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# Acceleration and deceleration models

**Rahmi Akçelik and Mark Besley**
**Akcelik & Associates Pty Ltd**

## 1 INTRODUCTION

Modelling of acceleration and deceleration distances and times associated with speed change cycles (stop - start and slow down - speed up manoeuvres) under normal driving conditions is essential for the analysis of operating cost, fuel consumption and pollutant emissions, as well as for determining geometric, stopped and queuing components of overall delay (Akcelik and Associates 2002). Similarly, modelling of acceleration and deceleration characteristics of individual vehicles is a key issue in relation to the accuracy of microsimulation models (Akçelik and Besley 2002).

A polynomial model of acceleration and deceleration profiles was derived for estimating instantaneous acceleration and deceleration rates, and the model was calibrated using extensive real-life driving data representing general driving conditions, i.e. involving a wide range of speeds change cycles on different road types (Akçelik and Biggs 1987). The model parameters were adjusted for use in intersection analysis on the basis of acceleration and deceleration rates reported in the literature in order to obtain more realistic values representing stop-start conditions at intersections. Further adjustments have been made to model parameters recently for use in aaSIDRA version 2 making use of acceleration time and distance information derived from queue discharge characteristics at signalised intersections (Akcelik and Associates 2002).

This paper describes the acceleration and deceleration models, and emphasises the research needs for better model calibration considering different vehicle types (cars, buses, trucks), specific traffic facilities (roundabouts, signalised and sign-controlled intersections, signalised and unsignalised pedestrian crossings, freeways), different traffic demand levels (light, medium, heavy), road types (urban, rural roads, city centre, suburban), and a wide range of initial and final speeds.

*Figure 1* shows an example of time-distance and speed-time diagrams representing the acceleration and deceleration manoeuvres of a vehicle stopping and starting at traffic signals.

**Corresponding author:** Rahmi Akçelik
Director, Akcelik & Associates Pty Ltd,
P O Box 1075 G, Greythorn Victoria, Australia 3104
Tel: +61 3 9857 9351, Fax: + 61 3 9857 5397
rahmi@akcelik.com.au

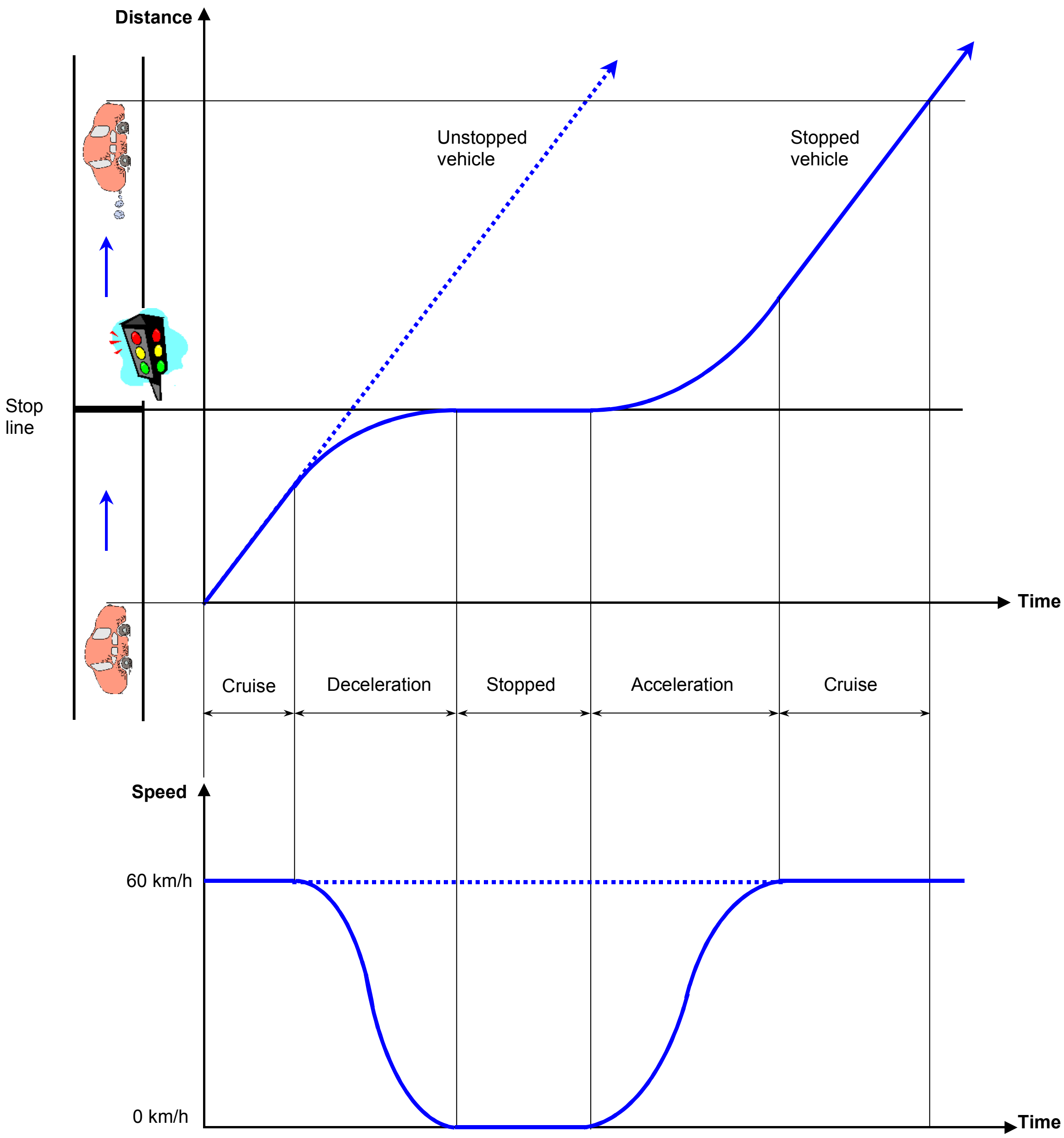


*Figure 1 - Time-distance and speed-time diagrams showing the acceleration and deceleration manoeuvres of a vehicle stopping and starting at traffic signals*

## 2 ACCELERATION AND DECELERATION MODELS

Acceleration and deceleration distances and times, together with the initial and final speeds during acceleration and deceleration manoeuvres, are the key information for modelling acceleration and deceleration of vehicles. The expressions used in aaSIDRA 2 for estimating acceleration and deceleration distances, times and rates are given below. Default values of model parameters ($p_1$ to $p_7$) are given in the aaSIDRA User Guide (Akcelik and Associates 2002).

Acceleration and deceleration distances, times and rates for various initial and final speeds for light vehicles are shown in *Figures 2 to 4*.

### Acceleration distance

$$L_a = m_a (v_i + v_f)\, t_a / 3.6 \quad (1)$$

where

$L_a$ = acceleration distance (m),
$v_i$ = initial speed in acceleration (km/h),
$v_f$ = final speed in acceleration (km/h),
$t_a$ = acceleration time (seconds), and
$m_a$ = a model parameter given by:

$$m_a = p_1 + p_2 v_f - p_3 v_i^{p_7} \quad (2)$$
*subject to* $m_{amin} \le m_a \le m_{amax}$

$$m_{amin} = [v_i + \rho_{amin} (v_f - v_i)] / (v_i + v_f) \quad (2a)$$

$$m_{amax} = [v_i + \rho_{amax} (v_f - v_i)] / (v_i + v_f) \quad (2b)$$

$$\rho_a = (v_a - v_i) / (v_f - v_i) \quad (3)$$
*subject to* $\rho_{amin} \le \rho_a \le \rho_{amax}$

$$\rho_{amin} = 0.400 \quad (3a)$$

$$\rho_{amax} = 0.700 \quad (3b)$$

where

$v_a$ = average speed during acceleration (km/h) given by:

$$v_a = 3.6\, L_a / t_a \quad (3c)$$

### Deceleration distance

$$L_d = m_d (v_i + v_f)\, t_d / 3.6 \quad (4)$$

$L_d$ = deceleration distance (m),
$v_i$ = initial speed in deceleration (km/h),
$v_f$ = final speed in deceleration (km/h),
$t_d$ = deceleration time (seconds), and
$m_d$ = a model parameter given by:

$$m_d = p_1 + p_2 v_i - p_3 v_f^{p_7} \quad (5)$$
*subject to* $m_{dmin} \le m_d \le m_{dmax}$

$$m_{dmin} = [v_f + \rho_{dmin} (v_i - v_f)] / (v_i + v_f) \quad (5a)$$

$$m_{dmax} = [v_f + \rho_{dmax} (v_i - v_f)] / (v_i + v_f) \quad (5b)$$

$$\rho_d = (v_d - v_f) / (v_i - v_f) \quad (6)$$

*subject to* $\rho_{dmin} \leq \rho_d \leq \rho_{dmax}$

$$\rho_{dmin} = 0.400 \quad (6a)$$

$$\rho_{dmax} = 0.700 \quad (6b)$$

where

$v_d$ = average speed during deceleration (km/h) given by:

$$v_d = 3.6\ L_d / t_d \quad (3c)$$

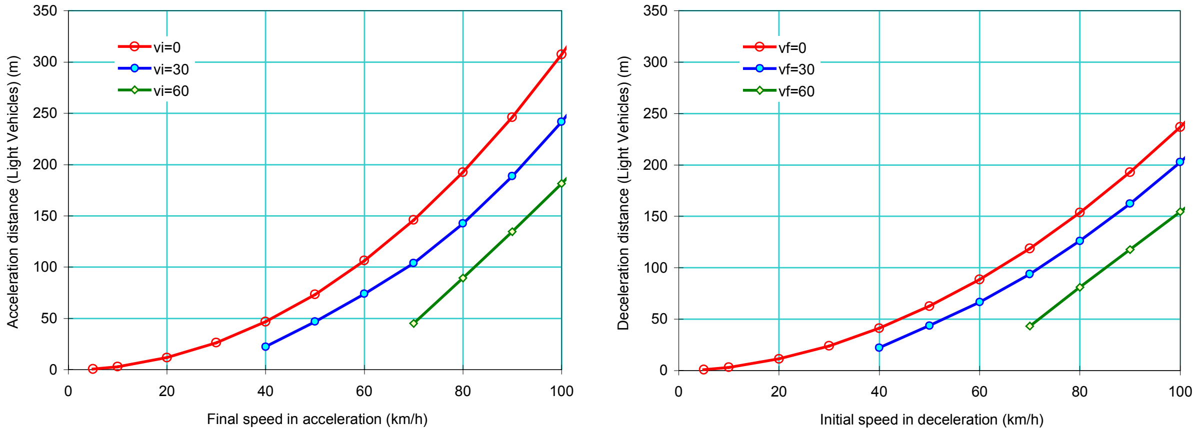


***Figure 2 - Acceleration and deceleration distances for various initial and final speeds ($v_i$ ,$v_f$) for light vehicles***

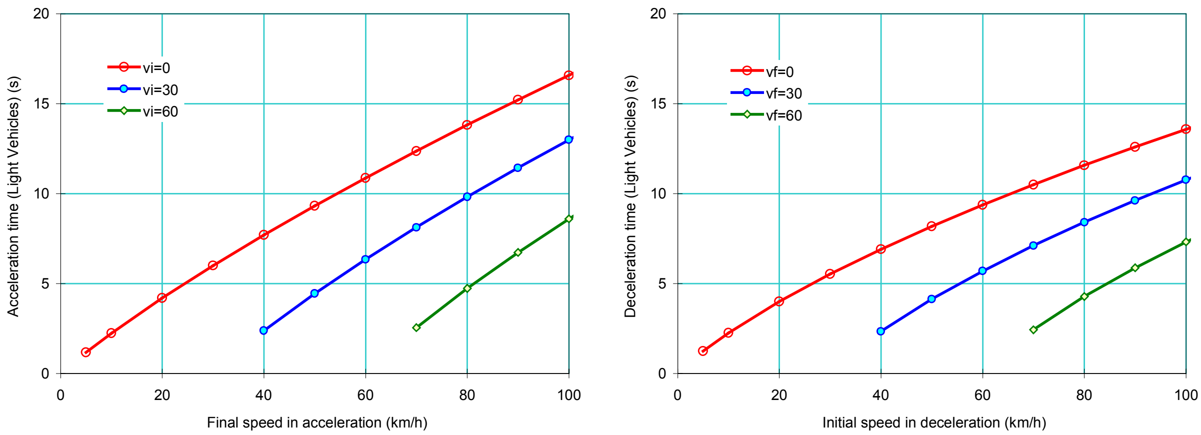


***Figure 3 - Acceleration and deceleration times for various initial and final speeds ($v_i$ ,$v_f$) for light vehicles***

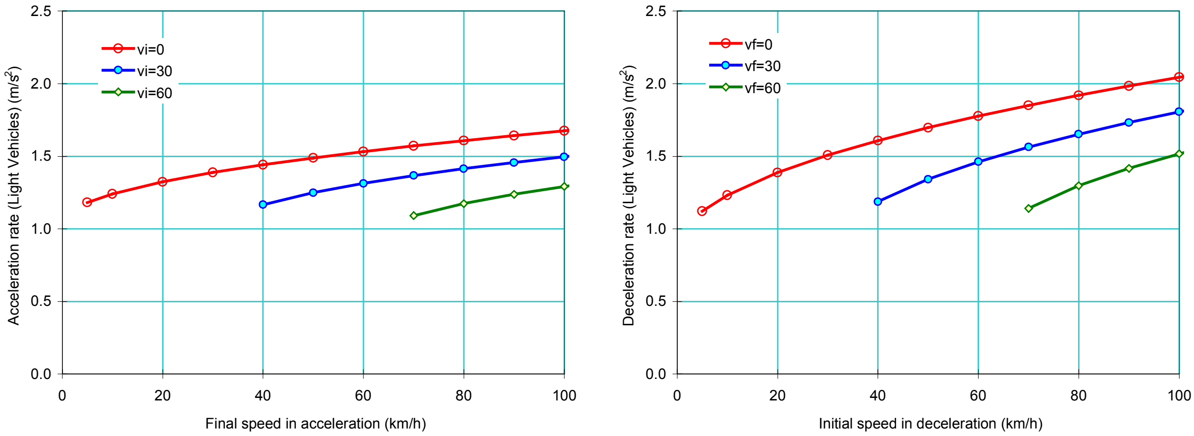


***Figure 4 - Average acceleration and deceleration rates for various initial and final speeds ($v_i$, $v_f$) for light vehicles***

## Acceleration time and rate

$$t_a = (v_f - v_i) / a_{aa} \qquad \text{where } v_f > v_i \text{ and } a_{ad} < 0 \qquad (7)$$

where

$t_a$, $v_f$, $v_i$ = as in *Equation (1)*, and
$a_{aa}$ = average acceleration rate (m/s$^2$) calculated as follows:

$$a_{aa} = f_{aLV} [p_1 + p_2 (v_f - v_i)^{0.5} - p_3 v_i] / 3.6 \qquad (8)$$

*for light vehicles*

$$a_{aa} = f_{aHV} [p_1 + p_2 \, PWR \, (v_f - v_i)^{0.5} + PWR^{0.5} (p_4 - p_3 v_i) - (p_5 v_f^{0.5} + p_6 Gr)/ PWR] / 3.6 \qquad (9)$$

*for heavy vehicles*

where

$f_{aLV}$ = adjustment factor for light vehicle acceleration rates
$f_{aHV}$ = adjustment factor for heavy vehicle acceleration rates,
Gr = approach grade (per cent)
PWR = power to weight ratio calculated from:

$$PWR = 1000 \, P_{max} / M_{HV} \qquad (9a)$$

where

$P_{max}$ = maximum rated engine power,
$M_{HV}$ = heavy vehicle mass in kg.

Default heavy vehicle parameters (representing 5 heavy vehicle classes as used in aaSIDRA 2) are $M_{HV}$ = 11000 kg, $P_{max}$ = 130, PWR = 11.8.

### Deceleration time and rate

$$t_d = (v_f - v_i) / a_{ad} \qquad \textit{where } v_f < v_i \textit{ and } a_{ad} < 0 \qquad (10)$$

where

$t_d$, $v_f$, $v_i$ = as in *Equation (4)*, and
$a_{ad}$ = average deceleration rate (m/s$^2$) calculated as follows:

$$a_{ad} = - f_{dLV} [p_1 + p_2 (v_i - v_f)^{0.5} - p_3 v_f] / 3.6 \qquad (11)$$

*for light vehicles*

$$a_{ad} = - f_{dHV} [p_1 + p_2 \, PWR \, (v_i - v_f)^{0.5} - p_3 v_f - p_4 M_{HV}^{0.5} + p_5 v_i + p_6 Gr] / 3.6 \qquad (12)$$

*for heavy vehicles*

where

$f_{dLV}$ = adjustment factor for light vehicle deceleration rates
$f_{dHV}$ = adjustment factor for heavy vehicle deceleration rates,
$M_{HV}$ = heavy vehicle mass in kg, and
Gr = approach grade (per cent).

### Acceleration and deceleration profiles

The polynomial model of acceleration and deceleration profiles described by Akçelik and Biggs (1987) can be expressed as follows:

$$a(t) = r \, a_m \, \theta \, (1 - \theta^m)^2 \qquad (13)$$

$$v(t) = v_i + 3.6 \, r \, a_m \, t_{a/d} \, \theta^2 \, [0.5 - 2 \, \theta^m / (m+2) + \theta^{2m} / (2m+2)]$$

*subject to m > 0*

$$L(t) = v_i \, t / 3.6 + r \, a_m \, t^2_{a/d} \, \theta^3 \, [1/6 - 2 \, \theta^m / ((m+2)(m+3)) + \theta^{2m} / ((2m+2)(2m+3))]$$

where

- $a(t)$ = acceleration or deceleration rate at time t (m/s$^2$) ($a > 0$ for acceleration, $a < 0$ for deceleration)
- $v(t)$ = speed at time t (km/h)
- $L(t)$ = distance at time t (m)
- $a_m$ = maximum acceleration or deceleration rate (m/s$^2$) ($a_{ma} > 0$ for acceleration, $a_{md} < 0$ for deceleration)
- $\theta$ = time ratio ($\theta = t / t_{a/d}$, i.e. $\theta = t / t_a$ or $\theta = t / t_d$)
- t = time since the start of acceleration (seconds)
- $t_{a/d}$ = acceleration or deceleration time ($t_a$ or $t_d$) (seconds)
- $v_i$ = initial speed (km/h)
- m = model calibration parameter, and
- r = model parameter given by:

$$r = [(1+2\,m)^{2+1/m}] / 4\,m^2 \qquad (13a)$$

The maximum acceleration rate ($a_m$) and the time when it is reached ($\theta_m$) can be determined from:

$$a_m = a_a / (r\,q) \qquad (14)$$

$$\theta_m = (1 + 2\,m)^{-1/m} \tag{14a}$$

where $a_a$ is the average acceleration rate (m/s$^2$) calculated from *Equation (8) or (9)* for acceleration or *Equation (11) or (12)* for deceleration, parameter r is calculated from *Equation (13a)* and parameter q is calculated from:

$$q = m^2 / [(2\,m + 2)\,(m + 2)] \tag{14b}$$

The model is calibrated by determining parameter m using known acceleration distance and time, and initial and final speed values.

Recent calibration of the aaSIDRA 2 acceleration models made use of acceleration time and distance information derived from queue discharge characteristics at signalised intersections (Akçelik and Besley 2002).

*Figures 5 and 6* show the acceleration rate, speed and distance profiles based on the polynomial model for the cases of a light vehicle (i) accelerating from zero initial speed, $v_i = 0$, to a final speed of $v_f = 60$ km/h (acceleration distance, $L_a = 106.4$ m, average acceleration rate, $a_{aa} = 1.53$ m/s$^2$, maximum acceleration rate, $a_{ma} = 2.69$ m/s$^2$, acceleration time, $t_a = 10.9$ s, with parameters $m_a = \rho_a = 0.587$ and $v_a = 35.2$ km/h), and (ii) decelerating from an initial speed of $v_i = 60$ km/h to zero final speed, $v_f = 0$ (deceleration distance, $L_d = 88.5$ m, average deceleration rate, $a_{ad} = -1.78$ m/s$^2$, maximum deceleration rate, $a_{md} = -3.09$ m/s$^2$, deceleration time, $t_d = 9.4$ s, with parameters $m_d = \rho_d = 0.566$ and $v_d = 34.0$ km/h).

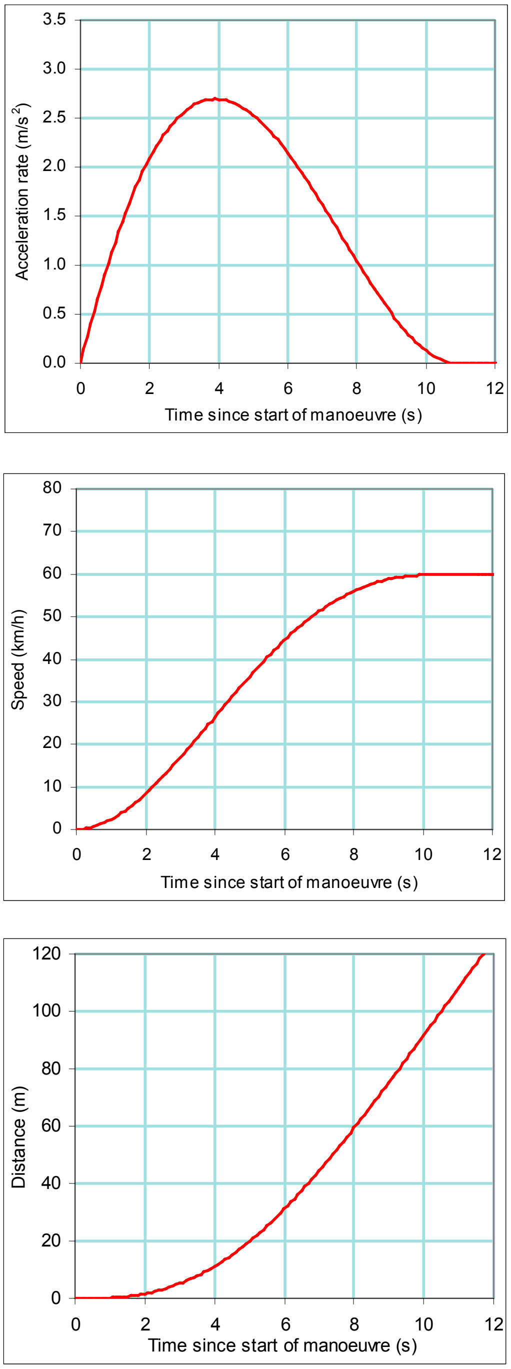


*Figure 5 - Polynomial model: Acceleration, speed and distance profiles for a vehicle ACCELERATING from zero initial speed to a final speed of 60 km/h*

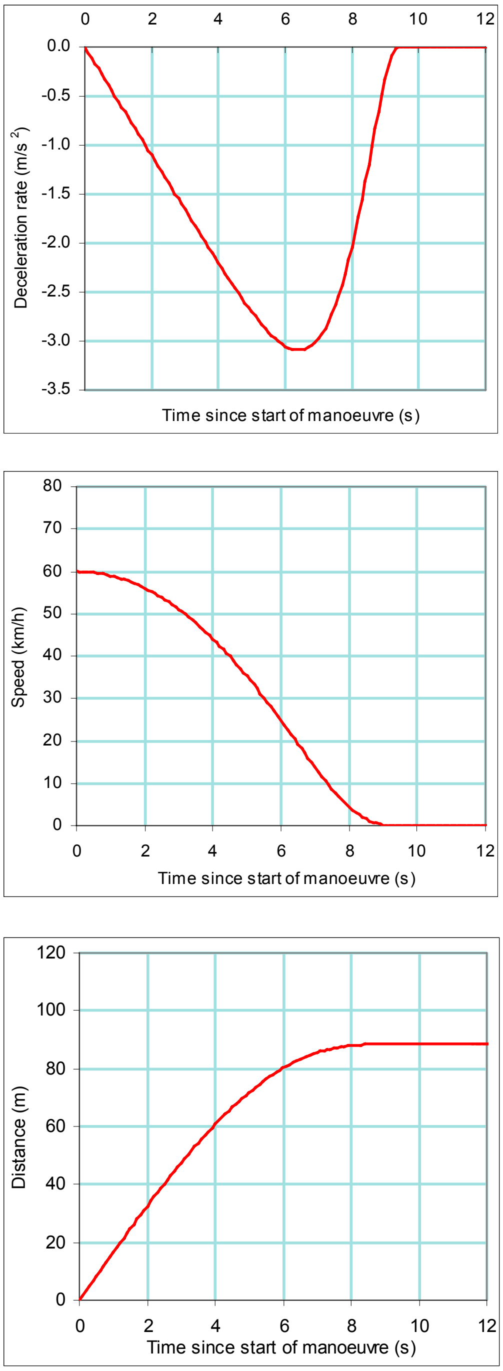


*Figure 6 - Polynomial model: Acceleration, speed and distance profiles for a vehicle DECELERATING from an initial speed of 60 km/h to zero final speed*

## 3 CONCLUSION

Further research is recommended towards improved calibration of acceleration and deceleration models described in this paper considering:

(i) specific traffic facilities (roundabouts, signalised and sign-controlled intersections, signalised and unsignalised pedestrian crossings, freeways),

(ii) different vehicle types (cars, buses, trucks),

(iii) different traffic demand levels (light, medium, heavy), and

(iv) different road types (urban, rural roads, city centre, suburban).

Such research would result in acceleration and deceleration models that are more representative of acceleration and deceleration capabilities of the current vehicle fleet, and would reflect driver behaviour in current traffic conditions.

This would benefit both analytical and microsimulation models towards achieving better estimates of geometric delay, operating cost, fuel consumption and pollutant emissions.



## REFERENCES

AKCELIK & ASSOCIATES (2002). *aaSIDRA User Guide*. Akcelik and Associates Pty Ltd, Melbourne, Australia.

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