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SPEED - FLOW AND BUNCHING MODELS FOR UNINTERRUPTED FLOWS

Rahmi AKÇELIK

Director, Akcelik & Associates Pty Ltd
P O Box 1075G, Greythorn Vic 3104, Australia,
Phone: +613 98574943, Fax: +613 98577462, rahmi.akcelik@sidrasolutions.com

ABSTRACT

Akçelik's time-dependent speed - flow model based on queuing theory concepts is used to develop alternative versions of the HCM speed - flow models for basic freeway segments, multilane highways and urban streets. The corresponding travel time - flow models show that higher-quality facilities have lower levels of bunching delays. A version of the speed - flow model that describes in-stream vehicle interactions and resulting queuing in terms of traffic bunching characteristics is introduced. Speed - flow and headway distribution models for uninterrupted traffic streams are integrated using a common *traffic delay parameter*. A new model of the proportion of bunched vehicles is proposed for the bunched exponential model of headway distribution. The driver response time parameter at capacity flow is discussed. A model for forced flow conditions is developed, and unsaturated and forced flow conditions are contrasted in relation to determining headway distributions.

1. INTRODUCTION

The author (Akçelik 2002a,b, 2003) discussed the speed - flow models given in the Highway Capacity Manual (TRB 2000; Reilly, et al 1990; Schoen, et al 1995) for basic freeway segments, multilane highways and urban streets, and suggested that the HCM speed - flow models have some features not consistent with traffic flow characteristics related to in-stream vehicle interaction and queuing considerations. The HCM models imply that the rate of reduction in speed with increased flow is greater, in other words, traffic delays are larger and increase at a faster rate, for higher-quality facilities. This characteristic of the HCM speed - flow models is in contrast with travel time - flow models for different road classes used for transport planning purposes (Akçelik 1991, 1996). Higher traffic delays for higher-quality facilities do not appear to be consistent with *queuing* mechanisms inherent to in-stream vehicle interactions (Blunden 1971, 1978; Davidson 1966). It is expected that such physical - environmental characteristics as wider lanes, a larger number of lanes, more lateral clearance and lower interchange or access point density represent higher-quality facilities with lower frequency (intensity) of delay-producing elements and situations.

A time-dependent speed - flow model developed by the author has been used in various applications successfully, including transport planning software systems, and has been referred to as *Akçelik's function* in the literature (Akçelik 1991, 1996; Akçelik, Besley and Roper 1999; Akçelik, Roper and

Besley 1999; Akcelik and Associates 2004; Dowling and Alexiadis (1997); Dowling, Singh and Cheng 1998; Singh 1999; Dowling, et al 2005; Sinclair Knight Merz 1998). This function is based on queuing theory concepts, providing a smooth transition between a steady-state queuing delay function for undersaturated conditions and a deterministic delay function for oversaturated conditions. Thus, it allows for estimation of *travel speed*, *travel time* and *travel delay* for both undersaturated and oversaturated conditions. The author used this function to develop alternative versions of the HCM speed - flow models for basic freeway segments, multilane highways and urban streets that are consistent with expected relationships between traffic delay and physical - environmental characteristics of uninterrupted traffic facilities (Akçelik 2002a,b, 2003). In the context of uninterrupted flows, *travel delay* will be referred to as *traffic delay*.

This paper introduces an explicit model that describes in-stream vehicle interactions and resulting *queuing* in terms of traffic *bunching* characteristics. For this purpose, the bunched exponential model of the distribution of vehicle headways is used (Akcelik and Associates 2004; Akçelik 1994; Akçelik and Chung 1994; Cowan 1975; Luttinen 1999, 2003; Troutbeck 1989). A new model of the proportion of bunched vehicles is proposed. The model uses the delay parameter of Akçelik's speed - flow model as a bunching parameter, thus linking the bunching and speed - flow models towards a more integrated framework for modeling uninterrupted traffic streams. This bunching model has been implemented in the SIDRA intersection analysis software (Akcelik & Associates 2004).

The paper also discusses the *driver response time* parameter at capacity flow. A model for *forced flow* conditions is then developed using a variable (linear) driver response - spacing relationship. Unsaturated and forced flow conditions are contrasted for the purpose of determining headway distributions. Application of the new bunching and speed - flow models to roundabout circulating streams is discussed in Akçelik (2003).

2. UNINTERRUPTED TRAVEL SPEED CONCEPT

The average uninterrupted travel speed (see *Figure 1*) can be expressed as:

$$v_u = 3600 / t_u = 3600 / (t_f + d_{tu}) \tag{1}$$

where v_u is the average uninterrupted travel speed at a given flow rate (km/h), $t_u = t_f + d_{tu}$ is the uninterrupted travel time per unit distance, (seconds/km), d_{tu} is the traffic delay (uninterrupted travel delay) per unit distance (seconds/km), $t_f = 3600 / v_f$ is the free-flow travel time per unit distance (seconds/km), and v_f is the free-flow speed (km/h).

Figure 2 shows the speed - flow, travel time - flow and traffic delay - flow relationships for uninterrupted movements. Region A in *Figure 2* represents unsaturated (undersaturated) conditions with arrival (demand) flow rates below capacity ($q_a \le Q$) that are associated with uninterrupted travel speeds, v_u between v_f and v_n ($v_f \ge v_u \ge v_n$) where v_f is the free-flow speed and v_n is the speed at capacity. With increasing flow rate in Region A, speeds are reduced below the free-flow speed due to traffic delays resulting from interactions between vehicles. Region B in *Figure 2* represents the forced (saturated) flow conditions with flow rates reduced below capacity ($q < Q$) which are associated with further reduced speeds ($v < v_n$) as observed at a reference point along the road. In this region, flow rates (q) are reduced flow rates due to forced flow conditions, not demand flow rates (q_a).

Region C in *Figure 2* represents oversaturated conditions, i.e. arrival (demand) flow rates above capacity ($q_a > Q$) cause large reductions in travel speeds ($v < v_n$) due to large queuing delays. These speeds can be observed by travel through the total section (along distance L_t), e.g. by an instrumented car. In this case, the flow represents the demand flow rate which can exceed the capacity value as measured at a point upstream of the queuing section.

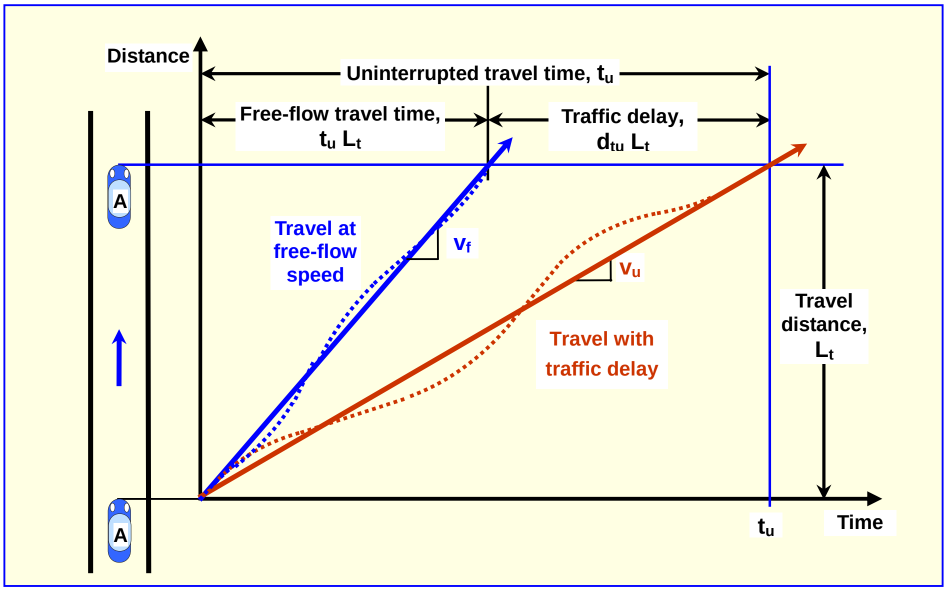

Figure 1: Definition of free-flow and uninterrupted travel speed

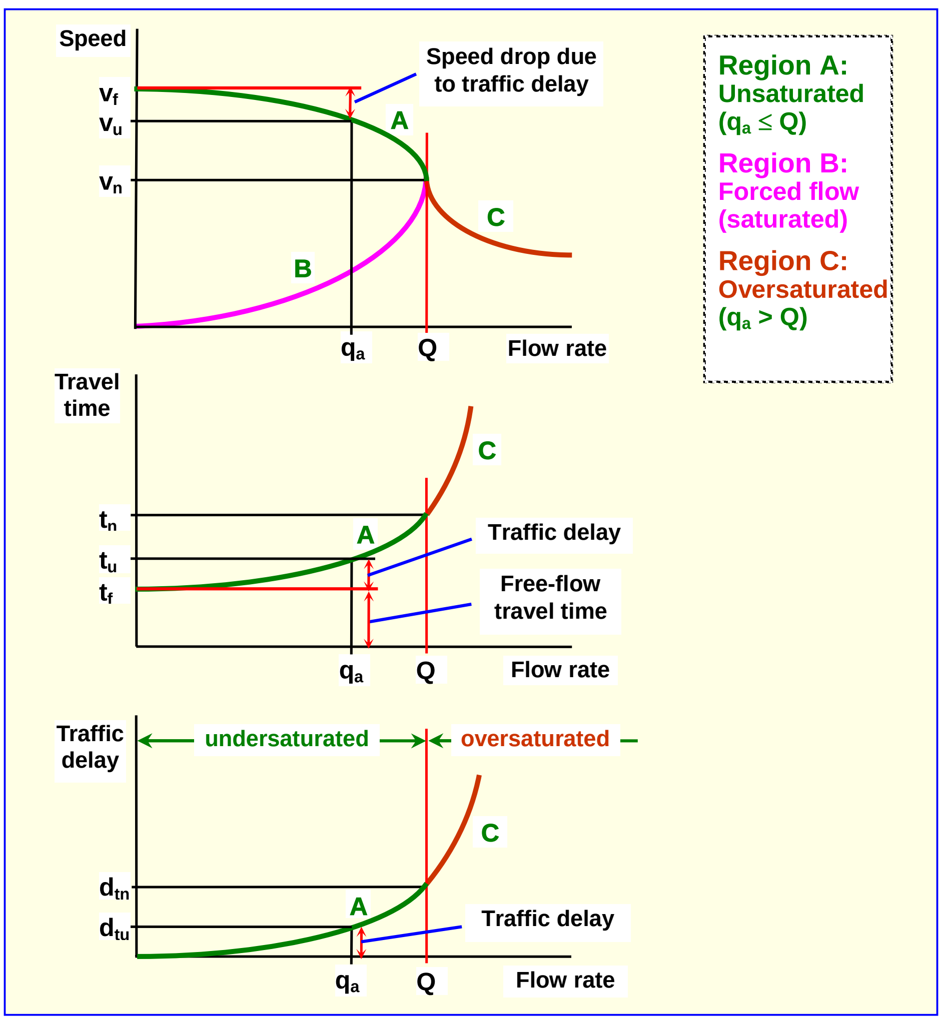

Figure 2: Speed, travel time and delay as a function of flow rate for uninterrupted traffic streams

3 BUNCHING MODELS

The following model developed by the author was introduced in aaSIDRA 2.1 for the prediction of proportion free (unbunched) vehicles in a traffic stream:

$$\varphi = (1 - \Delta\, q_a) / [1 - (1 - k_d)\, \Delta\, q_a] \qquad \textit{subject to } \varphi \geq 0.001 \qquad (2)$$

where k_d is the *bunching* parameter, which is the same as the *traffic delay* parameter in the speed-flow model, Δ is the average intrabunch headway (s), and q_a is the flow rate (veh/s). The minimum value of the proportion unbunched (0.001) in *Equation (2)* is used for computational reasons.

The following exponential bunching model (Akçelik and Chung 1994), and Tanner's (1962, 1967) linear bunching model (equivalent to *Equation (2)* when k_d = 1.0) should also be noted:

$$\varphi = e^{-b\, \Delta\, q_a} \qquad (3)$$

$$\varphi = 1 - \Delta\, q_a \qquad (4)$$

where b is a constant, Δ and q_a are as in *Equation (2)*.

Previously, it was recommended that the intrabunch headway should be selected on the basis of the best headway distribution prediction (Akçelik and Chung 1994). Although this is still an important objective, the intrabunch headway is treated as the average headway at capacity flow by definition (Δ = 3600 / Q where Q is the capacity in veh/h). General-purpose values of parameters b, k_d and Δ for use in *Equations (2) to (4)* are given in *Table 1*. *Extra bunching* to allow for the effect of upstream signals, which is used in aaSIDRA for roundabout approach streams, could be used for all uninterrupted streams.

The bunching model and the bunched exponential model of headway distribution apply for *unsaturated flow* conditions (flow rate below capacity), i.e. for Region A in *Figure 2*. Under *forced flow* conditions for (Region B in *Figure 2*), all vehicles are bunched with intrabunch headways larger than the minimum intrabunch headway due to lower speeds and spacings of vehicles. This is discussed in *Section 6*, and implications of the forced flow conditions on headway distributions are discussed in *Section 7*.

The values of the traffic delay / bunching parameter k_d given in *Table 1* were determined on the basis of exponential models used previously for uninterrupted streams (Akçelik and Chung 1994) and using data given in SR 45 (Troutbeck 1989) for roundabout circulating streams. Resulting speed - flow relationships were also considered in selecting appropriate values of the parameter. *Figure 3* shows the proportion unbunched for one-lane, two-lane and three-lane uninterrupted streams using the bunching model based on the traffic delay parameter (*Equation 2)* with parameters (k_d and Δ) given in *Table 1*.

Table 1: Parameter values for estimating the proportion of free (unbunched) vehicles in a traffic stream

Total number of lanes	Uninterrupted traffic streams				Roundabout circulating streams			
	Δ	3600/Δ	b	k_d	Δ	3600/Δ	b	k_d
1	**1.8**	2000	0.5	**0.20**	2.0	1800	2.5	2.2
2	**0.9**	4000	0.3	**0.20**	1.0	3600	2.5	2.2
> 2	**0.6**	6000	0.7	**0.30**	0.8	4500	2.5	2.2

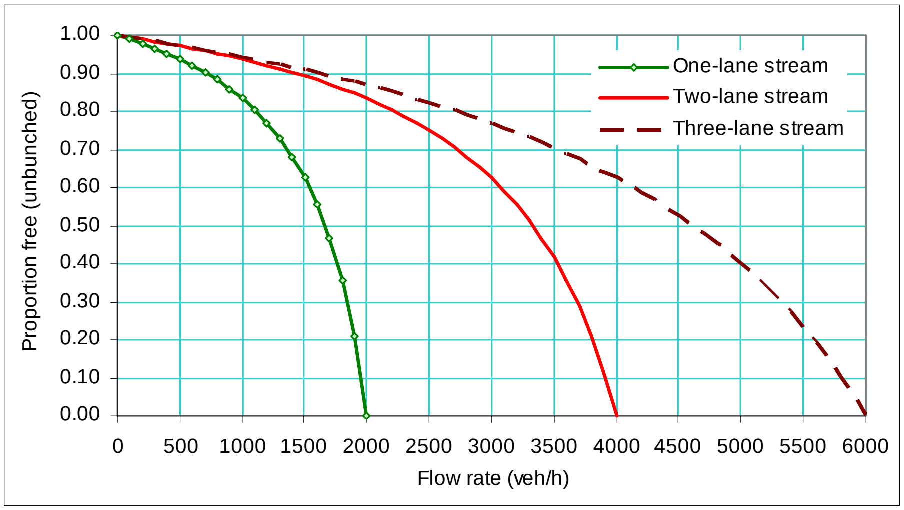

Figure 3: Proportion unbunched for one-lane, two-lane and three-lane uninterrupted streams using the traffic delay (k_d) and intrabunch headway (Δ) parameters given in *Table 1*

4 TRAVEL DELAY, TRAVEL TIME AND TRAVEL SPEED

The steady-state travel delay model for an uninterrupted stream corresponding to the bunching model given by *Equation (2)* is:

$$d_{tu} = 3600\ k_d\ x / [Q\ (1 - x)] \qquad (5)$$

where d_{tu} is the average travel delay for an uninterrupted stream (s/km), k_d is the traffic delay / bunching parameter as in *Equation (2)*, Q is the capacity in veh/h (Q = 3600 / Δ) and x is the degree of saturation: $x = q_a / Q = \Delta\ q_a / 3600$ where q_a is the arrival (demand) flow rate in veh/h.

The corresponding bunch size (n_b) and the corresponding queue size ($n_q = n_b - 1$, considering that the leader of a bunch of vehicles is not queued) are given by:

$$n_b = [1 - (1 - k_d)\ x] / (1 - x) \qquad (6)$$

$$n_q = k_d\ x / (1 - x) \qquad (7)$$

Using the time-dependent form of the travel delay model given by *Equation (5)* in *Equation (1)*, the travel time - flow and the corresponding speed - flow functions are given by the following equations (Akçelik 2002a,b) which assume no initial queued demand for the analysis period:

$$t_u = t_f + 900\ T_f\ \{ x - 1) + [(x - 1)^2 + 8\ k_d\ x / (Q\ T_f)]^{0.5} \} \qquad (8)$$

$$v_u = v_f / \{1 + 0.25\ v_f\ T_f\ [(x - 1) + ((x - 1)^2 + 8\ k_d\ x / (Q\ T_f)\ )^{0.5}]\} \qquad (9)$$

where t_u is the uninterrupted travel time per unit distance (s/km) at a given degree of saturation, v_u is the uninterrupted travel speed (s/km), t_f is the free-flow travel time per unit distance (travel time at x = 0) (s/km), T_f is the duration of the analysis period (h), (T_f = 0.25 h is specified in the HCM), Q is the capacity (veh/h), Q = 3600 / Δ, and x is the degree of saturation.

The speed - flow model should normally be used for single-lane streams although they could be used for multi-lane streams (lane groups) as a rough approximation. Speed - flow and bunching parameters for the uninterrupted stream models proposed by Akçelik (2002a,b, 2003) for the HCM *basic freeway segment*, *multilane highway* and *urban street* classes are given in *Table 2*. The speed - flow models given in *Table 2* were developed using the same speed ratio (ratio of speed at capacity to free-flow speed, v_n / v_f) for all classes in a facility type (v_n / v_f = 0.85 for freeways, 0.82 for multilane highways and 0.80 for urban streets). *Figure 4* shows the speed - flow graphs for freeways using parameters given in *Table 2* (labeled as "Akcelik") and the HCM speed - flow models.

Table 2: Parameters for speed - flow and bunching models for single-lane uninterrupted streams: HCM basic freeway segments, multilane highways and urban streets

	Basic Freeway Segments				Multilane Highways				Urban Streets			
Facility class	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
Free-flow speed, v_f (km/h)	**120**	**110**	**100**	**90**	**100**	**90**	**80**	**70**	**80**	**65**	**55**	**45**
Traffic delay / bunching parameter, k_d	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.10**	**0.12**	**0.15**	**0.14**	**0.21**	**0.29**	**0.42**
Intrabunch headway, Δ (s)	**1.500**	**1.532**	**1.565**	**1.600**	**1.636**	**1.714**	**1.800**	**1.895**	**1.946**	**2.000**	**2.057**	**2.118**
Capacity, Q (veh/h)	**2400**	**2350**	**2300**	**2250**	**2200**	**2100**	**2000**	**1900**	**1850**	**1800**	**1750**	**1700**
Speed at capacity, v_n (km/h)	**102.0**	**93.5**	**85.0**	**76.5**	**82.0**	**73.8**	**65.6**	**57.4**	**64.0**	**52.0**	**44.0**	**36.0**
Speed ratio, v_n / v_f	**0.85**	**0.85**	**0.85**	**0.85**	**0.82**	**0.82**	**0.82**	**0.82**	**0.80**	**0.80**	**0.80**	**0.80**
Average spacing at capacity, L_{hn} (m)	**42.5**	**39.8**	**37.0**	**34.0**	**37.3**	**35.1**	**32.8**	**30.2**	**28.9**	**34.6**	**39.8**	**47.2**
Response time to stop from speed at capacity, t_{rf} (s)	**1.25**	**1.26**	**1.27**	**1.27**	**1.33**	**1.37**	**1.42**	**1.46**	**1.55**	**1.52**	**1.48**	**1.42**

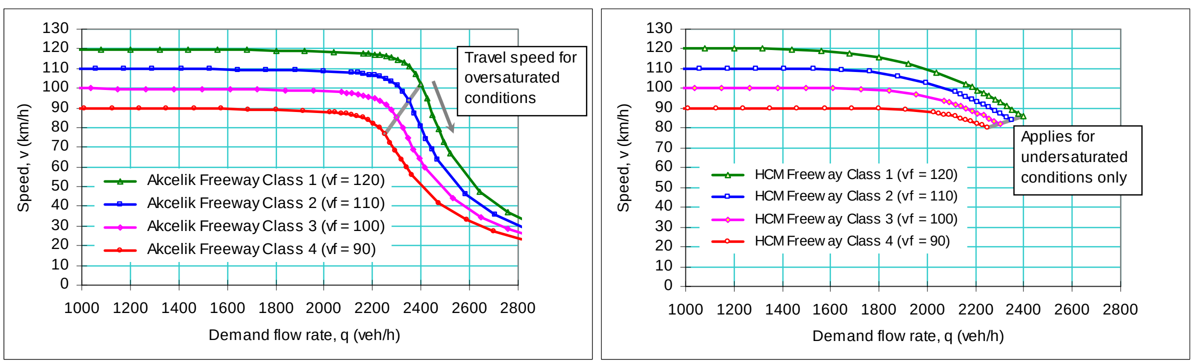

Figure 4: Speed - flow models for single-lane uninterrupted streams: HCM basic freeway segment classes (see *Table 2*)

5 DRIVER RESPONSE TIME

The *driver response time at capacity* (as it applies to bunched vehicles) shown in *Table 2* is calculated from

$$t_{rn} \quad = \ (3.6 / v_n) \, (L_{hn} - L_{hj}) = h_n - 3.6 \, L_{hj} / v_n \qquad (10)$$

where t_{rn} is the driver response time to stop from speed at capacity (s), v_n is the speed at capacity (km/h), $h_n = \Delta$ is the headway at capacity, i.e. the intrabunch headway (s), $L_{hn} = \Delta \, v_n / 3.6$ is the spacing at capacity (m), and L_{hj} is the jam spacing (m).

The basis of *Equation (10)* is shown in *Figure 5* (this is similar to Fig. 13.6 in Drew 1968). For simplicity, this assumes that the leading and following vehicles (A and B) have the same braking distance ($L_{bA} = L_{bB}$) and the spacing at capacity (L_{hn}) is sufficient for the following vehicle to stop leaving a gap of L_{hj} - L_v behind the leading vehicle when they both stop (L_v = vehicle length). Therefore, the response time represents a stopping condition that is reasonably comfortable. Note that, for a given headway at capacity, higher speed at capacity means a shorter response time, and a larger jam spacing means a shorter response time. *Equation (10)* can also be applied to saturation headways at signals as a stopping condition, giving driver response times equivalent to those obtained from queue discharge models.

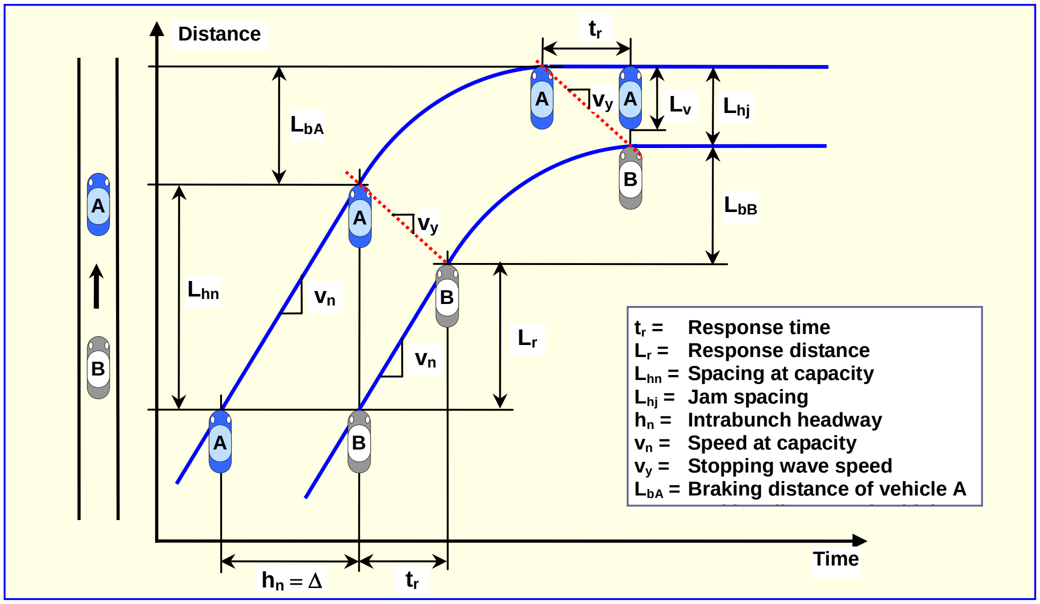

Figure 5: Derivation of the driver response time for vehicles driving with intrabunch headway considering safe stopping conditions

The response times in *Table 2* (in the range 1.25 to 1.55 s) were calculated using a jam spacing value of L_{hj} = 7.0 m (larger values may be more appropriate for freeways). In ARR 341 (Akçelik, Roper and Besley 1999), calibration of "Model 4+5" for a basic freeway segment gave v_f = 101 km/h, v_n = 90 km/h (v_n / v_f = 0.89), k_d = 0.09, Δ = 1.44 s (Q = 2500 veh/h), L_{hn} = 36.0 m and L_{hj} = 15.0 m ("Model 4" corresponds to *Equation 9*). The corresponding response time from *Equation (10)* is t_r = 0.84 s. Application of *Equation (10)* to other calibrated freeway models reported in ARR 341 gave response times in the range 0.75 to 1.12 s. Overall, the response times are similar to response times for saturation headways at signalized intersections which were determined to be in the range 0.84 to 1.39 s (Akçelik, Besley and Roper 1999, Akçelik and Besley 2002). In terms of the implied driver response times, the speed - flow and bunching models can be considered to be reasonable given that, in capacity conditions, drivers would be more alert with relatively low response times.

Consistent with *Equation (10)*, the spacing, headway and speed at capacity are related as follows:

$$L_{hn} = L_{hj} + t_{rn} \, v_n / 3.6 \qquad (11)$$

$$h_n = \Delta = t_{rn} + 3.6 \, L_{hj} / v_n \qquad (12)$$

$$v_n = 3.6 \, L_{hj} / (h_n - t_{rn}) \qquad (13)$$

Equation (12) for headway at capacity is essentially the same as the formula for saturation headway at signalized intersection derived from queue discharge characteristics (Akçelik, Besley and Roper 1999, Akçelik and Besley 2002). The *stopping wave speed* shown in *Figure 5* is given by:

$$v_y = 3.6 \, L_{hj} / t_{rn} = 3.6 \, L_{hj} / (h_n - 3.6 \, L_{hj} / v_n) \qquad (14)$$

6 FUNDAMENTAL RELATIONSHIPS FOR FORCED FLOW CONDITIONS

The speed - flow model given in *Section 4* can be used for regions A and C in *Figure 2*. A speed - flow model, and associated models for other fundamental traffic flow relationships, for region B in *Figure 2*, i.e. for ***forced flow*** conditions can be derived using the driver response time parameter. Under forced flow conditions when speed drops below the value at capacity ($v \le v_n$) because vehicle spacing is reduced below the value at capacity ($L_h \le L_{hn}$), the bunching models given in *Section 3* no longer apply. The spacing under these conditions can be expressed as:

$$L_h = L_{hj} + t_r\, v / 3.6 \qquad \textit{for } L_h \le L_{hn}\ (v \le v_n) \qquad (15)$$

Speed and headway are found from:

$$v = 3.6\, (L_h - L_{hj}) / t_r \qquad \textit{for } L_h \le L_{hn}\ (v \le v_n) \qquad (16)$$

$$h = 3.6\, L_h / v = L_h\, t_r / (L_h - L_{hj}) \qquad \textit{for } L_h \le L_{hn}\ (v \le v_n) \qquad (17)$$

where v is in km/h and h is in seconds.

This can be used to develop fundamental traffic flow relationships for forced flow conditions by making assumptions about the driver response time. For example, a linear driver response - spacing model can be used assuming that drivers become more alert as spacing decreases and more relaxed as the spacing increases:

$$t_r = p_1 + p_2\, L_h \qquad \textit{subject to } 0.5\text{ s} \le t_r \le 2.5\text{ s} \qquad (18)$$

where parameters p_1 and p_2 are derived to achieve a given t_{rn} at capacity ($t_r = t_{rn}$ when $L_h = L_{hn}$) and minimum headway at capacity ($dh/ dL_h = 0$ for $h = h_n$ when $L_h = L_{hn}$):

$$p_1 = t_{rn}\, [1 - L_{hn}\, L_{hj} / (L_{hn}^2 - L_{hn}\, L_{hj})] \qquad (19a)$$

$$p_2 = t_{rn}\, L_{hj} / (L_{hn}^2 - L_{hn}\, L_{hj}) \qquad (19b)$$

Using t_r from *Equation (18)*, the following speed, headway and spacing relationships are found:

$$v = 3.6\, (L_h - L_{hj}) / (p_1 + p_2\, L_h) \qquad (20)$$

$$h = 3.6\, L_h / v = L_h\, (p_1 + p_2\, L_h) / (L_h - L_{hj}) \qquad (21)$$

$$L_h = (L_{hj} + p_1\, v / 3.6) / (1 - p_2\, v / 3.6) \qquad (22)$$

Thus, the linear response time - spacing model implies a hyperbolic spacing - speed function. Density (k) in veh/km and flow rate (q) in veh/h are related to spacing (L_h) in metres and headway (h) in seconds by $k = 1000 / L_h$ and $q = 3600 / h$. The forced-flow model given by *Equations (20) to (22)* can also be expressed in terms of density and flow:

$$v = 3.6\, (1 / k - 1 / k_j) / (p_1 / 1000 + p_2 / k) \qquad \textit{for } k \ge k_n\ (v \le v_n) \qquad (23)$$

$$q = 3600\, (1 - k / k_j) / (p_1 + 1000\, p_2 / k) \qquad \textit{for } k \ge k_n \qquad (24)$$

Other parameters such as occupancy time, space time, etc described in previous publications (Akçelik, Besley and Roper 1999, Akçelik, Roper and Besley 1999) can also be calculated as a result. For the freeway case described in ARR 341 (see *Section 5* of this paper), $p_1 = 0.240$ and $p_2 = 0.0167$ were found. Using the new model for forced flow conditions (replacing "Model 5" of ARR 341), the speed - flow, speed - density, spacing - speed and occupancy - speed relationships together with measured values for the ARR 341 case (both unsaturated and forced flow conditions) are shown in *Figure 6*. Average vehicle length = 4.35 m and detection zone length = 2.0 m were used for calculating occupancy (time occupancy ratio) values.

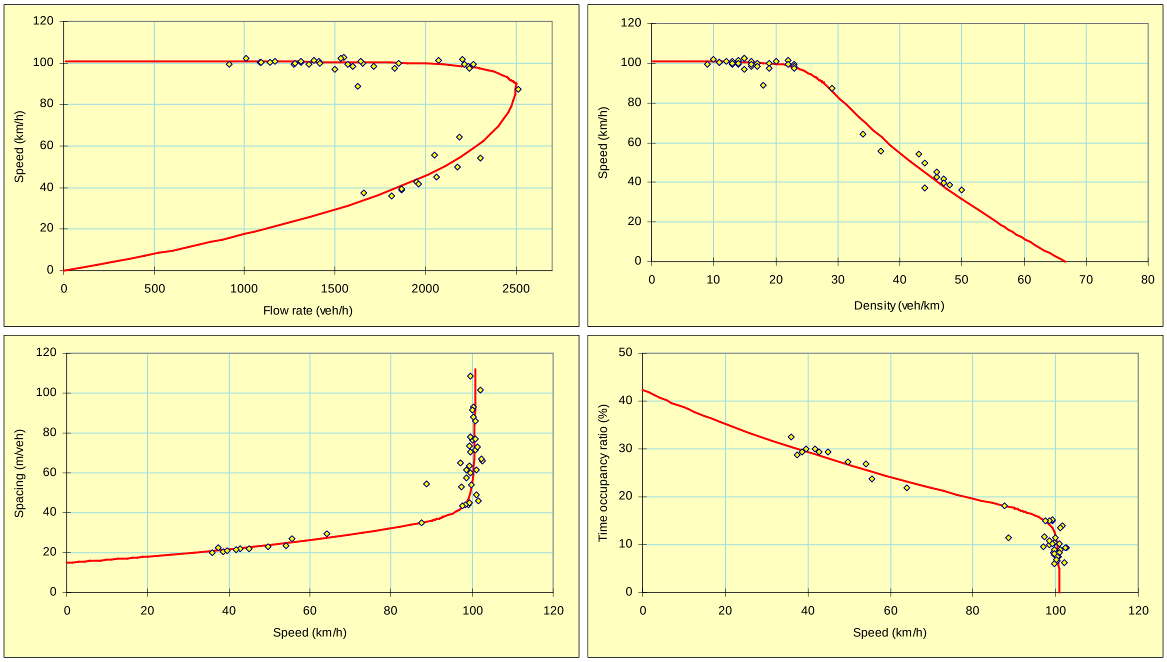

Figure 6 - Estimated and measured speed - flow, speed - density, spacing - speed and occupancy - speed values for the freeway basic segment data collected in Melbourne (ARR 341)

7 CONCLUDING REMARKS

While the proposed bunching, speed - flow and other associated models described in this paper appear to give reasonable results, they are recommended for further investigation. Alternative models developed to overcome various counter-intuitive characteristics of the HCM speed - flow models were derived on the basis of using the same speed ratio in *Akçelik's* functions. Further alternatives can be derived using similar principles. However, further analyses are recommended to determine fundamental characteristics of speed - flow and related models for uninterrupted traffic facilities using real-life data. It would also be interesting to compare the HCM and *Akçelik's* functions with the speed - flow relationships implied by various microsimulation models (Akçelik and Besley 2001).

The variable driver response time - spacing model (linear and other forms rather than a constant response time) should be explored for use in car-following models.

Further research into the proposed bunching model and its relationship with Akçelik's speed - flow model using real-life data is recommended. The unsaturated and forced flow conditions should be distinguished in the calibration and application of the bunching model for modeling headway distributions. The bunching model and the bunched exponential headway distribution model should be applied for *unsaturated* flow conditions. Under these circumstances, all headways larger than the intrabunch headway are considered to be unbunched in the headway distribution model. According to the model, when the flow rate reaches capacity, all vehicles are bunched as they travel at the intrabunch (minimum) headway at a capacity speed. The traffic stream is considered to be under *forced (saturated) flow* conditions when the average speed drops below the capacity speed and the average vehicle spacing drops below the spacing at capacity. Under these conditions, all vehicles should be considered to be bunched although headways are *larger* than the minimum intrabunch headway. The bunched exponential model is no longer valid under forced flow conditions, and using headway data collected under these circumstances would result in a biased bunching model.

Care should also be taken in applying the bunched headway model to vehicle platoons departing from queues at signalized intersection approaches. These vehicles cross the signal stop line at saturation headway and speed, and then accelerate towards the free-flow speed after clearing the intersection area (Akçelik and Besley 2002). Headways of vehicles in such platoons may be larger than the capacity (intrabunch) headway as they travel downstream, but these vehicles are still under forced flow conditions (at least partly) and the application of the bunched exponential model of headway distribution may become problematic. Similar considerations apply to roundabout circulating streams (Akçelik 2003).

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