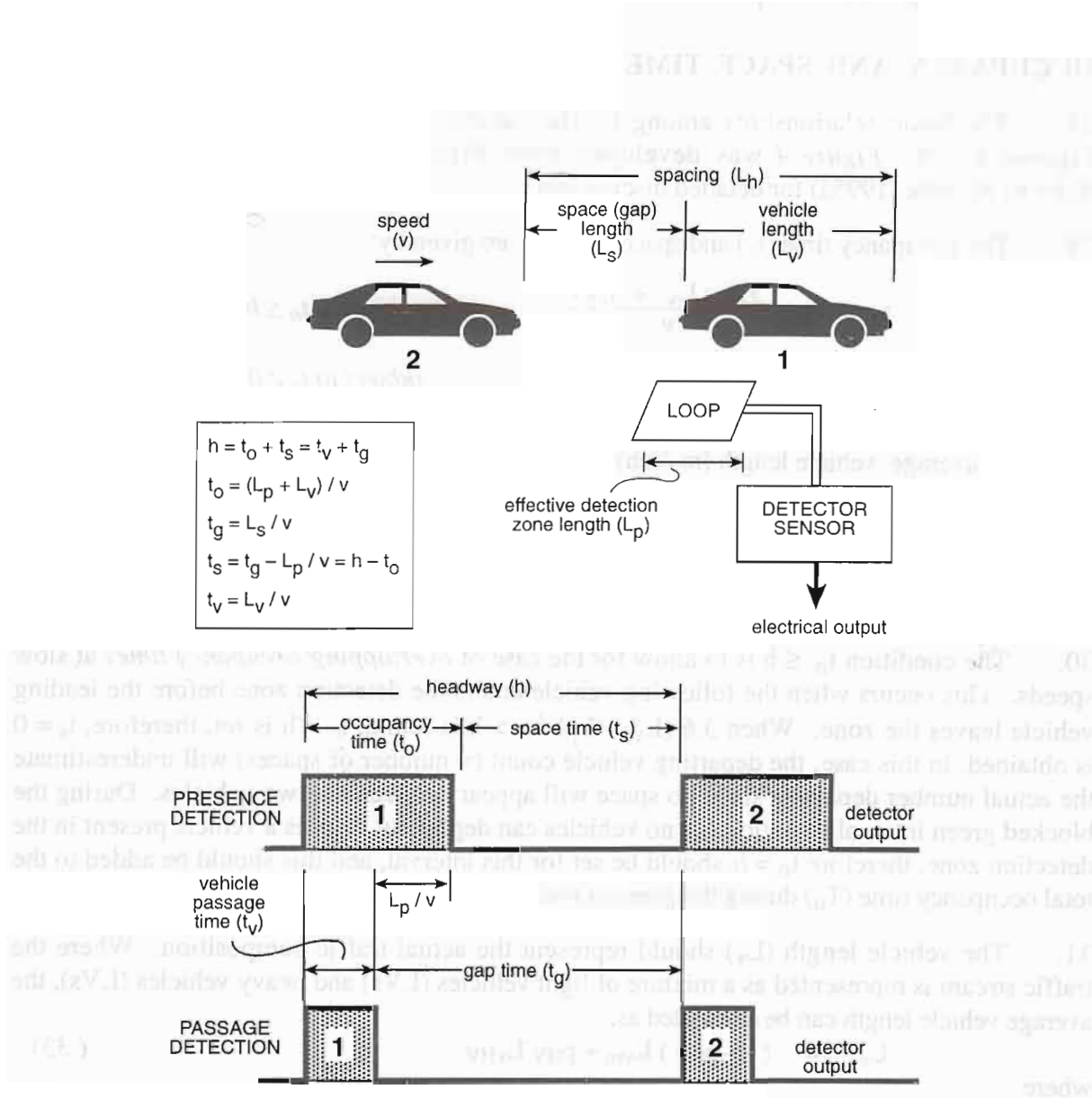


Fig. 4 – Traffic variables in adaptive signal control with presence and passage detection

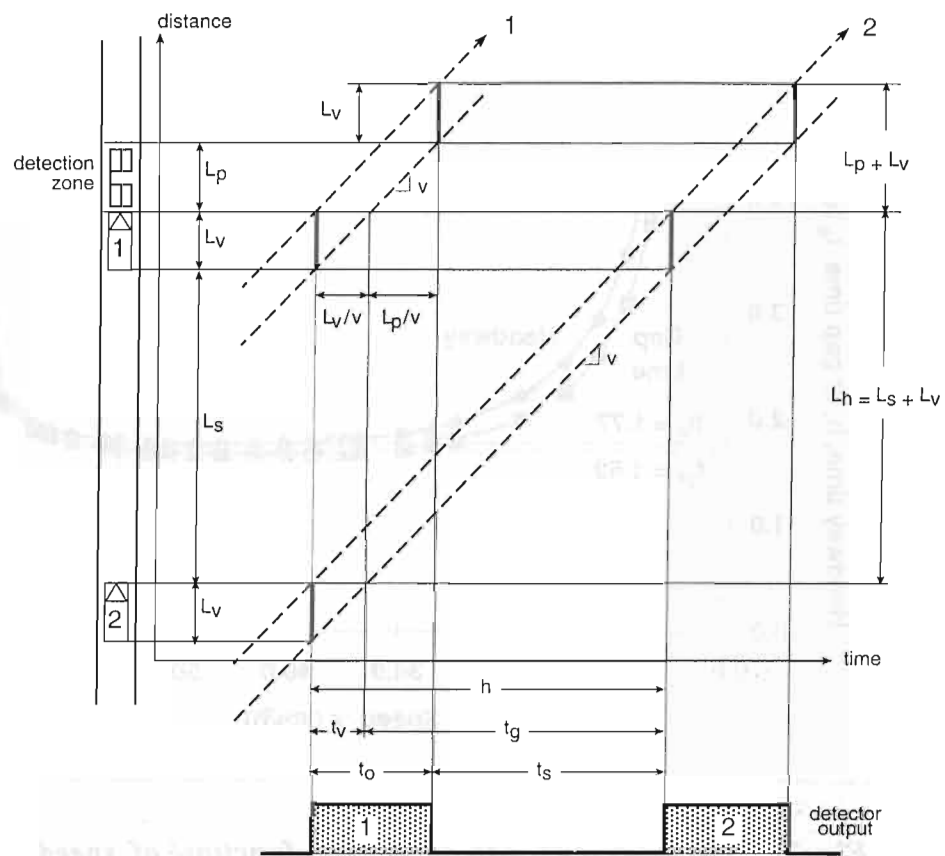


Fig. 5 – Time-distance diagram explaining the basic relationships of traffic variables in actuated control

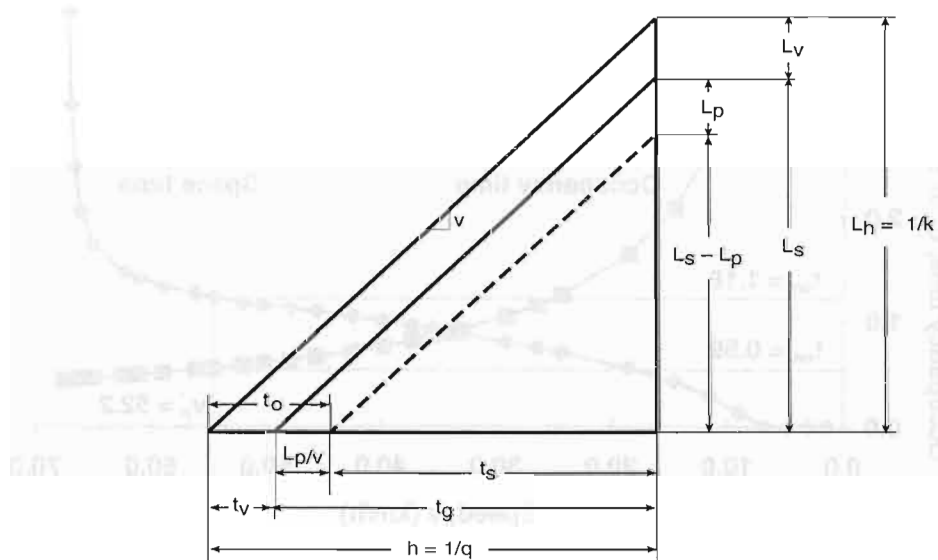


Fig. 6 – The relationships between vehicle spacing, headway, space length, gap time, space time and speed

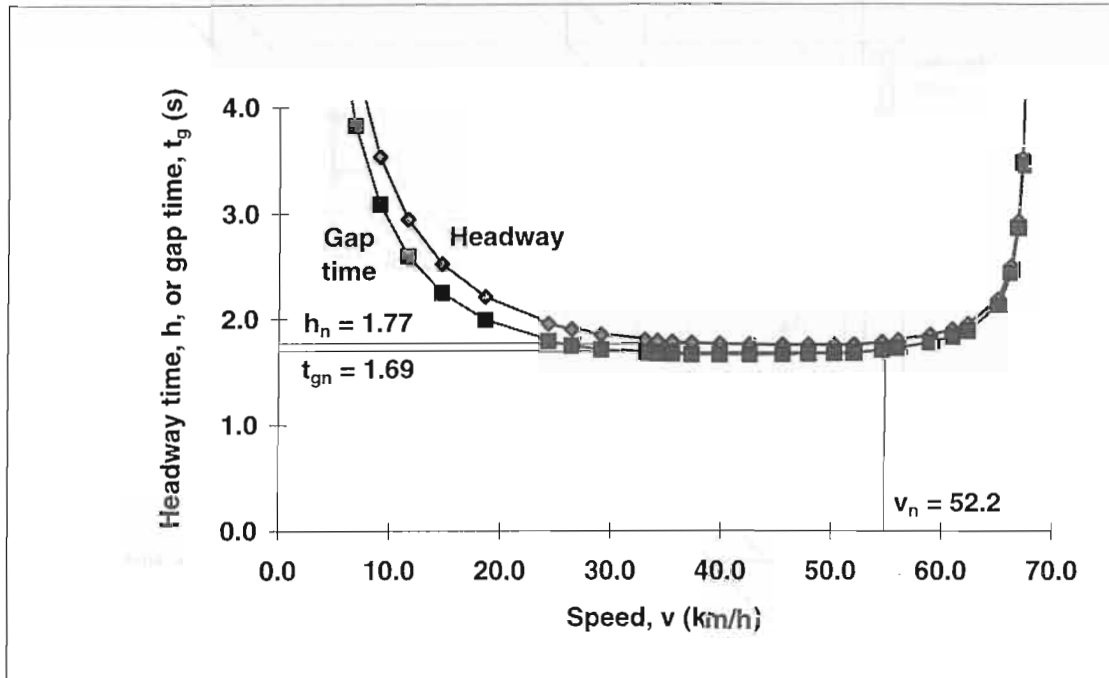


Fig. 7 – Headway and gap time as a function of speed
(corresponding to Figure 2)

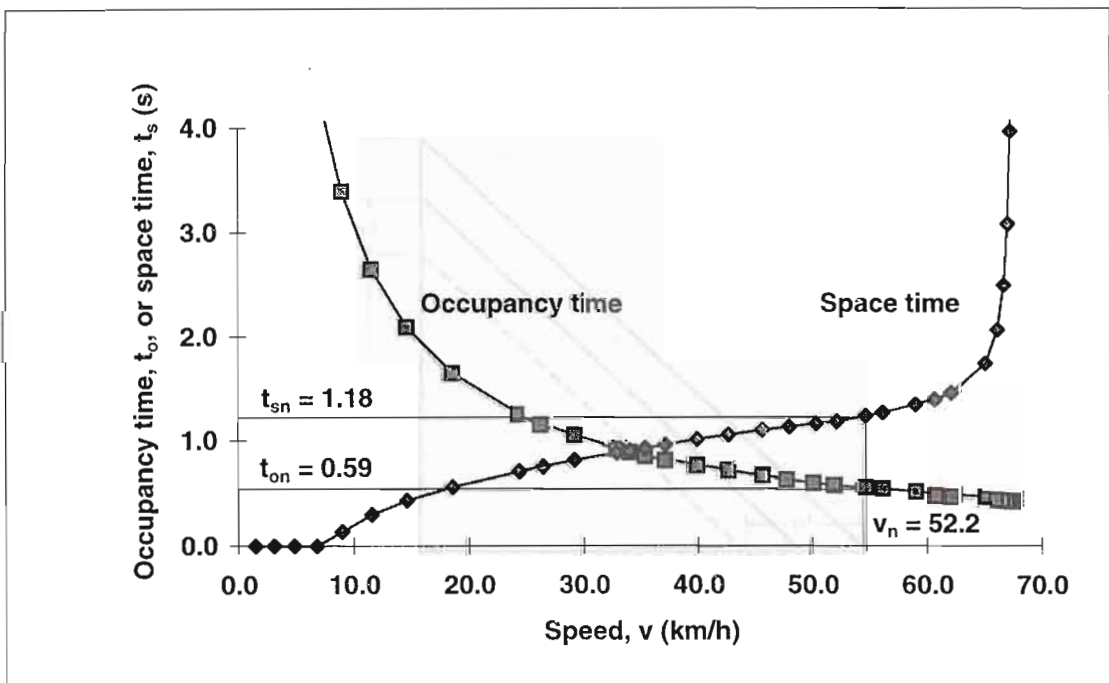


Fig. 8 – Occupancy and space time as a function of speed
(corresponding to Figure 2)

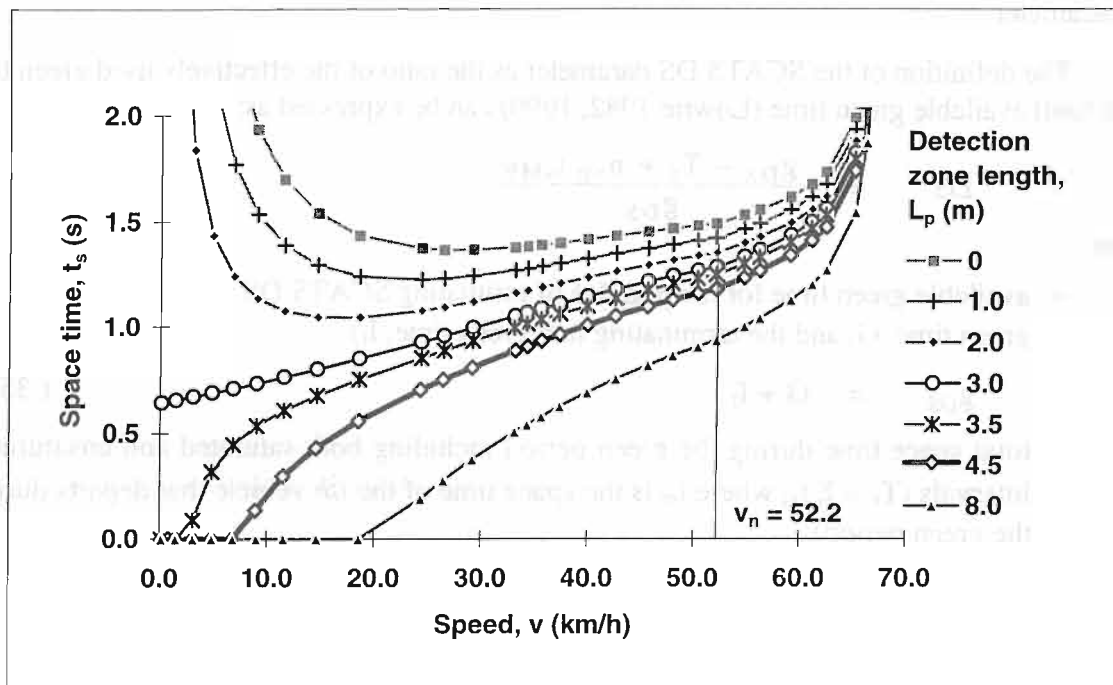


Fig. 9 – The relationship between space time and speed as a function of the detection zone length (corresponding to Figure 2)

SCATS DS PARAMETER

34. The SCATS DS parameter is the basic strategic control parameter in the operation of the SCATS dynamic signal control system including the control of an isolated intersection (Lowrie 1982, 1990). The DS parameter is defined as "the ratio of the effectively used green time to the total available green time", and is formulated in terms of presence loop occupancy and volume data measured by the system. Lowrie (1982, 1990) qualifies the SCATS DS parameter as "analogous to degree of saturation". In this section, an estimate of the DS parameter is calculated using the saturation flow and saturation speed estimates derived from the exponential queue discharge flow and speed models. This allows a discussion of the DS parameter in relation to the *degree of saturation* used in the traditional traffic signal analysis methods (Akçelik 1981, AUSTROADS 1993, TRB 1994, Webster and Cobbe 1966). The purpose is to demonstrate the validity of fundamental relationships with reference to a well-known control parameter used in practice. This information would be useful in the validation of analytical and simulation models used for the analysis of SCATS operation. The reader is also referred to an earlier paper by Fehon and Moore (1982) who discussed the relation between the speed - flow - concentration variables and the SCATS DS parameter.

35. The SCATS method determines the DS parameter from on-line measurements carried out all day using a complicated procedure. The *DS estimate* used in this paper is an analytical construct that does not directly relate to the DS parameter measured by SCATS. The emphasis in the discussion presented here is the on *meaning* of the DS parameter rather than the measured DS parameter and its role on actual SCATS operations which is outside the scope of this paper.

The SCATS method also applies various adjustments to the basic DS parameter for use in its control algorithms. These adjustments will be ignored in discussing the basic meaning of the DS parameter.

36. The definition of the SCATS DS parameter as the ratio of the effectively used green time to the total available green time (Lowrie 1982, 1990) can be expressed as:

$$DS = \frac{g_{DS} - T_s + n_{vg} t_{sMF}}{g_{DS}} \quad (34)$$

where

g_{DS} = available green time for the purpose of estimating SCATS DS (sum of the displayed green time, G , and the terminating intergreen time, I_t):

$$g_{DS} = G + I_t \quad (35)$$

T_s = total space time during the green period including both saturated and unsaturated intervals ($T_s = \sum t_{si}$ where t_{si} is the space time of the i th vehicle that departs during the green period),

n_{vg} = number of vehicles that depart during the green period, and

t_{sMF} = average space time at SCATS maximum flow, s_{MF} .

TOTAL NUMBER OF VEHICLES DEPARTING DURING THE GREEN PERIOD

37. The number of vehicles that depart during the green period (n_{vg}) can be calculated from Equation (29). However, allowance should be made for overlapping occupancy times (zero space times) due to low departure speeds which can occur at the start of green or because of downstream queue interference or similar reasons.

TOTAL SPACE TIME DURING THE GREEN PERIOD

38. The total space time, T_s , can be calculated using the average space times during queue clearance (t_{ssa}) and after queue clearance (t_{su}):

$$T_s = n_{vs} t_{ssa} + n_{vu} t_{su} \quad (36)$$

where n_{vs} and n_{vu} are the numbers of vehicles that depart during the saturated and unsaturated parts of the green period calculated from Equations (27) and (28).

39. The average occupancy and space times during the saturated and unsaturated intervals of the green period (i.e. during and after queue clearance) can be estimated using the following equations (see Equations 31 and 32):

$$t_{osa} = \frac{3.6 (L_v + L_p)}{v_{sa}} \quad \text{subject to } t_{osa} \leq h_{sa} \quad (37)$$

$$t_{ssa} = h_{sa} - t_{osa} \quad \text{subject to } t_{ssa} \geq 0 \quad (38)$$

$$t_{ou} = \frac{3.6 (L_v + L_p)}{v_u} \quad \text{subject to } t_{ou} \leq h_u \quad (39)$$

$$t_{su} = h_u - t_{ou} \quad \text{subject to } t_{su} \geq 0 \quad (40)$$

where

t_{osa} and t_{ssa} are the average occupancy and space times during the saturated part of the green period,

t_{ou} and t_{su} are the average occupancy and space times during the unsaturated part of the green period,

$L_v + L_p$ is the sum of vehicle and detection zone lengths,

v_{sa} is the average speed during the saturated part of the green period, calculated from *Equation (21)*,

v_u is the average speed during the unsaturated part of the green period, calculated from *Equation (23)*,

h_{sa} is the average departure headway during the saturated part of the green period

$$h_{sa} = \frac{3600}{q_{sa}} \quad (41)$$

where q_{sa} is the average flow rate during the saturated part of the green period determined using n_{vs} (total number of vehicle departures during the saturated part of the green period) from *Equation (30)*:

$$q_{sa} = \frac{n_{vs}}{G_s} \quad (42)$$

where G_s is the duration of the saturated part of the green period (for simpler analysis, the saturation flow rate can be used for this purpose, $q_{sa} = s$),

h_u is the average departure headway during the unsaturated part of the green period calculated from

$$h_u = \frac{3600}{q_u} \quad (43)$$

where q_u is the average departure flow rate during the unsaturated part of the green period determined as $q_u = q_a$ (the average arrival flow rate). With platooned arrivals, q_u is the average arrival flow rate during the green period.

40. For improved estimates of the average space time during the saturated part of the green time (t_{ssa}), and therefore the total space time (T_s) for use in calculating DS, the possibility of overlapping occupancy times (zero space times) at the start of green could be allowed for by ignoring the first 5 to 10 seconds of the displayed green period (calculating v_{sa} and n_{vs} accordingly). If this method is adopted, it should also be applied to the estimation of the average space time at SCATS Maximum Flow.

41. The average space time for all vehicles departing during the green period can be calculated from:

$$t_{sg} = \frac{T_s}{n_{vg}} \quad (44)$$

where T_s is the total space time from *Equation (36)* and n_{vg} is the total number of vehicles departed during the green period calculated from *Equation (29)*.

TOTAL OCCUPANCY TIME DURING THE GREEN PERIOD

42. The total occupancy time, T_o , during the green period can be calculated using the average occupancy times during queue discharge (t_{osa}) and after queue clearance (t_{ou}), and also allowing for the presence of demand for the duration of any blocked green interval (include l_b):

$$T_o = n_{vs} t_{osa} + n_{vu} t_{ou} + l_b \quad (45)$$

See *Equations (37) and (39)* for t_{osa} and t_{ou} , and *Equations (27) and (28)* for n_{vs} and n_{vu} .

43. The average occupancy time for all vehicles departing during the green period can be calculated from:

$$t_{og} = \frac{T_o}{n_{vg}} \quad (46)$$

where T_o is the total occupancy time from *Equation (45)* and n_{vg} is the total number of vehicles from *Equation (29)*.

AVERAGE HEADWAY DURING THE GREEN PERIOD

44. The average headway for all vehicles departing during the green period can be calculated from:

$$h_{dg} = t_{og} + t_{sg} = \frac{T_o + T_s}{n_{vg}} \quad (47)$$

Note that $T_o + T_s = G + l_e$ is obtained using *Equations (36) and (45)*. This corresponds to $g_{DS} = G + I_t$ in *Equation (34)*.

AVERAGE SPACE TIME AT SCATS MAXIMUM FLOW

45. An estimate of the SCATS Maximum Flow (MF) parameter, s_{MF} , is required for determining the corresponding average space time (t_{sMF}) for use in DS calculation. A DS estimate based on analytical considerations can be calculated from:

$$s_{MF} = \frac{f_{MF} q_m (G_{max} - \frac{1 - e^{-m_q G_{max}}}{m_q}) + 3600 n_e}{G_{max} + I_t} \quad (48)$$

where f_{MF} is a factor for higher values of the maximum queue discharge flow rate observed in individual signal cycles, q_m is the maximum queue discharge flow rate (per hour) for a traffic stream consisting of passenger car units (light vehicles) only, G_{max} is the maximum green time, I_t is the terminating intergreen time, and n_e is the number of vehicles that depart during the terminating intergreen period.

46. The estimate of s_{MF} from *Equation (48)* can be used as a substitute for the real-life SCATS MF value when it is not known in general analytical or simulation work. This estimate is based on the assumption that s_{MF} in real-life corresponds to long green times and when there are no heavy vehicles. Similarly, the average speed corresponding to s_{MF} can be estimated as:

$$v_{MF} = v_m \left[1 - \frac{1 - e^{-m_v (G_{max} + I_t)}}{m_v (G_{max} + I_t)} \right] \quad (49)$$

where v_m is the maximum queue discharge speed for a traffic stream consisting of passenger car units (light vehicles) only.

47. Thus, the average occupancy and space times at SCATS Maximum Flow (t_{oMF} and t_{sMF}) can be calculated from:

$$t_{oMF} = \frac{3.6 (L_{vm} + L_p)}{v_{MF}} \quad \text{subject to } t_{oMF} \leq h_{MF} \quad (50)$$

$$t_{sMF} = h_{MF} - t_{oMF} \quad \text{subject to } t_{sMF} \geq 0 \quad (51)$$

where L_{vm} is the average vehicle length for passenger car units (m/pcu), v_{MF} is average speed from Equation (49), and h_{MF} is the average headway corresponding to s_{MF} :

$$h_{MF} = \frac{3600}{s_{MF}} \quad (52)$$

using s_{MF} from Equation (48).

SCATS DS ESTIMATE AND TRADITIONAL DEGREE OF SATURATION

48. Traditionally, the degree of saturation used in traffic signal analysis (Akçelik 1981, AUSTROADS 1993, TRB 1994, Webster and Cobbe 1966) is calculated from:

$$x = \frac{q_a c}{s g} \quad (53)$$

where g is the effective green time (s), c is the cycle time (s), q_a is the demand (arrival) flow rate (veh/s), and s is the saturation flow rate (veh/s). Using the queue discharge method described in Section 5, s can be estimated from Equation (13).

49. The percentage difference between the DS estimate obtained using the method given in previous sections and the degree of saturation from Equation (53) can be calculated from:

$$\text{DIF\%} = 100 \frac{\text{DS}}{x} - 1 \quad (54)$$

50. It is emphasised that DS in Equation (54) is an estimate of the basic DS parameter, not the DS value measured by the real-life SCATS system. Similarly, x is an approximate value since the mathematically correct degree of saturation using the exponential queue discharge model should use the cycle capacity as

$$s g = \int_0^{G-l_b} q_s dt + n_e \quad (55)$$

instead of (sg) in Equation (53).

EXAMPLE

51. An example is given in Table 2 comparing the SCATS DS estimate with the traditional degree of saturation. This example uses the basic parameters for Site 7 in Table 1, but assumes 5 per cent heavy vehicles. The demand flow rate is 800 veh/h. The cycle time is $c = 90$ s, and the maximum green time is $G_{\max} = 70$ s. Various other input parameters and results from an Excel spreadsheet application of the method are shown in Figure 10. The results for varying green splits (given $c = 90$ s) are given in Table 2.

52. In all cases shown in *Table 2*, $q_n = 1941$ veh/h, $m_q = 0.365$, $v_n = 52.2$ km/h, $m_v = 0.099$, $s = 1939$ veh/h, $s_{MF} = 1965$ veh/h ($f_{MF} = 1.05$ used), $v_{MF} = 45.3$ km/h, $t_{sMF} = 1.16$ s, $v_u = 67.3$ km/h, $t_{su} = 4.03$ s as seen in *Figure 10*. For the medium degree of saturation case ($g = 55$ s, $r = 35$ s), the traditional degree of saturation and the DS estimate are seen to be $x = 0.675$ and $DS = 0.685$ with a difference of $DIF = 1.6$ %. With cars only, $x = 0.645$, $DS = 0.682$ and $DIF\% = 5.8$ %, and with 15 per cent heavy vehicles, $x = 0.739$, $DS = 0.696$ and $DIF\% = -5.8$ % difference were found. *Figure 11* shows the correlation between x and DS values with more data points obtained using $R = 46$ s, $G = 50$ s, $c = 100$ s, $G_{max} = 50$ s, and arrival flow rates in the range $q_a = 500$ veh/h to 950 veh/h ($p_{HV} = 0, 0.05$ and 0.10). The results in *Table 2* and *Figure 11* indicate good correspondence between the SCATS DS estimate and the traditional degree of saturation. Note that the actual SCATS Maximum Flow (MF) parameter for Site 7 (reported on the day following the survey at this site) was 2034 veh/h compared with the s_{MF} estimate of 1965 veh/h used in this example.

Table 2

**SCATS DS estimates for varying green splits with a cycle time of 90 s
for the example shown in *Figure 10* (Site 7, $q_a = 800$ veh/h, $p_{HV} = 0.05$)**

Case	g	r	g_s	t_{ssa}	T_s	n_{vg}	DS	x	DIF%
Low x	70	20	14.0	1.03	57.92	20.00	0.542	0.530	2.1 %
Medium x	55	35	24.6	1.13	42.28	20.00	0.685	0.675	1.6 %
High x	44	46	32.3	1.17	30.85	20.00	0.845	0.844	0.2 %
Oversat.	36	54	36.0	1.17	22.75	19.40	1.031	0.992	- 3.8 %

SCATS DS using exponential queue discharge flow and speed models		
Input	Site = 7	
Loop length, L_p	4.5	Queue disch. flow & speed
Car length, L_{vm}	4.0	Max. flow, q_m
HV length, L_{vHV}	10.0	$m_q = mv / (L_{hj}/L_{hn}) = 0.365$
Space length in que, L_{sj}	3.00	Max. speed, v_m
Arr. flow, q_a	800	Param. mv (regression)
Prop. Heavy Veh, p_{HV}	0.05	Unint. pcu speed, v_{fm}
Heavy Vehicle factors		Displayed red time, R
Sat. flow, f_{HV}	2.0	Displayed green time, G
Speed (queue), f_{vsHV}	1	G_{max}
Speed (unint), f_{vuHV}	1	G_{min}
Residual queue, nr	0.0	Yellow time, t_y
		All-red time, t_{ar}
Blocked green time, l_b	0.0	t_1 for sat. flow
Blockage factor for q_s	1.000	End departures, ne
Blockage factor for v_s	1.000	
Optional user-specified parameters (will take effect if non-zero)		
Sat. flow, s_m	0	Aver. veh. length = 4.3
SCATS Max. Flow, s_{MF}	0	Spacing in que, L_{hj} = 7.3
SCATS MF factor, f_{MF}	1.05	k_j = 137.0
Speed, v_{sm}	0	Max. flow, q_n = 1941
SCATS MF speed, v_{MF}	0	Max. speed, v_n = 52.2
Unsat. green speed, v_u	0	$L_{hn} = v_n/q_n = 26.9$
Start loss, l_s	0	kn = 37.2
End gain, l_e	0	$kn/k_j = L_{hj}/L_{hn} = mv/m_q = 0.27$
		mv (regression) = 0.099
		$m_q = mv / (L_{hj}/L_{hn}) = 0.365$
Max. queue discharge flow		Unsat. green interval
q_n (veh/h) =	1941	$q_u = q_a$ (veh/h) = 800
q_n (veh/s) =	0.539	$q_u = q_a$ (veh/s) = 0.222
h_n =	1.85	h_u = 4.50
v_n =	52.2	v_u = 67.3
t_{on} =	0.61	t_{ou} = 0.47
t_{sn} =	1.25	t_{su} = 4.03
SCATS Maximum Flow		Saturated green interval
s_{MF} (veh/h) =	1965	q_{sa} (veh/h) = 1746
s_{MF} (veh/s) =	0.546	q_{sa} (veh/s) = 0.485
h_{MF} =	1.83	h_{sa} = 2.06
v_{MF} =	45.3	v_{sa} = 34.1
t_{oMF} =	0.68	t_{osa} = 0.93
t_{sMF} =	1.16	t_{ssa} = 1.13
		Free flow speed, v_f = 67.8
		Unint. speed at q_a , v_u = 67.3
		$G = G_{max}$ if $G < G_{max}$ = 54.9
		Eff. green time, g = 55.0
		Eff. red time, r = 35.0
		Cycle time, c = 90.0
		Intergreen time, l_t = 6.0
		$G + l_t$ = 60.9
		$G_{max} + l_t$ = 76.0
		$R + t_y - l_e$ = 32.3
		Sat. flow (pcu/h), s_m = 2036
		Satf. factor, f_c = 1.050
		Sat. flow, s = 1939
		s (veh/s) = 0.539
		s_{MF}/s_m = 0.97
		s_{MF}/s = 1.01
		ns = 3.95
		Start loss, l_s = 2.7
		End gain, l_e = 2.8
		Flow ratio, $y = q_a/s$ = 0.413
		Sat. eff. green, g_s = 24.6
		Unsat. eff. grn, g_u = 30.45
		G_s = 27.2
		G_u = 27.7
		T_h = 57.74
		T_o = 15.46
		T_s = 42.28
		hdg = 2.89
		tog = 0.77
		tsg = 2.11
		$nvs = s^*g_s$ = 13.23
		nvu = 6.77
		nvg = 20.00
		$nvg*t_{sMF}$ = 23.12
		q_a*c = 20.00
		sg = 29.63
		Q = 1185
		x = 0.675
		DS = 0.685
		$DIF\%$ = 1.6

Fig. 10 – Example for the estimation of SCATS DS parameter

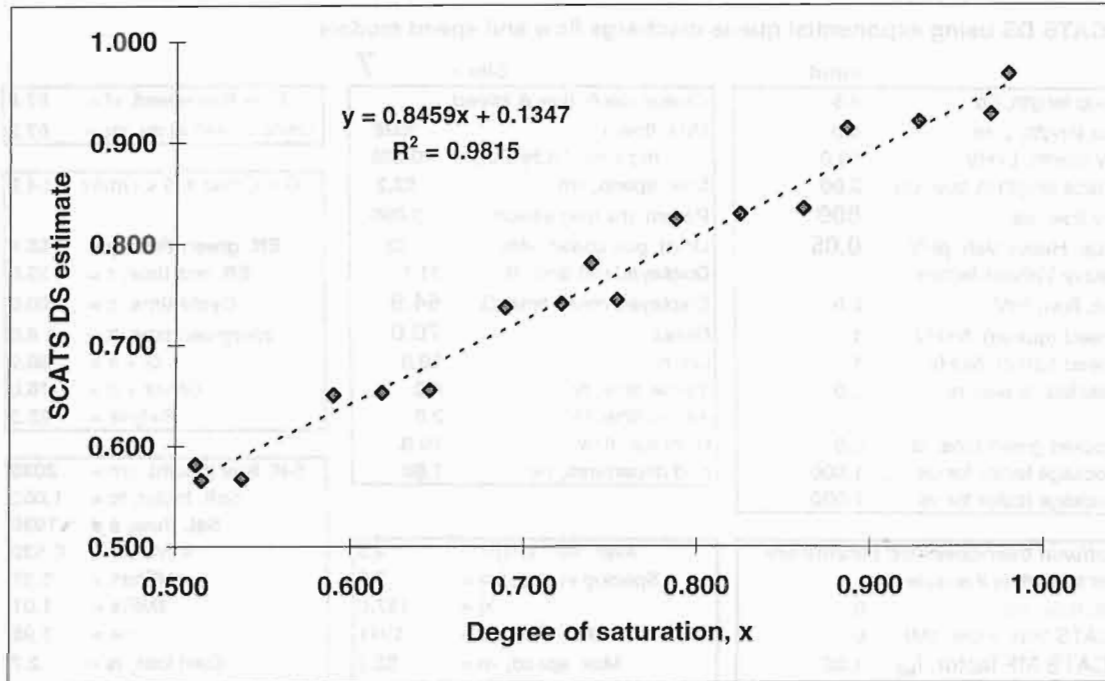


Fig. 11 – Correlation between the SCATS DS estimates and traditional degree of saturation values

CONCLUSION

53. Traditional analysis methods that use a constant saturation flow estimation may not be adequate for the analysis of the more complex operation of adaptive systems. The exponential queue discharge flow and speed models described in this paper provide more realistic estimation of departure headways, speeds, occupancy and space times for the evaluation of adaptive control algorithms using presence detection. Research is in progress for the analysis of the traditional vehicle-actuated and SCATS Master Isolated control methods using the method described in this paper.

54. The analysis and an example comparing the SCATS DS estimate with the traditional degree of saturation demonstrate the validity of fundamental relationships with reference to a well-known control parameter used in practice. This also indicates the soundness of the SCATS DS parameter which is one of the most important parameters in SCATS operation. It should be emphasised that the discussion presented in this paper is relevant to the meaning of the basic DS parameter rather than the role of this parameter in actual SCATS operation which is outside the scope of this paper.

55. It is recommended that investigations of speed-flow relationships are undertaken including measurements of jam spacing, with a view to obtaining reliable estimates of space times, and choosing appropriate detector loop lengths for control and monitoring purposes.

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