A Review of Gap-Acceptance Capacity Models

Rahmi Akçelik

ABSTRACT

Gap-acceptance capacity models apply to the analysis of minor movements at two-way stop and give-way (yield) sign-controlled intersections, entry streams at roundabouts and opposed (permitted) turns at signalised intersections. The same modelling principles apply to all these cases with different model parameters representing the intersection geometry, control and driver behaviour at different traffic facilities. This paper presents a review of some well-known analytical models that use bunched exponential and simple negative exponential distribution of headways in the opposing stream. Different bunching models are considered including the latest model used in SIDRA INTERSECTION. The capacity estimates from different models are compared.

Revision on 14 July 2011: A new "shifted delay parameter" model for bunching added (Appendix A).

1   INTRODUCTION

A review of various gap-acceptance capacity models that use bunched exponential and simple negative exponential distribution of headways in the opposing stream is presented. The model used in the SIDRA INTERSECTION software (Akcelik and Associates 2007) is described in detail and capacity estimates from different models are compared. Different bunching models including the latest model used in SIDRA INTERSECTION are described.

The paper updates information given in earlier papers which introduced the gap-acceptance model used in SIDRA INTERSECTION (Akçelik 1994) and described the calibration of arrival headway distributions (Akçelik and Chung 1994a,b). The reader is also referred to a more recent paper (Akçelik 2006) which discussed the relationship between speed - flow functions and the bunched exponential model of headway distribution for uninterrupted traffic streams.

In this paper, the model used in the SIDRA INTERSECTION software will be referred to as the "SIDRA model".

Author: Rahmi Akçelik
Director, Akcelik & Associates Pty Ltd,
Email: rahmi.akcelik@sidrasolutions.com
Phone: +61 3 9857 4943, Fax: +61 3 9857 7462
Mail: P O Box 1075 G, Greythorn Vic 3104, Australia
Website: www.sidrasolutions.com

The SIDRA model for unsignalised intersections was derived using a traffic signal analogy concept. The model uses the bunched exponential model of headway distribution. The signal analogy concept treats block and unblock periods in a priority (major) traffic stream (as defined in the traditional gap acceptance modelling) as red and green periods in a way similar to the modelling of signal-controlled traffic streams. In addition to modelling gap-acceptance capacity, this method enables the modelling of the average back of queue, proportion queued and queue move-up rate for the entry (minor) traffic stream in a manner consistent with models for traffic signals. This helps to fill the gap in modelling queue length, proportion queued and stop rate (major stops and queue move-ups separately) for unsignalised intersections, and presents a methodological advantage in that the same conceptual framework is employed in models for different types of intersection (Akçelik 1994; Akçelik and Chung 1994b).

The commonly-used average cycle-based queue length based on traditional queuing theory methods incorporates all queue states including zero queues. The back of queue is a more useful statistic since it is relevant to the design of appropriate queuing space (e.g. for short lane design).

The SIDRA capacity and performance models were developed using the bunched exponential model of arrival headway distribution for all types of intersection. This model is more realistic than the commonly-used simple exponential and shifted exponential models. However, the capacity and performance models are also applicable to simple negative exponential and shifted negative exponential distributions. The calibration of performance models was carried out using data generated by the microscopic simulation model MODELC incorporating a bunched exponential model of headways (Chung, Young and Akçelik 1992a,b).

For capacity and performance modelling, a lane-by-lane method is adopted generally, and therefore, the arrival headway distribution in a single lane of the approach road is considered. However, in modelling capacity of entry streams, the headway distribution of total traffic demand in all lanes of the priority (major) traffic stream is adopted with different values of minimum headway and bunching parameters for single-lane and multi-lane cases. When there are several opposing (higher priority) traffic streams, all opposing streams are combined as one stream and treated using appropriate multi-lane stream parameters.

2   ARRIVAL HEADWAY DISTRIBUTIONS

The estimation of arrival headways is fundamental to the modelling of gap acceptance processes for estimating capacities of sign-controlled traffic streams, roundabout entry streams and filter (permitted) turns at signalised intersections (e.g. Akçelik 1981; Akçelik and Troutbeck 1991; Troutbeck 1986, 1988, 1989, 1990, 1991a,b, 1993).

This paper considers a class of exponential arrival headway distribution models known as negative exponential (M1), shifted negative exponential (M2) and bunched exponential (M3). The bunched exponential distribution of arrival headways (M3) was proposed by Cowan (1975, 1984, 1987) and used extensively by Troutbeck (1986, 1988, 1989, 1990, 1991a,b, 1993) for estimating capacity and performance of roundabouts and other unsignalised intersections. A special case of the model was previously used by Tanner (1962, 1967) for unsignalised intersection analysis. A detailed discussion of the M3 model and the results of its calibration using real-life data for single-lane traffic streams and simulation data for multi-lane streams are given in Akçelik and Chung (1994a). Further discussions on the M3 model and gap-acceptance models in general can be found in Luttinen (1999, 2003).

The negative and shifted negative exponential distributions (M1 and M2) are extensively discussed and used in the literature as models of random arrivals. The shifted negative exponential model (M2) is normally used for single-lane traffic only. The bunched exponential distribution (M3) offers improved accuracy in the prediction of small arrival headways (up to about 12 seconds), which is important for most urban traffic analysis applications.

The cumulative distribution function, F(t), for the bunched exponential distribution of arrival headways, representing the probability of a headway less than t seconds, is:

$$F(t) \;=\; 1 - \varphi\, e^{-\lambda\,(t-\Delta)} \qquad\qquad \textit{for } t \geq \Delta \qquad\qquad\qquad (2.1)$$
$$\qquad\qquad\quad 0 \qquad\qquad\qquad\qquad\quad \textit{for } t < \Delta$$

where

Δ = average intrabunch (minimum) headway (seconds),

φ = proportion of free (unbunched) vehicles, and

λ = a model parameter calculated as:

$$\lambda \;=\; \varphi\, q\, /\, (1 - \Delta\, q) \qquad\qquad \textit{subject to } q \leq 0.98\,/\,\Delta \qquad\qquad (2.1a)$$

where q is the arrival flow rate (veh/s).

According to the model, the traffic stream consists of:

(i)   *bunched vehicles* with all intrabunch headways equal to the minimum arrival headway, Δ (proportion of bunched vehicles = $1 - \varphi$), and

(ii)   *free vehicles* with headways greater than the minimum arrival headway, Δ (thus, the proportion of free vehicles, φ, represents the unbunched vehicles with randomly distributed headways).

The average intrabunch headway corresponds to the average headway at capacity ($\Delta = 3600\,/\,Q$ where Q is the capacity in veh/h). Previously, it was recommended that the intrabunch headway should be selected on the basis of the best headway distribution prediction (Akcelik and Chung 1994a). Although this is still an important objective, the intrabunch headway may be treated as the average headway at capacity flow by definition.

The M1 and M2 models can be derived as special cases of the M3 model through simplifying assumptions about the bunching characteristics of the arrival stream as shown below.

Negative exponential (M1) model:

$$\Delta \;= 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2.2a)$$
$$\varphi \;= 1.0$$

Therefore:

$$\lambda \;= q \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2.2b)$$

Shifted negative exponential (M2) model:

$$\varphi \;= 1.0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2.3a)$$

Therefore:

$$\lambda \;= q\,/\,(1 - \Delta\, q) \qquad\qquad \textit{subject to } q \leq 0.98\,/\,\Delta \qquad\qquad (2.3b)$$

The maximum value of $\Delta\, q = 0.98$ is used for computational reasons.

Thus, models M1 and M2 assume no bunching ($\varphi = 1$) for all levels of arrival flows. On the other hand, model M3 can be used either with a known (measured) value of φ, or more generally, using a bunching model that estimates the value of φ as a function of the arrival flow rate. The bunching models are discussed in the next section.

3   BUNCHING MODELS

A summary of various bunching models exist in the literature is presented in this section.

The following exponential model was used in SIDRA INTERSECTION Version 2.0 and earlier versions for the prediction of proportion free (unbunched) vehicles in a traffic stream (Akçelik and Chung 1994a):

$$\varphi \quad = e^{-b \, \Delta \, q} \tag{3.1}$$

where b is a constant, Δ is the average intrabunch headway (s) and q is the flow rate (veh/s).

For the purpose of this paper, the M3 model with estimates of φ obtained from *Equation (3.1)* will be referred to as the **M3A model**.

An empirical relationship of a similar form $\varphi = \exp(-b' \, q)$, where $b' = 6$ to 9, was used by Brilon (1988) based on previous work by Jacobs (1979). The same empirical relationship was used by Sullivan and Troutbeck (1993).

The following "delay parameter" model for bunching was introduced in a SIDRA INTERSECTION Version 2.1 to replace the exponential model given by *Equation (3.1)*:

$$\varphi \quad = (1 - \Delta \, q) / [1 - (1 - k_d) \, \Delta \, q] \qquad \textit{subject to 1.0} \geq \varphi \geq \textit{0.10} \tag{3.2}$$

where k_d is the bunching delay parameter (a constant), Δ is the average intrabunch headway (s), and q is the flow rate (veh/s). The minimum value of $\varphi_{min} = 0.10$ is used for computational reasons.

This bunching model was derived using a method that integrated speed - flow and headway distribution models for uninterrupted traffic through the use of a common *traffic delay* parameter (k_d). The method is discussed in detail in Akçelik (2006).

For the purpose of this paper, the M3 model with estimates of φ obtained from *Equation (3.2)* will be referred to as the **M3D model**.

Values of parameters b, k_d and Δ for use in *Equations (3.1) and (3.2)* are given in *Table 1*. In all gap-acceptance cases, SIDRA INTERSECTION determines the effective number of lanes considering all opposing movements before selecting appropriate parameters from *Table 1*.

The values of the bunching delay parameter k_d given in *Table 3.1* were determined on the basis of exponential models used previously for uninterrupted streams (Akçelik and Chung 1994a) and using data given in SR 45 (Troutbeck 1989) for roundabout circulating streams. Resulting speed-flow relationships were also considered in selecting appropriate values of the parameter.

Refer to Appendix A for a new "shifted delay parameter" model. 14 July 2011

Table 1

Parameter values for estimating the proportion of free (unbunched) vehicles in a traffic stream

Total number of lanes	Uninterrupted traffic streams				Roundabout circulating streams			
	Δ	$3600 / \Delta$	b	k_d	Δ	$3600 / \Delta$	b	k_d
1	1.8	2000	0.5	0.20	2.0	1800	2.5	2.2
2	0.9	4000	0.3	0.20	1.0	3600	2.5	2.2
> 2	0.6	6000	0.7	0.30	0.8	4500	2.5	2.2

The following linear model of the proportion of free vehicles was used by Tanner (1962, 1967):

$$\varphi = (1 - \Delta q) \tag{3.3a}$$

This model is seen to be a special case of *Equation (3.2)* which is obtained when $k_d = 1.0$.

The M3 model with estimates of φ obtained from *Equation (3.3a)* will be referred to as the **M3T model**. For this model:

$$\lambda = q \tag{3.3b}$$

AUSTROADS (1993) roundabout guide uses the following linear model for roundabout circulating streams (also see Akçelik and Besley 2005):

$$\varphi = 0.75 (1 - \Delta q) \tag{3.4}$$

All bunching models given above assume that the proportion of free vehicles decreases (the proportion of bunched vehicles increases) with increasing arrival flow rate. All models except the AUSTROADS (1993) model predict zero bunching ($\varphi = 1.0$) at very low flows. All models except the exponential model (*Equation 3.1*) assume $\varphi = 0$ at $q = 1 / \Delta$.

Figure 1 shows the proportion unbunched for one-lane, two-lane and three-lane uninterrupted streams using the bunching model given by *Equation (3.2)*. *Figure 2* shows the proportion unbunched for one-lane, two-lane and three-lane roundabouts using the using the bunching model given by *Equation (3.2)* together with the Australian roundabout survey data for single-lane multi-lane roundabouts. *Figures 3 and 4* show the proportion unbunched (measured and estimated by alternative models) for single-lane and two-lane circulating streams at roundabouts.

The bunching model and the bunched exponential model of headway distribution apply for *unsaturated flow* conditions (flow rate below capacity). Under *forced flow* conditions, all vehicles are bunched with intrabunch headways larger than the minimum intrabunch headway due to lower speeds and spacings of vehicles. This is discussed in Akçelik (2006).

Extra bunching to allow for the effect of upstream signals, which is used in SIDRA INTERSECTION for roundabout approach streams, could be used for all uninterrupted streams.
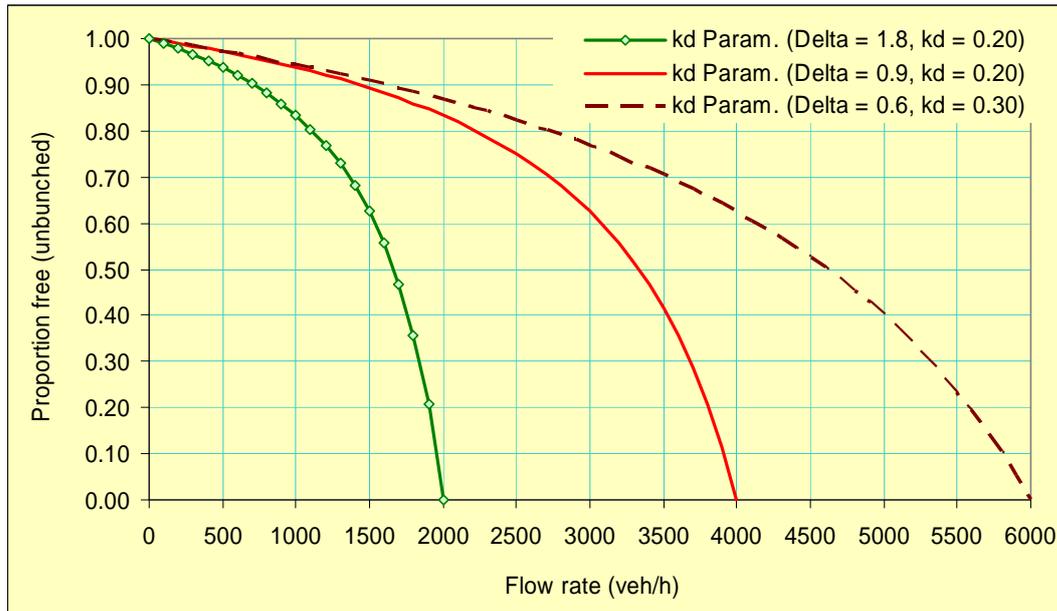
Figure 1 - Proportion unbunched for one-lane (Δ = 1.8 s), two-lane (Δ = 0.9 s) and three-lane (Δ = 0.6 s) uninterrupted streams using the bunching model based on traffic delay parameter
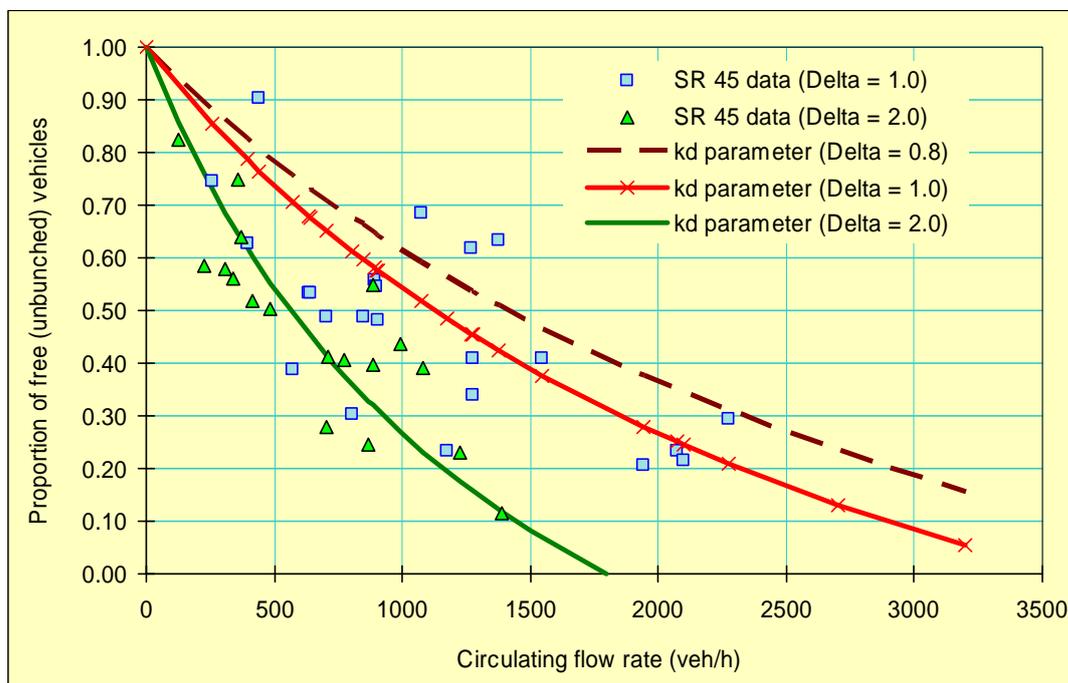

Figure 2 - Proportion unbunched for one-lane (Δ = 2.0 s), two-lane (Δ = 1.0 s) and three-lane (Δ = 0.8 s) roundabouts using the bunching model based on traffic delay parameter
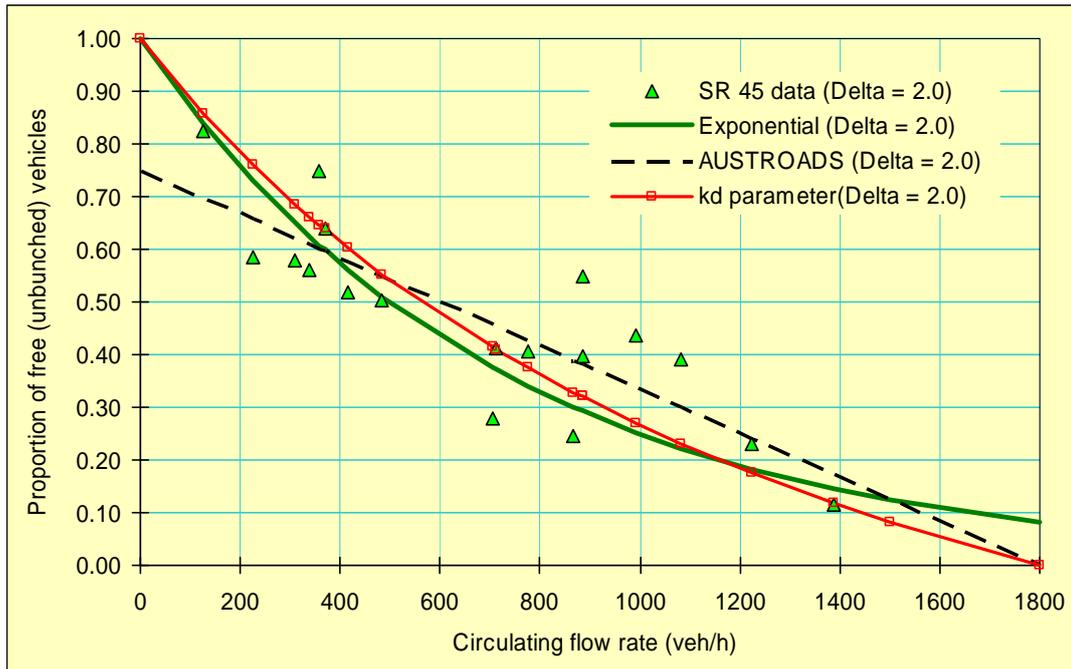
Figure 3 - Proportion unbunched for single-lane circulating streams at roundabouts as a function of the circulating flow rate (measured and estimated by alternative bunching models)
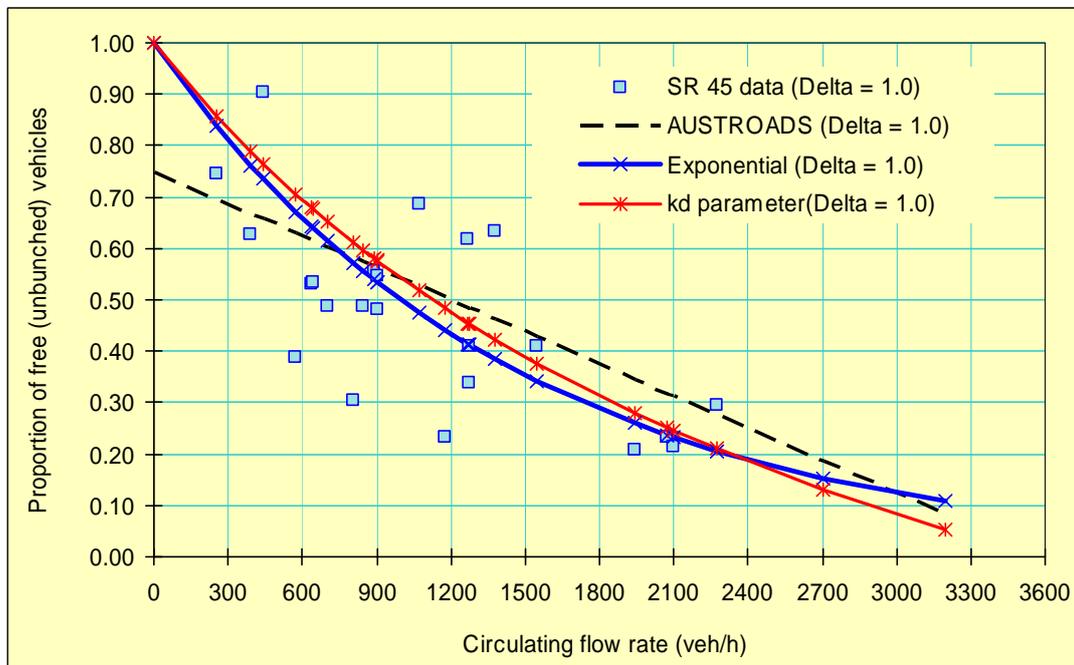

Figure 4 - Proportion unbunched for two-lane circulating streams at roundabouts as a function of the circulating flow rate (measured and estimated by alternative bunching models)

4 UNSIGNALISED INTERSECTION ANALYSIS BY TRAFFIC SIGNAL ANALOGY

A method for treating the traditional gap-acceptance modelling used for roundabouts and sign-controlled intersections by analogy to traffic signal operations was conceived by Akçelik (1991). The underlying assumptions are shown in *Figure 5* which depicts an entry (minor) stream at an unsignalised intersection giving way to an uninterrupted opposing (major) stream.

The method presented here derives equivalent average red, green and cycle times (r, g, c) for the gap-acceptance process considering average durations of block and unblock periods (t_b, t_u) in major streams as used in the traditional gap acceptance modelling. The equivalent average red, green and cycle times are referred to as *effective blocked time*, *effective unblocked time* and *gap-acceptance cycle time*, respectively.
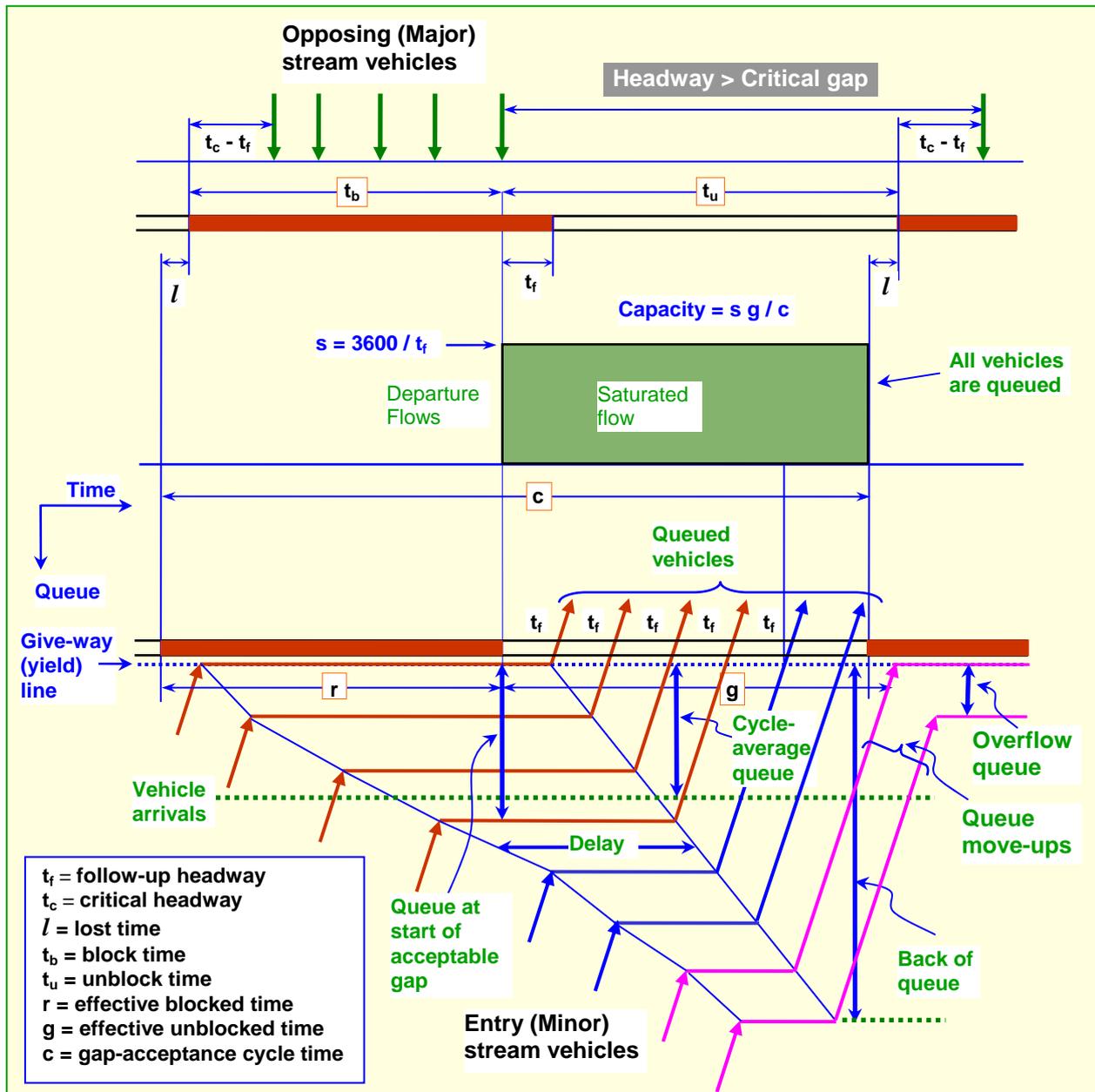

Figure 5 - Gap-acceptance capacity signal analogy concept

Block periods correspond to continuous periods of no acceptable gap, i.e. consecutive major stream headways less than the mean critical gap (t_c). Unblock periods correspond to headways equal to or greater than the critical gap, $h_i \geq t_c$, where h_i is the *ith* acceptable headway (gap) in the major stream. In accordance with the definition used in the traditional gap-acceptance theory, the duration of the unblock period is $t_{ui} = h_i - t_c$ (where $h_i \geq t_c$). This relationship can be explained by assuming that (i) the first minor stream vehicle departs t_f seconds after the start of the acceptable headway, and (ii) there cannot be any departures during the last ($t_c - t_f$) seconds of the acceptable headway. Parameter t_f represents the follow-up (saturation) headway.

The *effective unblocked time* (equivalent green time), g_i, includes the first t_f seconds of the acceptable headway (or unblock period). However, it is shorter than the unblock period by an amount called lost time (l_i) which cannot be used for any vehicle departures. This is because the number of vehicles (n_i) that can depart during an acceptable headway is assumed to be an integer: $g_i = n_i t_f$. Therefore, $g_i = t_{ui} + t_f - l_i = h_i - (t_c - t_f) - l_i$. The average value of the lost time is $l = 0.5 t_f$. This was confirmed by simulation results (using MODELC).

Similarly, the *effective blocked time* (equivalent red time), r_i, is related to the *ith* block period through $r_i = t_{bi} - t_f + l_i$.

The *gap-acceptance cycle time* is the sum of effective blocked and unblocked times, and is also equal to the sum of gap-acceptance block and unblock periods: $c_i = r_i + g_i = t_{bi} + t_{ui}$.

The *average capacity per cycle* is obtained as s g $= g / t_f$ where g is the effective unblocked time and t_f is considered to be a saturation headway ($s = 1 / t_f$ in veh/s, or $s = 3600 / t_f$ in veh/h). The entry stream capacity based on the gap-acceptance process can then be expressed as $Q_g = s g / c$, or $Q_g = s u$ where $u = g / c$ is the unblocked time ratio (equivalent green ratio), as in the case of signalised intersections.

The estimates of the average values of block and unblock periods (t_b, t_u), the effective blocked and unblocked times (r, g), the gap-acceptance cycle time (c), and the corresponding gap-acceptance capacity are given by *Equations (4.1) to (4.7)*.

All capacity and performance calculations are carried out for individual lanes of entry (minor) movements, but traffic in all lanes of the major (conflicting) movement is treated together as one stream. When there are several conflicting (higher priority) streams at sign-controlled and signalised intersections, all conflicting streams are combined as one stream. The resulting total opposing flow rate, q_m may be expressed in passenger car units (pcu) allowing for the effect of heavy vehicles in the opposing stream(s). In the following equations, q_m is in veh/s or pcu/s.

Average durations of block and unblock periods (seconds):

$$t_b \quad = e^{\lambda (t_c - \Delta_m)} / (\varphi \, q_m) - (1 / \lambda) \tag{4.1a}$$

$$t_u \quad = 1 / \lambda \tag{4.1b}$$

where λ is from *Equation (2.1a)*.

Average effective blocked and unblocked times (seconds):

$$r \quad = t_b - t_f + l = e^{\lambda (t_c - \Delta_m)} / (\varphi_m \, q_m) - (1 / \lambda) - t_f + l \tag{4.2a}$$

$$g \quad = t_u + t_f - l = (1 / \lambda) + 0.5 \, t_f \tag{4.2b}$$

where $l = 0.5 t_f$.

Average gap acceptance cycle time (seconds):

$$c \quad = r + g = e^{\lambda (t_c - \Delta_m)} / (\varphi_m q_m) \tag{4.3}$$

Unblocked time ratio:

$$u \quad = g / c = (1 - \Delta_m q_m + 0.5 \varphi_m q_m t_f) e^{-\lambda (t_c - \Delta_m)} \tag{4.4}$$

Entry stream saturation flow rate, s (veh/h):

$$s \quad = 3600 / t_f \tag{4.5}$$

Gap-acceptance capacity (veh/h):

$$\begin{aligned} Q_g \quad &= Q_g = s\, u = (3600 / t_f)\, u \\ &= (3600 / t_f)\, (1 - \Delta_m q_m + 0.5 \varphi_m q_m t_f)\, e^{-\lambda (t_c - \Delta_m)} \end{aligned} \tag{4.6}$$

Entry stream capacity (veh/h):

$$Q \quad = max\, (Q_g, Q_m) \tag{4.7a}$$

where Q_m is the minimum capacity (veh/h) given by:

$$Q_m \quad = min\, (q_e, 60\, n_m) \tag{4.7b}$$

where q_e is the entry stream flow rate (veh/h), and n_m is the minimum number of entry stream vehicles that can depart under heavy major stream flow conditions (veh / min).

When there are several opposing (higher priority) streams, the total major stream flow (q_m) is calculated as the sum of all conflicting stream flows and parameters Δ_m and φ_m are determined accordingly. *Equations (4.1) to (4.7)* should be used for $q_m > 0$. For $q_m = 0$, the following estimates should be used:

$$r = 0,\ g = c,\ u = 1.0,\ \text{and}\ Q_g = 3600 / t_f \tag{4.8}$$

A comparison of the capacities estimated by *Equation (4.6)* with those simulated by MODELC given in Akçelik and Chung (1994b) indicated very good match between the analytical and simulation model estimates.

An example of effective blocked and unblocked times and the gap-acceptance cycle time as a function of the major stream flow rate is given in *Figure 6* for the case of a simple gap acceptance situation with a four-lane uninterrupted major stream with $t_c = 6.0$ s, $t_f = 3.6$ s. The bunched exponential model with the intrabunch headway, $\Delta_m = 0.6$ s and delay parameter, $k_d = 0.3$ (from *Table 1*) was used for *Figure 6*. The unblocked time ratio as a function of the major stream flow rate for the same example is shown in *Figure 7*.
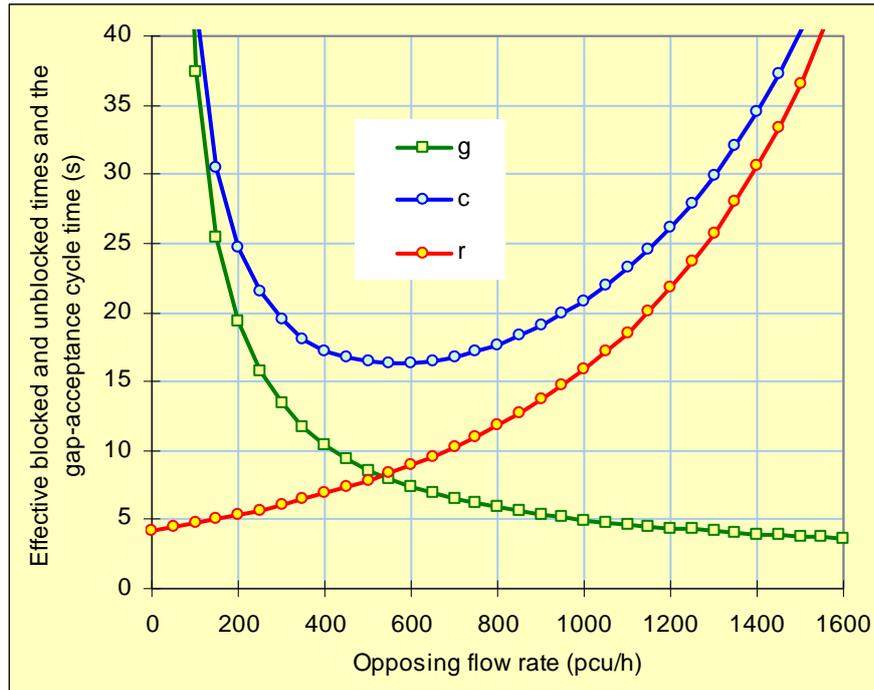
Figure 6 - The effective blocked and unblocked times and the gap-acceptance cycle time as a function of the major stream flow rate: four-lane uninterrupted major stream with t_c = 6.0 s, t_f = 3.6 s, Δ_m = 0.6 s and k_d = 0.3 (M3D bunched headway distribution model)
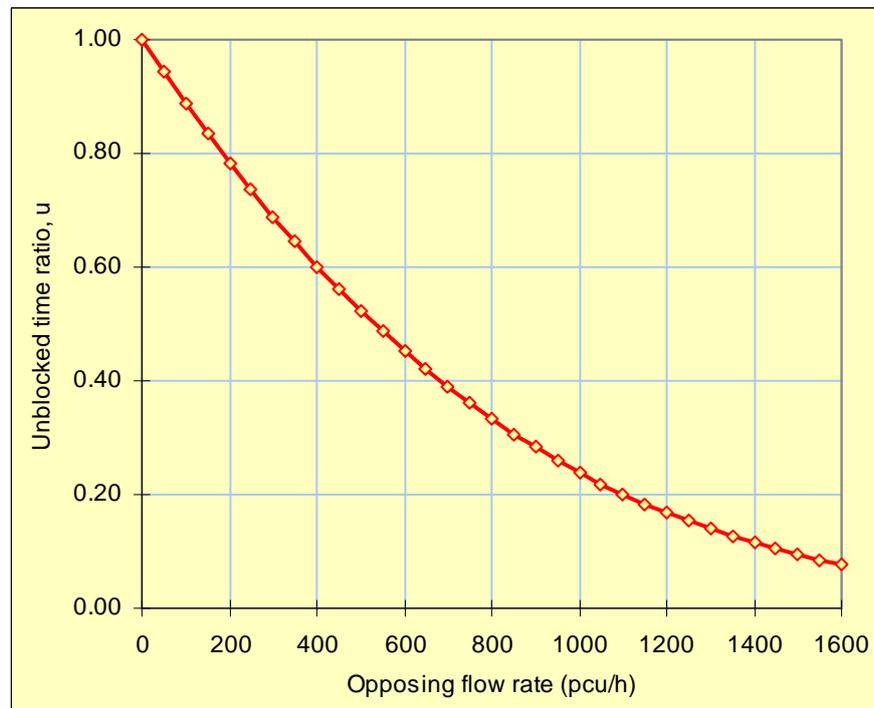

Figure 7 - The unblocked time ratio as a function of the major stream flow rate corresponding to Figure 6

5   GAP-ACCEPTANCE CAPACITY MODELS

Many different gap-acceptance capacity models exist in the literature. Further models can be generated using different arrival headway distributions (*Section 2*) in the gap-acceptance capacity model based on traffic signal analogy (*Equation 4.6*). The models are presented in this section, and model comparisons are given in *Section 6*.

For easy reference, the models are named in the form "Author name (indicating a different function form) - Headway Distribution symbol", e.g. Akçelik - M3D.

The following conditions apply to the key gap-acceptance parameters used in the capacity equations given in this section (e.g. see Luttinen 2003, Section 2.3):

(i)   follow-up headway and critical gap: $t_f < t_c$ (a general rule of thumb is $t_f / t_c = 0.6$);

(ii)   follow-up headway and intrabunch headway: $t_f > \Delta$ (otherwise the entry stream saturation flow rate would be higher than the opposing flow capacity); and

(iii)   follow-up headway, critical gap and intrabunch headway: $t_f + \Delta > t_c$ (otherwise, *priority sharing* between entering and circulating vehicles applies, and a correction is needed to the gap-acceptance capacity formula based on *absolute priority* of circulating stream vehicles (in the SIDRA INTERSECTION roundabout model, low critical gap values are quite common especially at high circulating flow rates).

For discussions on *priority sharing*, refer to Troutbeck and Kako (1997), Troutbeck (1999, 2002) and (Akçelik 2004). The process of *priority emphasis* (opposite of priority sharing) in the case of unbalanced flow patterns at roundabouts is discussed in Akçelik (2004).

The opposing flow rate, q_m in the equations given in this section is in vehicles per *second* or passenger car units per *second* (veh/s or pcu/s). The latter assumes an increased opposing flow rate to allow for heavy vehicles. The capacity values, Q_g estimated by these equations are in vehicles per hour (veh/h).

Each model implies an unblocked time ratio, u which could be determined from $u = Q_g / (3600 / t_f)$. In the case of the Akçelik models, the unblocked time ratio is modelled explicitly as seen from *Equations (4.4) and (4.6)*.

Capacity Models Based on Traffic Signal Analogy

The gap-acceptance capacity models based on *Equation (4.6)*, derived by Akçelik (1994) using the traffic signal analogy concept, are expressed below for different arrival headway distributions.

Akçelik - M3D Model

For the Akçelik - M3D model, the bunched exponential distribution is used with the bunching model to determine φ_m from *Equation (3.2)* using Δ_m and k_d values given in *Table 1*:

$$Q_g \quad = (3600 / t_f) (1 - \Delta_m q_m + 0.5 \, \varphi_m q_m t_f) \, e^{-\lambda \, (t_c - \Delta_m)} \tag{5.1}$$

The SIDRA INTERSECTION software uses this model. Version 2.0 and the earlier versions of the software used the *Akçelik - M3A Model*, i.e. *Equation (5.1)* with the exponential bunching model to determine φ_m from *Equation (3.1)* using Δ_m and b values given in *Table 1*.

Akçelik - M3T Model

For the Akçelik - M3T model, the bunched exponential distribution is used with the Tanner bunching model to determine φ_m from *Equation (3.3a)* with Δ_m values given in *Table 1*:

$$Q_g = (3600 / t_f) (1 - \Delta_m q_m) (1 + 0.5 q_m t_f) e^{-q_m (t_c - \Delta_m)} \tag{5.2}$$

Akçelik - M1 Model

For the Akçelik - M1 model, the simple negative exponential model of headway distribution (M1) is assumed using $\Delta_m = 0$, $\varphi_m = 1.0$ and $\lambda = q_m$ as in *Equation (2.2b)*:

$$Q_g = (3600 / t_f) (1 + 0.5 q_m t_f) e^{-q_m t_c} \tag{5.3}$$

Akçelik - M2 Model

For Akçelik - M2 model, the shifted negative exponential model of headway distribution (M2) is assumed using $\varphi_m = 1.0$ and $\lambda = q_m / (1 - \Delta_m q_m)$ as in Equation (2.3b):

$$Q_g = (3600 / t_f) (1 - \Delta_m q_m + 0.5 q_m t_f) e^{-q_m (t_c - \Delta_m) / (1 - \Delta_m q_m)} \tag{5.4}$$

Results for this group of models is given in *Figure 8* for the example used for *Figure 6*.
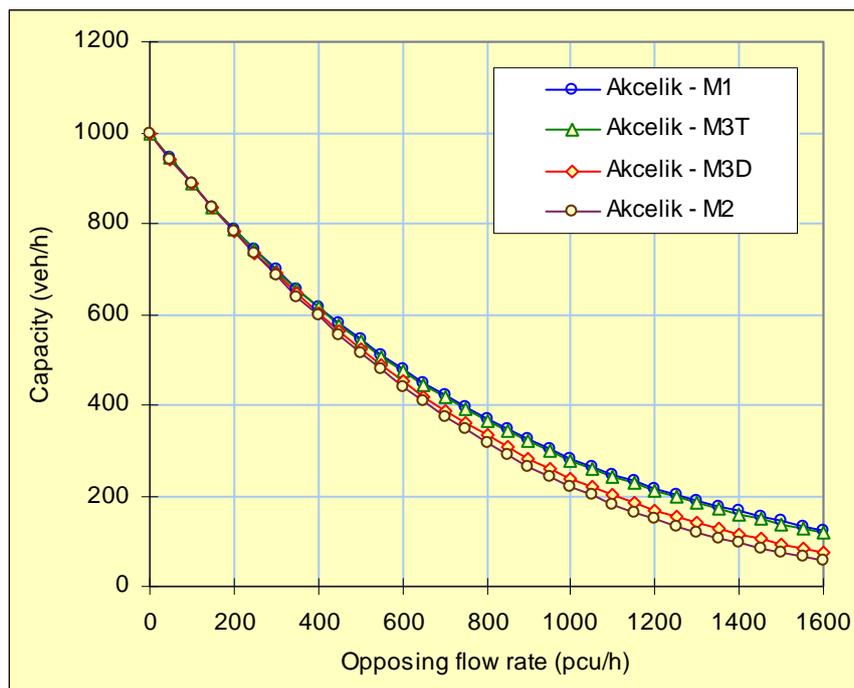

Figure 8 - Capacity as a function of the major stream flow rate estimated using Akçelik model with arrival headway distributions M1, M2, M3T and M3D for the same gap acceptance example as in Figure 6

Models similar to Akçelik Gap-Acceptance Capacity Model

The following models used in the literature are considered for comparison with those based on traffic signal analogy (*Equations 5.1 to 5.4*).

Siegloch - M1 Model

The Siegloch (1973) capacity model, which is used in the German guidelines (Brilon 1988, Brilon and Grossman 1991), assumes a negative exponential model of arrival headways (M1), and is given as:

$$Q_g = (3600 / t_f) \ e^{-q_m t_o} \tag{5.5a}$$

where t_o is the zero-gap parameter given by:

$$t_o = t_c - 0.5 \ t_f. \tag{5.5b}$$

The concept of zero-gap parameter is related to a gap-acceptance survey method attributed to Siegloch (Brilon and Grossman 1991, TRB 1997, Brilon, Koenig and Troutbeck 1997). The method is shown by means of an example in *Figure 9*. This relatively simple method requires queued conditions of the entry (minor) stream since the critical gap and follow-up headway parameters are relevant to capacity estimation. The method is implemented as follows:

- Make observations during times when there is, without interruption, at least one vehicle queuing in the minor street. A reasonably high number of queued vehicles is needed for a reliable regression.

- Record the number of vehicles, n, entering each main stream gap (headway) of duration t (including n = 0 cases).

- For each of the gaps accepted by n vehicles, compute the average of the accepted gaps t (circles in the graph).

- Find the linear regression of the average gap (headway) values as a function of the number of vehicles:

$$t = t_o + t_f \ n \tag{5.6a}$$

  where

$$t_o = t_c - 0.5 \ t_f \tag{5.6b}$$

  therefore

$$t_c = t_o + 0.5 \ t_f \tag{5.6c}$$

  and t_f is given by the regression directly.

In the example shown in *Figure 9*, $t_o = 3.73$ s, $t_f = 2.31$ s, and $t_c = 4.89$ s.

The Siegloch capacity model is seen to be similar to the Akçelik - M1 model *(Equation 5.3)*. The use of t_o instead of t_c, and omission of the factor $(1 + 0.5 \ q_m \ t_f)$ tend to compensate, and *Equations (5.3) and (5.5a)* give close values.

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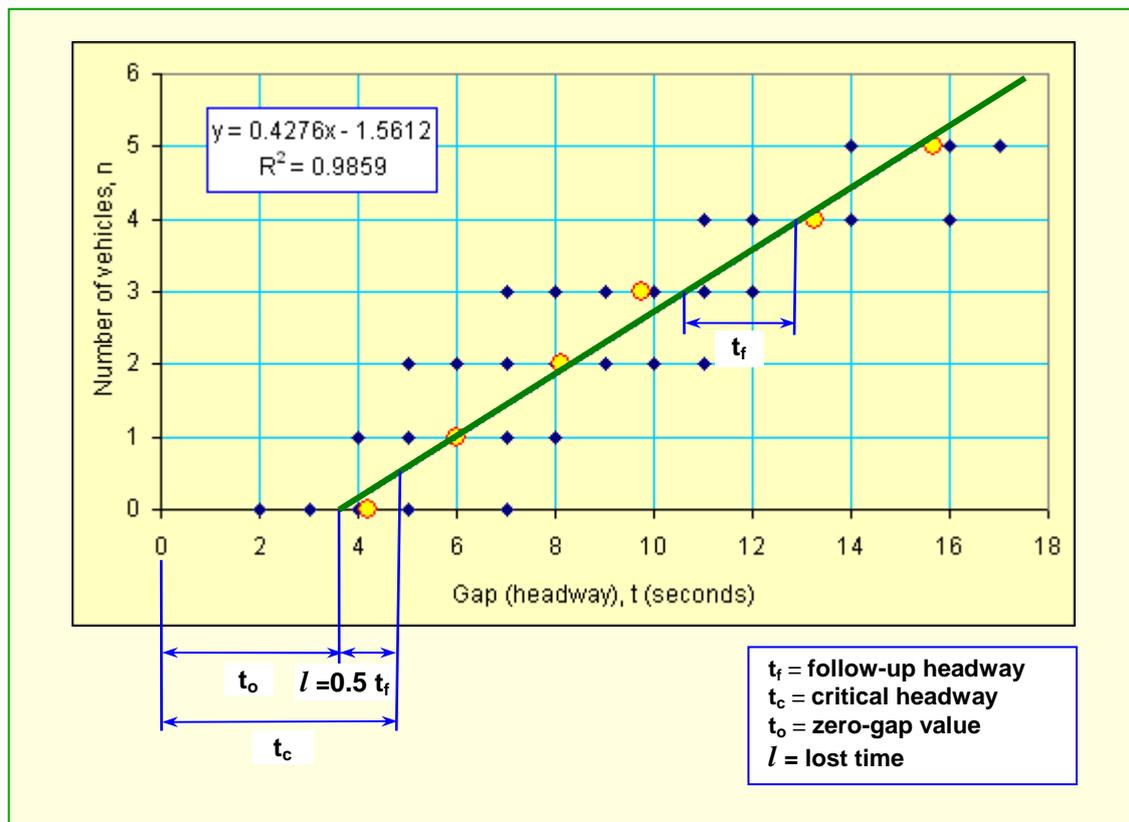**www.sidrasolutions.com**

Figure 9 - Critical gap and follow-up headway survey method

McDonald and Armitage - M3T Model

McDonald and Armitage (1978) used a gap-acceptance survey method (Fig. 1 in McDonald and Armitage 1978), which is similar to the Siegloch method described above. They used the concept of saturation flow (q_s) and lost time (L) for estimating roundabout capacities with a degree of traffic signal analogy. However, they did not equate the saturation flow with (3600 / t_f), and their lost time definition (L) is rather different from the lost time (l) used in this paper. Their survey method gives a saturation flow close to (3600 / t_f) and the lost time they measure (L) is identical to the zero-gap (t_o) parameter used by Siegloch (1973). Thus, putting $l = 0.5\ t_f$ as in *Equation (4.2b)*, the Siegloch/McDonald-Armitage method can be related to the method described in this paper through $t_o = t_c - l$.

Putting $q_s = 3600 / t_f$ and $L = t_o$, McDonald and Armitage (1978) capacity formula can be expressed as:

$$Q_g \quad = (3600\ /\ t_f)\ (1 - \Delta_m\ q_m)\ e^{-q_m\ (t_o\ -\ \Delta_m)} \tag{5.6}$$

This is similar to *Equation (5.2)* based on the M3T model, differences being similar to those noted for the Siegloch formula (*Equation 5.3*) and *Equation (5.5a)*.

Jacobs - M2 Model

Jacobs (1979) capacity model based on a shifted negative exponential distribution (M2 model) as described by Brilon (1988) is:

$$Q_g = (3600 / t_f) (1 - \Delta_m q_m)\ e^{-\lambda (t_o - \Delta_m)} \tag{5.7}$$

where $\lambda = q_m / (1 - \Delta_m q_m)$.

This is seen to be similar to *Equation (5.4)*, again, differences being similar to those noted for the Siegloch formula (*Equation 5.3*) and *Equation (5.5a)*.

Results for this group of models are given in *Figure 10* for the example used for *Figure 6*.
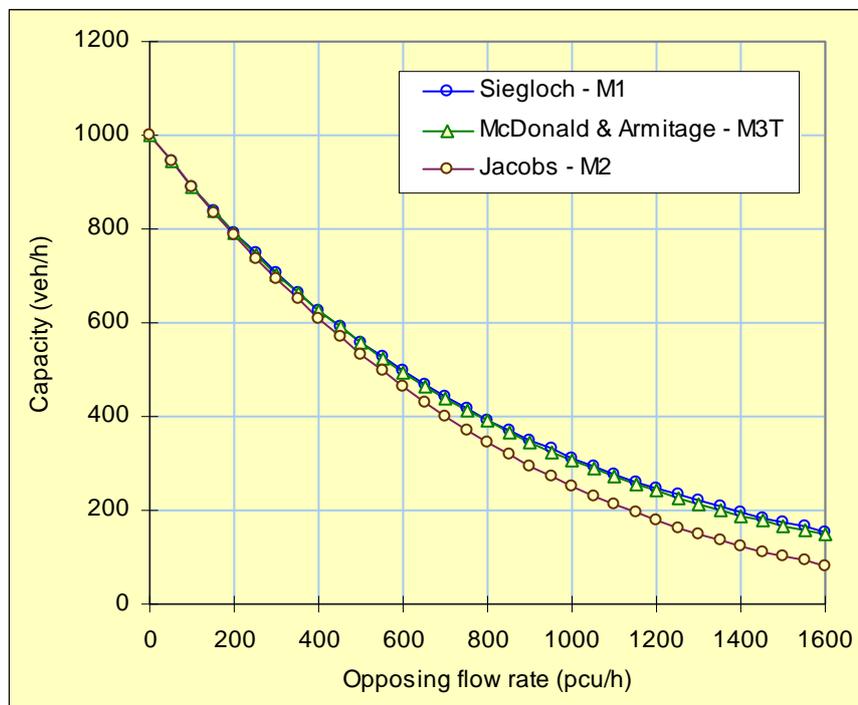

Figure 10 - Capacity as a function of the major stream flow rate estimated using various models with arrival headway distributions M1, M2 and M3T for the same gap acceptance example as in Figure 6

Traditional Models

A more traditional capacity formula based on gap-acceptance modelling (Tanner 1962, 1967; Troutbeck 1986, 1989) can be expressed in the following general form:

$$Q_g = 3600 \; \varphi_m \; q_m \; e^{-\lambda \, (t_c - \Delta_m)} / (1 - e^{-\lambda \, t_f}) \qquad \text{for } q_m > 0 \qquad (5.8)$$
$$= (3600 / t_f) \qquad\qquad\qquad\qquad\qquad \text{for } q_m = 0$$

Various capacity formulae found in the literature can be generated from *Equation (5.7)* by applying different arrival headway distributions. These are presented below. These models are given the names "Traditional - M1","Traditional - M3T" due to their historical development, rather than using a specific author name. However, it should be noted that this general function form was developed by Troutbeck (1986, 1989). The model has been used in the AUSTROADS (1993) roundabout model with the bunching model from *Equation (3.4)*.

Although a capacity model could also be derived for the shifted negative exponential headway distribution with the Traditional model function form using $\varphi_m = 1.0$ and $\lambda = q_m / (1 - \Delta_m \, q_m)$ as in *Equation (2.3b)*, this model is not considered for comparisons in this paper.

Traditional - M3D Model

For the Traditional - M3D model, *Equation (5.8)* is used with the bunched exponential distribution using the bunching model to determine φ_m from *Equation (3.2)* with parameters Δ_m and k_d from *Table 1*.

Traditional - M3T Model

The Traditional - M3T model is the model developed by Tanner (1962, 1967). This model can be derived *Equation (5.8)* by using the bunched exponential distribution with the Tanner bunching model to determine φ_m from *Equation (3.3a)*:

$$Q_g = 3600 \; q_m \; (1 - \Delta_m \, q_m) \; e^{-q_m \, (t_c - \Delta_m)} / (1 - e^{-q_m \, t_f}) \qquad \text{for } q_m > 0 \qquad (5.9)$$
$$= (3600 / t_f) \qquad\qquad\qquad\qquad\qquad\qquad \text{for } q_m = 0$$

This model was used for roundabout capacity estimation in the older AUSTROADS (1988) capacity guide, which recommended $\Delta_m = 2.0$ s for single-lane circulating flow and $\Delta_m = 0$ for multi-lane circulating flows. The latter is equivalent to the Traditional - M1 model given below.

Traditional - M1 Model

The Traditional - M1 model is based on the simple negative exponential model of headway distribution (M1), and can be derived from *Equation (5.8)* by using $\Delta_m = 0$, $\varphi_m = 1.0$ and $\lambda = q_m$ as in *Equation (2.2b)*:

$$Q_g = 3600 \; q_m \; e^{-q_m \, t_c} / (1 - e^{-q_m \, t_f}) \qquad\qquad \text{for } q_m > 0 \qquad (5.9)$$
$$= (3600 / t_f) \qquad\qquad\qquad\qquad\qquad\qquad \text{for } q_m = 0$$

This model is specified for unsignalised intersections in the AUSTROADS (1988) capacity guide, as well as the more recent AUSTROADS (2005) guide for intersections at grade.

The model is also used in the HCM (TRB 2000, Chapter 17) as the *potential capacity* for Two-Way Stop Control (used in the HCM version of SIDRA INTERSECTION). It should be noted that HCM applies *impedance factors* that reduce the potential capacity for entry streams that give way to movements which themselves are subject to gap-acceptance. In determining the opposing flow rates, HCM also applies other factors to increase some opposing flow rates (the

flow rate of opposed turns from the major road are doubled), therefore decreasing the potential capacity significantly.

Results for this group of models are given in *Figure 11* for the example used for *Figure 6*.
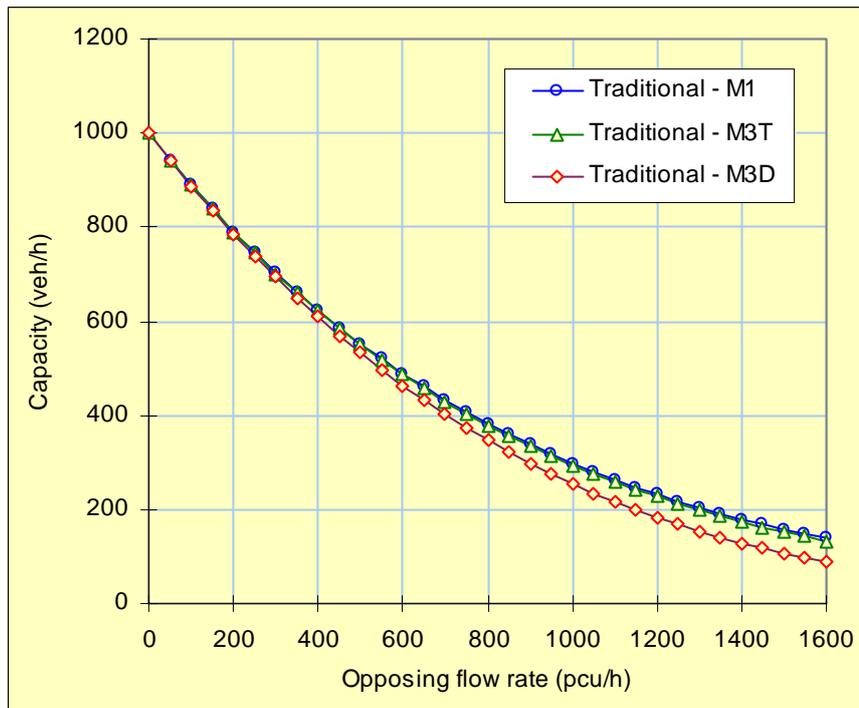

Figure 11 - Capacity as a function of the major stream flow rate estimated using various traditional models with arrival headway distributions M1, M3T and M3D for the same gap acceptance example as in Figure 6

6 CAPACITY MODEL COMPARISONS

Comparisons of some of the gap-acceptance capacity models given in *Section 5* are presented in this section. Comparisons are made using the basic gap-acceptance capacity model without allowance for the minimum capacity at high opposing flow rates. No adjustment for heavy vehicle effects is implied except the use of the opposing flow rate in passenger car units. Staged crossing or upstream signal effects are not considered.

The intrabunch headway values, Δ_m given in *Table 1* are used in all models that use this parameter for comparisons.

Comparisons of capacity models are given in *Figure 12* for the example used for *Figure 6* (a four-lane uninterrupted major stream with $t_c = 6.0$ s, $t_f = 3.6$ s, $\Delta_m = 0.6$ s and $k_d = 0.3$ for the bunched exponential model M3D).

The results shown in *Figure 8* and *Figures 10 to 12* indicate that:

(a) there is little difference between models for low major stream flows but the difference increases with the increasing opposing flow rate;

(b) the differences among models which use the same arrival headway distribution are negligible;

(c) the headway distribution M1 tends to give the highest capacity estimates whereas the headway distribution M2 tends to give the lowest capacity estimates, and the M3D distribution tends to give lower estimates than the M1 and M3T distributions;

(d) the headway distributions M1 and M3T tend to give close results; and

(e) the impact of the assumption about the arrival headway distribution is significant at high major stream flow levels.
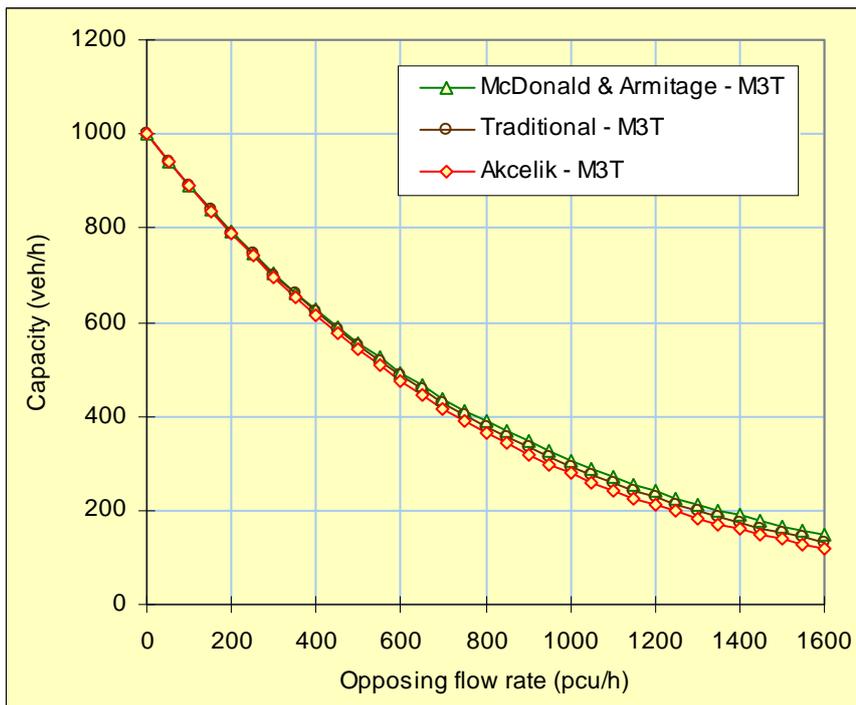
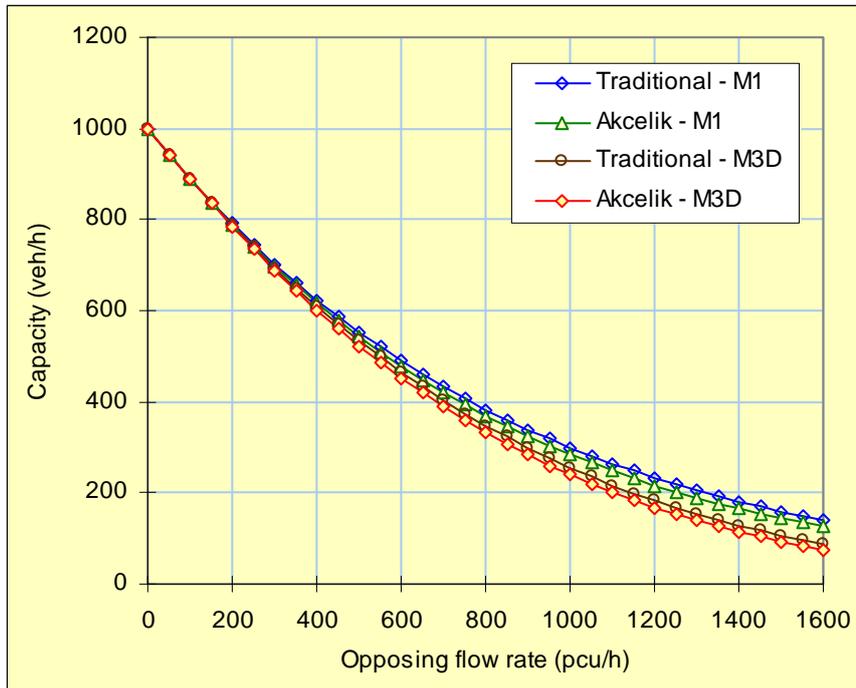Further discussion is given in *Section 7*.

Figure 12 - Capacity as a function of the major stream flow rate estimated using different models with M3D, M3T and M1 arrival headway distributions for the same gap acceptance example as in Figure 6

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7 CONCLUDING REMARKS

A summary of various gap-acceptance capacity models and comparisons of capacity estimates from these models have been presented in previous sections. The results show that there is little difference between models for low major stream flows, the differences among models which use the same arrival headway distribution are negligible, but differences in capacity estimates increase at high opposing flow rates.

The difference between the Traditional - M1 model used by the AUSTROADS (1988, 2005) and the Akçelik - M3D model used in SIDRA INTERSECTION is of interest. The capacity estimates from the Traditional - M1 model are higher at high opposing flow rates. The Traditional - M1 model assumes random arrivals with no bunching in contrast with the bunched headways model used by the Akçelik - M3D model. The assumption of no bunching cannot be supported especially at high opposing flow rates where vehicles are highly bunched. Another aspect of the Traditional - M1 model is that it is not sensitive to the number of opposing movement lanes, which is a shortcoming since the same opposing flow rate in more lanes means better gap-acceptance opportunities.

The amount of bunching estimated by the bunching model (*Section 3*) is an important parameter in estimating capacity using the bunched headway distribution models (M3D, M3A, M3T). Capacity increases with increased proportion bunched. The use of extra bunching for sign control applications (similar to the method used in SIDRA INTERSECTION for roundabouts) would allow model calibration. For example, the use of extra bunching of 15 per cent, results in the Akçelik - M3D model to estimate capacity values close to those from the Traditional - M1 model for high opposing flow rates for the example given in this paper.

While the differences between capacity estimates from different models at high opposing flow rates are small in terms of absolute values, they are large in terms of percentage (relative) values. For example, at an opposing flow rate of 1200 pcu/h for the example given in this paper, the capacity estimate is 167 veh/h for the Akçelik - M3D model is and 232 veh/h for the Traditional - M1 model, hence a difference of 65 veh/h. This corresponds to 39 per cent relative to the Akçelik - M3D model estimate. In a case where the entry movement flow rate is 200 veh/h, these results mean degrees of saturation of 1.2 (oversaturated) vs 0.9 (undersaturated). High percentage differences create uncertainties for the practitioners in the analysis of sign-controlled intersections, in particular because there are also uncertainties in the selection of critical gap and follow-up headway parameters.

While various documents, e.g. HCM (TRB 2000) and AUSTROADS (2005) provide guidance in the selection of the critical gap and follow-up headway parameters, they do not cover all possible factors affecting these parameters.

Research is recommended on capacity models for sign-controlled intersections in order to:

- assess alternative models using real-life capacity data,
- further calibrate headway distributions (bunching) models using real-life data representing different conditions,
- develop more detailed models for the estimation of critical gap and follow-up headway parameters as a function of the intersection geometry and traffic characteristics such as number of lanes, movement type, heavy vehicles, grade (for entry and major stream movements), major road speed, stop-sign vs give-way sign, restricted sight distance, and

delay time experienced by entry stream vehicles (or opposing flow level similar to the roundabout model in SIDRA INTERSECTION).

It should also be noted that the traffic signal analogy method described in *Section 4* (used in SIDRA INTERSECTION) provides parameters (r, g, c, u) for use not only in the capacity model but also directly in performance equations for unsignalised intersections (delay, average and percentile values of back of queue, queue move-up rate, effective stop rate, proportion queued, queue clearance time, and so on). This provides adoption of a consistent modelling framework for the comparison of different types of intersections. This is not generally available, for example HCM (TRB 2000) uses the back of queue for signalised intersections while it uses the cycle-average queue based on traditional queuing theory for two-way stop control.

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APPENDIX A

SHIFTED DELAY PARAMETER MODEL FOR BUNCHING

A recent paper by Vasconcelos, Seco, and Silva (2011) proposed the following *shifted linear* model for bunching:

$$\varphi = (1 - \Delta q) / (1 - A) \qquad \text{for } q > A / \Delta \qquad (A.1)$$
$$= 1.0 \qquad \text{otherwise}$$

For roundabouts in Portugal, Vasconcelos, et al (2011) found $A = 0.356$ with $\Delta = 2$ s.

The model can also be expressed as:

$$\varphi = (1 - \Delta q) / (1 - \Delta q_o) \qquad \text{for } q > q_o \qquad (A.2)$$
$$= 1.0 \qquad \text{otherwise}$$

where q_o is the limiting flow rate (veh/s) above which $\varphi < 1.0$.

For roundabouts in Portugal, $A = 0.356$ and $\Delta = 2$ s correspond to $q_o = 0.178$ veh/s (641 veh/h).

The shifted linear model gives the Tanner linear model (*Equation 3.3a*) for $q_o = 0$.

The following *shifted delay parameter (k_d)* model, as a variation of the original delay parameter model expressed by *Equation (3.2),* can be used instead of the shifted linear model:

$$\varphi = [(1 - \Delta q) / (1 - \Delta q_o)] / [1 - (1 - k_d) \Delta (q - q_o)] \qquad \text{for } q > q_o \qquad (A.3)$$
$$\text{subject to } 1.0 \geq \varphi \geq 0.10$$
$$= 1.0 \qquad \text{otherwise}$$

As in *Equation (3.2),* the minimum value of $\varphi_{min} = 0.10$ is used for computational reasons.

The shifted delay parameter model (*Equation A.3*) is equivalent to the original delay parameter model (*Equation 3.2*) for $q_o = 0$. The model becomes the same as the shifted linear model (*Equation A.2*) for $k_d = 1.0$.

Figure A.1 shows an example of proportion unbunched (free) for a one-lane stream ($\Delta = 2.0$ s) for the models described above (φ_{min} not applied).
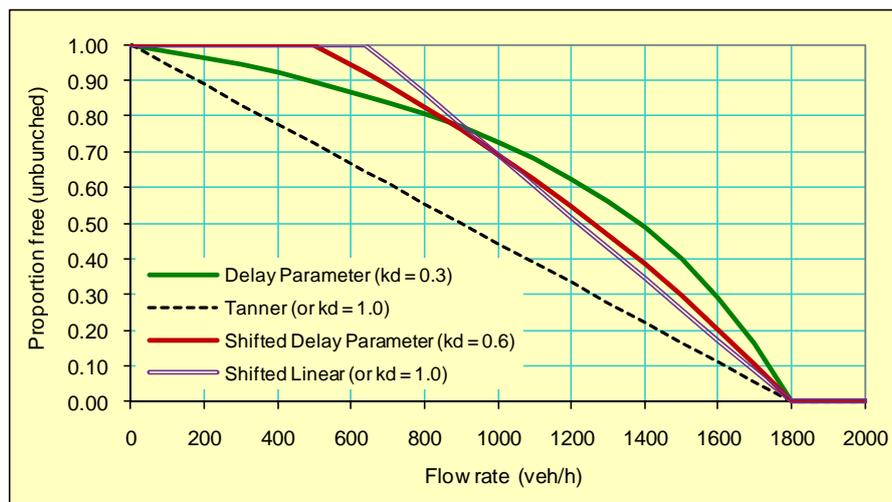

Figure A.1 - Proportion unbunched for a one-lane stream ($\Delta = 2.0$) for linear, shifted linear, delay parameter and shifted delay parameter models