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TECHNICAL NOTE

Progression Factors in the HCM 2000 Queue and Delay Models for Traffic Signals

Author:

Rahmi Akçelik

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Summary

This Technical Note recommends that the following conditions be used in implementing the HCM 2000 progression factors for delay and back of queue. The symbols used below are as follows: PF is the delay progression factor, PF_2 is the queue progression factor, R_p is the platoon ratio, P is the proportion of arrivals on green, u is the g/C for the lane group, and y_L is the lane group flow ratio per lane (arrival flow / saturation flow).

(i) $PF \geq 1.0$ *and* $PF_2 \geq 1.0$ *for Arrival Types 1 and 2,*

(ii) $PF \leq 1.0$ *and* $PF_2 \leq 1.0$ *for Arrival Types 4 to 6,*

(iii) $P \leq 0.95$ $(R_p \leq 0.95 / u)$ *for both PF and* PF_2*,*

(iv) $R_p \leq 0.95 / y_L$ *for both PF and* PF_2*,*

(v) $PF_2 = 1.0$ *for* $y_L \geq u$ $(X_L \geq 1)$*,*

(vi) $R_p \geq (1 - 0.95 (1 - u) / y_L) / u$ *for both PF and* PF_2*, and*

(vii) $R_p = 1.0$ $(P = u)$*, therefore* $PF = 1.0$ *and* $PF_2 = 1.0$ *for* $y_L \geq 0.95$*.*

(viii) *If Conditions (iii), (iv) and (vi) create inconsistent constraints on* R_p *and P, set* $R_p = 1.0$ *and* $P = u$*, therefore* $PF = 1.0$ *and* $PF_2 = 1.0$*.*

The numbering of conditions listed above reflects the discussion given in *Section 3* of this Technical Note. An order and further suggestions for their implementation are given in *Section 6*. The progression factor equations are given in *Section 2*. The issue of large values of progression factors that may result under certain circumstances is discussed in *Section 4*. A list of *Notations and Basic Relationships* is given at the end of the document.

The reasons for each condition are discussed in *Section 3*, including numerical examples. In summary:

Condition (i) applies a logical constraint on PF and PF_2 calculations for Arrival Types (AT) 1 and 2 (Very Poor or Unfavourable progression), simply stating that these arrival types must not produce a PF or PF_2 less than 1.0, given that they represent performance worse than AT 3 (Random arrivals) which will always produce PF or PF_2 = 1.0.

Condition (ii) is similar to *Condition (i)*. It applies a logical constraint on PF and PF_2 calculations for AT 4, 5 and 6 (Favourable, Highly Favourable, or Exceptional progression), simply stating that these arrival types must not produce a PF greater than 1.0, given that they represent performance better than AT 3.

Condition (iii) is a logical condition based on the definition of P and R_p, and negative values of PF and PF_2 can result if it is not applied.

Condition (iv) is needed to prevent the denominator of PF_2 going to zero or below, in which case the PF_2 value would be very large, undefined or negative. It places an upper bound on R_p and P. This has a logical equivalent that limits how many vehicles the defined arrival type can expect to process during the *green period* in relation to the saturation flow rate. Although this denominator does not explicitly appear in the PF formula for the delay model, the same logic applies for the maximum flow during green, so the constraint must be applied for PF as well.

Condition (v) essentially states that the PF_2 value is equal to 1.0 at saturation, and it cannot change from this value for oversaturated conditions. This is because the coordination effects virtually disappear under these conditions and the oversaturation effects are handled by the second term of the queue model.

Condition (vi) is needed to limit the demand during the *red period* dictated by the R_p, placing a lower bound on R_p and P, in a similar fashion to *Condition (iv)*.

Condition (vii) is used when the average flow rate is so high that it may not be possible to meet *Conditions (iv) and (vi)* simultaneously. This condition requires that $R_p = 1.0$ (P = u), and therefore PF = 1.0 and PF_2 =1.0, are used, i.e. the case is treated as Arrival Type 3, when the flow ratio (y_L) exceeds a practical limit.

Condition (viii) is applicable when *Conditions (iii), (iv) and (vi)* create inconsistent constraints that cannot all be met simultaneously. In this case, $R_p = 1.0$ (P = u), therefore PF = 1.0 and PF_2 = 1.0, are set, and therefore, the case is treated as Arrival Type 3.

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Correspondence: Rahmi Akçelik
Director, Akcelik & Associates Pty Ltd,
P O Box 1075 G, Greythorn Victoria, Australia 3104
Tel: +61 3 9857 9351, Fax: + 61 3 9857 5397
rahmi@akcelik.com.au

www.aatraffic.com

1. Introduction

The signalised intersection chapter of the latest "HCM 2000" edition of the US Highway Capacity Manual (TRB 2000, Chapter 16) introduced a *back of queue* model, which was developed by the author. A paper by Viloria, Courage and Avery (2000) presented a detailed discussion on the HCM 2000 queue model and comparisons with the aaSIDRA and various other queue models for signalised intersections.

This technical note discusses the progression factors that are used in the HCM 2000 queue and delay models in order to allow for the effects of platooned arrivals generated by coordinated signals. The purpose is to propose refinements to the application of various conditions on the parameters used in the progression factor equations, and discuss the cases of large progression factors and large back of queue values.

The two progression factors for the delay and queue models (PF and PF_2, respectively) are derived using the same method, and therefore use the same concept, basic simplifying assumptions about the platooned arrival patterns, and the same traffic parameters. These progression factors have been used in the aaSIDRA software package (Akcelik & Associates 2000) since 1995. For detailed background information on this subject, refer to previous publications by the author (Akçelik 1995, 1996).

The discussion given here will use the HCM 2000 notation, and needs to be read in conjunction with Chapter 16 (Signalized Intersections), Appendix G of HCM 2000. A full list of *Notations and Basic Relationships* is given before the reference list at the end of this document.

2. Progression Factors

The progression factors PF and PF_2 are derived using a simple *platooned arrivals* model that assumes different arrival flow rates (veh/h) during the green and red periods (v_{Lg}, v_{Lr}) as shown in *Figure 1* where r = effective red time, g = effective green time, c = cycle time, g_s = saturated part of the green period (queue clearance time), g_u = unsaturated part of the green period, and $v_L = (v_{Lr}\, r + v_{Lg}\, g) / C$ is the average arrival flow rate (veh/h) during the signal cycle.

In HCM 2000, the queue progression factor PF_2 is given by Equation (G16-8) in Chapter 16, Appendix G. It is used in the first term of the back of queue model (Equation G16-7) to adjust the "uniform" back of queue value for progression effects. The delay progression factor is given by Equation (16-10) in Chapter 16 for use in the first term of the delay model (Equation 16-9) to adjust the "uniform" delay value for progression effects.

For simplicity, HCM 2000 Equations (G16-7) and (G16-8) will be expressed here using u = g/C (lane group *green time ratio*), $y = v_L/s_L$ (lane group *flow ratio* per lane) and $q_L = v_L/3600$ (lane group *flow rate* per lane in vehicles per second). The delay equation will not be given since the focus of this paper is on the new HCM 2000 queue model. However, the HCM expression for the delay progression factor (PF) will be given since the two progression factors are interrelated and most of the discussion presented here applies to both factors. Similar notations will be used in both expressions.

First-Term Back of Queue

$$Q_1 = PF_2\, Q_u = PF_2\, \frac{q_L\, C\, (1-u)}{1-[\min(1.0, X_L)\, u]} \qquad (1)$$

where

Q_1 = first-term back of queue (veh),
Q_u = first-term back of queue for uniform (non-platooned) arrival flows (veh),
PF_2 = queue progression factor (*Equation 2*),
q_L = lane group flow rate per lane (veh/s), $q_L = v_L/3600$,

v_L = lane group flow rate per lane (veh/h),
C = cycle time (s),
u = green time ratio (g/C), and
X_L = lane group degree of saturation, i.e. v_L/c_L ratio where c_L is the lane group capacity per lane (veh/h).

From *Equation (1)*, the value of the first-term back of queue at capacity (X_L = 1.0) is:

$$Q_{1m} = PF_{2(X_L=1)}\, q_L\, C \quad (1a)$$

where $PF_{2(X_L=1)}$ is the value of PF_2 at capacity. It is shown below that $PF_{2(X_L=1)}$ = 1.0, and therefore:

$$Q_{1m} = q_L\, C = (v_L/3600)\, C \quad (1b)$$

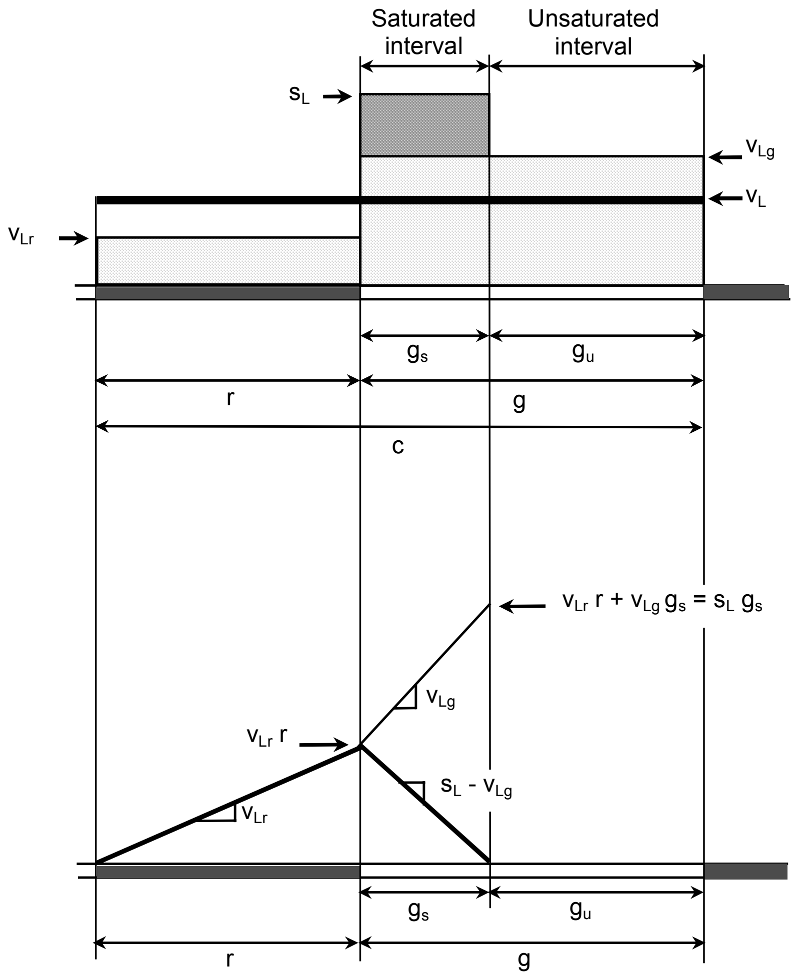

Figure 1 - Simple platooned arrivals model as the basis of HCM progression factors

Queue Progression Factor

$$PF_2 = \frac{(1-P)(1-y_L)}{(1-u)(1-R_p y_L)} = \frac{(1-R_p u)(1-y_L)}{(1-u)(1-R_p y_L)} \quad (2)$$

subject to conditions

(i) $PF_2 \geq 1.0$ for Arrival Types 1 and 2,
(ii) $PF_2 \leq 1.0$ for Arrival Types 4 to 6,
(iii) $P \leq 1.0$ ($R_p \leq 1 / u$),
(iv) $R_p < 1 / y_L$, and
(v) $PF_2 = 1.0$ for $y_L \geq u$ ($X_L \geq 1$).

where

PF_2 = queue progression factor,
u = green time ratio (g/C),
y_L = lane group flow ratio, i.e. v_L/s_L ratio where v_L is the lane group flow rate per lane (veh/h) and s_L is the lane group saturation flow rate per lane (veh/h), and
R_p = platoon ratio:

$$R_p = v_{Lg} / v_L = P / u \quad (2a)$$

where v_{Lg} is the arrival flow rate (veh/h) during the green period ($v_{Lg} = R_p v_L$), v_L is the average arrival flow rate (veh/h) during the signal cycle, and P is the proportion of traffic arriving during the green period (see the notation list for further explanation of R_p and P).

Delay Progression Factor

$$PF = \frac{(1-P)\, f_{pA}}{(1-u)} = \frac{(1-R_p\, u)\, f_{pA}}{(1-u)} \quad (3)$$

subject to conditions

(i) $PF \geq 1.0$ for Arrival Types 1 and 2,
(ii) $PF \leq 1.0$ for Arrival Types 4 to 6, and
(iii) $P \leq 1.0$ ($R_p \leq 1 / u$).

where R_p and u are as in *Equation (2)*, and

PF = delay progression factor, and
f_{pA} = an additional adjustment factor.

Equation (3) is derived from the following full expression for delay progression factor (Akçelik 1995), ignoring the additional parameter f_{pA}:

$$PF = \frac{(1-R_p\, u)\,[1-u+y_L\,(1-R_p)](1-y_L)}{(1-u)^2\,(1-R_p\, y_L)} \quad (3a)$$

This expression contains ($1 - y_L$) in the numerator and ($1 - R_p\, y_L$) in the denominator, as in *Equation (2)* for queue progression factor. Therefore *Equation (3a)* for PF would require *Conditions (iv) and (v)* as in the case of *Equation (2)* for PF_2. Note that the value of PF from *Equation (3a)* at $X_L = 1.0$ ($y_L = u$) is equivalent to the constant values of PF given by *Equation (3),* ignoring the effect of f_{pA}. Therefore, there is no need for a constraint on PF from *Equation (3)* for $X_L > 1.0$ similar to *Condition (v)* for PF_2. Conditions applicable to PF and PF_2 equations are discussed in detail in *Section 3*.

Table 1

Arrival types, platoon ratios (R_p), and the proportion arriving during green (P) for various green time ratios

Arrival Type	Progression Quality	Platoon Ratio (R_p)		Proportion arriving during green ($P = R_p u$)				f_{pA}
		Range	Default	u = 0.20	u = 0.40	u = 0.60	u = 0.80	
1	Very poor	≤ 0.50	0.333	0.067	0.133	0.200	0.266	1.00
2	Unfavourable	> 0.50 - 0.85	0.667	0.133	0.267	0.400	0.534	0.93
3	Random arrivals	> 0.85 - 1.15	1.000	0.200	0.400	0.600	0.800	1.00
4	Favourable	> 1.15 - 1.50	1.333	0.267	0.533	0.800	1.000	1.15
5	Highly favourable	> 1.50 - 2.00	1.667	0.333	0.667	1.000	1.000	1.00
6	Exceptional	> 2.00	2.000	0.400	0.800	1.000	1.000	1.00

Table 1 shows the six *Arrival Types* (ATs) given by HCM 2000 for determining platoon ratios (default values of R_p) when the green time ratio (u) and the proportion of traffic arriving during the green period (P) are not known. The values of P corresponding to the default values of R_p for various green time ratios (u) are given in *Table 1*.

If the proportion of traffic arriving during the green period (P) and the green time ratio (u) are known, the platoon ratio (R_p) is calculated from *Equation (2a)*. In this case, an Arrival Type can be assigned according to the ranges of R_p given in *Table 1*.

3. Conditions Applicable to Progression Factor Equations

Conditions (i) to (v) included in *Equations (2) and (3)* are based on those used in aaSIDRA version 1.0 (Akcelik & Associates 2000). These are discussed below, their practical meanings are explained and various refinements are recommended. Three additional conditions are introduced (Conditions vi to viii). An order and further suggestions for the implementation of the full set of conditions are given in *Section 6*.

Conditions (i) and (ii)

These are used to ensure that the progression factors are consistent with the definitions of *progression quality* relative to random arrivals (see *Table 1*). HCM 2000 recommends using a maximum PF value of 1.0 for Arrival Type 4 "as a practical matter". *Condition (i)* applies a minimum PF value of 1.0 for Arrival Types 1 and 2, and *Condition (ii)* applies a maximum PF value of 1.0 for Arrival Types 4, 5 and 6.

Numerical conditions that result in the violation of these conditions for Arrival Types other than Arrival Type 4 may occur, for example, $R_p = 0.667$ for AT = 2 with a green time ratio of u = 0.10 gives PF = 0.964 < 1.0. Although occurrence of such numerical conditions are limited, it is advisable that full set of *Conditions (i) and (ii)* are used in implementing the progression factor equations.

Condition (iii)

Condition $\boldsymbol{R_p \leq 1 / u}$ is needed to avoid negative values of PF and PF_2. It is equivalent to $\boldsymbol{P \leq 1.0}$, i.e. the proportion of traffic arriving during the green period should not exceed 100 per cent. If $R_p = 1 / u$, this will result in $P = R_p u = 1.0$ and therefore PF = 0 and $PF_2 = 0$. This is not a desirable condition since, in reality, it is very likely that there will be some vehicles that arrive during the red period! It is therefore recommended that a more strict condition such as $\boldsymbol{R_p \leq 0.95 / u}$ is adopted. This is equivalent to the use of a $\boldsymbol{P_{max} = 0.95}$, or the condition $\boldsymbol{P \leq 0.95}$ (95 per cent of traffic arriving during the green period and 5 per cent during the red period).

Example 1: Consider the case of a movement with arrival flow rate, v_L = 1083 veh/h and saturation flow rate, s_L = 1900 veh/h (therefore y_L = 0.570), receiving *exceptional* progression quality (AT = 6) with a green time ratio of u = 0.60 (g = 60 s, C = 100 s). From *Table 1*, default R_p = 2.000. Therefore, $P = R_p u = 1.200$ is found. However, the condition $R_p \leq 1 / u$ means that $R_p = 1 / 0.60 = 1.667$ should be used as the maximum value, resulting in $P = R_p u = 1.000$ (no vehicles arriving during red), therefore PF = 0 and $PF_2 = 0$. Similarly, the recommended condition $R_p \leq 0.95 / u$ would result in R_p = 1.583 (AT = 5), and $P = R_p u = 0.950$ (5 % arriving during red). This means that AT = 6 is not a realistic assumption in this case. AT = 5 is more appropriate, and PF = 0.125 and PF_2 = 0.551 are found as a result.

Condition (iv)

Condition $\boldsymbol{R_p < 1 / y_L}$ is needed for $(1 - R_p y_L) > 0$ in the denominator of *Equation (2)*. The condition is equivalent to $\boldsymbol{v_{Lg} < s_L}$, i.e. the arrival flow rate during the *green period* should be less than the saturation flow rate. This equivalence can be seen by putting $R_p = v_{Lg} / v_L$ (by definition) and $y_L = v_L / s_L$.

Although $(1 - R_p y_L)$ does not appear in *Equation (3*), this condition is relevant to the delay progression factor as seen from the implied requirement $\boldsymbol{v_{Lg} < s_L}$, otherwise an unrealistic platoon ratio will be used in calculating PF.

It is recommended that a maximum acceptable value of $\boldsymbol{v_{Lg} / s_L}$ is used, e.g. $\boldsymbol{(v_{Lg} / s_L)_{max} = 0.95}$ may be appropriate for a more strict condition. *Condition (iv)* would then be expressed as $\boldsymbol{v_{Lg} \leq 0.95\, s_L}$, or $\boldsymbol{R_p \leq 0.95 / y_L}$.

Example 2: Consider the same movement as in *Example 1*, i.e. v_L = 1083 veh/h, s_L = 1900 veh/h (y_L = 0.570), u = 0.60, but receiving *highly favourable* signal coordination (AT = 5). From *Table 1*, default R_p = 1.667. This means $P = R_p u = 1.000$. Applying *Condition (iii) as* $P \leq 0.95$, we need to set P = 0.950 and R_p = 0.950 / 0.600 = 1.583 as in *Example 1*. As a result, Condition $\boldsymbol{R_p < 0.95 / y_L}$ is also satisfied since R_p (= 1.583) < 0.95 / y_L (= 1.667). The platoon ratio of R_p = 1.583 implies an arrival flow rate during the green period, v_{Lg} = 1.583 x 1083 = 1714 veh/h, which is less than $\boldsymbol{0.95\, s_L}$ = 1805 veh/h. PF = 0.125 and PF_2 = 0.551 are found as in *Example 1*.

Condition (v)

Condition $\boldsymbol{PF_2 = 1.0 \text{ for } y_L \geq u}$ is related to the value of queue progression factor, PF_2 at capacity, $PF_{2(X_L=1)}$. This is obtained using $y_L = u$ (i.e. $v_L / s_L = g/C$ for $X_L = 1.0$) in *Equation (2)*, which results in:

$$PF_{2(X_L=1)} = 1.0 \qquad (4)$$

This condition requires that, for arrival flow rates above capacity ($X_L > 1.0$), the values of PF_2 must be restricted to $PF_{2(X_L=1)} = 1.0$. This is consistent with the use of a constant value of Q_u (the first-term back of queue for uniform arrivals) for $X_L > 1.0$ as seen from in *Equations (1), (1a) and (1b)*.

While relevant to the delay progression factor PF from the *Equation (3a)*, there is no need for a similar condition on *Equation (3)* since this simplified form of the equation used in the HCM gives constant values.

Condition (vi)

Similar to *Condition (iv)*, a constraint on the flow rate during the *red period* is also considered. This is $v_{Lr} < s_L$, i.e. the arrival flow rate during the red period should be less than the saturation flow rate (both in veh/h), or more strictly $v_{Lr} \le 0.95\ s_L$.

The arrival flow rate during the red period, v_{Lr} can be calculated from $v_{Lr} = v_L (1 - R_p u) / (1 - u)$. From this, the required condition can be expressed as $R_p \ge (1 - 0.95 (1 - u) / y_L) / u$. This is equivalent to $(1 - P) \le 0.95 (1 - u) / y_L$, i.e. proportion of traffic arriving during the red period must not exceed a maximum value, or $P \ge 1 - 0.95 (1 - u) / y_L$, i.e. proportion of traffic arriving during the green period must not be less than a minimum value.

As in the case of *Condition (iv)*, this condition should be applied to both delay and queue progression factors.

Example 3: Consider the same movement as in *Example 1*, i.e. v_L = 1083 veh/h, s_L = 1900 veh/h (y_L = 0.570), u = 0.60, but with *very poor* signal coordination (AT = 1). From *Table 1*, default R_p = 0.333. Proportion arriving during green is calculated as $P = R_p u = 0.200$, and therefore *Condition (iii)* is satisfied ($P \le 0.95$). Similarly, *Condition (iv)* is satisfied since R_p (= 0.333) < 0.95 / y_L (= 1.667). However, this platoon ratio implies an excessive arrival flow rate during the red period: v_{Lr} = 1083 x (1 - 0.200) / (1 - 0.60) = 2166 veh/h > s_L (= 1900 veh/h). For $v_{Lr} \le 0.95\ s_L$, the minimum value of proportion arriving during green is $P = 1 - 0.95 (1 - u) / y_L = 0.333$. Setting P = 0.333, we find $R_p = P / u = 0.556$ (AT = 2). This implies $v_{Lr} = 0.95\ s_L$ = 1805 veh/h, and means that AT =1 is not a realistic assumption in this case. AT = 2 is more appropriate, and PF = 1.667 and PF_2 = 1.049 are found as a result.

Condition (vii)

This is related to circumstances when the average flow rate is very high. Violation of both Conditions (iv) and (vi), i.e. $v_{Lg} > 0.95\ s_L$ and $v_{Lr} > 0.95\ s_L$, implies that the average flow rate $v_L > 0.95\ s_L$. Applying both conditions ($v_{Lg} = v_{Lr} = 0.95\ s_L$), $v_L = 0.95\ s_L$ is implied, which means that R_p = 1.0, therefore AT = 3. The condition $v_L > 0.95\ s_L$ means a very high flow ratio of $y_L > 0.95$. This would correspond to highly oversaturated conditions at signalised conditions. The use of AT = 3 is acceptable under such oversaturated circumstances.

Note that the principle of the conservation of arrival flows is satisfied when Condition (iv) or (vi) is applied individually. When applying these saturation flow related conditions, we do not lose any vehicles since we reduce the number of vehicles arriving in the red or green period and make them arrive in the other period without changing the average flow rate. In other words, we do not change the number of vehicles arriving per cycle, but we change the numbers arriving during green and red periods. The two conditions are applied simultaneously only for the purpose of progression factor calculations, and the average flow rate should not be reduced below $0.95\ s_L$ for other purposes because of the principle of conservation of flows.

Thus, when $y_L > 0.95$ (or $v_L > 0.95\ s_L$), Conditions (iv) and (vi) cannot be met simultaneously, and this case should be treated as Arrival Type = 3, i.e. the platoon ratio and proportion arriving green should be set to $R_p = 1.0$, $P = u$, therefore $PF = 1.0$ and $PF_2 = 1.0$.

Condition (viii)

A further condition needs to be considered in order to make allowance for the situation when *Conditions (iii), (iv) and (vi)* create inconsistent constraints on R_p and P that cannot all be met simultaneously. As in the case of *Condition (vii)*, this case should be treated as Arrival Type 3, i.e. the platoon ratio and the proportion arriving during the green period should be set to $\boldsymbol{R_p = 1.0}$ ***and*** $\boldsymbol{P = u}$, therefore $\boldsymbol{PF = 1.0}$ ***and*** $\boldsymbol{PF_2 = 1.0}$.

4. Large Values of Progression Factors

Under *very poor* and *unfavourable* progression conditions (AT = 1 and 2), manifested by low values of proportion of traffic arriving during green (P), and with large green time ratios (u), large values of progression factors PF and PF_2 may result from *Equations (2) and (3)*. *Figures 2 and 3* show graphs of such progression factors that can be obtained under AT =1.

For example, P = 0.100 and u = 0.800 (therefore R_p = 0.125, AT = 1) will give PF = 4.500. For the same example, a low flow ratio of y_L = 0.040 gives PF_2 = 4.342, and a high flow ratio of y_L = 0.600 gives PF_2 = 1.263. Note that PF_2 always reduces with increasing y, reaching PF_2 = 1.0 when y_L = u (X_L = 1.0) as seen from *Equation (4)* and *Figure 3*.

Such large PF and PF_2 values represent the effect of poor progression quality realistically. An easy test can be applied to the model in this respect. For this purpose, consider the worst-case scenario, which is when all vehicles arrive during the red period (P = 0, R_p = 0, AT = 1). Using R_p = 0 and f_{pA} = 1.0 (AT =1) in *Equations (2) and (3)*, the progression factors for this case are found as:

$$PF_{2m} = \frac{(1-y_L)}{(1-u)} \qquad (5)$$

$$PF_m = \frac{1}{(1-u)} \qquad (6)$$

These equations show that the largest values of PF_m and PF_{2m} will be obtained with largest green time ratios (u) as stated above. For example, u = 0.8 will result in PF_m = 5.0. Condition PF_{2m} = 1.0 applies for y_L = u in *Equation (5)* as in *Equation (2)*.

The queue length at the start of the green period is (v_L r /3600) for random arrivals (P = u, R_p = 1.0) and (v_L C /3600) when all vehicles arrive during the red period (P = 0, R_p = 0). This is shown for a fully-saturated signal cycle in *Figure 4* (P = 0, AT = 1). The case of random arrivals (distributed uniformly throughout the signal cycle (P = u, AT = 3) is also shown in the bottom part of *Figure 4*. Since v_L C = s_L g (y_L = u, X_L = 1.0) in this fully-saturated case, PF_2 = 1.0 and the first-term back of queue, Q_1 = v_L C / 3600 are found.

For random arrivals, the first-term delay (*uniform* delay) from HCM Equation (16-11) is d_{1u} = 0.5 r. This can be calculated from the lower triangular area in *Figure 4* as d_{1u} = (0.5 v_L r) C / (v_L C) = 0.5 r. Similarly, the first-term delay in the worst-case platooned arrivals scenario can be calculated from the upper triangular area in *Figure 4* as d_1 = (0.5 v_L C) C / (v_L C) = 0.5 C. The delay progression by definition is the ratio of these two delay values, PF = d_1 / d_{1u} = 0.5 C / (0.5 r) = C / r = C / (C - g) = 1 / (1 - u) as in *Equation (6)*.

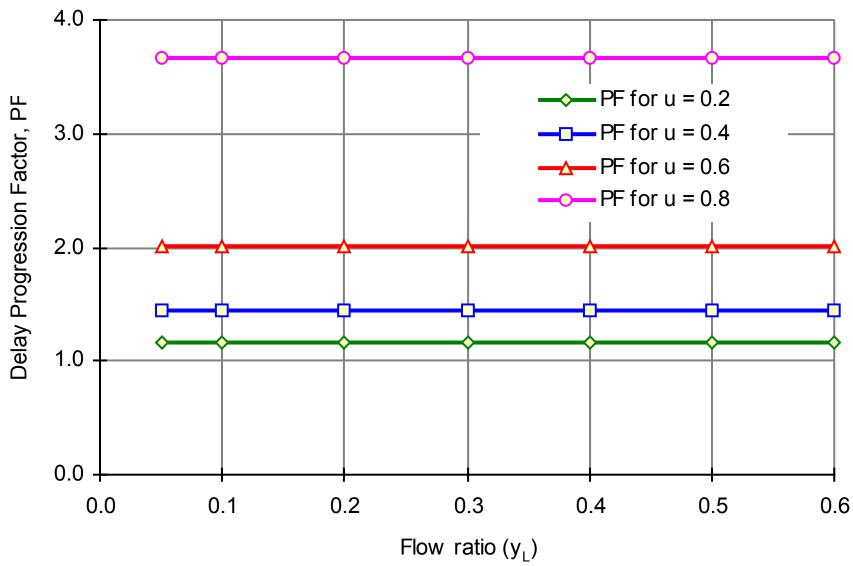

Figure 2 - Delay progression factor for Arrival Type = 1 (very poor progression) as a function of flow ratio (y_L) for various green time ratios (u)

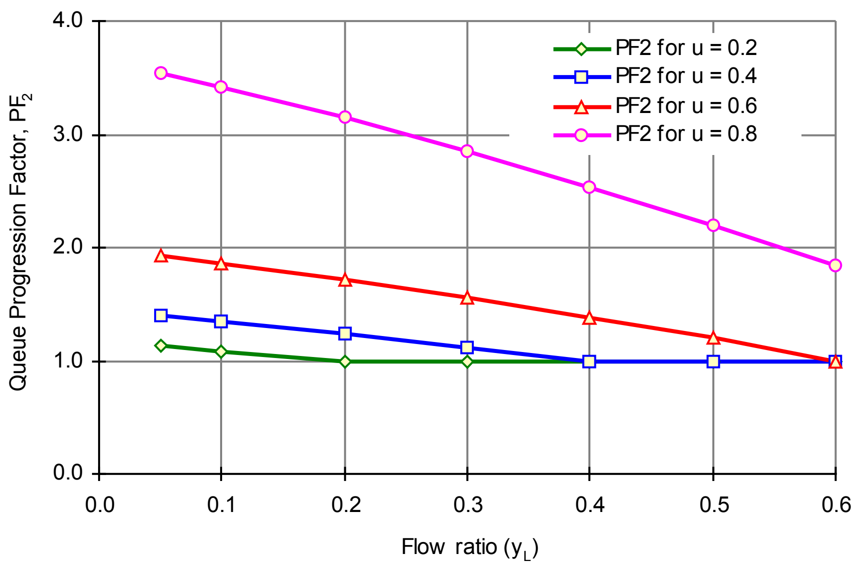

Figure 3 - Queue progression factor for Arrival Type = 1 (very poor progression) as a function of flow ratio (y_L) for various green time ratios (u)

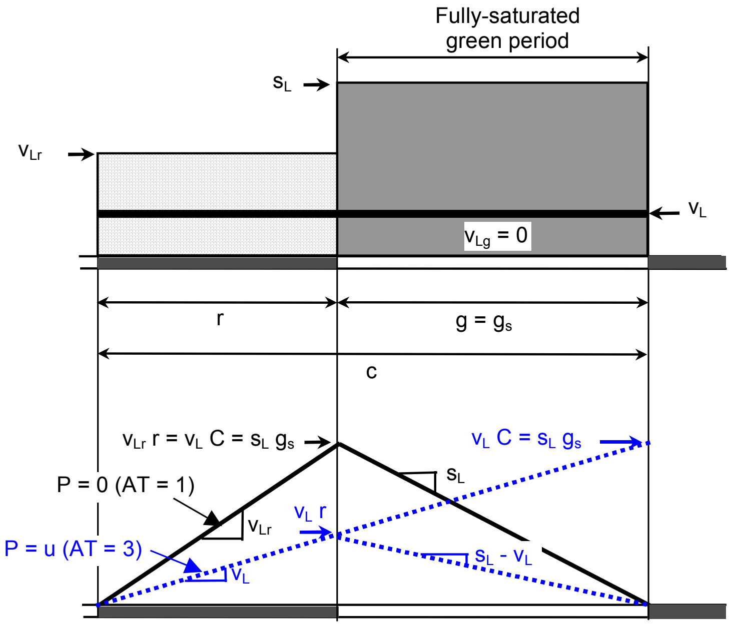

Figure 4 - Worst-case scenario for platooned arrivals (all vehicles arriving during the red period) and the non-platooned arrivals case

5. Conclusion

It is recommended all the conditions discussed in *Section 3* are used in implementation of the HCM 2000 progression factors. These are summarised as follows:

- *(i)* $PF \geq 1.0$ and $PF_2 \geq 1.0$ *for Arrival Types 1 and 2,*
- *(ii)* $PF \leq 1.0$ and $PF_2 \leq 1.0$ *for Arrival Types 4 to 6,*
- *(iii)* $P \leq 0.95$ ($R_p \leq 0.95 / u$) *for both PF and* PF_2*,*
- *(iv)* $R_p \leq 0.95 / y_L$ *for both PF and* PF_2*,*
- *(v)* $PF_2 = 1.0$ *for* $y_L \geq u$ ($X_L \geq 1$)*,*
- *(vi)* $R_p \geq (1 - 0.95\ (1 - u) / y_L) / u$ *for both PF and* PF_2*, and*
- *(vii)* $R_p = 1.0$ ($P = u$)*, therefore* $PF = 1.0$ *and* $PF_2 = 1.0$ *for* $y_L \geq 0.95$.
- *(viii)* *If Conditions (iii), (iv) and (vi) create inconsistent constraints on* R_p *and P, set* $R_p = 1.0$ *and* $P = u$*, therefore* $PF = 1.0$ *and* $PF_2 = 1.0$.

The numbering of conditions listed above reflects the discussion presented in this paper rather than an order for their implementation. It is recommended that the above conditions be implemented in the following order.

1. Apply *Condition (vii)* for both PF and PF_2 to satisfy the maximum value of the average arrival flow rate. If $R_p = 1.0$ *and* $P = u$ *(therefore* $PF = 1.0$ *and* $PF_2 = 1.0$*)* have been set as a result of this

condition, and the case is being treated as Arrival Type 3, finish the process as there is no need to check any other conditions.

2. Apply *Condition (iii)* for both PF and PF_2 to satisfy the maximum value of the proportion of traffic arriving during the green period.

3. Apply *Condition (iv)* for both PF and PF_2 to satisfy the maximum arrival flow rate during the green period.

4. Apply *Condition (vi)* for both PF and PF_2 to satisfy the maximum arrival flow rate during the red period.

5. Apply *Condition (viii)* when *Conditions (iii), (iv) and (vi)* create inconsistent constraints that cannot all be met simultaneously. If $R_p = 1.0$ *and* $P = u$ *(therefore* $PF = 1.0$ *and* $PF_2 = 1.0$*)* have been set as a result of this condition, and the case is being treated as Arrival Type 3, finish the process as there is no need to check further conditions.

6. Apply *Condition (v)* for queue progression factor PF_2 only (use the value at capacity for arrival flow rates above capacity). This is satisfied for the delay progression factor PF, which is a constant.

7. Apply *Condition (i)* if the Arrival Type is 1 or 2, or apply *Condition (ii)* if the Arrival Type is 4, 5 or 6. If one of these conditions is violated for the delay or queue progression factor, set $PF = 1.0$ or $PF_2 = 1.0$ as applicable.

In software packages implementing these conditions, it would be useful to give a warning message when these conditions come into effect, thus modifying user input. This would help the user to understand that some input specifications have been changed by software, possibly implying a different Arrival Type.

It is emphasised that, although some conditions do not appear to be relevant to the progression factor for delay calculations (PF) on the basis of numerical requirements, they should be applied to both queue and delay progression factors due to their practical meanings as constraints on arrival flow rates that can be achieved during the red and green periods.

The simple platooned arrivals model used in the HCM is realistic in predicting large values of progression factors that may result under *very poor* and *unfavourable* progression conditions (low values of proportion of traffic arriving during green) and with large green time ratios. Unrealistic limits should not be placed on progression factors predicted by the HCM equations since they provide valuable information regarding the combination of flow and signal coordination conditions that result in large back of queue values.

Notations and Basic Relationships

HCM 2000 and this document	aaSIDRA User Guide	Description and relationships
c_L	Q_e	Lane group capacity per lane (veh/h) $c_L = s_L\, g/C$ for interrupted traffic (where s is in veh/h)
C	c	Average cycle time (seconds) $C = r + g$
g	g	Average effective green time (seconds)
g_s, g_q	g_s	Saturated portion of the green period, or queue clearance time (seconds)
g_u	g_u	Unsaturated portion of the green period (seconds) $g_u = g - g_s$
P	P_G	Proportion of traffic arriving during the green period $P = q_{Lg}\, g \,/\, (q_L\, C) = v_{Lg}\, g \,/\, (v_L\, C) = R_p\, u$
P_R	P_R	Proportion of traffic arriving during the red period $P_R = q_{Lr}\, r \,/\, (q_L\, C) = v_{Lr}\, r \,/\, (v_L\, C) = 1 - P = 1 - R_p\, u$
PF	PF_1	Progression factor for delay
PF_2	PF_2	Progression factor for back of queue
q_L	q_L	Lane group arrival (demand) flow rate per lane (veh/s) $q_L = v_L\, /3600 = (v_{Lg}\, g + v_{Lr}\, r) \,/\, (3600\, C)$
q_L C	q_a c	Number of arrivals (veh) per cycle as measured at the back of the queue $q_L\, C = v_L\, C\, /3600 = (v_{Lg}\, g + v_{Lr}\, r) \,/\, 3600$
Q	N_b	Average back of queue (vehicles)
Q_1, Q_2	N_{b1}, N_{b2}	First and second terms of the average back of queue formula
Q_{1m}		The value of the first-term back of queue at capacity ($X_L = 1.0$) $Q_{1m} = q_L\, C = v_L\, C\, /3600$
Q_u	N_{bu}	First-term back of queue for uniform flow conditions (veh), i.e. when the arrival flow rate is equal for red and green periods ($v_{Lg} = v_{Lr} = v_L$), and therefore the proportion arriving during the green period equals the green time ratio, $P = u = g/C$, and the platoon ratio equals one ($R_p = 1.0$)
r	r	Average effective red time (seconds) $r = C - g$
R_p	P_A	Platoon ratio: the ratio of the average arrival flow rate during the green period to the average arrival flow rate during the signal cycle $R_p = q_{Lg} \,/\, q_L = v_{Lg} \,/\, v_L = P \,/\, u$

Continued >>>

HCM 2000 and this document	aaSIDRA User Guide	Description and relationships
s_L	s	Lane group saturation flow rate per lane (veh/h)
u, g/C	u	Green time ratio $u = g/C$
v_L		Lane group arrival (demand) flow rate per lane (veh/h) $v_L = (v_{Lg}\ g + v_{Lr}\ r) / C$
v_{Lg}	q_{ag}	Arrival flow rate (veh/h) during the green period (for random arrivals: $v_{Lg} = v_L$) $v_{Lg} = R_p\ v_L$
v_{Lr}	q_{ar}	Arrival flow rate (veh/h) during the red period (for random arrivals: $v_{Lr} = v_L$) $v_{Lr} = v_L\ (C – R_p\ g) / r = v_L\ (1 – R_p\ u) / (1 – u)$
X_L	x	Lane group degree of saturation, i.e. the ratio of arrival (demand) flow rate to capacity $X_L = v_L\ / c_L = v_L\ C / (s_L\ g) = y_L / u$
y_L	y	Lane group flow ratio per lane, i.e. the ratio of arrival (demand) flow rate to the saturation flow rate $y_L = v_L/s_L$

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