Strategic Traffic Assignment: Models and Applications to Capture Day-to-Day Flow Volatility

by

Melissa Dee Duell, B.A., B.S., Ph.D.

A thesis presented in fulfilment of the requirements for the degree of

Doctor of Philosophy



UNSW
AUSTRALIA

School of Civil and Environmental Engineering

The University of New South Wales

Accepted in April 2015

We are all inventors, each sailing out on a voyage of discovery, guided each by a private chart, of which there is no duplicate. The world is all gates, all opportunities.

Ralph Waldo Emerson

STATEMENT OF ORIGINALITY

I hereby declare that this submission is my own work and to the best of my knowledge it contains no materials previously published or written by another person, or substantial proportions of material which have been accepted for the award of any other degree or diploma at UNSW or any other educational institution, except where due acknowledgement is made in the thesis. Any contribution made to the research by others, with whom I have worked at UNSW or elsewhere, is explicitly acknowledged in the thesis. I also declare that the intellectual context of this thesis is the product of my own work, except to the extent that assistance from others in the project's design and conception or in style, presentation, and linguistic expression is acknowledged.

Signed _____

Date _____

ACKNOWLEDGEMENTS

As the saying goes, it takes a village to write a PhD! …well, maybe that's not how the saying goes, but it seems to apply in this situation. While of course the final document is my own, I was fortunate enough to be surrounded by peers, colleagues, supervisors, friends, and family who were so supportive that I only occasionally felt that I was undertaking an impossible task. So an enormous *thanks* to all the people who helped me through this journey, my friends from near and far who are too numerous to name – you've made my PhD experience fun and fulfilling.

Next, thanks to my supervisors Travis and Lauren, who among much invaluable advice, pointed out that I probably wouldn't be very happy if I ran away to become a brick-layer in Alaska. Of course, the opportunity to move to Australia for my PhD has had immeasurable impact on my development as a human being. I don't have the words to express how grateful I am, how much it means to me, but I'm going to try anyway: thank you. For all the talks and career advice and life advice and giving me the opportunity to see the world and visit my family and the encouragement to pursue my dreams and everything else that won't fit into this short paragraph, all I can say is thank you, and: I was listening.

My family more than deserves their name in the spotlight for keeping my head from becoming too skewed by an engineer's sense of humor and keeping me sane through an international relocation (looking at you Cary, Hillarie, Colin!). Finally, thanks to my parents for helping me to through this monumental undertaking with

lots of late night (for them) phone calls, action-packed visits to exotic and not-so-exotic places (thanks Dad and Teresa!), and emergency apple crisp (thanks Terry!). In particular, thanks to my mom for her endless support and encouragement, for humoring me during the hard moments when I was certain *this time really was* the time I had failed a test or presentation and it was just the most terrible thing, even though of course, it wasn't, and also for being there through all of the good stuff that in the end far out-numbered the bad. Again, words fail me but I want every person who reads this thesis until the end of time to know: I have the greatest mom in the world.

Rock on –

Melissa Duell December 2014

Strategic Traffic Assignment: Models and Applications to Capture Day-to-Day Flow Volatility

Melissa Duell, Ph.D.

The University of New South Wales, April 2015

Supervisors:

Prof. S. Travis Waller

Dr. Lauren Gardner

ABSTRACT

Traffic assignment models continue to play a critical role in the transportation planning process. Furthermore, day-to-day traffic flow volatility is a well-acknowledged phenomenon that planners and researchers alike view as increasingly important. Consequentially, current research advances have been addressing more complex assignment models capable of representing various aspects of volatility. However despite the importance of accounting for volatility, deployed assignment models capable of large-scale application have continued relying on traditional assumptions of determinism and perfect information.

This research focuses on the impact of day-to-day demand uncertainty on equilibrium-based traffic models by advancing the concept of *strategic traffic assignment*. In the strategic user equilibrium (StrUE) model, the daily travel demand is treated as a random variable, and users are assumed to have knowledge about the day-to-day demand but are unaware of the specific traffic conditions they will experience during travel. Therefore, drivers make a strategic route choice to minimize their expected travel cost and follow that route independent of the experienced conditions. The result is an equilibrium assignment based on link flow proportions, as opposed to link flow volumes. Furthermore, as the day-to-day demand realization changes, the equilibrium flow proportions will remain the same. Thus, the resulting flows may appear volatile on a day-to-day basis, but can actually be represented by a higher level mathematical equilibrium.

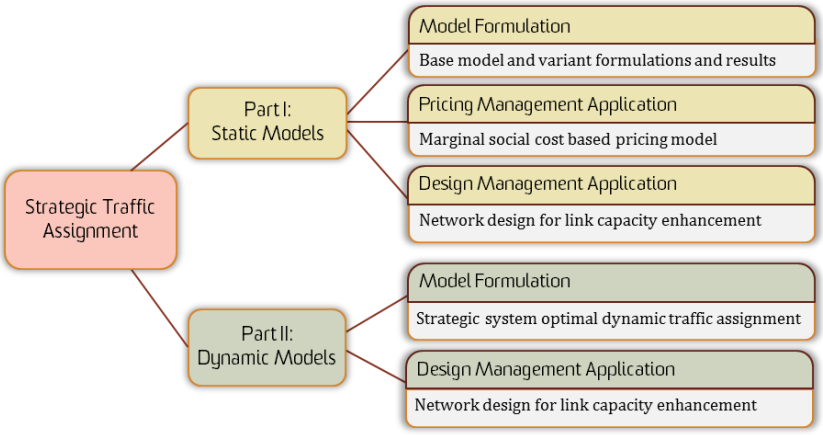


Figure 1. Research map summarizing the proposed work

The strategic concept has profound modelling implications. Part I of this thesis explores static models of strategic traffic assignment, including the user

equilibrium, the strategic system optimal, and the strategic system reliable models. Each of the models is formulated, and model assumptions and solution methods are discussed. The performance of each model is then demonstrated and compared on a number of test networks, ranging in size from small to large.

However, strategic traffic assignment is not only significant as a modelling approach, but also for the implications of the model in important network management applications. Therefore, this thesis implements the strategic traffic assignment model in two common transport problems: road pricing and capacity-enhancement network design. Each of these applications includes a novel model formulation, solution approach, and detailed demonstration.

Part II of this thesis introduces a dynamic strategic assignment model. While static equilibrium models are useful for many applications, particularly on in a macro-level setting, there are a number of important traffic characteristics they cannot capture due to their time invariant assumptions. Dynamic traffic assignment is a cutting edge extension to the basic models that provide a more realistic representation of traffic flow, although they are significantly more complex. In order to explore the strategic concept from multiple perspectives, Part II of this thesis proposes the strategic system optimal dynamic traffic assignment (StrSODTA) and explores a network design application.

StrSODTA is based on a single destination system optimal model that embeds the cell transmission approach to realistically propagate traffic through the network. The StrSODTA model retains the linear programming formulation and associated benefits, but due to the path based formulation, does face significant issues resulting from the number of constraints and corresponding computational complexity. While solving for network design in the static case is a challenging problem, in the StrSODTA model it is a simple extension with numerous implications.

Figure 1 presents a high-level summary of the proposed work. The core contribution of this research is to formulate and explore the implications of the strategic approach to accounting for day-to-day demand uncertainty, and furthermore to demonstrate the impact on practical transport planning applications.

LIST OF RELEVANT PUBLICATIONS

Duell, M., Gardner, L., Dixit, V. and Waller, S.T. Evaluation of a strategic road pricing scheme accounting for day-to-day and long term demand uncertainty**.** *Transportation Research Record: Journal of the Transportation Research Board,* 2467-02(1), 12-20*.*

Duell, M., and Waller, S.T. The implications of volatility in day-to-day travel flow and road capacity on traffic network design projects *Transportation Research Record: Journal of the Transportation Research Board* (in press).

Waller, S. T., Fajardo, D., **Duell, M**., & Dixit, V. (2013). Linear Programming Formulation for Strategic Dynamic Traffic Assignment. *Networks and Spatial Economics*, 13(4), 427-443.

Duell, M., Wen, T and S.T. Waller. (2013). A linear program network design model incorporating system optimal strategic dynamic traffic assignment behaviour. *Travel Behaviour and Society: Proceedings of the 18th Hong Kong Society of Transportation Studies Annual Conference*, 437-444.

Duell, M., Gardner, L., Dixit, V., and Waller, S.T. (2013) Strategic road pricing schemes accounting for day-to-day demand uncertainty. Proceedings of the 36th Australasian Transportation Research Forum in October 2013, Brisbane Australia.

Duell, M. Dixit, V., and Waller, S.T. (2013) Strategic system reliable formulation for traffic assignment. Proceedings of the OPTIMUM2013 Conference in April 2013.

Relevant publications which were not directly included in the thesis

Wen, T., Gardner, L.M., Dixit, V., **Duell, M**., and Waller, S.T. (2014). A Strategic User Equilibrium Model Incorporating Both Demand and Capacity Uncertainty. Proceedings of the 93rd Transportation Research Board Annual Meeting, 2014.

Rey, D., **Duell, M**. and Waller, S.T. (2014) Reduction methods for large scale linear programming models applied to strategic system optimal dynamic traffic assignment. 5th International Symposium on Dynamic Traffic Assignment, June 2014.

TABLE OF CONTENTS

Statement of Originality ii
Acknowledgements ii
Abstract iii
List of Relevant Publications viii
Table of Contents ix
List of Figures xii
List of Tables xv

Chapter 1 . Introduction 1

1.1 Introduction and Motivation 1
1.2 Contributions 6
1.3 Organisation 8

Chapter 2 . Time Invariant Strategic Modelling Approaches 13

2.1 Introduction 13
2.2 Literature Review 15
2.2.1 Background of transportation planning and modelling 15
2.2.2 Uncertainty and reliability in transport modelling 25
2.2.3 Strategic traffic assignment 28
2.3 Model Formulations 30
2.3.1 Strategic user equilibrium 31
2.3.2 Strategic system optimal formulation 34
2.3.3 Strategic system reliable formulation 38
2.4 Model assumptions and performance metrics 40
2.4.1 Strategic modelling assumptions 41
2.4.2 Link level performance metrics 43
2.4.3 System level performance metrics 45
2.5 Solution methodology 46
2.6 Demonstration of results 53
2.6.1 Description of test networks 54
2.6.2 Comparison Analytic and Simulation Results 58

2.6.3 System-Level Model Results 59
2.6.4 Link-Level Model Results 68
2.7 Concluding remarks 71

Chapter 3 . Network Management Application: Marginal Social Cost Pricing 72

3.1 Introduction 72
3.2 Background 76
3.3 Pricing Model Description 80
3.3.1 The StrUE MSC theoretical framework 80
3.3.2 The StrT application 85
3.3.3 Demonstration 89
3.4 Long Term Demand Uncertainty 93
3.4.1 Demonstration of long term demand scenario uncertainty 97
3.5 Model Demonstration 101
3.5.1 Results from evaluation of long term performance 101
3.5.2 Results from average demand based tolls 106
3.6 Concluding Remarks 108

Chapter 4 . Network Management Application: Link-Capacity-Based Design 110

4.1 Introduction 110
4.2 Background 113
4.3 Model Formulation 117
4.3.1 Strategic user equilibrium (StrUE) model 117
4.3.2 Strategic user equilibrium with capacity uncertainty model 119
4.3.3 Network design formulation 121
4.4 Solving the Model 125
4.4.1 Strategic assignment model 125
4.4.2 StrUEC model assumptions 128
4.4.3 Solving the design problem 132
4.5 Results and Discussion 137
4.5.1 Determining link capacity coefficient of variation 138

4.5.2 Small network demonstration 142
4.5.3 Medium network demonstration 147
4.6 Concluding Remarks 154

Chapter 5 . Dynamic Strategic Modelling Approaches 156

5.1 Introduction 156
5.2 Literature Review 160
5.2.1 Fundamentals of dynamic traffic modelling 162
5.2.2 Overview of the single destination SO-DTA model 163
5.2.3 Stochasticity in DTA 166
5.3 StrSODTA Model Formulation 170
5.3.1 Single Destination StrSO DTA Formulation 171
5.3.2 Multiple-Destination StrSO DTA Formulation 177
5.4 Model Demonstration 181
5.4.1 Solution approach 181
5.4.2 Description of test network 190
5.4.3 Single-destination model demonstration 192
5.4.4 Multiple-destination model demonstration 203
5.5 Concluding Remarks 210

Chapter 6 . Dynamic Management Application: Network Design 212

6.1 Introduction 212
6.2 Background 214
6.3 Formulation 216
6.4 Model demonstration 221
6.4.1 Model Demonstration: cell network 221
6.4.2 Model demonstration: medium networks 224
6.5 Concluding Remarks 230

Chapter 7 . Conclusion and Future Research Directions 232

Bibliography 239
Vita ..... **Error! Bookmark not defined.**

LIST OF FIGURES

Figure 1:1 Summary of fundamental thesis contribution 7

Figure 1:2 Framework of thesis research 9

Figure 2:1 Summary of research contribution 13

Figure 2:2 Summary of the strategic user equilibrium model framework 14

Figure 2:3 The four-step transport planning process 17

Figure 2:4 Summary of sources of uncertainty in the transport system 26

Figure 2:5 Summary of the strategic user equilibrium model framework 31

Figure 2:6 The Nguyen-Dupius network. 55

Figure 2:7 (a) Sioux Falls test network and (b) real Sioux Falls network (2014, OpenStreetMaps) 56

Figure 2:8 (a) Anaheim test network, and (b) real Anaheim network (2014, OpenStreetMaps) 57

Figure 2:9 Gold Coast, Queensland, Australia (2014 OpenStreetMaps) 58

Figure 2:10 Comparison of $\diamond S/\diamond E$ on the Gold Coast network 66

Figure 3:1 Summary of research contribution 76

Figure 3:2 Demonstration network and network parameters. 90

Figure 3:3 Analytical results on the demonstration network 91

Figure 3:4 Performance metrics on demonstration network 92

Figure 3:5 Results on the demonstration network under long term uncertainty: 98

Figure 3:6 Model evaluation under long term demand uncertainty: Expectation....99

Figure 3:7 Model under long term uncertainty: STD....100

Figure 3:8 Short term demand uncertainty tolling results: Sioux Falls....103

Figure 3:9 Long term evaluation of Sioux falls results, M....103

Figure 3:10 Long term uncertainty of Sioux Falls, STD....104

Figure 3:11 Anaheim network evaluation to day-to-day pricing scheme....104

Figure 3:12 Long term evaluation of Anaheim network, ΔM....105

Figure 3:13 Long term evaluation of S on Anaheim network....105

Figure 4:1 Summary of research contribution....113

Figure 4:2 Results for Nguyen Dupuis network and StrUE subproblem....144

Figure 4:3 Results for Nguyen Dupuis network and StrUEC subproblem....145

Figure 4:4 Results for Nguyen Dupuis where $CVstr = 0.3$ and $cova = 0.3 \forall a$....146

Figure 4:5 Example of genetic algorithm convergence....148

Figure 4:6 Results for the Sioux Falls network with (a) StrUE subproblem and (b) StrUEC subproblem....149

Figure 4:7 Sioux Falls network, long term NDP results for StrUE....152

Figure 4:8 Sioux Falls network, NDP results for StrUEC....152

Figure 4:9 Sioux Falls network, NDP results for StrSO....153

Figure 5:1 Example of dynamic profile of travelers in the Sydney network (Sydney Household Travel Survey)....157

Figure 5:2 Summary of research contribution....159

Figure 5:3 Cell-density diagram for $\xi = 1, m = (1,2), \phi = 1$: path density 194

Figure 5:4 Cell-density diagram for $\xi = 1, m = (1,2), \phi = 1$: total density 195

Figure 5:5 Cell-density diagram for $\xi = 1, m = (1,2), \phi = 1$: path density 196

Figure 5:6 Cell-density diagram for $\xi = 1, m = (1,2), \phi = 2$: path density 197

Figure 5:7 Cell-density diagram for $\xi = 1, m = (1,2), \phi = 2$: total density 198

Figure 5:8 Cell-density diagram $m = (4,2), \phi = 1$: path density 199

Figure 5:9 Cell-density diagram for $m = (4,2), \phi = 2$: path density 200

Figure 5:10 Size of LP on Sioux Falls network 207

Figure 6:1 Summary of research contribution 213

Figure 6:2 Demonstration cell network and cell parameters 222

Figure 6:3 Total system travel time for the StrSO DTA LP model under a varying budget 224

LIST OF TABLES

Table 2-1 Summary of a few notable equilibrium model variations 23

Table 2-2 Summary of demonstration network 54

Table 2-3 Overall Sioux Falls results: StrUE 60

Table 2-4 Overall Sioux Falls results: StrSO 60

Table 2-5 Overall Sioux Falls results: StrSR 62

Table 2-6 Overall Anaheim Results: StrUE 63

Table 2-7 Overall Anaheim Results: StrSO 63

Table 2-8 Overall Anaheim Results: StrSR 64

Table 2-9 Overall Gold Coast Results: StrUE 65

Table 2-10 Overall Gold Coast Results: StrSO 66

Table 2-11 Overall Gold Coast Results: StrSR 67

Table 2-12 Sioux Falls network: comparison of $\diamond Sij$ 70

Table 2-13 Anaheim network: comparison of $\diamond Sij$ 71

Table 3-1 Notation for the general strategic approach 81

Table 3-2 Additional notation for the chapter 3 pricing application 85

Table 3-3 Convergence results for E and AD for the StrUE and StrT assignment patterns. 90

Table 3-4 Notation regarding the long term demand uncertainty in Chapter 3 94

Table 3-5 Sioux Falls network: long term evaluation of StrT model 107

Table 3-6 Sioux Falls network: long term evaluation of average demand results 107

Table 4-1 Summary of genetic algorithm terms and problem-specific representation 133

Table 4-2 Methods of generating coefficient of variation for all links in the network 139

Table 4-3 Sensitivity analysis of link capacity volatility types 150

Table 4-4 Comparing the effects of random seeds on the Sioux Falls network 154
Table 5-1 Test Network Characteristics 191
Table 5-2 Expected demand for the single destination Nguyen Dupius case 192
Table 5-3 Time step sensitivity analysis of performance metrics 201
Table 5-4 Comparison of different time steps and the number of demand scenarios 201
Table 5-5 Comparison of $p1,4$ at departure time $\tau = 1$ 202
Table 5-6 Sensitivity analysis of k and dt 203
Table 5-7 Full expected demand for Nguyen Dupius 204
Table 5-8 Multiple-destination, perfectly correlated demand Nguyen Dupius 205
Table 5-9 Multiple-destination, uncorrelated demand, Nguyen Dupius 206
Table 5-10 Expected demand for Sioux Falls 206
Table 5-11 Sioux Falls, perfectly correlated demand results 208
Table 5-12 Sioux Falls, independent demand results 209
Table 6-1 Demand parameters for the demonstration network 223
Table 6-2 Departure time parameters for the demonstration network 223
Table 6-3 Expected demand for the Nguyen Dupius network 225
Table 6-4 Results for Nguyen Dupius network, correlated demand, where $\boldsymbol{\Xi = 3}$ 225
Table 6-5 Results for Nguyen Dupius network, independent demand, where $\boldsymbol{\Xi = 3}$ 226
Table 6-6 Expected demand for the Sioux Falls network 227
Table 6-7 Results for Sioux Falls network with correlated demand 227
Table 6-8 Design project results for the Sioux Falls network: perfectly correlated demand 228
Table 6-9 Results for Sioux Falls network with independent demand 229
Table 6-10 Design project results for the Sioux Falls network: independent demand 230

Chapter 1

Introduction

1.1 Introduction and Motivation

Transportation systems shape the fabric of contemporary society. Cities around the world rely on their transport system not only for the freedom that comes with personal mobility but also for bolstering the national economy, security, evacuation in the event of natural disasters, technological innovation, logistics and product delivery, and numerous other areas of interest. Advancements in the field of transportation have the potential to impact all facets of society, from the daily decision-making of the individual choosing how she gets to work to global issues that transcend national borders, such as the impact of vehicle emissions on climate change.

Due to its ubiquitous nature, it is not difficult to imagine the substantial amount of money that governments, private corporations, and citizens invest in transportation. In Australia, the Departure of Infrastructure and Transport had a 2013-14 budget of $6.8 billion. In the United States, the Department of Transportation had a 2014 budget of $77.2 billion, with a substantial percentage of that being invested in infrastructure projects. While there are a wide variety of expenses and investments that comprise the portfolio of those large budgets,

significant amounts are spent on forecasting future conditions in order to evaluate the impacts of a range of factors, including infrastructure projects but also unpredictable events such as changes in land use, technological innovations, policy decisions, and the state of the economy.

However, despite the large amounts of money and significant scope of model outcomes, the inherent uncertainty in the underlying transport system and corresponding models is often neglected. The lack of consideration of uncertainty may be due to a lack of transparent tools to inform practitioners and policy makers, in addition to the significant complexity of the problem at hand. Of particular interest to this thesis is the daily volatility in traffic flow that has a significant effect on how drivers make decisions and on transport system evaluation.

This thesis focuses on the area of network modelling, the foundation of which is described in Section 2.2. One method of examining flow patterns in a transport network is by using traffic assignment models. Given the network structure and origin-destination travel demands (which are estimated through additional modelling steps in the transport planning process), traffic assignment will determine link flow patterns throughout the network, which can then be used to estimate travel times and travel speeds. Total system travel time is the most commonly used metric of performance for transport models on a macroscopic scale because it is considered to be a measure of the congestion that costs travellers millions of dollars per year. There are two common formulations that describe the

assumptions behind the traffic assignment models: user equilibrium (UE) and system optimal (SO). The UE and SO formulations will be discussed in detail in Chapter 2.

Transport planning agencies around the world use traffic assignment models to rank potential infrastructure projects. These models depend upon the behavioural assumptions to describe individuals' route choice selection process; it follows that multi-million dollar decisions hinge upon the ability of equilibrium models to evaluate the impact of transport network changes. However, the questions transport planners are now asking of these models are not the questions that these models were originally created to answer. Therefore new models are needed to explicitly address the points in question, particularly in regard to uncertainties and volatility. Many of these questions are closely connected to the uncertainty inherent in the transport planning process, and in travel demand in particular. This thesis is motivated by the need to improve transport modelling and decision-making in regard to inherent network uncertainties.

This thesis introduces a novel formulation for traffic assignment that explicitly incorporates demand uncertainty and additionally captures more complex behaviour from users. It also addresses the issue of day-to-day flow volatility, which describes the recurrent changes in traffic conditions from one day to the next. This approach is based on the idea that a strategic approach is the best way to describe user behaviour. From the user perspective this means that a

person knows of the available routes to take and assigns a probability to each. As the person learns about the trip distribution of the network, they update these probabilities until an equilibrium based on the expected condition is reached. From the perspective of the system operator, this results in a certain proportion of the trips that utilize each available path. Because people update their expected portfolio of strategies based on the distribution of the trip demand, any given actualization of the traffic patterns will not seem to be in equilibrium. In this way, this novel formulation for a traffic assignment model is able to capture a more complex behaviour from users and to represent the volatility that planners observe in day-to-day traffic flow.

Transport planners use multiple models and modelling approaches in order to evaluate an infrastructure project or policy question. In general, these models are macroscopic in nature, meaning that they use average quantities to describe the system from a regional perspective (and traditionally cannot capture the volatility observed in traffic flow). The modelling approach and application described in Chapter 2, 3, and 4 is macroscopic in nature. However, growing in popularity in recent years is the microscopic traffic modelling approach, usually simulation based. Traffic microsimulators use mathematical representations and probabilistic elements of stopping distance and lane-changing behaviour, in addition to many more parameters, in order to represent traffic flow on a very fine-grained level. However, traffic microsimulation has a significant computational burden, requires very detailed input data, and does not account for how any changes may impact the

way in which travellers select their routes. As a result, traffic microsimulation is most suited to investigate projects with a limited localised impact, such as environmental impact analyses for proposed structures, parking studies, traffic signal timing plans, and numerous other projects.

As a compromise between the extreme detail of microsimulation and the aggregated estimation of macroscopic models lies a middle ground: mesoscopic traffic models. While these modelling conventions are not necessarily completely agreed between all traffic researchers, for the purposes of this thesis, models on a mesoscopic scale are considered as relative to their counterparts; a mesoscopic model captures a greater amount of detail than a macroscopic model, but does not preclude the computational burdens of microsimulation. Transport planners and practitioners often utilize multiple traffic models in conjugation in order to produce a more comprehensive evaluation of network behaviour. Therefore, in addition to the static transport planning model that is proposed in Chapter 2, Chapter 5 proposes a strategic dynamic traffic assignment model that can capture traffic characteristics such as spillback and shockwave propagation.

Transport policy is informed by decision-making tools that are built on planning models. Most planning organizations use regional models on multiple scales to make decisions about the future of the locale. However, it is well-acknowledged that underlying physical activities that comprise the transport system are immensely complicated, and furthermore the aggregate of numerous

stochastic processes including individual decision-making. Just like predicting the stock market, the agents are too numerous to predict with exact accuracy. Add in the sheer size of many regions of interest, the significance of the decisions being made, and the need of politicians for accountability and it becomes clear that transport planning is a very complex topic.

Therefore, researchers are constantly seeking improvements to transport planning models that account for more diverse aspects of the system at hand but that are also reliable, based on sound behavioural assumptions, and able to be scaled up to practically-sized problems. Due to numerous critical issues affecting cities all over the world, it is more important than ever before that planners have tools that inform robust policy. The research in this thesis aims to provide the foundation for such a tool.

1.2 Contributions

This thesis introduces the novel strategic-based framework that incorporates travel demand uncertainty and allows planners to quantify network reliability. Additionally, due to the straightforward assumptions at the foundation of the framework, the strategic approach can be scaled to practically-sized problems. Figure 1:1 emphasises the core contribution of this thesis.

Core Thesis Contribution

Formulate and explore the implications of the **strategic approach to accounting for day-to-day demand uncertainty** from both a static and dynamic perspective;

Furthermore, to demonstrate the implications for **practical transport planning applications**.

Figure 1:1 Fundamental thesis contribution

This research contains the first attempt to model the strategic framework comprehensively, on multiple scales and in the context of practical planning applications. The contributions of this research are summarised as follows:

Formulate the strategic modelling framework on multiple scales.

- Firstly, a set of time invariant models to capture the effects of day-to-day demand uncertainty on user route choice on a macroscopic scale;
- Secondly, a dynamic strategic linear programming approach that accounts for demand uncertainty and system optimal behaviour from a mesoscopic perspective.

Provide a robust means of measuring the tradeoff between travel time and reliability while providing a means to quantify the variance of travel time on a link and in the system as a whole.

- Due to the fundamental assumptions, the model can be applied to practically sized network problems.

Investigate the impacts of the strategic approach in the context of practical and relevant planning applications.

- Both static and dynamic approaches to ranking infrastructure projects in the context of the capacity enhancement network design problem, including mathematical formulations and solution approaches;
- A novel marginal social cost based pricing scheme and a method to evaluate the impact of long term demand uncertainty on network performance;

Propose a framework that includes a clear outline of all assumptions and solution methods, leading to numerous direct extensions.

- Each chapter includes a thorough discussion of its relevant research contributions and suggestions for future research directions.

The next section describes the structure of this thesis and includes a summary of the content of each chapter.

1.3 Organisation

This thesis introduces a novel approach to the traffic assignment problem, broadly classified as the strategic approach. These models are investigated in two separate, but related, approaches: Chapters 2, 3, and 4 explore time invariant time invariant strategic traffic assignment models. First, the base model and variants is introduced, formulated including a detailed discussion of assumptions, the solution method investigated, and then all models demonstrated and compared. This thesis places particular emphasis on the practicality of this approach, and so the next topic explores two realistic applications of the static strategic assignment models: first best pricing and capacity network design. Chapters 5 and 6 explores dynamic modelling approaches, beginning with the introduction of the strategic system optimal dynamic traffic assignment model and additionally investigating the

capacity enhancement network design problem. Figure 1:2 presents a summary of the research work in this thesis and the following paragraphs describe each chapter in greater detail.

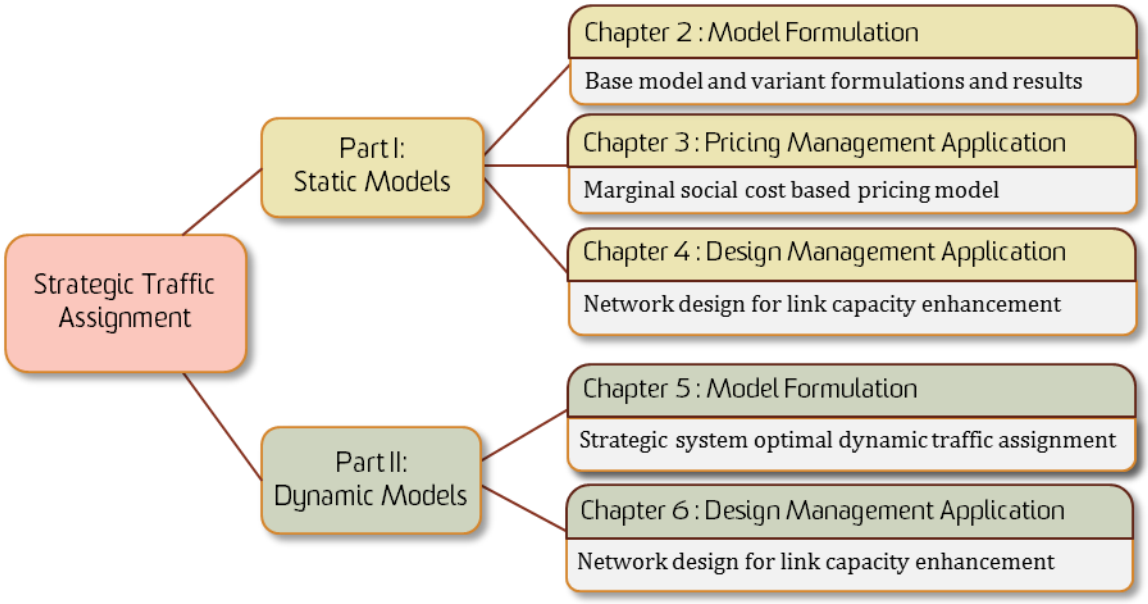


Figure 1:2 Framework of thesis research

Chapter 2: Time Invariant Modelling Framework. Time invariant modelling approaches to solve the traffic assignment problem are one of the most widely used transport planning tools. Chapter 3 introduces the three strategic models of interest in this work. First, background to transport planning models and relevant literature to uncertainty traffic assignment is reviewed. Then, the chapter discusses and formulates the models, reviews the model assumptions and the solution method. The models are then demonstrated on a number of test networks. Results reflect the tradeoff between travel time and reliability.

Chapter 3: Road Pricing Application. Planning models such as the StrUE model proposed in Chapter 2 are useful to practitioners because of their numerous practical applications. Road pricing schemes are commonly regarded as one of the primary management tools available to network operators to manage congestion and growth in an urban network. Chapter 3 introduces a first best, marginal-social cost pricing scheme based on the strategic approach, including model formulation and solution approach. Additionally, Chapter 3 highlights the importance of accounting for the performance of a model under long term planning demand uncertainty. Results show how modelling evaluations could be suboptimal if long term uncertainty is neglected.

Chapter 4: Network Design Problem. The ranking and evaluation of infrastructure projects is probably the most common planning application of traffic assignment models such as those described in Chapter 2. It is particularly important to incorporate the unpredictable effects of day-to-day demand uncertainty may have on the ranking of infrastructure projects; however, as discussed in Chapter 2, daily operational capacity may be another non-deterministic quantity that users consider when making a route choice. Therefore, Chapter 4 compares the strategic user equilibrium model introduced in Chapter 2 and the strategic user equilibrium with capacity uncertainty model, introduced by Wen et al (2014), for the ranking and evaluation of network design projects. This chapter formulates the model, discusses assumptions and performance measures, then discusses the genetic algorithm used as a solution method.

Chapter 5: Dynamic Modelling Framework. Planners commonly utilize a number of transport models on different scales, either in tandem or for comparison, in order to get a more holistic characterisation of a transport system. Therefore, Chapter 5 of this thesis focuses on a dynamic strategic traffic assignment model. Dynamic traffic assignment can capture important traffic phenomenon that is lacking in its static counterparts, such as queuing and shockwave propagation. However, introducing the element of time has a significant impact on the computational complexity of the problem, and thus, a majority of dynamic traffic assignment models are simulation-based and deterministic.

The model introduced in Chapter 5 of this thesis is based on a linear programming foundation that incorporates the cell transmission model and strategic path proportions across multiple demand scenarios. The linear programming formulation allows well-established solution approaches to be utilized, although the addition of demand scenarios increases the size of the linear program significantly. Chapter 5 discusses background to the dynamic traffic assignment problem, then formulates the strategic system optimal dynamic traffic assignment model. Chapter 5 also describes a solution method that incorporates static planning data with reasonable assumptions in order to provide a possible platform for rough comparison between the models introduced in this thesis. Results show the propagation of traffic in a simple scenarios, then compare the results on the Sioux Falls network.

0: Dynamic Network Design Problem. This thesis examines a network management application for the dynamic strategic system optimal linear programming model. Dynamic network design approaches that incorporate uncertainty are not common in the past literature due to the significant complexity of the problem; however, the dynamic model proposed in this thesis incorporates a linear programming formulation at its core, and therefore capacity expansion is a relatively simple extension (unlike the complex problem seen in Chapter 4). The network design problem is formulated and demonstrated on three test networks of various sizes. Results show that different design project selections based on different demand scenarios.

Chapter 7: Conclusion. Finally, the relevant conclusions from the thesis are summarized. One of the advantages of the strategic framework of focus in this thesis is the many avenues of future research. A number of extensions and advancements are discussed.

Chapter 2

Time Invariant Strategic Modelling Approaches

2.1 Introduction

Decisions made on the transport policy level have a significant impact on both the day-to-day lives of the public and the long-term vitality of the region served by the transport system. Therefore it is vital that planners have robust, transparent tools to inform decision-making that account for the range of diverse factors that affect the transport system. This thesis focuses on modelling approaches that account for the impact of uncertainty in day-to-day demand.

Chapter 2 Contributions

Formulation and solution methods for the **strategic user equilibrium**, **strategic system optimal**, and **strategic system reliable** models.

- Accounts for day-to-day demand uncertainty;
- Flexible framework, scalable to large networks

Demonstration and comparison of models on a variety of test networks

Figure 2:1 Summary of research contribution

Chapter 3 introduces three time-invariant models that encapsulate the strategic approach to modelling day-to-day flow volatility in traffic assignment. The strategic user equilibrium (StrUE) model captures the behaviour of users based on the economic concept of equilibrium, where routes are identified to minimize *individual* travel time. The strategic system optimal (StrSO) model identifies the user routes that will minimize the travel time on a *system* level. The StrSO reflects important system behaviour and is useful for applications such as marginal social cost pricing. Finally, the strategic system reliable (StrSR) model identifies user routes that minimize the variation of total system travel time, which is a measure of network reliability. The strategic models are outlined in Figure 2:2.

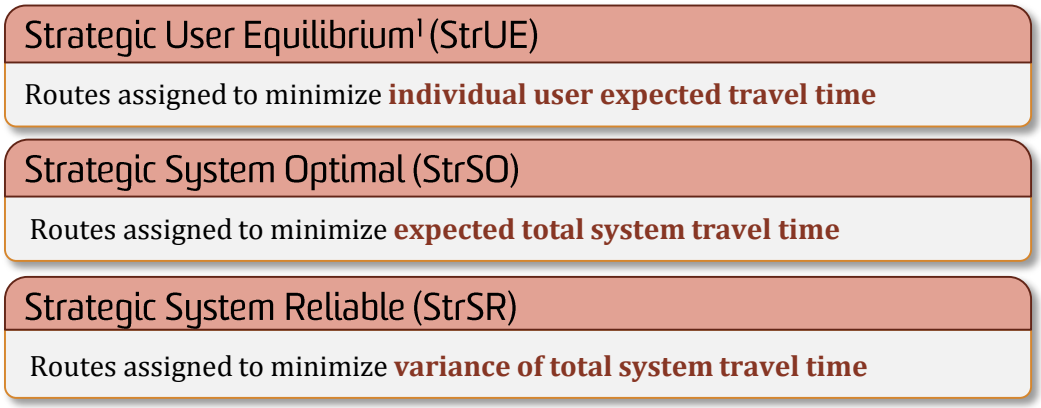


Figure 2:2 Summary of the strategic user equilibrium model framework

Together, the StrUE, StrSO, and StrSR models characterize and quantify important aspects of network uncertainty that may have significant policy implications for transport planners.

While traditional transport models have relied primarily on travel time as a performance metric, increasingly planners recognize the importance of alternative considerations such as reliability and travel time variability. Previous research has demonstrated that users value travel time variability as well as minimizing travel time (Senna, 1994). This research develops a novel framework to evaluate performance and reliability in a transportation network.

Chapter 2 is outlined as follows. First, Section 2.2 provides a background on transport modelling and the uncertainty in transport networks. Section 2.3 describes the models and provides the mathematical formulation of each. Then, Section 2.4 discusses important model assumptions and model performance metrics. Section 2.5 describes the solution approach in detail. Finally, Section 2.6 demonstrates the models on a number of test networks.

2.2 Literature Review

Models in transport planning have rich history in both research and practice. This section provides context for the strategic traffic assignment approach, including a review of the traditional traffic assignment problem and other approaches to modelling uncertainty and reliability in transport networks.

2.2.1 Background of transportation planning and modelling

Engineers and scientists use *models* to simplify and abstract a complex, changing world. Models serve a wide range of purposes; from a philosophical perspective,

some parties believe that the purpose of models is to enhance understanding and help explain understanding. Other parties believe that the utility of models lie in their predictive accuracy. Both of these frames of thoughts are represented in the transport modelling community. From the transport perspective, mathematical models are applied for a variety of purposes. Often the modeller seeks to identify the key connections between causes and effects in transport decision-making, either from the providers of transport services or by transport users.

Transport models require the representation of transport infrastructure and services, along with time and space, where travel demand manifests. This is commonly achieved through the use of transport networks. Networks have a wide variety of applications, such as the intricate networks of pipes that supply water in a city, the supply of electricity through the grid, communication networks (including phones, internet, data), and even more conceptual ideas such as a social network, which describes the interactions and connections of different groups of people. Transportation is a prime application of many network flow problems, such as the traffic assignment problem, the shortest path problem, and the vehicle routing problem. A road network is commonly represented as a set of links and nodes. Links represent roads, the physical space that vehicles must travel. Nodes may represent the connection between links. For a user to travel across a link, they incur some cost, generally a measurement of time. Generally, the capacity of a link is seen as finite.

The history of analytical transportation modelling dates to the 1950s. Throughout that time, the primary tool for forecasting future demand and performance of a transportation system has been the four-step model. The first three steps comprise the travel demand modelling process: trip generation, which examines data and other indications of where people live or work, or other activities that generate trips. This process results in an estimation of productions and attractions (of trips). The second step is trip distribution where the estimation from the previous step is transformed into trips. The third step is mode choice where the trips are divided among available modes, usually based on some measure of travel cost/impedance. The final step is traffic assignment, or route choice, in which the routes for trips (usually vehicle trips) are determined. Although it is important to note that the four-step model includes a feedback mechanism, for example where the travel costs determined through traffic assignment can be used to adjust the distribution of trips and mode choice, traffic assignment essentially yields link flows, and the corresponding system performance measures.

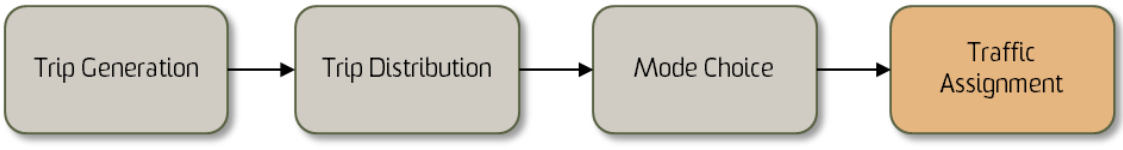


Figure 2:3 The four-step transport planning process

The core assumptions of traffic assignment are founded on the equilibrium principle. Equilibrium is an economic concept in which there is a “balance” between

the demand and supply, and it describes numerous markets for commodities, such as petrol. In the transport system, "demand" is users who wish to travel and the "supply" may be the infrastructure supplied by the transport planner. In the state of equilibrium, no trip-maker has incentive to behave differently than they already do. This descriptive principle is at the core of many approaches describing traffic on road networks: at a Wardropian equilibrium, no single user can decrease her travel time by unilaterally changing routes (Wardrop, 1952). For a network made up of N nodes and A arcs, the nonlinear mathematical programming formulation for traditional UE is that by Beckmann et al (1956) and is contained in Equations (2.1) - (2.4)

$$minimize \sum_{\forall(i,j)\in A} \int_0^{x_{ij}} t_{ij}(y)dy \qquad (2.1)$$

$$subject\ to$$

$$\sum_k f_k^{rs} = q^{rs} \qquad \forall r,s \qquad (2.2)$$

$$f_k^{rs} \geq 0 \qquad \forall k,r,s \qquad (2.3)$$

$$x_{ij} = \sum_r \sum_s \sum_k f_k^{rs} \delta_{ij,k}^{rs} \qquad \forall a \qquad (2.4)$$

Where $t_{ij}(x)$ is the travel time on link (i,j) given x_{ij} flow on the link, q^{rs} is the demand between nodes r and s $(r,s \in N)$, f_k^{rs} is the flow on path k between nodes r and s, and $\delta_{ij,k}^{rs}$ is an indicator equal to 1 if link (i,j) belongs to path k between nodes r and s, and 0 otherwise. Constraint (2.2) says that the path flow on a path will be equal to the demand, (2.3) is the non-negativity constraint for path flow, and (2.4) disaggregates path flow to link flow.

The objective function of this formulation has no intuitive meaning; rather, meaning can be found in the optimality conditions (not shown here) which specify that each user will take the *shortest path* (Sheffi, 1985). This is one of the most important implications for this formulation, and follows from the behavioural assumption discussed above. The gradient of the UE problem is the shortest path problem. Therefore gradient-based optimization methods can be used to solve. Most generally a transport network equilibrium problem can be solved by assigning all flow to the current shortest path, updating costs, reassigning the flow, iteratively. Eventually the equilibrium condition will be met where no user can change paths for a shorter travel time. Alternatively, the classic UE model has been formulated as a variational inequality, complementarity system, or fixed point problem. Section 2.5 contains a more thorough discussion of solution approaches.

The system optimal formulation does not reflect behavioural assumptions about users in the system, but rather the optimization problem that *minimizes total system travel time*. In this formulation, it would be possible for a single user to

change paths for a shorter travel time. Rather, the SO formulation is reflective of shortest *marginal cost* path where no user can change paths for a shorter marginal cost travel time, where the marginal cost represents the cost to the system of adding one user to the path (or link).

The objective function for this mathematical programming formulation is more intuitive: simply minimize the total time for each user on each link. The constraints for this program are the same as the UE problem above which reflects the fact that these formulations are closely related to each other. The SO mathematical program is shown below (Sheffi, 1985):

$$Minimize \sum_{\forall(i,j)\in A} x_{ij} t_{ij}(x) \tag{2.5}$$

s.t.

$$\sum_{k} f_k^{rs} = q_{rs} \qquad \forall r,s \in N \tag{2.6}$$

$$f_k^{rs} \geq 0 \qquad \forall k \in K, r,s \in N \tag{2.7}$$

$$x_{ij} = \sum_{r}\sum_{s}\sum_{k} f_k^{rs}\delta_{ij,k}^{rs} \qquad \forall (i,j) \in A \tag{2.8}$$

System optimal formulations have important implications for the system. The total travel time in this formulation is the best possible performance for a network, so it

serves as a lower bound for UE problems. It also describes the behaviour that system operators would enforce from users if they could. This is an important idea for tolling problems, where operators want to incentivize users to take the optimal paths in order to reduce overall network congestion.

Traditionally, both of these formulations require that user have perfect information (i.e., they know the shortest path), and that the demand q^{rs} be known. The former concern is addressed through stochastic equilibrium (discussed in Section 2.2.2). The latter concern is a primary motivator for this research. In reality travel demands are *not* known with any level of certainty, although planners do have some information about the demand. Furthermore, the equilibrium behaviour that is foundational to the UE model is not observed in everyday traffic flow, i.e., one can observe the same intersection every day and not see the same flow.

Both the popularity and the profound impact of the traditional UE problem are reflected by the numerous variations proposed over the years: stochastic user equilibrium (Mirchandani & Soroush, 1987), probabilistic user equilibrium (Lo & Tung, 2003), demand driven travel time reliability-based user equilibrium (Shao et al, 2006b), dynamic user equilibrium (Friesz et al, 1993; Han & Heydecker, 2006), late arrival penalised user equilibrium (Watling, 2006), user equilibrium with recourse (Unnikrishnan & Waller, 2009), mean-excess travel time user equilibrium (Chen & Zhou, 2010), traffic equilibrium under behavioural inertia (Xie & Liu,

2014), and more, summarized in Table 2-1. Note that the models are listed alphabetically. As appropriate, these models will be described in more detail below.

Table 2-1 Summary of a few notable equilibrium model variations

Model	Key Works	Summary	Assumptions	Additional notes
demand driven time travel reliability-based user equilibrium (DRUE)	Shao et al (2006a)	Route choice based on travel time budget which is average travel time plus extra buffer time; model extension accounts for user error in perception	Versions with and without accounting for error in user perception (Shao et al, 2006b)	Related to travel time budget approach of Lo et al (2006)
deterministic user equilibrium (DUE)	Wardrop (1952); Beckmann et al (1956); Sheffi (1985)	The travel time on all used paths between an OD pair are equal and minimum	Perfect information; risk neutral behaviour; fixed demand and capacity;	Difficult to observe in reality but widely used in practice
dynamic user equilibrium (also DUE)	Friesz et al (1993)	The travel time on all used paths for each OD pair and each departure time are equal and minimum	Time dependent; mostly deterministic and perfect information;	Difficult to formulate and solve
late arrival penalized user equilibrium (LAPUE)	Watling (2006)	Incorporates a schedule delay term in disutility function to penalise late arrival under fixed departure times	Incorporates mixed distributions of travel times	Unreliability based rule later incorporated in Chen and Zhou (2010)
mean excess travel time user equilibrium (METE)	Chen and Zhou (2010)	Route choice considers both travel time budget as reliability measure (travel time budget) and unreliability measure (late arrival penalty)	Mean excess travel time defined as conditional expectation of travel times beyond the travel time budget	travel time reliability and travel time budget

Model	Key Works	Summary	Assumptions	Additional notes
probabilistic user equilibrium (PUE)	Lo and Tung (2003); Lo et al (2006)	Travel time distributions of all used routes have the same mean, which is the minimum for that OD pair, and all used routes have a variability within an acceptable range	variation in travel time caused by link capacity degradation random variable	Lo et al (2006) expand to account for risk adverse behaviour and travel time budget
stochastic user equilibrium (SUE)	Daganzo and Sheffi (1977); Mirchandani and Soroush (1987)	Travel cost accounts for error in user perception	Commonly assume error follows logit or probit distribution	Can be extended to include numerous model variations; how to calibrate parameter
strategic user equilibrium (StrUE)	Dixit et al (2013); this thesis	Route choice based on minimizing expected cost; equilibrium based on fixed proportion assignment	Users have perfect knowledge of demand distribution; homogenous travellers;	Core assumptions facilitate large-scale application
user equilibrium with recourse (UER)	Unnikrishnan and Waller (2009)	Adaptive route choice based on policies that incorporate information gained during travel	Multiple possible network states and associated probabilities	Based on online-shortest-path routing problem
user equilibrium under behavioural inertia	Xie and Liu (2014)	Route choice based on concept of behavioural inertia, which describes the tendency to switch routes on the basis of ongoing experience	Network supply and demand changes and different level of behavioural inertia result in mixed equilibrium	Recent work

2.2.2 Uncertainty and reliability in transport modelling

Accounting for uncertainty in transport planning encompasses network analysis under a variety of possible scenarios rather than the unrealistic assumption of deterministic, or "expected" conditions. As Waller et al (2001) discuss, planning agencies often employ the implicit assumption that the expected performance of a system is equal to the system's performance at the expected value, an assumption that only holds for linear systems. Thus, using a single fixed point estimate of the expected future demand (and neglecting the impact of long term demand uncertainty) can lead to the significant overestimation of future system performance, which has further negative ramifications on project rankings.

In particular, incorporating uncertainty in modelling attempts to account for the impact of possible deviant model behaviour and how it may influence important policy decisions. Uncertainty in transport modelling is a well-explored topic, but there remains numerous, critical questions to be answered.

Transport modellers widely acknowledge that uncertainty may arise from a number of possible sources, as summarized in Figure 2:4. Commonly, these source are categorized an endogenous or exogenous, supply or demand, on the time frame of consideration, and additionally effects such as behaviour, which aren't related to travel decision, network infrastructure, or the time frame of consideration. Demand typically refers to travel by users of the system; supply typically refers to the

infrastructure that planners provide. The background of research relating to each is traced below.

	Demand	Supply
Short term	daily fluctuations; adverse weather; natural disaster evaluation	adverse weather; traffic incidents; driver behaviour; connectivity resulting from infrastructure degradation; construction
Long term	land use patterns; disruptive technologies; government policies	infrastructure development;



Behavioural Uncertainty
Error in user perception; different levels of risk adversity

Figure 2:4 Summary of sources of uncertainty in the transport system

Capacity degradations in traffic networks were first explored as a part of equilibrium analysis by Arnott et al (1991), who consider the impact of differing levels of information on traffic congestion, and Chen et al (2002), who examine capacity reliability, which is defined as the probability that the capacity of an arc will be adequate to meet the demand. Lo and Tung (2003) propose a probabilistic user equilibrium, where the equilibrium is based on the travel time distribution for an OD pair and a maximum variability condition. Lo et al (2006) extend the probabilistic user equilibrium to include users with different levels of risk aversion,

where users minimize a travel time budget and require a punctual arrival, where capacity was treated as a uniform random variable subject to traffic incidents. Lam et al (2008) propose a traffic assignment model as a fixed point problem that accounts for the impact of predictable, adverse weather patterns, including its effect on demand, capacity, and travellers' perception errors.

The uncertainty in travel demand has also been treated from a number of perspectives in the equilibrium modelling community, including: expected, elastic, stochastic, strategic, inertial. Asakura and Kashiwadani (1991) were among the first to consider network reliability as the consequence of daily variation in travel times. Bell et al (1999) use a sensitivity based techniques to analyse the impact of travel demand fluctuations. Bell (2000) proposed a game theoretical approach which accounts for the situation where users are extremely pessimistic about travel time. Yin and Ieda (2001) consider the issue of nonrecurrant congestion.

Others have employed methods formed on a statistical techniques. Clark and Watling (2005) consider the effect that demand uncertainty has on the travel time distribution of arcs. Watling (2006) extends this model to include a late arrival penalty, where the travel time of arcs follow a probability distribution, but utility is based on users' arrival time at the destination relative to a preferred schedule.

The assumption of perfect information is often managed through stochastic user equilibrium (SUE). SUE is well-established method of accounting for error in user perception. The most common application of stochastic user equilibrium

applies a discrete choice model (such as the logit distribution) to determining travel costs. Dial (1971) was among the first to address a stochastic user equilibrium problem using analytical methods. Daganzo and Sheffi (1977) and Sheffi and Powell (1982) incorporated the Wardropian equilibrium principle that explicitly treats user perception as stochastic. Hazelton (1998) proposed a conditional stochastic user equilibrium that is a generalized version of the SUE model with stochastic flows. Additionally, many of the models described above under demand and supply have been extended to account for a stochastic variation (Connors & Sumalee, 2009; Damberg et al, 1996)

While stochastic user equilibrium may appear to have close similarities to the strategic approach proposed here (particularly because discrete choice models are used to find the probability of users taking a particular path), strategic specifically addresses the uncertainty in demand (not user path choice), and the proportions emerge from uncertainty in the number of trips taken, not error in user perception.

Though these works do not have a direct influence on the currently proposed research, they have built a repertoire of important tools that lay the foundation for the strategic traffic assignment approach proposed here.

2.2.3 Strategic traffic assignment

This work advances the concept of strategic traffic assignment. This idea examines the behavioural aspect of travellers in a transport network: users may know of a

number of paths that are available for them to take, and as such develop a mixed strategy, which in essence assigns a specific probability of use to each path. To further clarify this idea, a strategy may be thought of as a plan that encompasses all possible outcomes (i.e., realizations of travel demand) and defines a course of action for each scenario.

The idea of travellers employing strategies was first introduced by Chriqui and Robillard (1975) in regards to transit passengers choosing an optimal subset of routes from overlapping lines to minimize delay. Marcotte and Nguyen (1998) formulated an approach for capacitated transit networks using hyperpaths based on the probability of a link being available when a user arrives. This approach was then refined by Marcotte et al (2004), the work that introduced strategic decision making by users in traffic assignment networks, rather than path-based route selection. Strategic in this context refers to the idea that users have a subset of possible paths from each node which are ordered by preference, depending on the availability of that link upon arrival at the previous node. They employ this idea in a capacitated network with rigid link costs. However, the availability of a given link depends on how many others employ a similar strategy, and is thus flow dependent. The objective function of this of this formulation then minimizes the expected delay, which is based on access probabilities at each link. The variational inequality formulation of this problem was solved and compared using five solution algorithms.

Hamdouch et al (2004) expanded the previous approach into dynamic capacitated network. Again, users chose routes according to strategies consisting of a subset of possible preferred routes based on arc availability. Here congestion effects were represented by queuing delays and the loading strategy was based on preference and classes that guaranteed first in first out requirements were met. Again, the authors formulated the problem using a variational inequality approach (minimized expected delay) that was solved using a heuristic algorithm based on the method of successive averages.

While the works discussed in this section introduce the concept of strategic assignment, it is important to note that they use the term with a significantly different intent. The strategies in the above approach are based on access probabilities when the user reaches a certain node. The strategic concept as used here reflects a mixed strategy approach on behalf of users (Aumann and Brandenburger, 1995) and captures a higher level equilibrium of path choice. Additionally, the works above assume that users adopt the strategic approach to minimize their travel time, as opposed to the *expected* travel time equilibration that is assumed in strategic as used here.

2.3 Model Formulations

This section presents the qualitative description and mathematical derivations and formulation for the StrUE, StrSO, and StrSR models.

2.3.1 Strategic user equilibrium

The StrUE model is based on the core assumption that travellers make their route choice based on knowledge of the day-to-day demand distribution. This knowledge results from past experience travelling in the network. However, on any given day of travel, a driver does not know what the travel conditions she will experience. Therefore, travellers employ a strategy to make a route choice. While this strategy could be given a variety of definitions, in approaches outlined in this thesis, the strategy is straightforward: users will choose the least expected cost path. The expectation of the path cost is a function of the total demand distribution (which users are assumed to know). Thus, route choices are made a priori, and users are assumed to remain on their chosen path regardless of the travel conditions they experience. This is considered the first stage of the model.

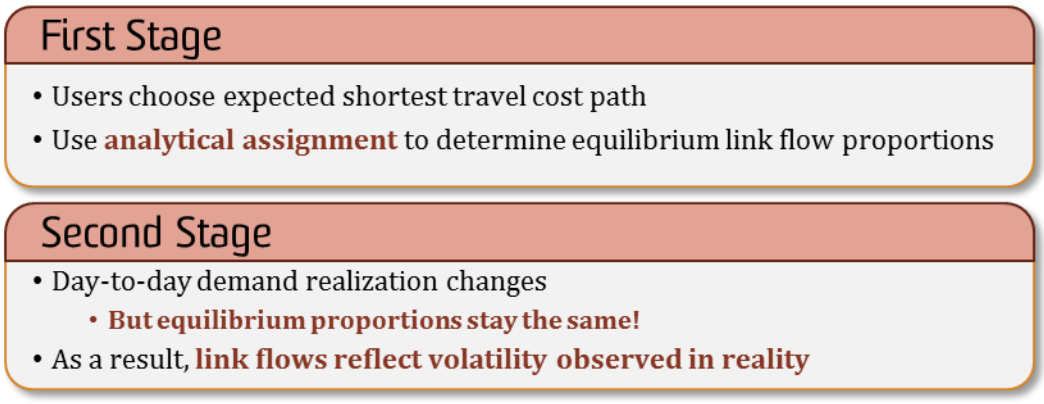


Figure 2:5 Summary of the strategic user equilibrium model framework

The modelling implications of the user behaviour in the first stage of the model are a mathematical equilibrium based on a fixed assignment pattern of link proportions. As an analogue to a Wardropian equilibrium, a strategic equilibrium implies the following: no user can unilaterally change routes for a lower *expected* travel time. The corollary of this statement is that no proportion of the demand on each path between an origin and a destination can change paths for a lower expected travel time on any path. However, the day-to-day travel realization changes according to some demand distribution. Thus, in the second stage of the model, link proportions remain fixed but total demand is changing, resulting in flows that vary reflecting disequilibrium similar to what is observed in traffic networks. The physical interpretation of this approach is flows that appear to be a disequilibrium for any given manifestation (for example, a daily demand) but are in fact defined by a higher level equilibrium based on the demand distribution.

The strategic approach has two main advantages: it accounts for demand uncertainty, which is an improvement over deterministic models, and it quantifies the variance in link travel time. This variance can be interpreted as a measure of reliability. Furthermore, in a network design problem, the planner can consider link variance and variance in total system travel time as part of the decision making process.

Strategic traffic assignment can be summarized by the following:

- Users have knowledge of the travel demand, but they do not know what the demand realization will be on a specific travel day;
- Therefore, they employ a **strategy** in which they choose the least *expected cost* path;
- The modelling implications of this behaviour is a mathematical equilibrium based on link flow *proportions* that remain fixed while the day to day demand realization is changes resulting in flow volatility.

Consider a directed graph $G = (V, A)$ where V is the set of nodes (vertices) and A is the set of links (edges), where an individual link is indexed by a. Let $r \in R$ index an origin and $s \in S$ index one destination from the set of destinations. Let W be the set of origin-destination pairs connecting origins r with destinations s, where q_{rs} indicates the proportion of total demand between origin r and destination s. The total demand is a random variable T with associated probability distribution$g(T)$. The travel cost on a link t_{ij} is a function of link flow, which is a function of the proportion of the total flow on the link p_{ij} and T. Furthermore let K_{rs} be the set of paths connecting origin r and destination s, and let f_k^{rs} represent the proportion of the total travel demand on that path. The StrUE model as previously introduced may then be written as:

$$minimize \quad z(p,T) = \int_0^{\infty} \sum_{(i,j)\in A} \int_0^{p_{ij}} t_{ij}(pT)\, g(T) dp dT \tag{2.9}$$

subject to

$$\sum_k f_k^{rs} = q^{rs} \qquad \forall r \in R, \forall s \in S \tag{2.10}$$

$$f_k^{rs} \geq 0 \qquad \forall r \in R, \forall s \in S, \forall k \in K \qquad (2.11)$$

$$p_{ij} = \sum_{r \in R} \sum_{s \in S} \sum_{k \in K} f_k^{rs} \delta_{ij,k}^{rs} \qquad \forall (i,j) \in A \qquad (2.12)$$

To ensure uniqueness of link flows, for each origin-destination, path flow proportion is assumed to be equal under all demand scenarios. Therefore, each path will be altered proportionally when the total origin-destination demand varies. The system performance metrics in the strategic approach can either be found through analytical derivations or simulation-based sampling methods, and will be detailed in the next section.

2.3.2 Strategic system optimal formulation

This section derives the formulation for the StrSO model. In a similar manner as the StrUE model, the StrSO can be interpreted as a two stage model. In the first stage, *routes are assigned* to minimize expected total travel time at a system level. The result is a set of optimal link flow proportions that remain fixed, in the same way that link flow is fixed in the deterministic approach. Then in the second stage of the model interpretation, travel conditions manifest and system performance metrics can be derived.

Strategic system optimal (StrSO) assignment is an analogue to the system optimum in traditional user equilibrium, where users are routed according to a

strategy to minimize *total system travel time*. StrSO represents the lower bound on network performance. Additionally, StrSO proportions are a component of the calculation of strategic marginal social cost-based tolls, which is the topic of Chapter 4.

For a system optimal mathematical program, the objective is to minimize total travel time. In the deterministic case, the total travel time (TSTT) is a straightforward calculation of the travel cost on each link $t_{ij}(x)$, multipled by the flow on the link that experiences that travel cost x_{ij}.

$$TSTT_{deterministic} = \sum_{(i,j)\in A} x_{ij} t_{ij}(x) \tag{2.13}$$

However, in the strategic case the flow on each link is defined as a proportion of the total system trips T, where the random variable T has a probability distribution defined as $g(T)$, as discussed in the previous section.

$$x_{ij} = p_{ij}T \rightarrow p_{ij} = \delta_{ij,k}^{rs} f^{rs} : \sum_{k\in K_{rs}} f_k^{rs} = 1 \tag{2.14}$$

Therefore, applying the same approach as Equation 3.1 will result in a calculation for the total travel time for any specific random variable T, not the expectation of the total system travel time.

$$TSTT(T) = \sum_{(i,j)\in A} p_{ij}T\left(t_{ij}(x)\right) \tag{2.15}$$

The StrSO approach seeks to minimize the *expected* total system travel time, E. The expected value of a random variable is defined as the integral of the random variable with respect to its probability measure. If X is a random variable defined in probability space, consisting of a sample space Ω, a set of events Σ, and a set of probabilities associated mapped to events P, then the expected value of X is defined as:

$$E[X] = \int_{\Omega} XdP = \int_{\Omega} X(\rho)P(d\rho) \tag{2.16}$$

When this integral exists, it defines the expectation of X. This property supports the expectation of the random variable for total trips as:

$$E[T] = \int_{0}^{+\infty} Tg(T) = M_1 \tag{2.17}$$

The total system travel time is a function of the link cost functions, which is a function of link proportion and total trips. However, the expectation of a measurable function of T, such as the TSTT, where the probability density function is $g(T)$, the following property holds true (citation).

$$E[g(T)] = \int_{-\infty}^{+\infty} w(T)g(T)dT \tag{2.18}$$

Link costs are defined as a function of link proportion and total trips. The total travel time is calculated using the expectation of the *total* cost on each link $\phi_{ij}(p, T)$. Using Equation 3.9, the total cost on each link may be defined as:

$$E[\phi_{ij}(p,T)] = \int_0^\infty p_{ij} t(p,T) g(T) dT \tag{2.19}$$

In the case of StrSO, the sample space is defined by the non-negative travel demand realization. The probability measure is the continuous probability density function describing the total travel demand $g(T)$. However, the expectation of total system travel time is the expectation of a function, not a random variable. The expectation of total system travel time is the expectation of a function, which is treated by the property in Equation 3.10. Therefore, the expectation of total system travel time may be defined as follows.

$$\begin{aligned} E = Exp\left(\sum_{(i,j)\in A} \phi_{ij}(p,T)\right) &= \sum_{(i,j)\in A} \int_0^\infty p_{ij} t_{ij}(p,T) g(T) dT \\ &= \int_0^\infty \sum_{(i,j)\in A} p_{ij} t_{ij}(p,T) g(T) dT \end{aligned} \tag{2.20}$$

The constraints to define the strategic solution space are the same in the StrSO program as in the StrUE program. Therefore, the mathematical program to define the StrSO flows, where the objective function is to minimize total travel time and the decision variables are path proportion f_{rs}^k and link proportions p_{ij}, is defined in Equations (2.21) - (2.24).

$$minimize \quad E(p,T) = \int_0^\infty \sum_{(i,j)\in A} p_{ij} t_{ij}(p,T) g(T) dT \tag{2.21}$$

subject to

$$\sum_k f_k^{rs} = q^{rs} \qquad \forall r \in R, \forall s \in S \tag{2.22}$$

$$f_k^{rs} \geq 0 \qquad \forall r \in R, \forall s \in S, \forall k \in K \tag{2.23}$$

$$p_{ij} = \sum_{r \in R} \sum_{s \in S} \sum_{k \in K} f_k^{rs} \delta_{ij,k}^{rs} \qquad \forall (i,j) \in A \tag{2.24}$$

Similar to the StrUE formulation, the formulation for StrSO is relatively straightforward, but useful. This program accounts for day-to-day uncertainty and can be scaled up for practical problems. As an additional note, the general formulation presented here needs an explicit definition for the travel cost function in order to become tractable and yield useful solutions. These assumptions will be the topic of Section 2.4.

2.3.3 Strategic system reliable formulation

Reliability is an increasingly important consideration for transport planners. Numerous works have shown the influence of travel time variability on traveller route choice (Asensio & Matas, 2008; Roughan et al, 2002; Senna, 1994). System variance represents a possible measurement of reliability that is not possible in a traditional traffic assignment approach. Strategic system reliable (StrSR) assignment represents a lower bound on the system variance of the road network performance. This powerful concept results from the strategic framework of the

assignment models proposed in this work, in which the day-to-day travel demand is assumed to follow a known distribution.

The strategic system reliable model assigns routes so as to minimize the *variance* of the total system travel time. The variance of a set measures how "spread out" the members of that set are. In a set consisting of system travel time realizations, the variance quantifies how much the total travel time deviates from the expected value. The variance of a continuous random variable X is defined as:

$$V[X] = Exp((E-\mu)^2) = \int_{-\infty}^{+\infty} (X-\mu)^2 f(T)dT \tag{2.25}$$

Where μ is the mean of the probability distribution. If X is a real valued random variable defined on Ω, it is more common to define the variance as:

$$V[X] = E(X^2) - \mu^2 \tag{2.26}$$

The variance of total system travel time is then defined as the square of the summation of the total link travel time on all links.

$$V[E(p,T)] = Exp\left(\left(\sum_{(i,j)\in A} \phi_{ij}(p,T)\right)^2\right) - Exp\left(\sum_{(i,j)\in A} \phi_{ij}(p,T)\right)^2 \tag{2.27}$$

Which may be rewritten as:

$$V[E(p,T)] = \left(\int_0^{\infty}\left(\sum_{(i,j)\in A} \phi_{ij}(p,T)g(T)dT\right)^2\right) - \left(\int_0^{\infty}\sum_{(i,j)\in A} \phi_{ij}(p,T)g(T)dT\right)^2 \tag{2.28}$$

The StrSR mathematical program is presented in Equations (2.29) - (2.31). The objective is to minimize the variance (which is identical to minimizing the standard deviation) of total travel time, as defined in Equation (3.20). The decision variables are f_{rs}^{k} which is the proportion of the total flow on each path for each OD pair and p_{ij} the proportion of the total flow on each link.

$$minimize \quad V[E(p,T)] \tag{2.29}$$

$$subject\ to$$

$$\sum_{k} f_k^{rs} = q^{rs} \qquad \forall r \in R, \forall\, s \in S \tag{2.30}$$

$$f_k^{rs} \geq 0 \qquad \forall r \in R, \forall s \in S, \forall k \in K \tag{2.31}$$

$$p_{ij} = \sum_{r \in R} \sum_{s \in S} \sum_{k \in K} f_k^{rs} \delta_{ij,k}^{rs} \qquad \forall (i,j) \in A \tag{2.32}$$

The next section discusses the assumptions and model performance metrics.

2.4 Model assumptions and performance metrics

As with all models, a number of assumptions are necessary in order to provide a method that can be solved and therefore analysed. The assumptions in this work are intended to provide analytical expressions, and furthermore, facilitate the application of the proposed framework to practically sized problems. In addition,

model performance metrics are the tool that measures model performance, an essential component of any modeling framework. Therefore, this section also defines and derives the strategic model performance metrics, which will then be demonstrated in the section containing the experimental results.

2.4.1 Strategic modelling assumptions

In order to solve the strategic assignment model, it is necessary to make an assumption regarding the distribution of the demand. This work assumes the demand follows a lognormal distribution, a continuous probability distribution where the logarithm of the random variable is normally distributed. A lognormal distribution is common distribution that has been found to be a good description of many natural growth processes, such as growth of living tissue, extreme values of rainfall data, and also in disciplines such as economics and finance. A number of previous works have also utilized the assumption of a lognormal demand distribution (Zhao & Kockelman, 2002; Zhou & Chen, 2008). An additional benefit of assuming a lognormal distribution for the travel demand is that the higher level moments of a lognormal distribution have a closed form solution:

$$M_{\beta} = e^{\beta\mu+\frac{1}{2}\beta^2\sigma^2} \tag{2.33}$$

Therefore, this work assumes a lognormal distribution for the total travel demand with random variable $T \sim LN(E_{str}, CV_{str})$, where E_{Str} is the total expected demand, the CV_{str} is the coefficient of variation of total trips, where:

$$CV_{str} = \frac{\sigma_{str}}{E_{str}} \tag{2.34}$$

The OD demand is assumed to be perfectly correlated, and therefore follows fixed, specified proportions. As discussed previously, this assumption is necessary to ensure uniqueness of link flows, which further ensures the uniqueness of the objective function. An interpretation to support this assumption may be weather conditions that affect the network as a whole. This assumption leads to a tractable model formulation and the ability for the model to be applied to practically sized problems; however, future work will investigate modelling for situations in which the demand is not perfectly correlated.

Travellers make their route choices based on knowledge of the distribution and the resulting expected travel costs. In order to solve the StrUE, StrSO, and StrSR models, a closed form assumption for the travel cost function is necessary. In transport planning, it is common to assume that the travel time on a link varies as a polynomial function of the ratio between the flow on the link and the capacity on the link. The lower bound on the travel cost is a free-flow travel time that is often considered to be a "middle of the night" travel when there are very few vehicles, or simply the length of a road segment divided by the posted speed limit. This work applies a variation of the BPR function; travel time t_{ij} is assumed to be a function of link flow proportion p_{ij} and the random variable for total demand T:

$$t_{ij}(p,T) = t_{ij}^{f}\left(1 + \alpha\left(\frac{p_{ij}T}{c_{ij}}\right)^{\beta}\right) \tag{2.35}$$

Where t_{ij}^{f} is the free flow travel time on link (i,j), c_{ij} is the capacity, α and β are BPR shaping parameters that are commonly assumed to be 0.15 and 4, respectively. For simplicity, this work assumes that the α and β parameters in the BPR function are the same on every link. The flow proportion on each link $p_{ij}$is an output from solving the StrUE model.

The assumptions made in the strategic modelling approach are summarized by the following:

- The total demand can be described by a lognormal distribution;
- The demand for all origin destination pairs is perfectly correlated;
- The travel cost can be described by the BPR function;
- Link cost functions are independent and therefore their co-variances are equal to zero.

2.4.2 Link level performance metrics

The strategic approach includes two methods for calculating all performance metrics. The first is to use analytical equations, which is possible due to the assumption of lognormal demand discussed in the previous section. The second method is to use a simulation-based approach that will be described in Section 2.5. The use of two methods is useful as a form of verification. Additionally, the

simulation method may be necessary for cases in which the analytical solutions don't exist. In this work, " ⋄ " implies that a quantity is analytically derived, while "⊙" indicates that the quantity was found through simulation, which will be detailed in Section 2.5. This section introduces the expected link travel time $\diamond E_{ij}$ and link standard deviation of travel time $\diamond S_{ij}$.

The StrUE model assumes that users will choose the expected shortest cost path, with the path cost is additive of the cost of each link comprising the path. The expected link cost $\diamond E_{ij}$ can be derived as follows.

$$\diamond E_{ij}(p,T) = \int_0^\infty t_{ij}(pT)g(T)dT = t_{ij}^f\left(1+\alpha\left(\frac{p_{ij}}{c_{ij}}\right)^\beta M_\beta\right) \tag{2.36}$$

Where M is the analytical moment of the demand distribution that is found as in Equation (2.33). One of the advantages of the strategic approach is that it provides a closed form expression to quantify the variance of a link. Equation (2.26) shows the general calculation for the variance of a quantity. Combining Equations (2.26) and (2.36), an expression for the variance of a link $\diamond V_{ij}$ can be derived.

$$\begin{aligned}\diamond V_{ij} = &\int_0^\infty t_{ij}^f\left(1+\alpha\left(\frac{p_{ij}T}{c_{ij}}\right)^\beta\right)^2 g(T)dT \\ &-\left(\int_0^\infty t_{ij}^f\left(1+\alpha\left(\frac{p_{ij}T}{c_{ij}}\right)^\beta\right)g(T)dT\right)^2\end{aligned} \tag{2.37}$$

$$\diamond V_{ij} = \frac{t_{ij}^{f2}\alpha^2}{c_{ij}^{2\beta}}\left(M_{2\beta}-M_\beta^2\right)p_{ij}^{2\beta} \tag{2.38}$$

$$\diamond S_{ij} = \sqrt{\diamond V_{ij}} \tag{2.39}$$

The next section describes the system level performance metrics.

2.4.3 System level performance metrics

This section presents the derivation and calculation of expected total system travel time $\diamond E$, the primary metric of interest for planners in particular, and the standard deviation of total system travel time, $\diamond S$. More about why these are important.

The expected total system travel time can be calculated by combining Equations (2.20)and (2.36).

$$\diamond E = \sum_{(i,j)\in A} \left(t_{ij}^{f} p_{ij} M_1 + \left(\frac{\alpha t_{ij}^{f}}{c_{ij}^{\beta}} \right) p_{ij}^{\beta+1} M_{\beta+1} \right) \tag{2.40}$$

In order to aid with the presentation of system performance metrics, consider the two parts of total system travel time as that resulting from sum of the free flow travel time on each link, F, and that resulting from the sum of the delays on each link, D.

$$F = \sum_{(i,j)\in A} t_{ij}^{f} p_{ij} \tag{2.41}$$

$$D = \sum_{(i,j)\in A} \left(\frac{\alpha t_{ij}^{f}}{c_{ij}^{\beta}} \right) p_{ij}^{\beta+1} \tag{2.42}$$

Using this notation, the expected total system travel time can then be written as:

$$\diamond E = FM_1 + DM_{\beta+1} \tag{2.43}$$

The standard deviation is more complex because we need to find the expectation of the sum of link travel times squared with respect to the total demand T. However, assuming that T is not link-dependent (implying that the power of β is the same on all links), then the total trips T can be factored out. Thus, the standard deviation can be calculated by finding F and D as a summation of values from each link, and then computing the final expression presented in Equation .

$$V[\diamond E] = F^2M_2 + D^2M_{2\beta+2} + 2FDM_{\beta+2} - \left(FM_1 + DM_{\beta+1}\right)^2 \tag{2.44}$$

$$\diamond S = \sqrt{V[\diamond E]} \tag{2.45}$$

While $\diamond S$ is somewhat nonstandard, it can still be calculated relatively easily using a single pass through the array of links.

2.5 Solution methodology

In order to scale the StrSO and StrSR models to a larger network, an algorithm based on the Frank Wolfe algorithm was developed, where the link costs are the gradient of the objective function, once again assuming the BPR function to describe link costs and that the demand fits a lognormal distribution. The link cost functions are convex, continuous, and strictly increasing with a symmetric Jacobian, and thus we can prove the Hessian is positive definite and the solution is unique.

Practical applications of user equilibrium models present a number of challenges, including collecting the appropriate field data (geometric network data, road speeds and capacities), preparing model inputs (i.e., interface between data and model), model calibration and validation of model output, the computational challenges of large scale problems, and interpreting and analysing model output. Research on computation methods to provide precise equilibrium flows has been an active topic for many decades. Some examples include the path-based gradient project method by Jayakrishnan et al (1994), the origin-based algorithm (Bar-Gera, 2002), Dial's bush-based Algorithm B (Dial, 2006; Nie, 2010), the bush-based local cost user equilibrium (Gentile, 2014), and the traffic assignment by alternative paired segments (TAPAS) algorithm (Bar-Gera, 2010). However, the Frank Wolfe method (Leblanc et al, 1975) remains one of the most popular approaches to solving the user equilibrium problem due to its straightforward implementation and low memory requirements. An adapted Frank Wolfe approach is implemented here; however, there are well-acknowledged issues relating to the convergence of the Frank Wolfe method that are discussed below. For practical planning issues that may the determinant of multi-million dollar projects, a more precise solution approach is recommended (although Frank Wolfe is still used in practice).

The Frank Wolfe method is a convex combination algorithm that considers the linear approximation of an objective function (Frank & Wolfe, 1956). Frank Wolfe is an iterative procedure consisting of two primary steps: minimize a linearised subproblem to find the decent direction, and then determining the

optimal stepsize to minimize a convex combination of the current solution. The objective function must be convex and differentiable. As previously mentioned, the Frank Wolfe approach is well-established as a means to solve the user equilibrium mathematical program, where the linear subproblem of the objective function is the shortest path problem.

The subproblem for the StrUE problem is the expected cost function. The subproblem of the StrSO and StrSR models are the gradient of the expected total system travel time and variance of total system travel time with respect to each link p_{ij}, respectively.

The gradient of $\diamond E$ is relatively simple to find for each link and represented the expected marginal cost function, contained in Equation (2.47).

$$\nabla(\diamond E) = \frac{\partial}{\partial p}\left(\sum_{(i,j)\in A}\left(t_{ij}^{f} p_{ij} M_1 + \left(\frac{\alpha t_{ij}^{f}}{c_{ij}^{\beta}}\right) p_{ij}^{\beta+1} M_{\beta+1}\right)\right) \tag{2.46}$$

$$\nabla(\diamond E) = \sum_{(i,j)\in A}\left(t_{ij}^{f} M_1 + \left(\frac{\alpha t_{ij}^{f}}{c_{ij}^{\beta}}\right) p_{ij}^{\beta} M_{\beta+1}\right) \tag{2.47}$$

In order to solve the StrSR model, the linearized subproblem of the Frank Wolfe method represents the marginal reliability function, which is calculated as the gradient of the StrSR objective (the system variance) with respect to the link proportion variable (the decision variable in the Frank Wolfe method). The general function is shown in Equation (2.48).

$$\nabla(\diamond S) = \frac{\partial}{\partial p}\left(F^2 M_2 + D^2 M_{2\beta+2} + 2FDM_{\beta+2} - \left(FM_1 + DM_{\beta+1}\right)^2\right) \qquad (2.48)$$

Where the summations F and D are both functions of functions of p_{ij}. The gradient of the system variance requires the use of the chain rule. For two generic functions $f(x)$ and $g(x)$, the chain rules states:

$$f\big(g(x)\big) = f'\big(g(x)\big)g'(x) \qquad (2.49)$$

In the case of the marginal reliability function, the inner function of p_{ij} is the travel cost function. The partial derivative of the travel cost function is displayed in Equation (2.50).

$$\frac{\partial}{\partial p_{ij}} t_{ij}(p,T) = t_{ij}^f \frac{\alpha(\beta+1)}{c_{ij}^\beta} p^\beta \qquad (2.50)$$

Finally, Equations (2.48), (2.49), and (2.50) can be combined

$$\begin{aligned}\nabla(\diamond S) = \; & 2FM_2 t_{ij}^t + 2M_{2\beta+2} D\left(\frac{\partial}{\partial p_{ij}} t_{ij}\right) \\ & + 2M_{\beta+2}\left(t_{ij}^f D + F\left(\frac{\partial}{\partial p_{ij}} t_{ij}\right)\right) \\ & - 2\big(M_1 F + M_{\beta+1} D\big)\left(M_1 t_{ij}^f + M_{\beta+1}\left(\frac{\partial}{\partial p_{ij}} t_{ij}\right)\right)\end{aligned} \qquad (2.51)$$

While this expression looks prohibitively complex, it can still can calculated by passing through an array of links only twice. The first computation is to sum the F and the D terms. On the second time pass through the array, the gradient with respect to $p_{ij}$is saved as the "cost" of the link.

The approach for solving the strategic models is described in Algorithm 2.1. A note about the pseudo-code of the algorithms in this work: the solution methods in this thesis were programmed by the author using the object oriented C++ programming language. Thus the general programming paradigm is influenced by C++ standard guides ("Google C++ Style Guide,"). Of course, reproducing C++ code for an illustrative algorithm is both tedious and unnecessary; however, the general approach of the pseudo-algorithms is intended to be functional and clear, but grounded in C++ paradigm. Functions that are intended to be "called" (and therefore linked to other functions) are named using the underscore, "_", i.e., `solve_strategic_model()` refers to Algorithm 2.1. In some places, just a short description is provided, as opposed to a detailed function. For example, Algorithm 2.1 does not include the code for the shortest path algorithm, which is a rich field of research unto itself. See Ahuja et al (1993) for more details. For clarity, details such as declaring variables and vectors are also excluded from the pseudo-algorithms.

Algorithm 2.1: Solve the strategic model pseudocode

```
INPUT: Network G = (N, A); g(E_S, CV_S);
    procedure solve_strategic(TYPE)
1:      K*_rs ← find shortest paths using free flow costs;
2:      while (update_relative_gap(p, p*, c) > 0.0001)
3:          //update proportion assignment
4:          for (i, j) ∈ A do
5:              p_ij ← p_ij + λ(p*_ij − p_ij);
6:          end for
7:          update costs for TYPE;
8:          //find shortest path given new costs
9:          for (r ∈ R) do
10:             find shortest path matrix for r;
11:             for (s ∈ S: rs ∈ W) do
12:                 find k*_rs;
13:                 for (i, j) ∈ k*_rs do
14:                     p*_ij += q_rs;
15:                 end for
16:             end for
17:         end for
18:         λ ←minimize objective function(TYPE)
19:     end while
20:     calculate performance metrics;
    end procedure
OUTPUT: ⋄ E; ⋄ S; ⊙E; ⊙S; p_ij ∀(i, j) ∈ A;
```

Note that a "relative gap" inspired termination criteria was selected, which measures how far the network is from equilibrium flows, i.e., the difference in the path costs. The relative gap is calculated as the difference between the cost of the “all or nothing” path assignment and the cost of the current link flows, outlined in Algorithm 2.2.

Algorithm 2.2: Solve the strategic model pseudocode

```
    sub-procedure update relative gap(p, p*, c)
1:      for (i, j) ∈ A do
2:          E*+= p*_ij * c_ij
3:          E+= p_ij c_ij
4:      end for
5:      return (E* − E)/E;
    end sub-procedure
```

As with other demand scenario based approaches, if the probability distribution of the total demand is unknown, a simulation method could be used to solve the model. In the case presented in this thesis, it is assumed that the total travel demand follows a lognormal distribution, and therefore the travel cost, objective functions, and system performance measures have analytical solutions, presented in the previous section. However, a simulation-based method is still useful for testing and for the purposes of verification. An example of the simulation method employed in this work is presented in Algorithm 2.3. Given the equilibrium proportions that are output from solving the strategic model, the expected total system travel time and the standard deviation of total travel time can be determined by sampling a random variable for total trips from the strategic demand distribution using a Monte Carlo based approach, and then calculating the appropriate performance metrics from the samples.

Algorithm 2.3: Solve the strategic model pseudocode

INPUT: A; $g(E_s, CV_s)$; $p_{ij}, \forall (i,j) \in A$; N demand samples

procedure simulation

E = running average of system travel time

for i = [1…N] **do**

sample_trips ← random sample from $g(E_s, CV_s)$;

for $(i,j) \in A$ **do**

$TSTT$ += $t_{ij}^{f} * (1 + \alpha * pow(p_{ij} * sample_trips / c_{ij}, \beta))$;

end for

$E \leftarrow E \cup TSTT$

end for

$\odot E \leftarrow average(E)$

$\odot S \leftarrow std(E)$

end procedure

OUTPUT: $\odot E$, $\odot S$

Note that the *Simulation Sub-Procedure* can be easily adapted to StrUE, StrSO, and StrSR by using the correct strategic proportions as input. Further note that all sampling results (e.g., $\odot E, \odot STD$) can be calculated as "running" averages to prevent unnecessary memory storage and computation time.

Combining the algorithms above, the strategic models can be solved and analysed. The next section presents results for each model on a variety of test networks.

2.6 Demonstration of results

The preceding sections outlined the concept, formulation, assumptions, and solution method for the StrUE, StrSO, and StrSR models. This section discusses the implementation of the models on four test networks of varying size and discusses

the implications of the model in regards to system level and link level performance metrics.

2.6.1 Description of test networks

The StrUE, StrSO, and StrSR models were tested on four networks. The first network was based on the Nguyen Dupius network, a small sized test network that is common in transport test problems. The Nguyen Dupius network consists of 13 nodes, 19 links, 2 origins, and 2 destinations. The capacity on all links is 2,200, and it is considered a congested network. The Nguyen Dupius network is useful for demonstration and test purposes; however, it is not large enough to capture significant effects of route choice. The Nguyen Dupius network is shown in Figure 2:6. Table 2-2 contains a summary of the data for the demonstration networks in this chapter.

Table 2-2 Summary of demonstration network

	Nguyen Dupius	Sioux Falls	Anaheim	Gold Coast
nodes	13	24	416	4054
links	19	76	914	9565
zones	4	24	38	1067
E(T)	6,265	360,000	106,076	121,921

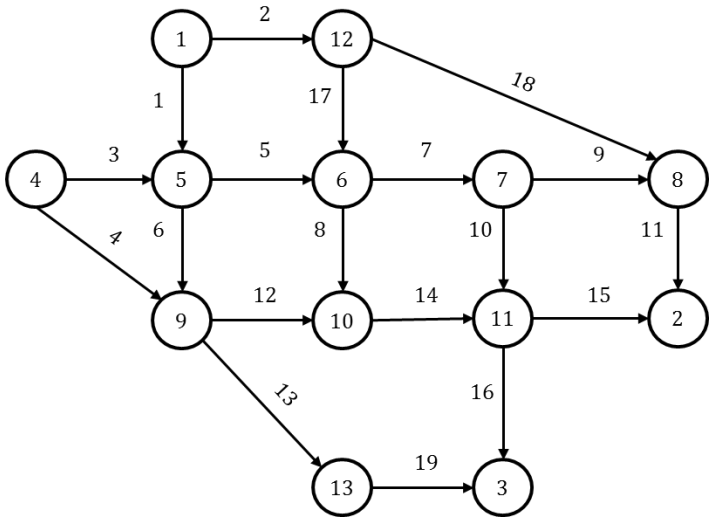


Figure 2:6 The Nguyen-Dupius network.

The second network of interest is the ubiquitous Sioux Falls network, which is commonly treated as a sort of benchmark case in the transport modelling community. This is likely due to the grid-based nature of the network. While the use of the Sioux Falls network (based on the same network data) is useful to create rough comparisons between different methods, it is not considered a realistic representation of Sioux Falls, South Dakota, USA. Figure 2:7 shows the Sioux Falls grid network that is used as a test problem, and the real Sioux Falls network in 2014 (from OpenStreetMaps).

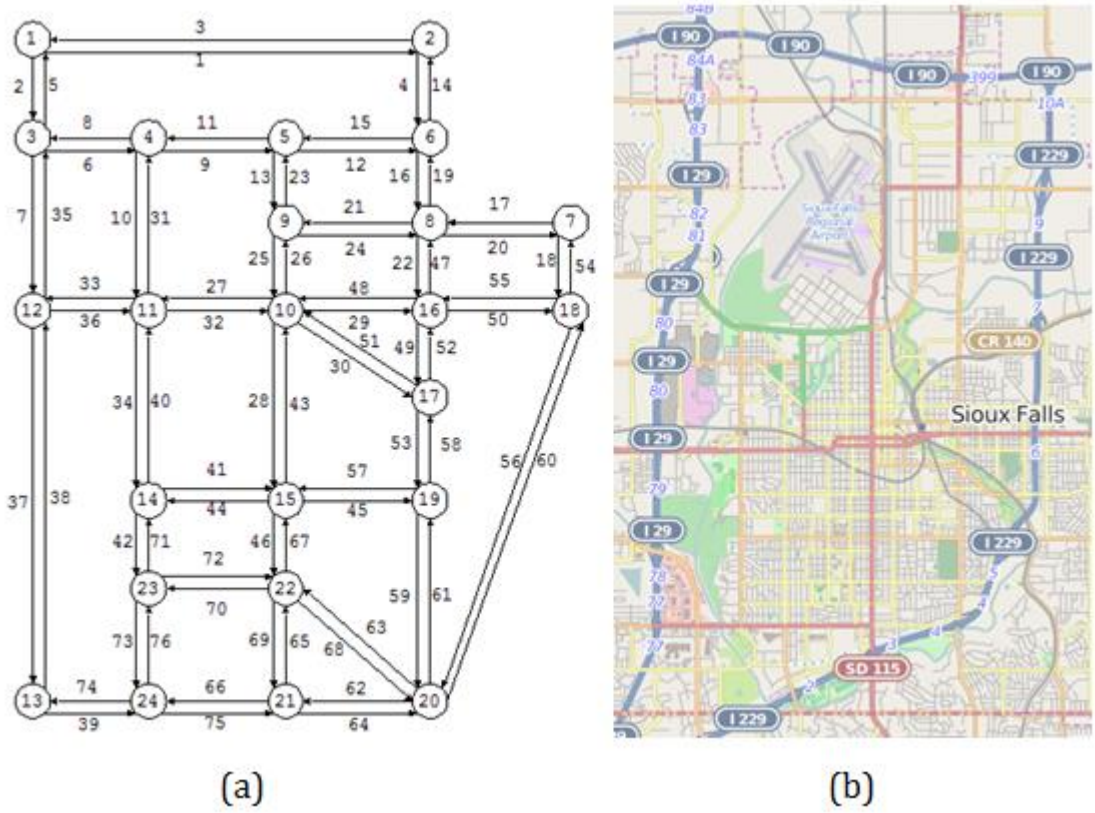


Figure 2:7 (a) Sioux Falls test network and (b) real Sioux Falls network (2014, OpenStreetMaps)

The third network is the Anaheim network, also a relatively common network in transportation test problems. The data roughly approximates the area, based on planning data from 1992. Given that Anaheim is located in the heart of Los Angeles, the basic infrastructure has probably not seen dramatic changes in the last twenty years, but of course, the network is not intended as realistic in this work.

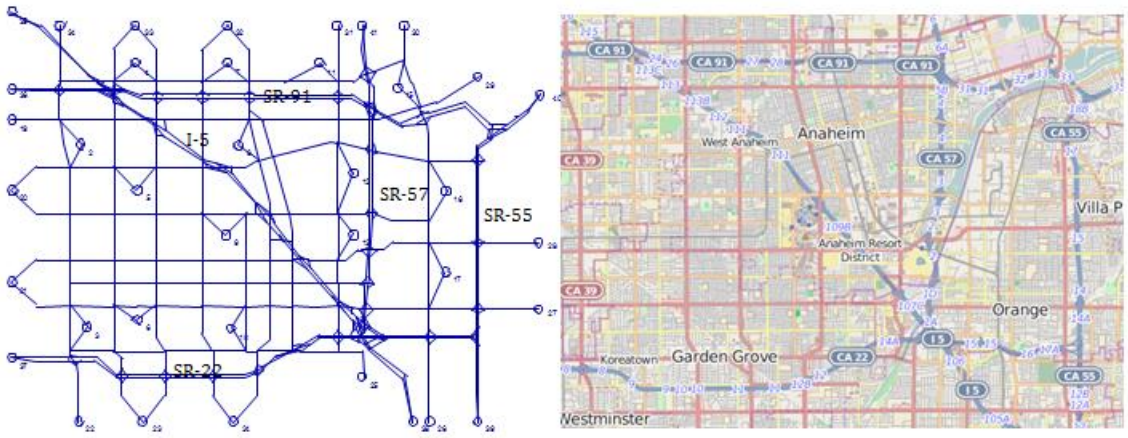


Figure 2:8 (a) Anaheim test network, and (b) real Anaheim network (2014, OpenStreetMaps)

Finally, this thesis presents the strategic modelling results on the Gold Coast network. The data for this network came from Bar-Gera, who reports that the data was provided by Veitch Lister Consultancy in Brisbane, Australia. The Gold Coast network is the largest demonstration network in this thesis, with 4054 nodes, 9565 links, and 1067 zones. While this is a realistic sized network, regional planning networks can become significantly larger (for example, the Sydney, Australia planning network has about 80,000 links and 3,000 zones).

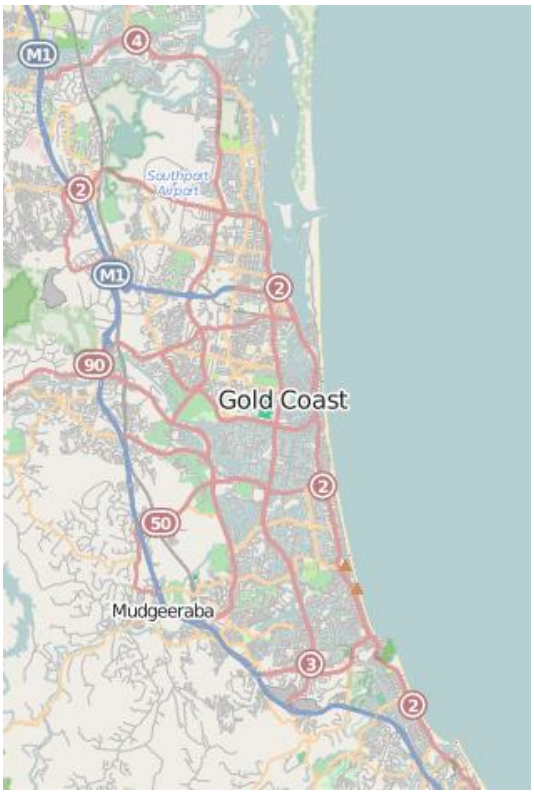


Figure 2:9 Gold Coast, Queensland, Australia (2014 OpenStreetMaps)

2.6.2 Comparing Analytic and Simulation Results

This section compares the results from the analytic method (Figure X) and the simulation based method. This comparison serves a dual purpose; firstly, convergence of analytical and simulation results is a simple check for the elimination of human error. Secondly, for alternate strategic approaches in which a distribution is not known, it would be possible to solve the strategic model using the simulation approach alone. This section validates the convergence of analytical and simulation results.

2.6.3 System-Level Model Results

This section presents the model results on the Sioux Falls, Anaheim, and Gold Coast networks. First the overall results for each network are presented comprehensively. Then, the data is compared and analysed in a more relatable way.

Table 2-3 presents the StrUE model results on the Sioux Falls network, where the coefficient of variation of the total travel demand CV_{str} varies from $0 \leq CV_{str} \leq 0.85$. Table 2-3 contains the four system performance metrics: $\diamond E_{strue}, \diamond S_{StrUE}, \odot E_{StrUE}, \odot S_{StrUE}$. Note that the number of simulation samples $N = 200{,}000$. This ensures that the convergence between the analytical and simulation results is acceptable.

The deterministic case is included in the first row of Table 2-3, where $CV_{str} = 0$. When day-to-day demand uncertainty is included in the model, the prediction of system metrics is higher in every case. The change in performance metrics is generally nonlinear with accordance to the travel cost function. When $CV_{str} > 0.25$, the $\diamond STD$ was greater than $\diamond E$. As CV_{str} increases, the $\diamond S$ increases very dramatically. This suggests that the best application for the StrUE model may be networks where the $CV_{str} < 0.5$.

Table 2-3 Overall Sioux Falls results: StrUE

CV_{str}	$\diamond E_{StrUE}$	$\diamond S_{StrUE}$	$\odot E_{StrUE}$	$\odot S_{StrUE}$
0.00	7.48E+06	0	7.48E+06	0
0.05	7.57E+06	1.22E+06	7.57E+06	1.22E+06
0.10	7.86E+06	2.69E+06	7.87E+06	2.69E+06
0.15	8.38E+06	4.74E+06	8.38E+06	4.75E+06
0.20	9.23E+06	8.04E+06	9.24E+06	7.96E+06
0.25	1.05E+07	1.39E+07	1.05E+07	1.40E+07
0.30	1.25E+07	2.55E+07	1.25E+07	2.68E+07
0.35	1.54E+07	4.96E+07	1.55E+07	5.23E+07
0.40	1.98E+07	1.03E+08	1.98E+07	9.72E+07
0.45	2.67E+07	2.31E+08	2.69E+07	1.93E+08
0.50	3.74E+07	5.50E+08	3.60E+07	3.02E+08
0.55	5.45E+07	1.39E+09	5.10E+07	7.51E+08
0.60	8.19E+07	3.66E+09	8.08E+07	1.61E+09
0.65	1.26E+08	1.00E+10	1.16E+08	2.11E+09
0.70	1.98E+08	2.85E+10	1.96E+08	6.48E+09
0.75	3.17E+08	8.28E+10	2.84E+08	9.11E+09
0.80	5.11E+08	2.46E+11	4.61E+08	1.76E+10
0.85	8.33E+08	7.43E+11	1.27E+09	3.13E+11

Table 2-4 contains the overall results for the StrSO model and sensitivity analysis based on the CV_{str}. The StrSO model is the lower bound on the StrUE model, so as expected, all performance metrics are reduced.

Table 2-4 Overall Sioux Falls results: StrSO

CV_{str}	$\diamond E_{StrSO}$	$\diamond S_{StrSO}$	$\odot E_{StrSO}$	$\odot S_{StrSO}$
0.00	7.20E+06	0	7.20E+06	0
0.05	7.29E+06	1.12E+06	7.29E+06	1.12E+06
0.10	7.57E+06	2.47E+06	7.58E+06	2.48E+06
0.15	8.10E+06	4.39E+06	8.11E+06	4.42E+06
0.20	8.93E+06	7.52E+06	8.93E+06	7.42E+06
0.25	1.02E+07	1.32E+07	1.03E+07	1.36E+07
0.30	1.21E+07	2.43E+07	1.20E+07	2.36E+07
0.35	1.51E+07	4.79E+07	1.52E+07	6.14E+07
0.40	1.95E+07	1.01E+08	1.95E+07	8.77E+07
0.45	2.64E+07	2.28E+08	2.67E+07	1.97E+08
0.50	3.72E+07	5.46E+08	3.86E+07	4.45E+08
0.55	5.43E+07	1.38E+09	5.42E+07	8.81E+08
0.60	8.17E+07	3.64E+09	8.70E+07	1.94E+09
0.65	1.26E+08	1.00E+10	1.15E+08	2.53E+09
0.70	1.98E+08	2.84E+10	1.95E+08	1.12E+10
0.75	3.16E+08	8.28E+10	2.66E+08	8.50E+09
0.80	5.11E+08	2.46E+11	4.23E+08	1.58E+10
0.85	8.33E+08	7.42E+11	6.81E+08	2.76E+10

Table 2-5 displays the results for the four performance metrics under the sensitivity analysis experiment, in which CV_{str} varies between $0.05 \leq CV_{str} \leq 0.7$, for the StrSR model. The case in which $CV_{str} = 0$ is not feasible in the case of the StrSR model. Additionally, the cases in which the $CV_{str} > 0.7$ were judged to show unrealistic amounts of variability (as seen in Table 2-3 and Table 2-4) and so were not included in the tabular results.

Table 2-5 Overall Sioux Falls results: StrSR

CV_{str}	$\diamond E_{StrSR}$	$\diamond S_{StrSR}$	$\odot E_{StrSO}$	$\odot S_{StrSO}$
0.05	7.30E+06	1117150	7.31E+06	1120140
0.10	7.59E+06	2.47E+06	7.59E+06	2.46E+06
0.15	8.11E+06	4.38E+06	8.10E+06	4.38E+06
0.20	8.95E+06	7.51E+06	8.92E+06	7.37E+06
0.25	1.02E+07	1.32E+07	1.02E+07	1.30E+07
0.30	1.22E+07	2.43E+07	1.21E+07	2.30E+07
0.35	1.51E+07	4.79E+07	1.51E+07	4.40E+07
0.40	1.95E+07	1.01E+08	1.93E+07	8.51E+07
0.45	2.64E+07	2.28E+08	2.64E+07	2.52E+08
0.50	3.72E+07	5.46E+08	3.66E+07	3.58E+08
0.55	5.43E+07	1.38E+09	5.21E+07	9.80E+08
0.60	8.17E+07	3.64E+09	7.82E+07	1.29E+09
0.65	1.26E+08	1.00E+10	1.22E+08	4.44E+09
0.70	1.98E+08	2.84E+10	1.67E+08	4.25E+09

Next, the tabular results are presented for the Anaheim network. Table 2-6 shows the results for the StrUE model, Table 2-7 shows the results for the StrSO model, and Table 2-8 shows the results for the StrSR model. A similar trend is displayed for the Anaheim network as was seen in the Sioux Falls network. As the CV_{str} increases, the difference between the StrUE and the StrSO performance metrics, particularly $\diamond E$ decreases.

Table 2-6 Overall Anaheim Results: StrUE

CV_{str}	$\diamond E_{StrUE}$	$\diamond S_{StrUE}$	$\odot E_{StrUE}$	$\odot S_{StrUE}$
0.00	1.32E+06	0	1.32E+06	0
0.05	1.33E+06	9.65E+04	1.33E+06	9.65E+04
0.10	1.34E+06	2.00E+05	1.34E+06	2.00E+05
0.15	1.36E+06	3.22E+05	1.36E+06	3.22E+05
0.20	1.38E+06	4.82E+05	1.39E+06	4.83E+05
0.25	1.43E+06	7.21E+05	1.43E+06	7.17E+05
0.30	1.50E+06	1.14E+06	1.50E+06	1.15E+06
0.35	1.59E+06	1.96E+06	1.59E+06	1.85E+06
0.40	1.74E+06	3.74E+06	1.74E+06	4.17E+06
0.45	1.95E+06	7.83E+06	1.95E+06	6.27E+06
0.50	2.27E+06	1.78E+07	2.30E+06	2.06E+07
0.55	2.78E+06	4.32E+07	2.69E+06	1.93E+07
0.60	3.55E+06	1.10E+08	3.66E+06	6.36E+07
0.65	4.76E+06	2.92E+08	4.89E+06	1.13E+08
0.70	6.73E+06	8.07E+08	5.89E+06	1.43E+08
0.75	9.94E+06	2.31E+09	1.07E+07	8.14E+08
0.80	1.52E+07	6.77E+09	1.38E+07	6.68E+08
0.85	2.37E+07	2.02E+10	3.18E+07	5.63E+09

Table 2-7 Overall Anaheim Results: StrSO

$\boldsymbol{CV_{str}}$	$\diamond \boldsymbol{E_{StrSO}}$	$\diamond \boldsymbol{S_{StrSO}}$	$\odot \boldsymbol{E_{StrSO}}$	$\odot \boldsymbol{S_{StrSO}}$
0.00	1.30E+06	0	1.30E+06	0
0.05	1.31E+06	8.86E+04	1.31E+06	8.85E+04
0.10	1.32E+06	1.83E+05	1.32E+06	1.83E+05
0.15	1.33E+06	2.90E+05	1.33E+06	2.91E+05

0.20	1.36E+06	4.25E+05	1.36E+06	4.23E+05
0.25	1.39E+06	6.18E+05	1.39E+06	6.11E+05
0.30	1.45E+06	9.42E+05	1.45E+06	9.37E+05
0.35	1.53E+06	1.58E+06	1.54E+06	1.92E+06
0.40	1.65E+06	2.99E+06	1.65E+06	2.95E+06
0.45	1.84E+06	6.34E+06	1.82E+06	4.78E+06
0.50	2.13E+06	1.47E+07	2.12E+06	1.12E+07
0.55	2.58E+06	3.64E+07	2.60E+06	3.03E+07
0.60	3.30E+06	9.54E+07	3.30E+06	3.62E+07
0.65	4.46E+06	2.61E+08	5.31E+06	5.82E+08
0.70	6.33E+06	7.38E+08	5.94E+06	1.81E+08
0.75	9.40E+06	2.14E+09	8.91E+06	3.03E+08
0.80	1.44E+07	6.36E+09	1.28E+07	1.05E+09
0.85	2.28E+07	1.92E+10	3.38E+07	7.29E+09

Table 2-8 Overall Anaheim Results: StrSR

CV_{str}	$\diamond E_{StrSR}$	$\diamond S_{StrSR}$	$\odot E_{StrSO}$	$\odot S_{StrSO}$
0.05	1.32E+06	86389.3	1.32E+06	8.63E+04
0.10	1.33E+06	1.78E+05	1.33E+06	1.78E+05
0.15	1.35E+06	2.83E+05	1.35E+06	2.84E+05
0.20	1.38E+06	4.13E+05	1.38E+06	4.16E+05
0.25	1.42E+06	5.99E+05	1.42E+06	5.96E+05
0.30	1.49E+06	9.09E+05	1.49E+06	9.37E+05
0.35	1.58E+06	1.52E+06	1.57E+06	1.42E+06
0.40	1.72E+06	2.87E+06	1.73E+06	2.98E+06
0.45	1.92E+06	6.10E+06	1.93E+06	5.93E+06
0.50	2.22E+06	1.43E+07	2.21E+06	9.99E+06
0.55	2.69E+06	3.57E+07	2.67E+06	1.81E+07

0.60	3.41E+06	9.42E+07	3.31E+06	3.34E+07
0.65	4.56E+06	2.59E+08	4.42E+06	7.73E+07
0.70	6.42E+06	7.34E+08	6.79E+06	3.44E+08

Finally, the strategic method results on the largest test network presented in this thesis are presented in Table 2-9, Table 2-10, Table 2-11. The Gold Coast network is relatively uncongested, revealed by an analysis of the expected link flows and standard deviations.

Table 2-9 Overall Gold Coast Results: StrUE

CV_{str}	$\diamond E_{StrUE}$	$\diamond S_{StrUE}$	$\odot E_{StrUE}$	$\odot S_{StrUE}$	Time (seconds)
0.00	1.19E+06	0	1.19E+06	0	460.48
0.05	1.19E+06	7.54E+04	1.19E+06	7.55E+04	437.20
0.10	1.20E+06	1.55E+05	1.20E+06	1.54E+05	464.94
0.15	1.21E+06	2.43E+05	1.21E+06	2.43E+05	478.86
0.20	1.22E+06	3.50E+05	1.22E+06	3.51E+05	577.91
0.25	1.25E+06	4.94E+05	1.25E+06	4.97E+05	614.55
0.30	1.28E+06	7.19E+05	1.28E+06	7.14E+05	641.58
0.35	1.33E+06	1.14E+06	1.33E+06	1.12E+06	823.85
0.40	1.40E+06	2.03E+06	1.40E+06	2.50E+06	938.35
0.45	1.51E+06	4.08E+06	1.51E+06	5.60E+06	1189.58
0.50	1.67E+06	9.02E+06	1.64E+06	5.89E+06	1451.23

Figure 2:10 shows an analysis comparing the scale of the expected total travel time $\diamond E_{StrUE}$ and the standard deviation $\diamond S_{StrUE}$. Even in an uncongested network like

the Gold Coast, there is a significant variation in total system travel time. When $CV_{str} > 0.4$, the variations is twice the total system travel time itself.

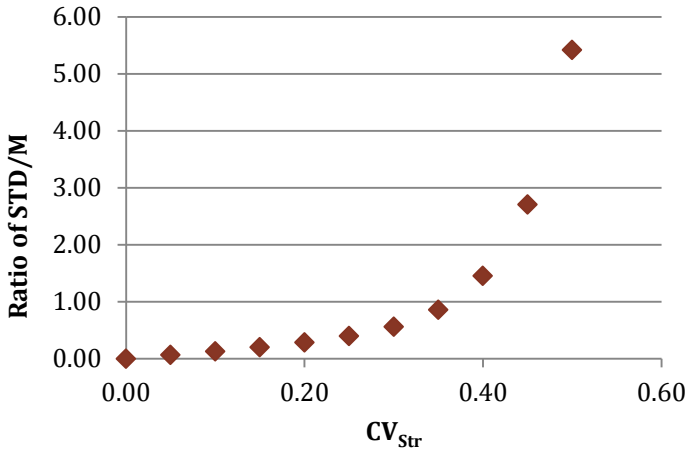


Figure 2:10 Comparison of $\diamond S/\diamond E$ on the Gold Coast network

Table 2-10 shows the sensitivity analysis results for StrSO on the Gold Coast network. The rightmost column also shows the computation time in seconds. The computation time increases as the network volatility increases because more iterations are necessary to converge the relative gap. Due to the fact that the Gold coast is a relatively uncongested network, the differences between StrUE and StrSO are not as substantial.

Table 2-10 Overall Gold Coast Results: StrSO

CV_{str}	$\diamond E_{StrSO}$	$\diamond S_{StrSO}$	$\odot E_{StrSO}$	$\odot S_{StrSO}$	Time
0.00	1.18E+06	0	1.18E+06	0	1874.14
0.05	1.18E+06	7.04E+04	1.18E+06	7.04E+04	1911.85
0.10	1.18E+06	1.43E+05	1.18E+06	1.44E+05	2131.89

0.15	1.19E+06	2.23E+05	1.19E+06	2.23E+05	2439.88
0.20	1.20E+06	3.14E+05	1.20E+06	3.14E+05	2778.05
0.25	1.22E+06	4.29E+05	1.22E+06	4.30E+05	3736.80
0.30	1.25E+06	5.97E+05	1.25E+06	6.16E+05	5249.95
0.35	1.29E+06	8.91E+05	1.29E+06	8.89E+05	7319.21
0.40	1.34E+06	1.51E+06	1.34E+06	1.65E+06	10011.30
0.45	1.43E+06	2.97E+06	1.42E+06	2.12E+06	15040.10
0.50	1.56E+06	6.60E+06	1.57E+06	8.63E+06	20095.60

Finally, Table 2-11 shows the sensitivity analysis results for the StrSR model on the Gold Coast network. The $\diamond E_{StrSr}$ is greater than the $\diamond E_{StrSO}$, but the $\diamond S_{StrSR}$ is less than the $\diamond S_{StrSO}$. Again, the results show a small percentage difference due to the characteristics of the Gold Coast network.

Table 2-11 Overall Gold Coast Results: StrSR

CV_{str}	$\diamond E_{StrSR}$	$\diamond S_{StrSR}$	$\odot E_{StrSR}$	$\odot S_{StrSR}$
0.05	1.19E+06	68884.4	1.19E+06	68753.6
0.10	1.20E+06	1.40E+05	1.20E+06	1.40E+05
0.15	1.21E+06	2.17E+05	1.21E+06	2.17E+05
0.20	1.22E+06	3.04E+05	1.22E+06	3.03E+05
0.25	1.25E+06	4.12E+05	1.25E+06	4.14E+05
0.30	1.28E+06	5.67E+05	1.28E+06	5.62E+05
0.35	1.32E+06	8.37E+05	1.32E+06	8.44E+05
0.40	1.39E+06	1.41E+06	1.39E+06	1.33E+06
0.45	1.48E+06	2.79E+06	1.48E+06	2.60E+06
0.50	1.60E+06	6.30E+06	1.62E+06	4.79E+06

The strategic framework quantifies the variation in total system travel time as a result of the variation in the total travel demand distribution. Analysis beyond the scope of this thesis will reveal the extent of calibration that is needed to apply the framework to realistic networks but this section demonstrates that the theory is sound.

2.6.4 Link-Level Model Results

While system level performance metrics are the primary values of interest to transport planners, it is often important to make link level estimations of performance as well. In particular, link level measures such as estimated travel time or flow are often used in the calibration process for standard traffic assignment models. However, caution such be employed regarding link level evaluations in static traffic assignment models. Excluding the concerning assumptions with regard to the representation of capacity, from a pure modelling perspective, link level results should be treated with caution. This is due to the solution methods for the modelling and the equilibrium principle itself; Bar-Gera (2010) suggests that in many equilibrium models, there may not be just a single equilibrium solution with regard to link flow.

Frank Wolfe is a commonly used solution approach due to its straightforward implementation and its undemanding computational requirements. However, as mentioned in Section 2.5, when it comes to the small changes that determine an equilibrium solution, Frank Wolfe displays what's

known as the "tailing" effect (Dial, 2006). Essentially, Frank Wolfe adjusts the link flows for all OD pairs by an identical amount, regardless of how close or far from an equilibrium solution that OD pair may be. This limits the effectiveness of the Frank Wolfe method. However, even methods that can achieve a higher level of precision (relation gap < 10^{-14}) still display the issue of multiple possible equilibrium solutions. While the equilibrium system performance metric remains the same (such as the total system travel time), the flows themselves may vary, although to what extent is still an open question for researchers.

However, link level performance metrics are still an important representation of model performance and a characteristic that is of interest to planners. Therefore, this section will briefly examine one of the link level metrics of performance, the standard deviation of travel time $\diamond S_{ij}$.

Table 2-12 shows a histogram comparing the link standard deviation $\diamond S_{ij}$ resulting for the StrUE and StrSO models where $CV_{str} = 0.15$, where the range of $\diamond S_{ij}$(minutes) is shown in leftmost column. The columns titled StrUE and StrSO indicate the number of links that were in the range indicated in the leftmost column. An examination of the results suggests that StrSO actually displayed a higher standard deviation on some links, particularly in the "7.5-10 minutes" bin, which contained 4 links in the StrUE model but 6 links in the StrSO model. While the system standard deviation is lower in the StrSO model, it is interesting to note that the impact on individual links could be counter-intuitive.

Table 2-12 Sioux Falls network: comparison of $\diamond S_{ij}$

Link STD	StrUE	StrSO
0	0	0
<1	22	20
1-2.5	14	17
2.5-5	16	17
5.0-7.5	18	14
7.5-10	4	6
10-12.5	2	2
>12.5	0	0

The Sioux Falls network is considered congested, with a maximum flow-to-capacity ratio of 2.57 and average ratio of 1.46. On the other hand, the Anaheim network has a maximum average-flow-to-capacity ratio of 1.88 and an average of 0.26 (due to the fact that it has many links that are not utilized). The links with a low average-flow-to-capacity ratio will also have a low standard deviation. Anaheim is less congested, and therefore the range of standard deviations will be much smaller, as seen in Table 2-13. A majority of the links in the Anaheim network have a standard deviation of less than half a minute. This result makes sense in a larger network, where only specific links may be the main cause of congestion.

Table 2-13 Anaheim network: comparison of $\diamond S_{ij}$

Link STD	StrUE	StrSR
<0.25	158	84
0.25-0.50	742	824
0.50-0.75	11	4
0.75-1.0	2	1
1.0-1.5	0	0
1.5-2.0	0	0
>2.0	1	1

2.7 Concluding remarks

This chapter introduced the three base, time-invariant strategic modelling approaches. A background and literature review provided a detailed problem context for the strategic modelling approach that accounts for day-to-day uncertainty in demand. Each model was discussed and formulated. Next, this chapter discussed a number of assumptions to make the model tractable and derived system performance metrics. A solution approach based on the Frank Wolfe method was described and outlined. Finally, the models were demonstrated and compared on four test networks.

One of the primary advantages of the strategic approach is that it can be extended to many practical problems that are relevant to practitioners and researchers alike. The next two chapters extend the base model to the tolling problem and the network design problem.

Chapter 3

Network Management Application: Marginal Social Cost Pricing

3.1 Introduction

Transport network road pricing is a topic of great interest to researchers and practitioners alike. It is one of the primary management tools available to road operators to improve network performance for the benefit of the system as a collective. Additionally, a well-planned tolling scheme will not only help relieve congestion, it can also produce a profit that will help operators expand and maintain infrastructure for a stronger, more reliable system.

However, the problem of road pricing becomes more complex when the inherent uncertainty in origin-destination (OD) trip demand in considered. While traditional deterministic models like the marginal social cost (MSC) approach can be easily solved, they may overestimate performance when factors such as the future planning demand vary from the forecasted value. Additionally, a

deterministic model does not capture the effect of day-to-day demand volatility on user route choice behaviour.

Road network pricing research has a well-established foundation in the literature. One common research topic is marginal social cost (MSC) pricing, based on the economic ideas of Pigou (1920). This pricing scheme assumes users behave in a "selfish" manner, seeking to minimize their own travel costs. Prices are then set on each link such that a user is charged a toll equivalent to the marginal impact of her using a given link (i.e., the increase in travel cost to everyone on a link resulting from a single additional user). This is also referred to as first best pricing, in which all links in a network are priced. Second best pricing represents an extension of this problem, in which a subset of the network links are tolled.

While the first best pricing problem can be easily solved, a complexity is introduced when demand uncertainty is considered. In the short term, users face a varying day-to-day travel demand. For longer term planning, unpredictable changes in land use, technology, and many other factors make demand forecasts difficult. These inherent network uncertainties must be accounted for in pricing models to ensure they are robust to future changes in travel demand. The success of a particular project relies on accurately predicting tolling profits. Around the world, a surprising number of failed tollway projects have consistently relied on poorly forecasted demand for modelling, and suffered the consequences (Bain, 2009).

A particular example of this can be found in the well-known case of the Sydney Cross City Tunnel (Phibbs, 2008). The Cross City Tunnel was a public-private partnership project intended to connect the eastern suburbs to the western suburbs of Sydney that opened in 2005. Unfortunately, the operating company went into receivership less than two years after the tunnel opened, and it is estimated that $220 million dollars of initial investment has been lost. While there are many complex factors that lead to the ultimate failure of any project, most agree that an important contributor to Cross City Tunnel case was the poorly forecasted demand values. It was estimated that a daily demand of 90,000 vehicles would use the tunnel, while the actualized number was closer to 30,000. Another complaint was that the toll was much too high and discouraged people from using the tunnel. While this is an extreme example, the importance of accounting for factors of uncertainty, particularly when it relates to the financing of an important public project, cannot be underestimated. A more detailed analysis of tollways in Australia and the impact of inaccurate demand forecasts can be found in Zheng and Chiu (2011).

Finally, it is worth noting that the methods proposed here are only one small part of the ultimate decision-making process for any tollway project. Non-technical factors such as a bias toward optimism and politically and/or economically motivated misrepresentation, as well as social attitudes towards congestion pricing, are all important factors that play a role in toll prices that are actually used in practice (Flyvbjerg, 2008) . In an interesting look at tollway projects in Australia,

Davidson (2011) includes a case study for a potentially representative case for the Go Between Bridge in Brisbane. In this paper he noted the frequent changes to forecasted demand values used during the modelling process and described ways in which this value was misused. Such practices indicate that accounting for demand uncertainty when modelling tollway projects is a challenging and timely problem recognized by practitioners, and is indeed one of the recommendations to improve toll modelling made by Davidson in the conclusion of his paper.

This work explores a first best tolling framework when the impact of short-term day-to-day demand uncertainty on user behaviour is included by implementing a variant of a strategic user equilibrium based assignment model, referred to as StrUE (Dixit et al, 2013). Under StrUE, users determine route choice based on the expected shortest cost path for a known distribution of the day-to-day demand. The strategic model output is a set of fixed link flow *proportions* that define link flow patterns. Then, on any given day, the actual link flow *volumes* will be a function of the strategic fixed proportions and the realized demand. Therefore, a particular demand realization will result in non-equilibrium link flows, representing the volatile network behaviour observed in reality. Using a marginal social cost based approach, this work proposes a tolling methodology that attempts to induce strategic system optimal (StrSO) behaviour from users in a strategic equilibrium with tolls (StrT) model.

The long-term uncertainty in planning demand also plays an important role; if the future planning demand scenario varies from the forecast, the performance of a tolling scheme may be overestimated. A robust pricing scheme will consistently estimate system performance for a range of possible future demand realizations. Therefore, this work proposes a procedure to evaluate the robustness of a tolling scheme, where possible future demand *scenario* realizations are sampled from a future planning demand distribution. The methodology introduced in this work isolates the effect of day-to-day demand uncertainty in the short-term from the effect of the long-term planning demand uncertainty, and presents a method to clearly compare the effects of accounting for each source on tolling scheme evaluation. Thus, this work demonstrates the importance of including both sources of uncertainty when evaluating the system performance of a tolling scheme. Figure 3-1 summarizes the research contributions of Chapter 3.

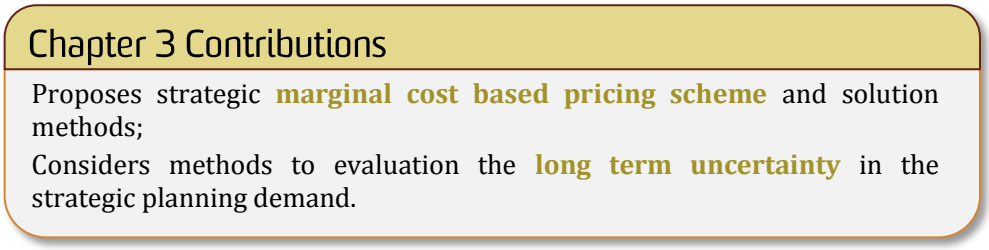


Figure 3:1 Summary of research contribution

3.2 Background

As previously noted, marginal social cost pricing based on Pigouvian (Pigou, 1920) taxes has a rich history in the literature (Yang & Huang, 1998). This method aims to

set tolls in such a way that a collective system optimal behaviour is induced, rather than drivers choosing routes unilaterally to minimize their own travel time (selfish behaviour) (Newbery, 1990; Yang & Huang, 1998). The tolling framework addressed in this work is classified as first best, which means that it is possible to toll every link in the network in order to achieve some objective. While maximizing social welfare by relieving congestion may be a common goal from public planning agencies, many other objectives have also been explored, among those aims that may represent the interests of private tolling agencies, such as: maximizing revenue, minimizing tolling locations, and minimizing the maximum toll collected (Hearn & Ramana, 1997).

Second best tolling scenarios, in which not all links in the network are available to be tolled because of political or social restrictions, have also been well-explored in the literature (Lawphongpanich & Hearn, 2004; Verhoef, 2002) . However, in order to introduce the impact of the StrT model, only schemes in which all links in the network are priced are considered in this work.

While the pioneering works on pricing road networks assumed travel demand and other network characteristics (such as link capacity) to be fixed values, the impact of uncertainties on transport models has become another popular topic in the literature. This is particularly important for tolling scenarios, because optimal prices that are calculated for an unrealized level of demand could have an unpredictable impact on network conditions, a fact that is further

discussed by Lemp and Kockelman (2009). It is commonly agreed that the main sources of uncertainty in a transport network result from the demand (Clark & Watling, 2005; Duthie et al, 2011), supply (Lo et al, 2006), and behavioural choices from travellers (Damberg et al, 1996). Boyles et al (2010) examined first best pricing while accounting for uncertainty in road capacity and further looked at the impact of supplying users with information about the state of the network. This work highlights the difference between tolling schemes that respond to network conditions and tolls that are intended to address recurring, predictable congestion. Each of these sources could impact optimal toll design in different ways. Researchers begin by analysing difference sources in isolation, but more complicated models like Gardner et al (2011) account for both uncertainty in demand and in supply may offer more realistic insights into the road network.

A number of works have approached the issue of travel demand uncertainty and its impact on tolling. Gardner et al (2008b) examine the impact of long-term demand uncertainty, such as that resulting from changes in land use, technology, and petrol prices, on robust tolling schemes, and evaluate a number of approaches to solve this problem. They show that MSC tolls that are calculated using an expected demand can result in suboptimal system performance, especially when the actual system performance differs significantly from what was forecasted. Gardner et al (2010) further explore a number of solution methods for solving a similar problem, finding that using an inflated demand scenario gave the most consistently robust results. Li et al (2008) propose a bi-level mathematical

programming formulation to solve for first best tolls aimed at increasing network reliability, where users' choices are determined using a multinomial logit model. Sumalee and Xu (2011) also examine the impact of stochastic demand by treating both network demand and link flows as random variables. This work addresses uncertainty in user behaviour by considering how different risk attitudes from users might impact pricing results, which is additionally a method of incorporating users' value of travel time reliability. Li et al (2012) extend this model to find the optimal tolls with the objective of minimizing emissions.

The work introduced here differs from previous contributions in its novel behavioural model to capture the strategic decisions of users. Strategic traffic assignment was described in Chapter 2 and finds equilibrium flows based on expected path costs. This model results in link volumes that will vary from day-to-day, thus accounting for short-term demand uncertainty that users face making day-to-day route choice decisions. Waller et al (2013) propose a linear formulation for a dynamic version of the strategic problem that finds optimal route flows across a discrete set of possible demand scenarios.

Chapter 3 extends the strategic assignment model to a StrT first best pricing application. Previous work has examined the impact of short-term demand uncertainty or long-term demand uncertainty on first best tolling in isolation, but rarely in combination. This work proposes a flexible framework to fill this gap.

3.3 Pricing Model Description

First, this section describes and formulates the strategic behavior based marginal social cost pricing model from a theoretical perspective. Next, the modeling assumptions are thoroughly discussed. Finally, this section provides a model analysis on a small example network.

3.3.1 The StrUE MSC theoretical framework

As previously discussed, the strategic route choice assignment model accounts for the day-to-day volatility in demand by assuming that users know the day-to-day demand distribution and make their choices strategically based on this knowledge. Travellers then follow a route choice decision based on expected cost regardless of manifested travel demand or experienced travel conditions, but the number of users traveling in each demand actualization will change based on the distribution of the total travel demand $g(T)$. The result of this approach is a fixed *proportion* of flow that will travel on each link; the actual link *flow* will then vary based on realizations from the day-to-day travel demand distribution. For completeness, Equations (3.1)-(3.4) show the mathematical formulation of the StrUE model, also presented in Chapter 2.

$$minimize\, z(f) = \int_0^{\infty} \sum_{(i,j)\in A} \int_0^{p_{ij}} t_{ij}(wT)\, g(T) dw dT \tag{3.1}$$

subject to

$$\sum_k f_k^{rs} = q_{rs} \qquad \forall r \in R, \forall s \in S \qquad (3.2)$$

$$f_k^{rs} \geq 0 \qquad \forall r \in R, \forall s \in S, \forall k \in K \qquad (3.3)$$

$$p_{ij} = \sum_{r \in R} \sum_{s \in S} \sum_{k \in K} f_k^{rs} \delta_{ij,k}^{rs} \qquad \forall (i,j) \in A \qquad (3.4)$$

As previously discussed, in order to ensure uniqueness of link flows, for each origin-destination, path flow proportion is assumed to be equal under all demand scenarios. Therefore, each path will be altered proportionally when the total origin-destination demand varies. The system performance measures in the strategic approach can either be found through analytical derivations or simulation-based sampling methods, and will be detailed in the Section 3.3.2. A reminder of notation for this section is contained in Table 3-1.

Table 3-1 Notation for the general strategic approach

$a \in A$	Index for link a in set of all network links A
$r \in R, s \in S$	Index for origin r in set of all origins R and destination s in set of all destinations S
p_a	Proportion of the total travel demand on link a
f_k^{rs}	Proportion of the total travel demand on path k connecting origin r and destination s
T	Random variable representing the total number of trips for all OD pairs
$g(T)$	Probability distribution for the day-to-day travel demand, representing number of trips T

M_k	The kth analytical moment of the demand distribution $g(T)$
$t_a(pT)$	Travel cost function on link a
$\tau_a \in \Phi$	The toll value on link a contained within set of tolls values Φ
$\delta_{a,k}^{rs}$	Indicator equal to 1 if link a in contained on path k connecting origin r and destination s and 0 otherwise

The purpose of a MSC based pricing scheme is to ensure that the traffic patterns that result from individual decision makers seeking to maximize their own utility from a myopic perspective can be "improved" to the social optimal through the implementation of tolls. The problem of setting optimal tolls in the strategic assignment scenario becomes significantly more complex than the deterministic case. This is in part due to the way each model handles the "individual" traveller. The first order output of the deterministic user equilibrium model is link flows, representing the number of individuals on each link. Traditional pricing schemes target the individual vehicle on a link by pricing the *individual* impact on system travel time. Furthermore, realistic applications of traditional tolling are also constrained by the individual, because they must charge a certain amount to each user on a road each day.

However, the first order output from the strategic approach is *proportions* on each link, and the link flows are an extension of this proportion that change based on the realization of the day-to-day travel demand. Thus, a pure MSC strategic pricing approach would target the proportion of flow on a link by pricing the proportional impact on system travel time; however, system travel time is a

product of random variable T and will be changing with each realization of the demand. It follows that the actual toll price on each link would also be changing with the realization of the total trips T. Therefore, in order to set a MSC pricing scheme that would result in perfect StrSO flow patterns, the network operator would need to have perfect knowledge of all demand realizations; obviously, this is unrealistic.

However, with a slight modification in approach, StrT can be derived to fit the more realistic data constraints of the problem. Therefore, the approach is based on the concept of an average daily demand total system travel time $AD(TSTT)$. In this method, the day-to-day demand realization is still a changing random variable T, but an average daily total travel time, defined as the proportion on a link multiplied by the first moment (i.e., the mean) of the demand distribution, is targeted in the pricing scheme. In the strategic case the tolls are set so that the system travel time for an average daily demand is minimized. The $AD(TSTT)$is calculated in Equation (3.6).

$$AD(TSTT) = \sum_{(i,j)\in A} \int_0^\infty (p_{ij}M_1)t_{ij}(pT)g(T)dT \tag{3.5}$$

$$= M_1 \sum_{(i,j)\in A} \int_0^\infty p_{ij}t_{ij}(pT)g(T)dT \tag{3.6}$$

Equation (3.5) shows that because the first moment of the demand distribution (M_1) is constant and a property of the system demand, minimizing $AD(TSTT)$ is

equivalent to minimizing the expected total system travel time $E(TSTT) = \sum_a \int_0^\infty p_a t_a(pT) g(T) dT$. To derive the tolls that should be implemented on each link so as to minimize $AD(TSTT)$, one must consider the integration by parts of the following term:

$$\sum_{(i,j)\in A} \int_0^\infty \int_0^{p_{ij}} p \frac{dt_{ij}(pT)}{dp} dp \; g(T) dT \tag{3.7}$$

$$= \sum_{(i,j)\in A} \int_0^\infty \left(p_{ij} t_{ij}(p_{ij} T) - \int_0^\infty t_{ij}(p,T) dp \right) g(T) dT \tag{3.8}$$

$$= \sum_{(i,j)\in A} \int_0^\infty \int_0^{p_{ij}} \left(\left(t_{ij}(p,T) + p \frac{dt_{ij}(p,T)}{dp} \right) dp \right) g(T) dT \tag{3.9}$$

$$= \sum_{(i,j)\in A} \int_0^\infty p_{ij} t_{ij}(p,T) g(T) dT \tag{3.10}$$

It is observed that minimizing the first part of the left hand side of the Equation (3.7) represents the StrUE objective and the minimizing the right hand side presented in Equation (3.8) represents minimizing $AD(TSTT)$. Therefore the marginal toll that needs to be applied on each link would be, p_a^* is the $AD(TSTT)$ flow pattern:

$$\tau_{ij} = \int_0^\infty \int_0^{p_a^*} p \frac{dt_{ij}(p,T)}{dp} dp g(T) dT \tag{3.11}$$

3.3.2 The StrT application

This section describes the specific notation, equations and assumptions made for the application of the StrT model and the MSC approach in this work. Table 3-2 contains a detailed summary of the notation introduced in this section.

Table 3-2 Additional notation for the chapter 3 pricing application

T	Day-to-day random variable for the demand following a lognormal distribution, $T \sim LN(\mu_s, \theta_s)$; assume a fixed proportion of demand for all OD pairs.
$E_s(T)$	The expected total number of trips, where $E(T) = e^{\mu_s + \theta_s^2/2}$
$Var_s(T)$	The variance of the total number of trips T, where $Var(T) = (e^{\theta_s^2} - 1)e^{2\mu_s + \theta_s^2}$
CV_S	The coefficient of variation of the day-to-day travel demand distribution equal to the ratio of the mean to the standard deviation: $\frac{E(T)}{\sqrt{Var(T)}}$
$g(E_s, CV_S)$	Convenient notation of the lognormal strategic day-to-day demand distribution with expected value of demand E_S and standard deviation of demand $COV_S * E_S$; assume that parameters μ_s and θ_s are found as above.
$\boldsymbol{p}$	Set of link flow proportions for all $a \in A$ output by a strategic assignment model
c_a	Capacity on link a in v/hr
$t_{f,a}$	Free flow travel time on link a $(minutes)$
α, β	Geometric link parameters for the BPR cost function equal to 0.15 and 4 respectively
$VOTT$	The value of travel time for network users; for simplicity, assumed to be $\$10/min$
$TSTT$	Abbreviation for total system travel time
n	Sample realized demand values where n: $T \sim LN(\mu_s, \theta_s)$
N	Total number of demand samples
E	A system performance measure representing **expected value of *TSTT*** (minutes)

AD	A system performance measure representing the expected value average demand system travel time based on the **average daily demand** (minutes)
STD	A system performance measure representing **standard deviation of $\boldsymbol{TSTT}$** (minutes)
R	A system performance measure representing **expected revenue** from a pricing scheme Φ (\$)
$\diamond(\cdot)$	Symbol meaning that value " $\cdot$ " is **analytically derived**, e.g., $\diamond E$ is the analytical $TSTT$
$\odot(\cdot)$	Symbol meaning that value " $\cdot$ " was obtained through **simulation testing**, e.g., $\odot E$ is the average $TSTT$ from n demand samples
$\Delta(\cdot,\cdot)$	The **percentage difference** between two system performance measures; e.g., $\Delta(\diamond E_{StrUE}, \odot E_{StrUE})$ is difference between the analytical and simulated E values resulting from $StrUE$ (%)

Regarding the day-to-day travel demand, this approach assumes a lognormal distribution with random variable $T \sim LN(\mu_s, \theta_s)$, and that the OD demand follows fixed, specified proportions. Travellers make their route choices based on knowledge of the distribution and the resulting expected travel costs. Additionally, as in the previous chapter, this work uses a modified version of the well-known BPR function to make the formulation presented in previous section tractable:

$$t_{ij}(p,T) = t_{ij}^{f}\left(1 + \alpha\left(\frac{p_{ij}T}{c_{ij}}\right)^{\beta}\right) \tag{3.12}$$

In order to derive the link toll values in this work, where the α and β parameters are the same for all links in the network, and $\bar{\boldsymbol{p}}$ is the optimal link proportion patterns resulting from the StrAD, the tolls can be represented as:

$$\tau_{ij} = t_f \alpha \beta M_\beta \left(\frac{\overline{p_{ij}}}{c_{ij}}\right)^\beta \tag{3.13}$$

The four assignment problems necessary in this approach (StrUE, StrSO, StrAD, and StrT) result in three possible system performance measures each. The value of a system performance measure will differ depending on the assignment problem.

The three system performance measures are: expected total system travel time E, average demand total system travel time AD, and standard deviation of total system travel time STD. Additionally, the StrT problem includes tolling and outputs expected revenue R. While this combination results in 14 possible system performance measures, not all of these combinations are necessary in order to evaluate the pricing model performance. This work focuses on E and STD.

Each of these performance measures can be analytically derived using the theoretical framework described in Section 3.3.1 and Section 2.3.1 and the assumptions about the demand distribution. Continuing from the previous chapter, the symbol "$\diamond$" indicates that a measure was calculated from the analytical equation. The three analytical performance measures can be found as:

$$\diamond E = \sum_{a \in A} \left(t_{ij}^f \left(p_{ij} M_1 + \alpha \frac{p_{ij}^{\beta+1}}{c_{ij}^\beta} M_{\beta+1} \right) \right) \tag{3.14}$$

$$\diamond AD = \sum_{(i,j) \in A} \left(t_f M_1 p_{ij} \left(1 + \alpha \frac{p_a^{\beta+1}}{c_{ij}^\beta} M_\beta \right) \right) \tag{3.15}$$

$$\diamond\ S = \left(F^2 M_2 + D^2 M_{2\beta+2} + 2FDM_{\beta+2} - \left(FM_1 + DM_{\beta+1}\right)^2\right)^{1/2} \tag{3.16}$$

$$\diamond\ R = \sum_{(i,j)\in A} p_{ij} M_1 \tau_{ij} \tag{3.17}$$

Additionally, system performance measures can be found through simulation testing, where random numbers are generated from the strategic demand distribution to represent demand realizations. Dixit et al (2013) show that analytical and simulation results converge. It was observed through empirical testing that a high number of demand samples N were necessary for the analytical and simulation results to reliably converge. This is a reflection of the complex behaviour of the $StrT$ assignment problem and the polynomial power β. In order to find a balance between computation and convergence reliability, a generous value of $N = 200{,}000$ (unless specified otherwise) is assumed for the remainder of this work.

The pseudo-code for finding the strategic marginal social cost based tolls is presented in Algorithm 4.1. The steps of this algorithm consist of solving the StrUE model to get the results for the base case (for the purposes of comparison), then solving the same network data for the StrSO case, then calculating the toll for each link, then solving the StrT model, then calculating the performance metrics.

Algorithm 3.1: Strategic pricing pseudo-code

INPUT: Network $G = (N, A)$; $g(E_S, CV_S)$;

procedure solve_StrT()

1: //solve for the base StrUE model

2: $p_{ij,StrUE} \forall (i,j) \in A, \diamond E_{StrUE}, \diamond S_{StrUE} \leftarrow$ *solve StrUE*$(g(E_S, CV_S))$;

3: $\odot E_{StrUE}, \odot S_{StrUE} \leftarrow$ *simulation sub-procedure*$(\boldsymbol{p_{StrUE}})$;

4: $\overline{\boldsymbol{p}}_{\boldsymbol{StrAD}} \leftarrow$ *solve StrAD*$(g(E_S, CV_S))$;

5: //calculate network tolls for distribution

6: **for** $(i,j) \in A$ **do**

7: $\tau_{ij} \leftarrow t_{f,ij} * \alpha * \beta * M_\beta * pow(\overline{p_{ij}} / c_ij, \beta)$;

8: $\Phi_s \leftarrow (\Phi_s \cup \tau_{ij})$;

9: **end for**

10: //apply tolls and solve new model

11: $\boldsymbol{p}_{\boldsymbol{StrT}}, \diamond E_{StrT}, \diamond S_{StrT} \leftarrow$ Solve_StrT$(g(E_S, CV_S), \Phi_s)$;

12: $\odot E_{StrT}, \odot S_{StrT} \leftarrow$ Simulation Subprocedure$(\boldsymbol{p}_{\boldsymbol{StrT}})$;

13: Calculate Δ;

end procedure

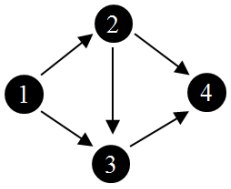
OUTPUT: Φ_s;

The next section provides a small network demonstration of the StrT pricing model and solution approach.

3.3.3 Demonstration

This demonstration focuses on clarifying the MSC StrT approach and studying the impact of the strategic day-to-day demand uncertainty on system performance. The demonstration network is similar to the well-known Braess's paradox network, in which the addition of a link between nodes two and three causes an increase in TSTT due to the difference in equilibrium versus system optimal behaviour. This network was chosen to capture the interaction between strategic user behaviour

and the presence of tolls. Figure 3:2 shows the demonstration network, network parameters, and demand. The initial demand lognormal distribution in this problem has parameters $E_s(T) = 20$ and $CV_S = 0.2$.



From link	*To link*	c_a	$t_{f,a}$
1	2	50	12
1	3	50	36
2	3	50	1.2
2	4	50	36
3	4	50	12

$$T \sim g(20, 0.2)$$

Figure 3:2 Demonstration network and network parameters.

The results from the analytical method compared to the simulation method converge closely, in part due to the high number of demand samples. Table 3-3 shows the analytical and simulation results for E and AD resulting from the $StrUE$ and the $StrT$ assignment problems. While the values of AD and E are not the same, solving the StrAD and the StrSO assignment problems will result in identical proportions.

Table 3-3 Convergence results for E and AD for the StrUE and StrT assignment patterns.

	$\diamond(\cdot)$	$\odot(\cdot)$	$\Delta(\diamond, \odot)$
$\boldsymbol{E_{StrUE}}$	1526	1531	0.31%
$\boldsymbol{AD_{StrUE}}$	1394	1397	0.20%

$\Delta(E_{StrUE}, AD_{StrUE})$	8.7%	8.8%	-
E_{StrT}	1066.87	1066	0.0%
AD_{StrT}	1026.51	1026.11	0.00%
$\Delta(E_{StrT}, AD_{StrT})$	3.78%	3.74%	-

Additionally, this demonstration illustrates the impact of variation in the day-to-day demand, quantified as the CV_S of the strategic demand distribution, on the system performance. In order to capture this effect, Algorithm 3.1 described in Section 3.3.2 was implemented using the same $E_S(T)$ but varying $0 \leq CV_S \leq 0.85$ in increments of 0.05. Figure 3:3 Analytical results on the demonstration network and Figure 3:4 display the results of $\odot E$ and $\odot STD$ from the untolled assignment StrUE and the assignment including tolls StrT from the varying CV_S experiment.

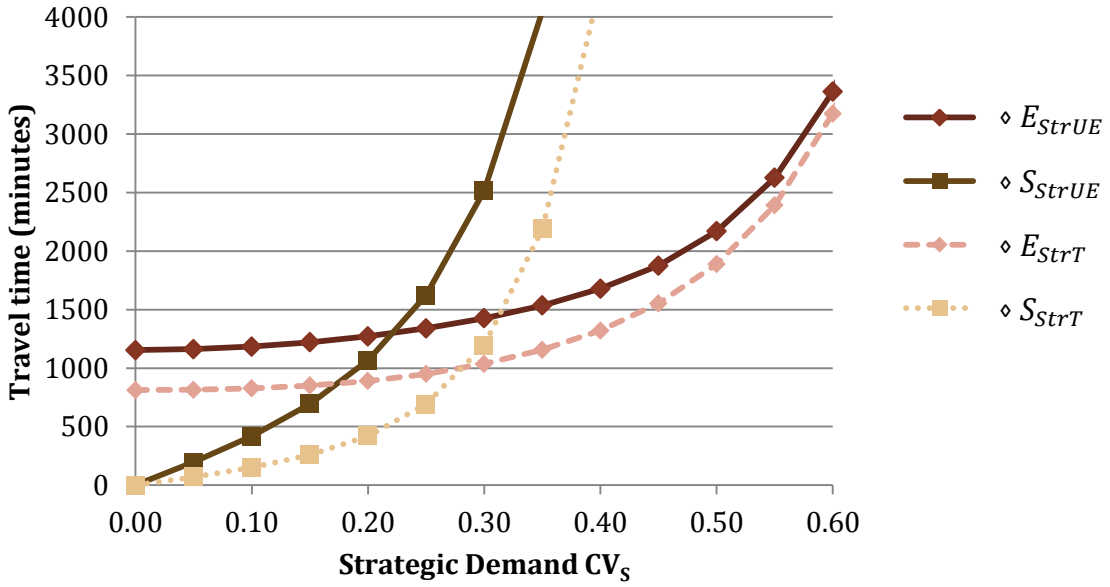


Figure 3:3 Analytical results on the demonstration network

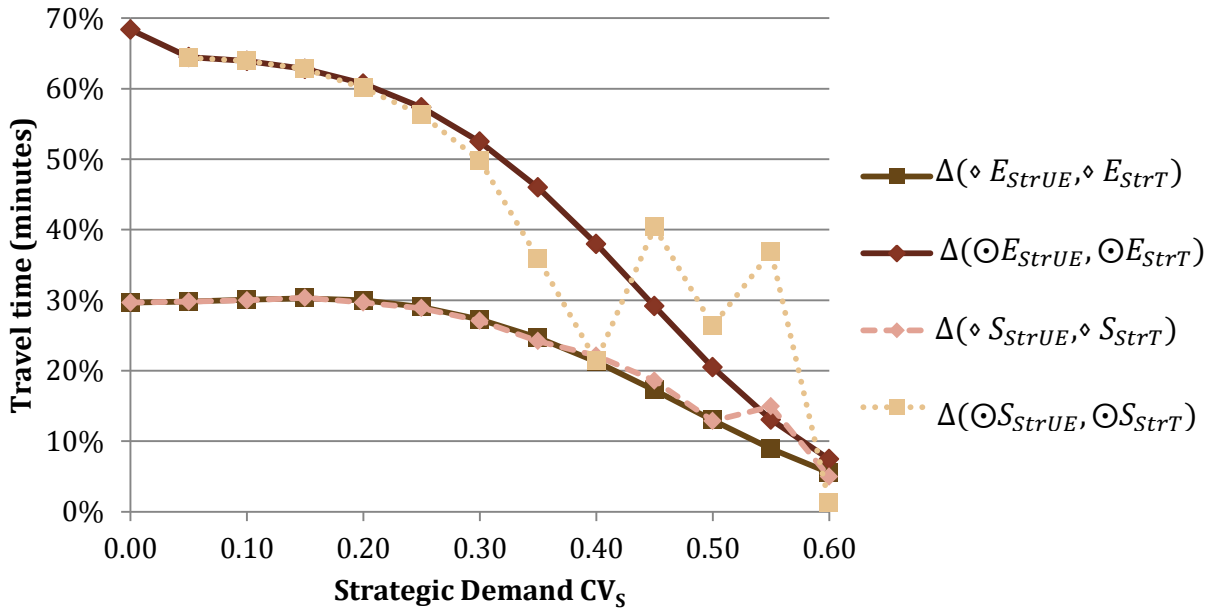


Figure 3:4 Performance metrics on demonstration network

Figure 3:3 and Figure 3:4 display the results from StrUE and StrT in two ways; Figure 3:3 shows the absolute results while Figure 3:4 shows the relative results. The horizontal axis in both Figure 3:3and Figure 3:4 show the CV_S of the strategic demand distribution. The vertical axis of Figure 3:3 shows the value of $\odot E$ and $\odot STD$ in minutes. The vertical axis of Figure 3:4 shows the percentage difference between the StrUE results and the StrT results, $\Delta(\odot StrUE, \odot StrT)$, for both E and STD. This is a reflection of system performance improvement that resulted from the implementation of tolls.

To order to facilitate visual comprehension, the results for the $\diamond$ metrics are in solid lines and the results for the $\odot$ are in dashed lines. Therefore, visually speaking, for any value of CV_S, the difference between the two solid lines in Figure

3:3 is equal to the point for that value of CV_S in Figure 3:4 and the same for the dashed lines.

For the case of $CV_S = 0.05$, Figure 3:3 shows $\odot E_{StrUE} = 1397\ minutes$ and $\odot E_{StrT} = 978\ minutes$. The difference between these two values is about 30%, which is the Δ value shown by the blue bar for $CV_S = 0.05$ in Figure 3:3. The 30% represents the reduction in expected TSTT due to the tolling scheme, which also reduced $\odot STD$ by 65%.

Figure 3:3 illustrates the relation between variation in day-to-day demand and network tolling behaviour. When $0.05 \leq CV_S \leq 0.3$, the addition of tolls consistently reduced $\odot E$ and $\odot STD$ in the network in a nonlinear fashion. However, when $0.4 \leq CV_S$, this relationship dismantles, and the $\odot STD$ for both StrUE and StrT becomes much greater than $\odot E$. Additionally, Figure 3:4 indicates that the relative differences between the tolled and untolled networks are smaller for higher CV_S values. Figure 3:3 is not scaled to include these values because an STD that is so much greater than the E value seems unrealistic. While of course, observations are network specific, results indicate that the strategic pricing model may be best applied in networks where the $CV_S < 0.4$.

3.4 Long Term Demand Uncertainty

While the strategic pricing approach accounts for the short term uncertainty in demand users face when making route choice decisions, planners must still be concerned regarding the uncertainty in the long-term future planning demand. In

the deterministic approach, the interpretation of this concept lies in the exact value of demand that is used to make planning decisions. In cases accounting for long-term uncertainty, the future realization of the travel demand may be different from the predicted planning value. Gardner et al (2008b) show that not accounting for possible variation in realized planning demand may result in overestimation of toll performance.

Table 3-4 Notation regarding the long term demand uncertainty in Chapter 3

ω	Possible long term (future) demand scenario realization ω
$\Omega(\mu_\Omega, \theta_\Omega)$	Distribution of long term (future) planning demand scenarios $\omega \sim N(\mu_\Omega, \theta_\Omega)$
CV_Ω	The coefficient of variation of the long term planning demand scenario distribution to the ratio of the mean to the standard deviation: $\frac{\mu_\Omega}{\theta_\Omega}$
Q	Number of long term demand scenario samples where $Q: \omega \sim N(\mu_D, \theta_D)$
$M(\cdot)$	The mean of a quantity obtained from set of Q planning demand samples; i.e., $M(\diamond E)$ is the long term expected analytical total system travel time
$STD(\cdot)$	The standard deviation of a quantity obtained through set of Q samples; i.e., $STD(\odot STD)$ is the standard deviation of a set of standard deviations of each demand scenario obtained through simulation

An analogous situation exists with the strategic approach. However, in the strategic approach the long-term planning uncertainty regards a future demand *scenario*. In each demand scenario, planners know that travellers will react strategically by using their knowledge of $g(E_S, CV_S)$ to make route choices, but the

planner does not know the exact value of E_S that will be realized. In order to have a reliable estimation of the performance of a pricing scheme, the network operator needs to test the impact of the long-term uncertainty associated with a strategic planning demand scenario. A robust pricing scheme will give reliable evaluations when the realized strategic demand scenario differs from the forecasted strategic planning demand scenario.

This section describes the necessary assumptions and the method to test the robustness of a pricing scheme that is applied to evaluate the impact of long-term demand uncertainty on the StrT model.

The system performance measures are similar to the approach without long-term demand uncertainty. However, due to the added sampling method, mean and standard deviation results for all strategic system performance measures can be found. This work places emphasis on results obtained through the simulation approach: $M(\odot E)$ is the simulation-based expected $TSTT$ including the impact of long-term planning demand scenario uncertainty, and $STD(\odot E)$ is the *long term* standard deviation of the simulation-based expected total travel time. The mean value of $\odot STD$ is a robust reflection of variation in the strategic demand scenario $TSTT$, while $STD(\odot STD)$ reflects the variation *of the variation within* future demand scenarios.

Finally, long-term measures of effectiveness are necessary. This study focuses on the change in $\odot E$ and $\odot STD$ between the "do nothing" StrUE scenario,

in which the long-term strategic demand is evaluated without tolls, and the strategic tolling scenario, StrT. The difference in travel time is denoted $\Delta(M(\odot E_{StrUE}), M(\odot E_{StrT}))$ and the reduction in future system variation in travel time is denoted $\Delta(STD(\odot E_{StrUE}), STD(\odot E_{StrT}))$.

The method for testing the robustness of a set of strategic marginal social cost based tolls follows:

Algorithm 4.2: Long term uncertainty evaluation pseudo-code

INPUT: A; $g(E_s, CV_s)$; $p_{ij}, \forall (i,j) \in A$; Q

procedure

1: **for** $i = [1,2,\dots,Q]$ **do**

2: $E_s' \leftarrow$ random sample from distribution $\Omega(\mu_\Omega, \theta_\Omega)$

3: $p'_{StrT}, , \diamond E_{StrUE}, \diamond S_{StrUE} \leftarrow$ *solve_strategic*$(g(E_S', CV_S)$, "StrUE");

4: $\odot E_{StrUE}, \odot S_{StrUE} \leftarrow$ *simulation sub-procedure*(p'_{StrT});

5: $RS(\diamond E) \leftarrow RS(\diamond E) \cup \diamond E$,

6: $RS(\odot E) \leftarrow RS(\odot E) \cup \odot E$,

7: $RS(\diamond S) \leftarrow RS(\diamond S) \cup \diamond S$,

8: $RS(\odot S) \leftarrow RS(\odot S) \cup \odot S'$

9: **end for**

end sub-procedure

OUTPUT: $M(\diamond E)$, $S(\diamond E)$,$M(\diamond S)$, $S(\diamond S)$, $M(\odot E)$, $S(\odot E)$, $M(\odot STD)$, $S(\odot STD)$

This procedure reflects a robust evaluation that accounts for the long-term uncertainty in demand. Note that this procedure can be easily adapted to evaluate impact of long-term uncertainty in $StrUE$ by setting network tolls $\Phi = 0$, or in $StrSO$ by solving for the appropriate assignment pattern in Line 3. Additionally,

this procedure will sample from two distributions (both Ω and g), so it is critical that adequate Q and N values are chosen to minimize sampling bias.

3.4.1 Demonstration of long term demand scenario uncertainty

The demonstration network from Section 3.3.3 is revisited in order to provide clarification between the impact of the day-to-day uncertainty resulting from the strategic approach, and the impact of long-term uncertainty in the strategic planning demand scenario.

The network parameters in Figure 3:2 remain the same, with the exception of E_S, which is no longer a known value. The future planning demand scenario in this demonstration has a mean of $\mu_\Omega = 20$ and $CV_\Omega = 0.2$, therefore demand realization $\omega \sim N(20,4)$, and for this demonstration, $Q = 1000$. Procedure B was then implemented to obtain an evaluation of tolling scheme Φ that reflected the impact of long-term demand uncertainty.

Figure 3 shows the results from the varying CV_S experiment, however, now the impact of planning demand scenario uncertainty is accounted for. Again, CV_S was varied from $0 \leq CV_S, \leq 0.6$ in increments of 0.05. CV_S is not affected by the uncertainty in the planning demand scenario. For each possible CV_S value, Procedure B was implemented to obtain $M(\odot E)$ and $M(\odot STD)$ in the StrUE and StrT models. The horizontal axis of Figure 3 shows each possible CV_S value. The vertical axis of Figure 3(a) shows the values of travel time resulting from the long-term planning demand scenario sampling, while the vertical axis of Figure 3(b)

shows the percentage reduction in $M(\odot E)$ and $M(\odot STD)$ resulting from the presence of tolls.

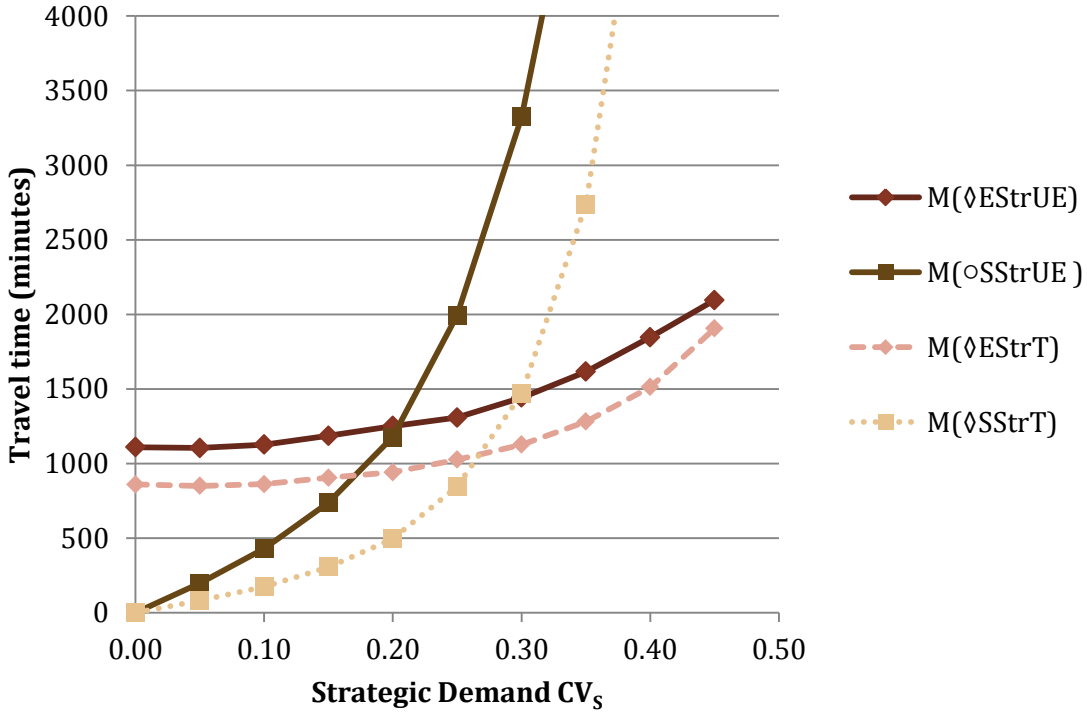


Figure 3:5 Results on the demonstration network under long term uncertainty:

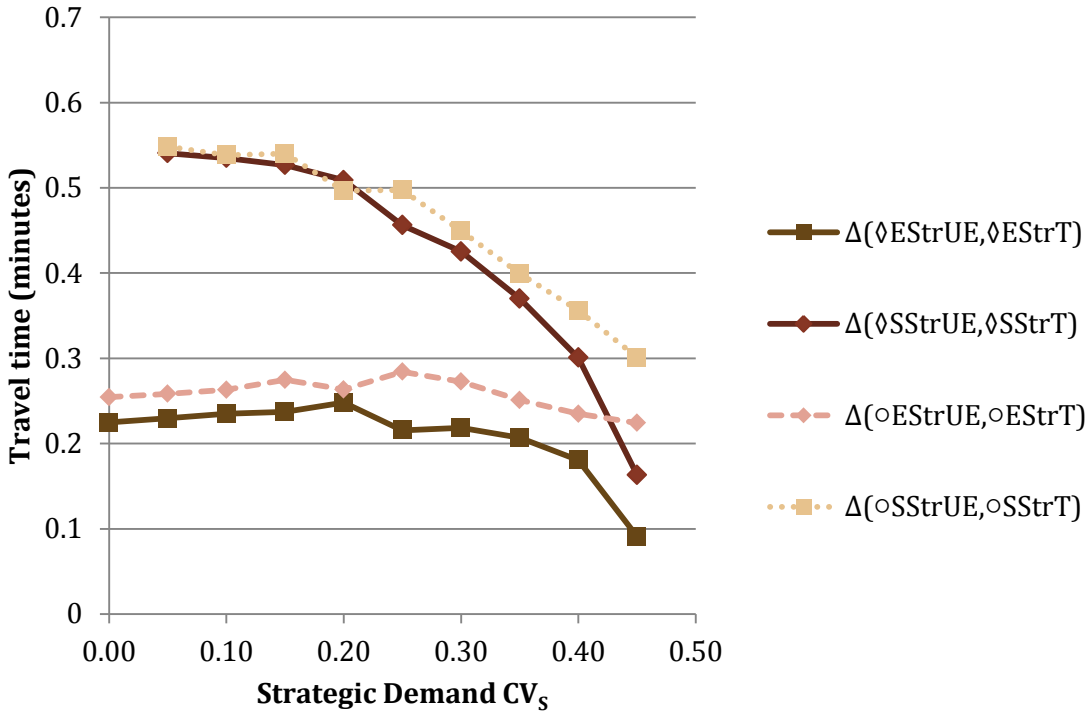


Figure 3:6 Model evaluation under long term demand uncertainty: Expectation

For the case of $CV_S = 0.05$, Figure 3(a) shows $M(\odot E_{StrUE}) = 1411\, minutes$ and $M(\odot E_{StrT}) = 1048\, minutes$. The difference between these two values is about 25%, which is the Δ value shown by the blue bar for $CV_S = 0.05$ in Figure 3(b). Again for the case of $CV_S = 0.05$, a robust evaluation of the StrT model results in 25% reduction in travel time and 54% reduction in standard deviation of travel time, as opposed to 30% and 65% respectively for the results without considering long-term uncertainty.

While Figure 3:6 shows similar behaviour to the results in Figure 3:4 (showing the same experiment but without the added consideration of long-term uncertainty), they are not identical. This implies that a network operator should

not rely on a pricing scheme without evaluating its robustness using a method like Procedure B, lest system performance measures be overestimated. In addition, the unrealistic behaviour observed when $0.4 \leq CV_S$ in Figure 4:3 is less prominent in Figure 4:5.

Figure 3:7 shows the long term standard deviation of the four metrics for the demonstration network. Once again, the horizontal axis shows CV_s varying in increments of 0.05 and the vertical axis shows $\Delta(\cdot,\cdot)$ parameter that reflects the difference between the values from the StrT model and the StrUE model.

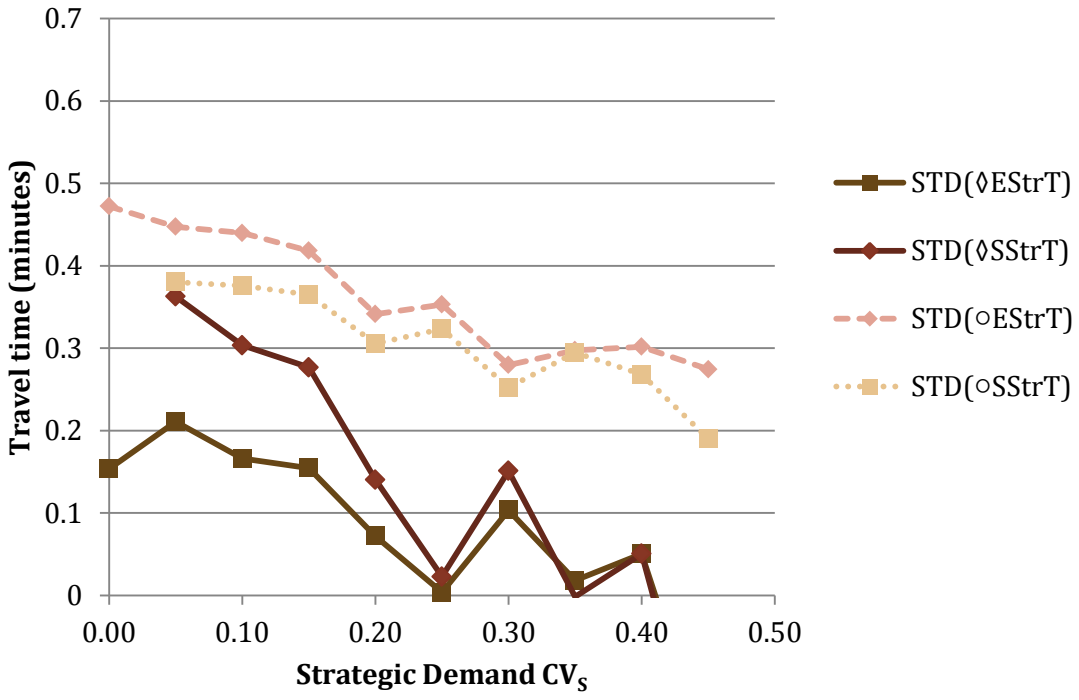


Figure 3:7 Model under long term uncertainty: STD

In Figure 3:7, the $\diamond STD$ and $\odot STD$ do not appear to have converged in the same manner. If the two measures were the same, then the long-term results would predict that all values have a similar standard deviation. The simulation procedure resulted in a greater reduction in performance metrics in the StrT model. In some cases the analytical results suggest that the $\diamond STD$ is not reduced at all. This is a non-intuitive result on the demonstration network. It is possible that this result is a reflection of computational significance. Investigating this measure on additional networks may shed light on this phenomenon. This is the topic of the next section.

3.5 Model Demonstration

Previous sections introduced the StrT model, a method for evaluating model performance under long term demand uncertainty, and demonstrations of each on a small network. This section scales the StrT model to the medium sized networks of Sioux Falls and Anaheim. Note that while the StrT model could be theoretically applied to networks such as the Gold Coast, the long term evaluation does require solving the model Q times (where Q needs to be a large number such as 1000) and therefore would be computationally prohibitive on a really large network.

3.5.1 Results from evaluation of long term performance

This section implements Algorithms 3.1 and 3.2 on the networks of Sioux Falls and Anaheim to demonstrate results and illustrate scalability of the proposed method. These are both well-known transportation network modelling test networks, the data for which was obtained from Bar-Gera (Bar-Gera, 2012). Sioux Falls consists of

24 nodes, 76 links, and 24 zones, while Anaheim consists of 416 nodes, 914 links, and 38 zones. All link parameters are as specified in known data, with the additional strategic demand parameter of $g(T: 360{,}600, CV_S)$ and future planning scenario parameter of $\Omega(\omega: 360{,}600{,}0.2)$ for Sioux Falls, and $g(T: 106176, CV_S)$ and $\Omega(\omega: 106176{,}0.2)$ for Anaheim. For these models, $N = 50{,}000$ and $Q = 1000$.

The experiment varying $0 \leq CV_S \leq 0.6$ in increments of 0.05 described in Sections 3.3.3 and Section 3.4.1 was repeated for both the case when *not* including long term planning scenario uncertainty, which yields performance measures $\Delta(\odot E)$ and $\Delta(\odot STD)$ reflecting the reduction in system travel time due to the addition of the tolls. The same experiment varying CV_S was then repeated for Procedure B to illustrate the different values for effectiveness that might be obtained when the robustness of tolls is included in the evaluation.

Figures 3:8 – 3:13 shows the results of this experiment for Sioux Falls and Anaheim, where the evaluation resulting from day-to-day demand uncertainty only are compared with the results from the model evaluation when accounting for long term demand uncertainty. The results for E and for S are included. The horizontal axis in these figures shows the varying CV_S in increments of 0.05. The vertical axis in both figures then represents the Δ values.

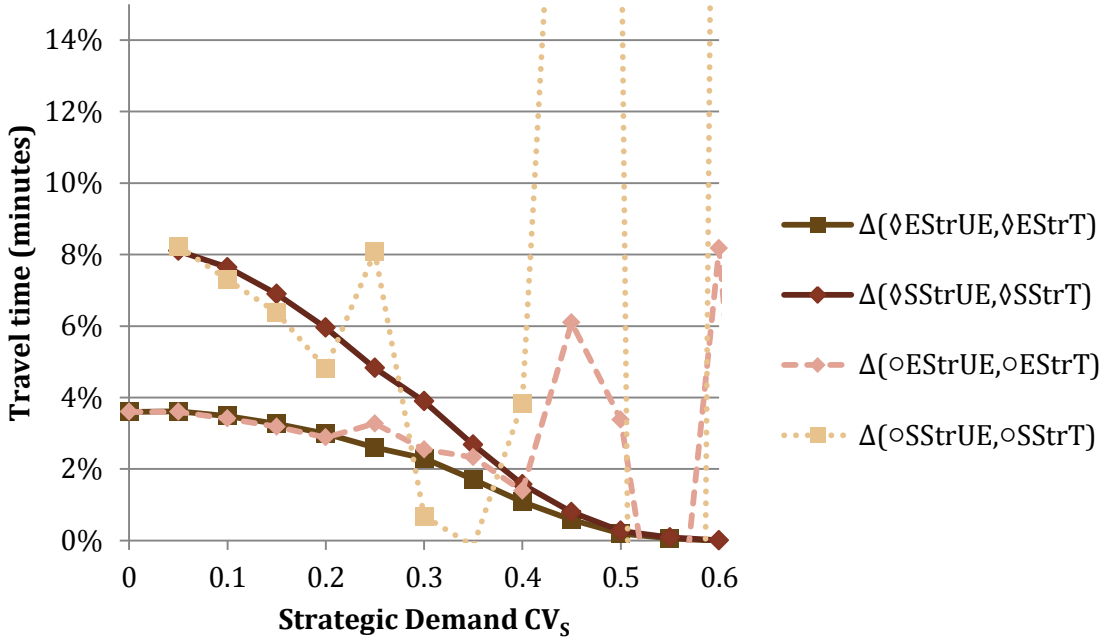


Figure 3:8 Short term demand uncertainty tolling results: Sioux Falls

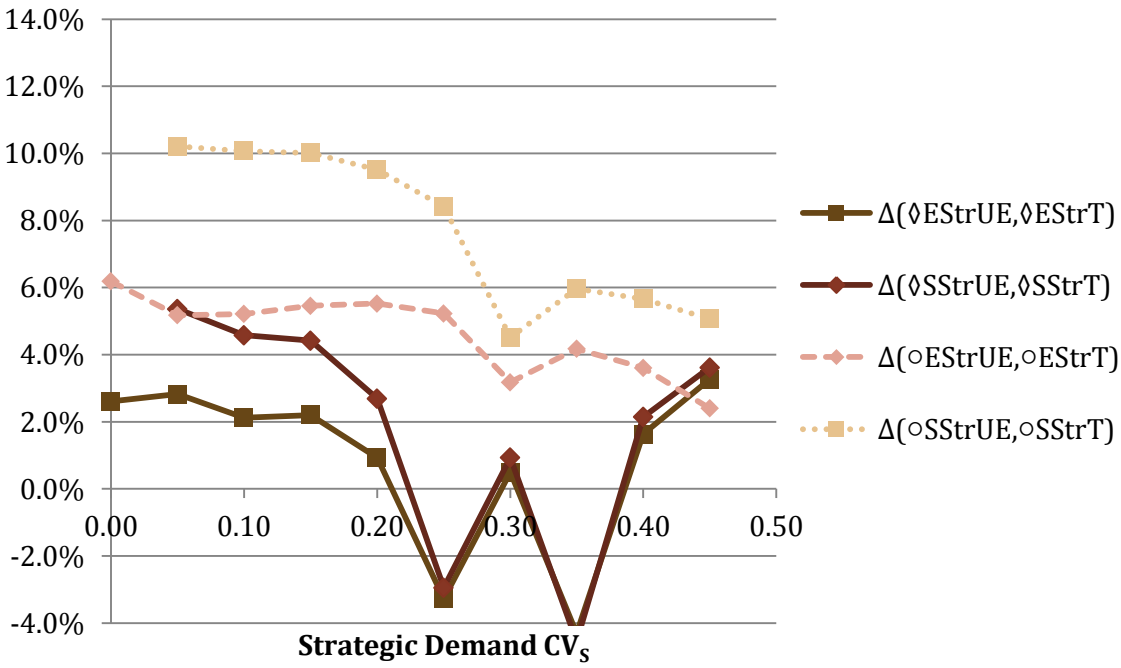


Figure 3:9 Long term evaluation of Sioux falls results, M

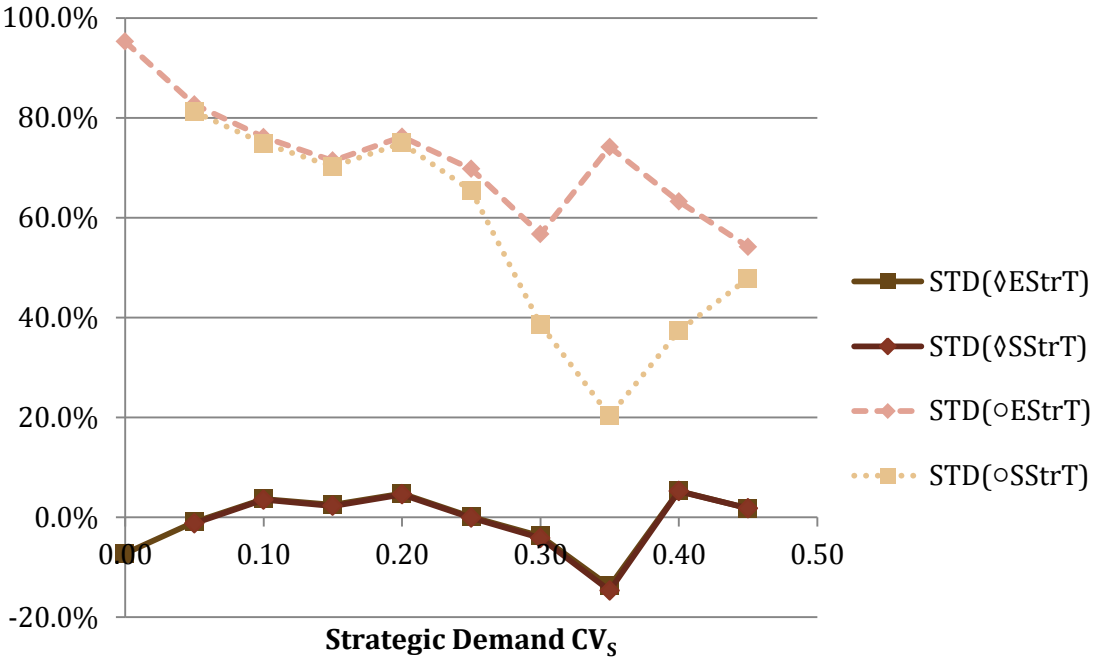


Figure 3:10 Long term uncertainty of Sioux Falls, STD

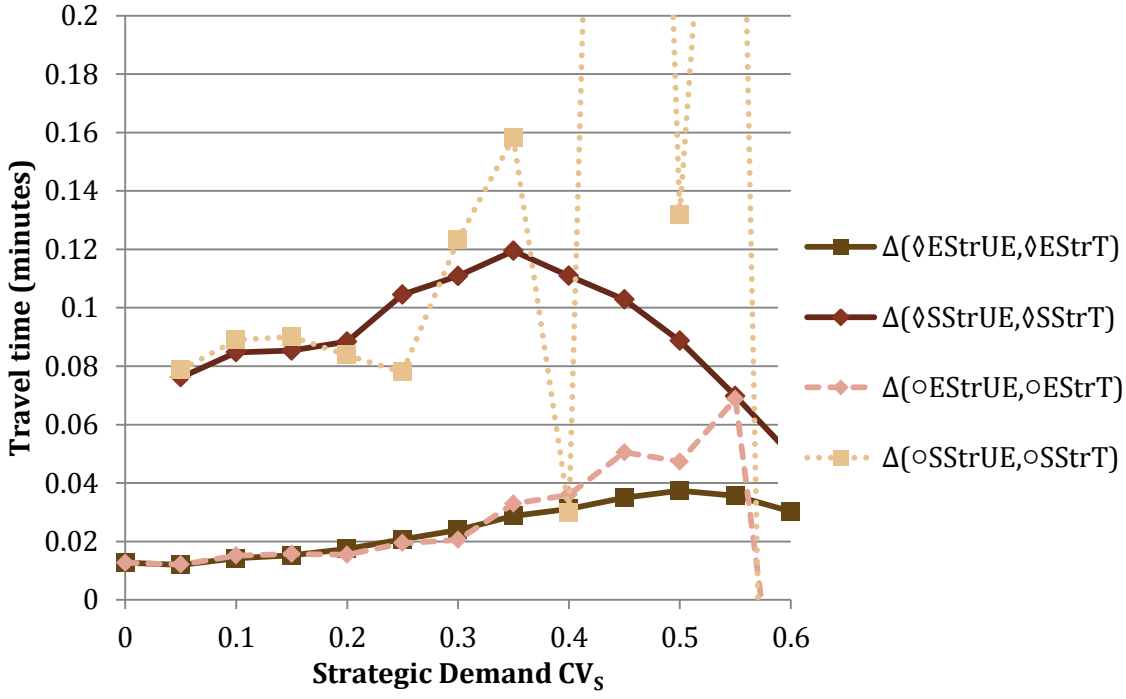


Figure 3:11 Anaheim network evaluation to day-to-day pricing scheme

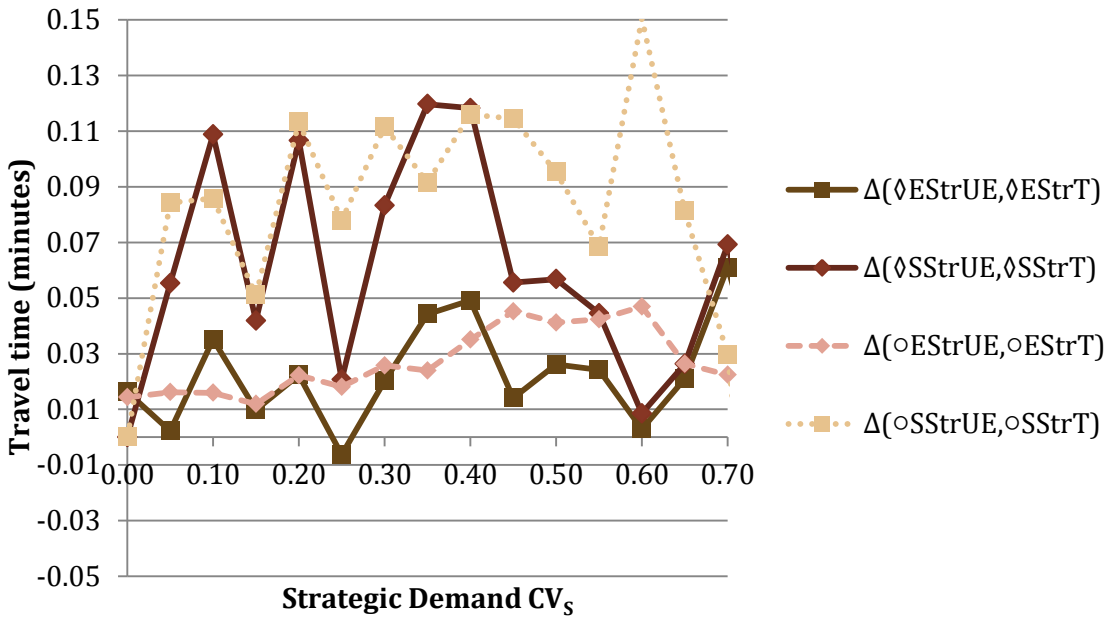


Figure 3:12 Long term evaluation of Anaheim network, ΔM

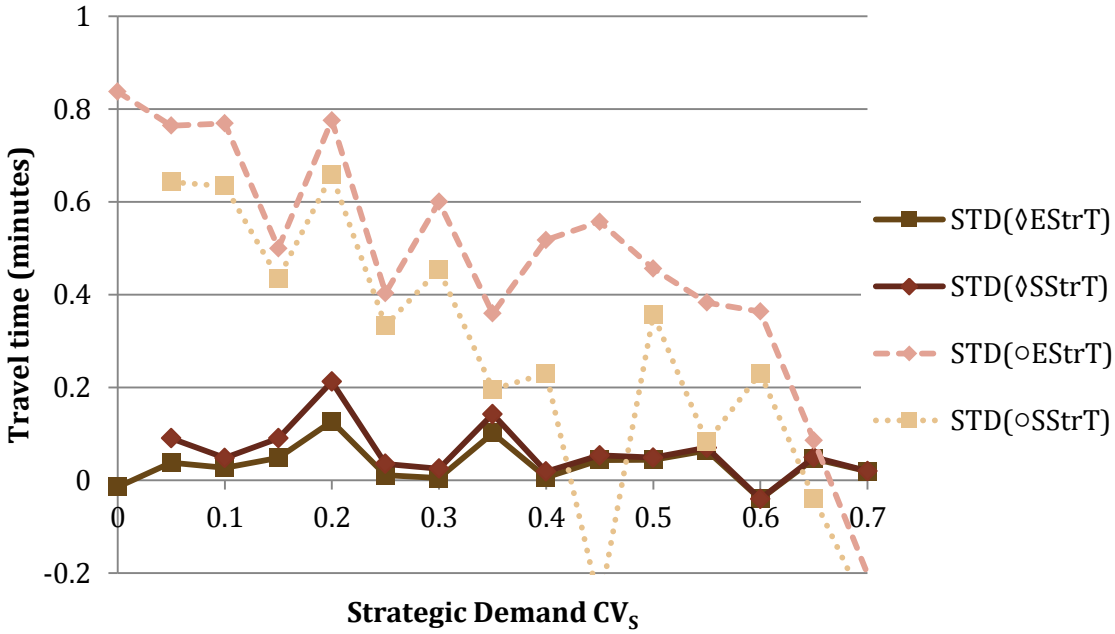


Figure 3:13 Long term evaluation of S on Anaheim network

Both of these figures suggest a number of observations about the behaviour of the StrT model. In both networks, when $\mathbf{0} \leq \boldsymbol{CV_S} \leq \mathbf{0.25}$, not accounting for planning

demand scenario uncertainty seems to *underestimate* system effectiveness. However, at larger values of $\boldsymbol{CV_S}$, the StrT model seems to dismantle and the results vary wildly. This may be an effect of sampling bias, but initial empirical observation indicates that the system performance can vary widely and model convergence is a complicated issue. Nonetheless, this outcome clearly shows that ignoring future planning scenario uncertainty can result in incorrect predictions of tolling scheme performance, and supports the need for further research.

3.5.2 Results from average demand based tolls

Figure 3:9 through Figure 3:13 shows the StrT model performance considering long-term demand scenario uncertainty; however, it is also important to consider the case where tolls are determined based on an average demand (i.e., short-term demand uncertainty is *not* included in the toll setting process). The same experiment from Section 5.1 was performed for the set of tolls determined based on deterministic conditions, assuming that $CV_S = 0$, representing the average demand. On the Sioux Falls and Anaheim networks, results for $M(\odot E)$ and $M(\odot STD)$ were similar for the cases where tolls were determined based on average demand versus strategic demand. However, the results for $STD(\odot E)$, a measure of system volatility, differed substantially. In the Anaheim network, for the case of $CV_S = 0.25$, average demand tolls resulted in $\Delta(STD(\odot E_{StrUE}, \odot E_{StrT}) = 75\%$ and strategic demand tolls resulted in $\Delta(STD(\odot E_{StrUE}, \odot E_{StrT}) = 62\%$, while Sioux Falls showed a similar pattern. These results illustrate that neglecting short-

term demand uncertainty may result in an overestimation of toll performance with regards to system robustness.

Table 3-5 shows a sensitivity analysis comparing the analytical and simulation results on the Sioux Falls network, under long term demand scenario uncertainty (using Algorithm 3.2). The rightmost column indicates the CV_s. Table 3-5 shows the performance metrics, i.e., the amount the tolling scheme reduced the metric of interest. Table 3-6 shows the results where the tolling scheme was based on the average demand scenario, i.e., CV_s=0.

Table 3-5 Sioux Falls network: long term evaluation of StrT model

	$\boldsymbol{\Delta(M(\diamond E))}$	$\boldsymbol{\Delta(STD(\diamond E))}$	$\boldsymbol{\Delta(M(\diamond S))}$	$\boldsymbol{\Delta(STD(\diamond S))}$
0.1	0.9%	-2.9%	3.0%	-3.7%
0.2	1.5%	1.2%	3.1%	1.3%
0.3	2.2%	-2.8%	3.2%	-3.1%
0.4	9.7%	9.2%	11.1%	9.3%
	$\boldsymbol{\Delta(M(\odot E))}$	$\boldsymbol{\Delta(STD(\odot E))}$	$\boldsymbol{\Delta(M(\odot S))}$	$\boldsymbol{\Delta(STD(\odot S))}$
0.1	5.5%	83.4%	10.6%	81.9%
0.2	5.4%	80.2%	9.4%	78.8%
0.3	5.0%	86.3%	7.4%	40.9%
0.4	3.8%	68.1%	5.3%	13.6%

Table 3-6 Sioux Falls network: long term evaluation of average demand results

	$\boldsymbol{\Delta(M(\diamond E))}$	$\boldsymbol{\Delta(STD(\diamond E))}$	$\boldsymbol{\Delta(M(\diamond S))}$	$\boldsymbol{\Delta(STD(\diamond S))}$
0.1	2.1%	3.7%	4.6%	3.5%
0.2	0.9%	4.8%	2.7%	4.6%
0.3	0.5%	-3.8%	0.9%	-4.2%
0.4	1.6%	5.3%	2.1%	5.2%
	$\boldsymbol{\Delta(M(\odot E))}$	$\boldsymbol{\Delta(STD(\odot E))}$	$\boldsymbol{\Delta(M(\odot S))}$	$\boldsymbol{\Delta(STD(\odot S))}$

0.1	5.2%	76.2%	10.1%	74.9%
0.2	5.5%	76.2%	9.5%	75.0%
0.3	3.2%	56.7%	4.5%	38.6%
0.4	3.6%	63.3%	5.7%	37.3%

3.6 Concluding Remarks

This work introduced a strategic marginal social cost based pricing methodology. The strategic tolling model (StrT) approach accounts for the influence of day-to-day demand volatility on user route choice behaviour, and sets tolls such that users are "priced" for the marginal impact of their myopic route choice on system travel time.

However, network operators must be aware of the additional uncertainty in the long term planning demand scenario; that is, a future strategic demand scenario realization in which the expected value of total trips E_S differs from the forecasted value. A procedure to evaluate the robustness of a strategic pricing scheme was proposed. Initial results show that if both sources of uncertainty are not included in an evaluation of a strategic pricing approach, performance of a tolling scheme could be underestimated or overestimated, and it is not intuitive how the system will behave.

This work contains an introduction to a strategic pricing approach, and has juxtaposed two sources of demand uncertainty in order to clearly differentiate between them. There are a number of research directions that emerge from the comparison. In particular, the use of two sampling distributions may result in unknown convergence behaviour that requires further investigation. Additionally,

the use of Bayesian statistical inference to describe the prior probability distribution of the strategic day-to-day travel demand may present an interesting avenue of research. Finally, the strategic pricing approach to the next-best pricing problem has been left for future research.

Chapter 4

Network Management Application: Link-Capacity-Based Design

4.1 Introduction

As previously discussed in Chapter 1, one of the primary purposes of transport planning models is the ranking and evaluation of infrastructure design projects. Planning tools, such as traffic assignment models based on the Wardropian equilibrium principle, can capture the effect that improvements in the network have on route choice in vehicle travellers. However, traditional models do not account for the inherent uncertainty in these methods, leading to an important question for researchers and practitioners alike: how do optimal project designs change in the face of non-deterministic network parameters?

Uncertainties in network modelling are well-established phenomena in traffic settings, as evidenced by the multitude of literature reviewed in Chapter 2. However traditional equilibrium-based network design approaches are primarily

deterministic and therefore make a single prediction that is usually interpreted as an average, rather than any specific manifestation of network conditions. While there are important reasons to use such traditional models (model stability, uniqueness, tractability), this approach will almost certainly misrepresent real network conditions, particularly in networks that deviate significantly from the average. To complicate the matter, network assignment models are often used to evaluate the effects of changes in the network, such as infrastructure design. In such situations, deviant model behaviour is particularly important to capture due to its unpredictable impact on design projects.

This chapter focuses on the network design problem (NDP) where the planner wishes to determine the optimal links to which to add capacity in order to improve a stated network performance measure while accounting for day-to-day network flow volatility. One of the difficulties in this problem arises in predicting vehicle user reactions to changes in network infrastructure. Additionally, NDP models must serve the varying, and at times opposing, needs of different users; planners are interested in the network from a macroscopic perspective, while daily travellers may view the transport system myopically, i.e., individual people just want to get to work on time. Viable NDP approaches need to provide reliable, cost effective, and equitable designs that consider the benefits and potential impacts from multiple perspectives.

This work applies the strategic assignment approach introduced in Chapter 2, in which total travel demand in treated as a random variable. In this chapter, the uncertainty in day-to-day capacity is additionally considered. Day-to-day capacity applies the same concept of a demand that varies each day due to variations in factors such as user behaviour, except to *capacity*. The concept of link capacity is inherently a dynamic quantity that is adapted to static parameters for the time invariant traffic assignment problem. The concept of capacity may seem deterministic; a road can only hold a finite number of vehicles. However, static capacity is an agglomeration of density, the number of vehicles a road can contain at a given time, and flow, which is the number of vehicles that can flow through a road. This concept of static capacity is influenced by external factors such as weather, and also factors such as driver behaviour, the presence of parking, or even semi-recurrent incidents, such a driver double-parking to make a quick trip into the shops, or a delivery van blocking a lane. The idea of day-to-day capacity attempts to account for these random variations.

This chapter implements the strategic user equilibrium with capacity uncertainty (StrUEC) introduced by Wen et al (2014). In the StrUEC model, link capacity is treated as an independent random variable with a known probability distribution. This work assumes that link capacity and demand are independent random variables, and that the capacity distribution on each link is independent from all other links. The StrUEC model will be described in more detail in Section 0.

Chapter 4 proposes a novel formulation for the NDP that integrates the strategic user equilibrium (StrUE) model to capture user behaviour in the face of day-to-day variation in demand and the strategic user equilibrium with capacity (StrUEC) model to represent the day-to-day variation in link capacity. Additionally, model results examine the impact of uncertain modelling parameters on design project selection and evaluation. Finally, designs are compared for a when the model includes demand uncertainty, capacity uncertainty, both source of uncertainty, or based on the StrUE, StrSO, or StrSR models.

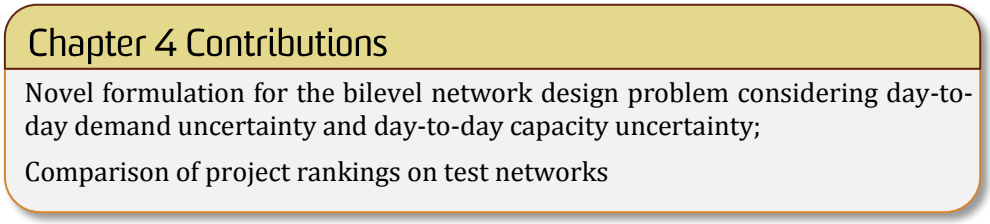


Figure 4:1 Summary of research contribution

4.2 Background

This work focuses on incorporating two sources of uncertainty into the network design problem. While accounting for different sources of uncertainty makes the NDP still more complex, it is essential that researchers develop approaches to quantify how those uncertainties impact infrastructure projects. However, network design models and algorithms have a rich history in the literature (Boyce & Janson, 1980; Dantzig et al, 1979; Leblanc, 1975; Magnanti & Wong, 1984). As such an active field, only selected relevant works are discussed here; see Yang and Bell (1998) for an overview and historical developments, (Chen et al, 2011) for a review

of uncertainty in the NDP specifically, and Wismans et al (2011a) for an in depth review of NDP applications using a dynamic approach.

Most generally, network design is conceptually simple: the problem of finding the optimal location(s) to enhance a network given a limited "budget." In this work, such enhancements are generally vehicle capacity improvements that can have a variety of interpretations, from the discrete additions (e.g., lanes, roads) to projects that may have a more continuous nature (e.g., optimized signal timing plans, other projects such as widening of shoulders, elimination of parking, etc). The NDP is traditionally formulated as a bi-level mathematical programming problem, where the upper level represents the "planner's" perspective that measures the impact in the network due to the change, and the lower level represents the users' reaction to those changes (Yang & Bell, 1998). Due to the nonconvex cost function resulting from the addition of capacity, the NDP can't be solved by traditional optimization techniques and heuristic methods are necessary. A few previous examples of bilevel network design formulations include multi-objective signal timing (Sun et al, 2003), accounting for long term demand uncertainty (Ukkusuri et al, 2007), total travel time reliability with stochastic route choice (Sumalee et al, 2006), optimal toll pricing strategies (Gardner et al, 2008a), examining the impact of environmental justice considerations (Duthie & Waller, 2008), minimizing emissions (Ferguson et al, 2012; Sharma & Mathew, 2011), and health impacts (Jiang & Szeto, 2014).

It is essential to account for uncertainty in in network design decisions, particularly because infrastructure project ranking is one of the primary purposes of network equilibrium models, like those outlined in Section 2.2. In fact, Lo and Tung (2003) approach the capacity reliability problem from the design perspective from the onset.

In other cases, equilibrium models are separately applied to the network design problem. Davis (1994) proposed a stochastic user equilibrium approach that leads to a more tractable model. Chootinan et al (2005) propose a network design problem based on stochastic user equilibrium that maximizes capacity reliability. Sumalee et al (2006) propose a network design model maximizes the network reliability, which is the probability that the total travel time with be less than a threshold. They include error in user perception, and Poisson-distributed demand. Yin and Lawphongpanich (2007) propose a continuous network design approach where the demand belongs to a convex set, as opposed to a probability distribution. Ukkusuri and Patil (2009) network design that accounts for dynamic through the use of multiple periods and demand uncertainty and elasticity and furthermore emphasise the importance of flexibility in network investment decisions. Szeto et al (2010) propose a network design framework that incorporates change of land use over time.

Uncertainty in transport network modelling is usually viewed as arising from demand, capacity, and user behaviour. Previous research has looked at the

strategic behaviour from users in terms of hyperpaths that are formed due to the possibility of being unable to enter capacitated links (Marcotte et al, 2004). The current work also employs strategic user behaviour in the sense that people will choose a route choice based on a range of possible network conditions they may encounter during travel, but the underlying modelling approach is based on the strategic user equilibrium (StrUE) introduced by Dixit et al (2013). StrUE finds equilibrium flow proportions based on expected path costs, and is detailed in Section 2.3. The output from the strategic assignment approach is link volumes that will vary from day-to-day, thus accounting for short-term demand uncertainty that users face making day-to-day route choice decisions.

This work extends the strategic assignment model to form the subproblem for a network design scenario focused on link capacity additions. Previous work has examined the impact of short-term demand uncertainty and link capacity uncertainty, but less often in combination. Example include Lam et al (2008), who account for the impact of reliability considerations on user route choice due to variations in capacity and demand and the concept of an effective travel time, and Siu and Lo (2008), who propose an equilibrium formulation based on the travel time budget among heterogeneous commuters.

This work proposes a novel approach to address this gap.

4.3 Model Formulation

This work captures users' reactions to day-to-day demand uncertainty using the strategic assignment model described in Chapter 3. However, the "capacity" of links as employed by most static traffic assignment approaches is another non-deterministic quantity that users consider when selecting a route and should be included in the evaluation design projects. Therefore, Section 4.3.2 describes the strategic behaviour approach that also accounts for the variability in capacity. Finally, Section 4.3.3 formulates the bilevel network design model incorporating strategic route choice assignment.

4.3.1 Strategic user equilibrium (StrUE) model

For completeness, the StrUE model is briefly recounted here. StrUE is a novel traffic assignment model that seeks to capture the day-to-day volatility in traffic flow by assuming that users choose a minimize expected cost route; however, the actual experienced cost of a route depends on the number of people who choose to travel on any given day, which is a random variable with a known probability distribution. The strategic concept is simple, but powerful; the model introduces variability of link flow while maintaining the ability to scale up to large size networks. The mathematical formulation is recounted below.

The notation previously introduced is recounted here. Consider a directed graph $G = (V, A)$ where V is the set of nodes (vertices) and A is the set of links

(edges), where an individual link is indexed by a. Let $r \in R$ index an origin and $s \in S$ index one destination from the set of destinations. Let W be the set of origin-destination pairs connecting origins r with destinations s, where q_{rs} indicates the proportion of total demand between origin r and destination s. The total demand is a random variable T with associated probability distribution $g(T)$. The travel cost on a link is $t_a(p, T)$, which is a function of the proportion of the total flow on the link p_a: $\sum_{a \in A} p_a = 1$, and T. Furthermore let K_{rs} be the set of paths connecting origin r and destination s, and let f_k^{rs} represent the proportion of the total travel demand on that path. Finally, let $\delta_{a,k}^{rs}$ be the incidence matrix that is equal to 1 if link a is on path k between origin r and destination s and 0 otherwise. The StrUE model as previously introduced may then be written as:

$$\text{minimize} \int_0^{\infty} \sum_{(i,j) \in A} \int_0^{p_{ij}} t_{ij}(wT)\, g(T) dw dT \tag{4.1}$$

subject to

$$\sum_k f_k^{rs} = q^{rs} \qquad \forall r \in R, \forall s \in S \tag{4.2}$$

$$f_k^{rs} \geq 0 \qquad \forall r \in R, \forall s \in S, \forall k \in K \tag{4.3}$$

$$p_{ij} = \sum_{r \in R} \sum_{s \in S} \sum_{k \in K} f_k^{rs} \delta_{ij,k}^{rs} \qquad \forall (i,j) \in A \tag{4.4}$$

The StrUE model formulation provides a straightforward framework that can be applied to practically sized problems by modifying well established solution methods. As discussed in Section 2.3, to ensure uniqueness of link flows, for each origin-destination, path flow proportion is assumed to be equal under all demand scenarios. Therefore, the equilibrium flow on each path will vary in a proportional manner when the total origin-destination demand varies. The system performance metrics in the strategic approach can be found through analytical derivations or simulation-based sampling methods, and will be detailed in the next section.

4.3.2 Strategic user equilibrium with capacity uncertainty model

In both the deterministic approach and the StrUE model, capacity serves as a model input and assumed to be a fixed value. Capacity is inherently a static representation of a dynamic concept. It is often intended to be a proxy to capture the effects of congestion, where the travel time increases as the ratio of flow to capacity on a link increases. In spite of this, the capacity of a road will fluctuate due to factors such as driving behavior and adverse weather conditions, phenomena that is captured in this work by the concept of "day-to-day" capacity. Drivers may consider this fluctuating capacity when making route selections; therefore, it is important to consider its impact on network design project rankings.

The StrUEC model, introduced by Wen et al (2014), accounts for the day-to-day volatility in capacity and vehicle users' reaction to knowledge of that volatility.

The StrUEC approach assumes that the inverse capacity $C_{inv,a}$ on each link is a random variable with a known probability distribution $h_a(C_{inv})$. Vehicle users have knowledge of the capacity distribution on all links and choose the expected least cost route, where the expected cost is based on the probability distribution of both the demand and the capacity distribution on each link. Wen et al show the uniqueness of the model assignment solution.

Continuing the notation previously introduced, the StrUEC model seeks the set of link flow proportions that satisfy the mathematical program in Equation (4.5). The difference between the objective function for the StrUE model in (4.1) and the StrUEC model in (4.5) is that the expected cost is now a function of flow proportion, total demand, and link capacity.

$$\begin{aligned} &\text{minimize } z_C(p) \\ &= \int_0^{\infty} \int_0^{\infty} \sum_{(i,j)\in A} \int_0^{p_{ij}} t_{ij}(w, T, C_{inv})\, g(T) h_{ij}(C_{inv}) dw dC_{inv,ij} dT \end{aligned} \qquad (4.5)$$

subject to

Constraints [2] – [4]

In traditional network design, the capacity of a link is increased in vehicles per hour, which will lower total system travel time and make the link more attract to drivers. However in network design with uncertain capacity, the *expected* capacity of the link, which is one of the parameters of the distribution of the link capacity

$h_{ij}(C_{inv})$, is increased because capacity is not a deterministic quantity. There might be other possibilities where the coefficient of variation or the variance of the capacity is decreased (for example policy targeting illegal parking practices). For simplicity, the most straightforward interpretation is utilized in this work. Alternate possibilities will be the topic of future research.

4.3.3 Network design formulation

The network design problem with uncertainty is formulated as a bilevel nonlinear mathematical programming problem. The upper level seeks to minimize the planning objective accounting for volatility in the network, for example, expected total system travel time or standard deviation of travel time, both of which are a function of proportion of flow on each link and capacity changes in the transportation network. The lower level represents drivers' reactions to changes in the road network, modelled by the StrUE or StrUEC approaches presented in Section 3.1 and 3.2.

This work focuses on ranking and evaluating design projects in a traffic network, although principles similar to those discussed here would apply to other NDP applications. Let S be a predetermined set of possible network design scenarios indexed by s, each of which is defined by the amount of capacity or expected capacity ρ to add to some number of links such that the total cost of improving the links is below the budget B in order to minimize objective $w \in \Omega$. δ_{ij}^{s}

is a binary decision variable equal to 1 if link (i,j) is an optimal location to add capacity in project scenario s. Note that links which are not available to be improved by the amount $p \in P_S$ will be constrained such that $\delta_{ij}^{s} = 0$. Additionally, the binary constraint on δ^{s} could be relaxed, in which case this would be a continuous network design formulation. The binary approach was utilized here because it puts realistic bounds on the solution space, which is already quite large.

The upper level problem represents the "planner's" perspective, who seeks the optimal links to which to add capacity for each design scenario in order to minimize an objective $s_{B,p}^{w}$. The upper level decision variables also impact the lower level problem, which is the strategic traffic equilibrium approach that accounts for different sources of network uncertainty. For each design scenario $s_{B,p}^{w}$, the formulation to minimize the objective *w* follows:

$$minimize\ w \tag{4.6}$$

subject to

$$\sum_{(i,j)\in A} \rho\delta_{ij}^{s} \leq B_s \tag{4.7}$$

$$\delta_{ij}^{s} \in \{0,1\} \quad \forall (i,j) \in A \tag{4.8}$$

subject to

$StrUE\ or\ StrUEC$

The defining point of this formulation is which objective function should be utilized for Equation (4.6). There are two basic system objectives that are of interest in this work: expected total system travel time E and the standard deviation of total system travel time STD. Due to the assumptions in the strategic assignment model, there are two approaches to solving for these objective measures. The first is to use the analytical equations that are derived based on a travel cost function and the distribution of the random variables. The expression for $\diamond E$, which was introduced in Duell et al (2014), is shown in Equation (4.9).

$$\diamond E = \int_0^\infty \sum_{(i,j)\in A} p_{ij} t_{ij}(p, T, \delta^s) g(T) dT \tag{4.9}$$

The analytical expression for the total system travel time where link inverse capacity is a random variable given in Equation (4.10).

$$\diamond E_C = \int_0^\infty \int_0^\infty \sum_{(i,j)\in A} p_{ij} t_{ij}(p, T, C_{inv}, \delta^s) h_{ij}(C_{inv}) g(T) dT \tag{4.10}$$

Note that in this work, the $\diamond$ symbol indicates that a quantity is analytically derived, as opposed to using a simulation procedure that involves sampling from the distribution of the random variable, which is denoted as $\odot$. The simulation based expected total system travel time is $\odot E$ and $\odot S$. A simulation based procedure may be necessary for cases where no analytical form exists. A procedure to determine the simulation values was provided in Algorithm 2.3.

The standard deviation of total system travel time is denoted as $\diamond STD$, and it is the expected value of the square of the total system travel time minus the square of the expected value of total system travel time. Note that expectation is denoted $Ex(X)$ in this work. Equations (4.11) and (4.12) contain the $\diamond V$ for StrUE and StrUEC models respectively.

$$\diamond V = Ex_T\left(\left(\sum_{(i,j)\in A} p_{ij}t_{ij}(p,T,\delta^s)\right)^2\right) - \left(Ex_T\left(\sum_{(i,j)\in A} p_{ij}t_{ij}(p,T,\delta^s)\right)\right)^2 \quad (4.11)$$

$$\diamond V_C = Ex_{T,C(ij)}\left(\left(\sum_{(i,j)\in A} p_{ij}t_{ij}(p,T,C(ij)_{inv},\delta^s)\right)^2\right) - \left(Ex_{T,C(ij)}\left(\sum_{(i,j)\in A} p_{ij}t_{ij}(p,T,C(ij)_{inv},\delta^s)\right)\right)^2 \quad (4.12)$$

The next section outlines the assumptions to provide tractable forms of Equations (4.9) – (4.12), as well as the solution method for the bilevel program proposed in Equations (4.6) – (4.8).

In order to evaluate a design scenario, this work employs two performance metrics to measure the relative impact of each design scenario. For the performance metrics, $\Delta(\cdot,\cdot)$ is used to indicate the percentage difference between two quantities. The decrease in expected total system travel time is the percentage difference between the E_0, the system travel time in the base case with no design

changes, and E_s, the expected system travel time that results from design scenario s, with the same principle applying to the case of $\diamond STD$.

$$\Delta E_s = \Delta(\diamond E_s, \diamond E_0) = 1 - \frac{E_s}{E_0} \tag{4.13}$$

$$\Delta STD_s = \Delta(\diamond STD_s, \diamond STD_0) = 1 - \frac{STD_s}{STD_0} \tag{4.14}$$

4.4 Solving the Model

This section details the assumptions and methodology to solve the bilevel network design model presented in Section 4.3.3. The upper level of the model is solved using a heuristic based on natural evolution known as a genetic algorithm. The strategic assignment submodel presented in Sections 4.3.1and 0 is solved using a straightforward approach based on the well-known Frank Wolfe method. Additionally, the strategic assignment approach both with and without capacity uncertainty both require a number of assumptions in order to provide a tractable form of the model to solve analytically. All solution methods and assumptions are detailed Sections 4.4.1-4.4.3.

4.4.1 Strategic assignment model

In order to solve the strategic assignment model, as in Chapters 3 and 4, this approach assumes a lognormal distribution for the total travel demand with random variable $T \sim LN(E_{str}, CV_{str})$, where E_{Str} is the total expected demand, the

$CV_{str}$is the coefficient of variation of total trips, and that the OD demand follows fixed, specified proportions. Travellers make their route choices based on knowledge of the distribution and the resulting expected travel costs. In order to solve the StrUE and StrUEC models that are the subproblem of this work, we assume that travel cost varies with flow according to a variation of the BPR function, where flow is a function of link proportion p_{ij} and the random variable for total demand T:

$$t_{ij}(p,T) = t_{ij}^{f}\left(1 + \alpha\left(\frac{p_{ij}T}{c_{ij} + \delta_{ij}^{s}n_s}\right)^{\beta}\right) \tag{4.15}$$

Where t_{ij}^{f} is the free flow travel time on link a, c_a is the capacity, α and β are BPR shaping parameters that are commonly assumed to be 0.15 and 4, respectively. For simplicity, this work assumes that the α and β parameters in the BPR function are the same on every link. The flow proportion on each link $p_{ij}$is an output from solving the StrUE model. The StrUE model assumes that users will choose the expected shortest cost path, where the expected link cost is can be derived as in Equation (4.7).

$$E_{ij}(p,T) = \int_{0}^{\infty} p_{ij}t_{ij}(pT)g(T)dT = t_{ij}^{f}\left(1 + \alpha\left(\frac{p_{ij}}{c_{ij}}\right)^{\beta} M_{\beta}\right) \tag{4.16}$$

Where M is the analytical moment of the demand distribution that is found as:

$$M_\beta = e^{\beta\mu+\frac{1}{2}\beta^2\sigma^2} \tag{4.17}$$

The expected total system travel time is an important metric for planners, especially for the ranking of design projects. For simplicity, unless stated otherwise, we assume the link capacity c_{ij} includes the additional projects and will leave out the $\delta_{ij}^s n_s$ from the travel cost function. Using the Equations 10 and 14, the $\diamond E$ can be calculated as:

$$\begin{aligned} \diamond E &= \int_0^\infty \sum_{(i,j)\in A} p_{ij} t_{ij}(p,T) g(T) dT \\ &= \sum_{(i,j)\in A} \left(t_{ij}^f p_{ij} M_1 + \left(\frac{\alpha t_{ij}^f}{c_{ij}^\beta} \right) p_{ij}^{\beta+1} M_{\beta+1} \right) \end{aligned} \tag{4.18}$$

In order to aid with the presentation of system performance metrics, consider the two parts of total system travel time as that resulting from sum of the free flow travel time on each link F and that resulting from the sum of the delays on each link, D.

$$F = \sum_{(i,j)\in A} t_{ij}^f p_{ij} \tag{4.19}$$

$$D = \sum_{(i,j)\in A} \left(\frac{\alpha t_{ij}^f}{c_{ij}^\beta} \right) p_{ij}^{\beta+1} \tag{4.20}$$

Using this notation, the expected total system travel time can then be written as:

$$\diamond E_{str} = F M_1 + D M_{\beta+1} \tag{4.21}$$

The standard deviation is more complex because we need to find the expected value with respect to the total demand T, of the sum of link travel times squared. However, assuming that T is (so the power of β is the same on all links), then the total trips T can be factored out and the standard deviation calculated by summing each different quantity on each link, and then computing the final expression presented in Equation (21).

$$\diamond V = F^2 M_2 + D^2 M_{2\beta+2} + 2FDM_{\beta+2} - \left(FM_1 + DM_{\beta+1}\right)^2 \tag{4.22}$$

$$\diamond S = \sqrt{\diamond V} \tag{4.23}$$

While $\diamond S$ is somewhat nonstandard, it can still be calculated relatively easily using a single pass through the array of links. Next we consider the differences in calculating strategic model performance metrics when including link capacity as a random variable.

4.4.2 StrUEC model assumptions

Additionally, this work considers the model in which capacity is a random variable that users consider for when making a route choice decision. In order to capture the variation in day-to-day capacity, we assume that capacity follows a gamma distribution. The inverse of capacity therefore follows an inverse gamma distribution $C \sim Inv_\Gamma(k, \frac{1}{\theta})$, where k and θ are the distribution shaping and scaling

parameters respectively and specific to link a. Assume that the expected capacity on a link is c_{ij}, and the standard deviation is c_{ij}^{std}. Furthermore, we assume that capacity distributions of each link are independent from one another and independent from the demand.

As input to the StrUEC model we consider the coefficient of variation on link ij as $cov_{ij} = \left(\frac{c_{ij}^{std}}{c_{ij}}\right)$ and we calculate the link distribution parameters as:

$$k_{ij} = \frac{c_{ij}}{\theta_{ij}} = \frac{1}{cov_{ij}^2} \tag{4.24}$$

$$\theta_{ij} = \frac{c_{ij}}{k_{ij}} = c_{ij} \times cov_{ij}^2 \tag{4.25}$$

When capacity is a random variable, the expected cost can be rewritten using the moment of the link specific capacity distribution, first presented in Wen et al (2014).

$$\begin{aligned} E_{ij}(p,T) &= \int_0^\infty \int_0^\infty p_{ij} t_{ij}(p,T,C_{inv}) h_{ij}(C_{inv}) g(T) dT \\ &= t_{ij}^f \left(1 + \alpha p_{ij}^\beta M_\beta L_{ij,\beta}\right) \end{aligned} \tag{4.26}$$

Where $L_{ij,\beta}$ is the β^{th} moment of the inverse gamma link capacity distribution that is computed as:

$$L_{ij,\beta} = \frac{1/\theta^\beta}{\prod_{n=1}^{\beta}(k-n)} \tag{4.27}$$

Note that this definition does place constraints on the feasible values of cov_{ij}.

$$\begin{aligned} \diamond E_C &= \int_0^\infty \int_0^\infty \sum_{(i,j)\in A} p_{ij} t_{ij}(p, T, C_{inv}) h_{ij}(C_{inv}) g(T) dT \\ &= \sum_{(i,j)\in A} \left(t_{ij}^f p_{ij} M_1 L_{ij,1} + \alpha t_{ij}^f p_{ij}^{\beta+1} M_{\beta+1} L_{ij,\beta} \right) \end{aligned} \tag{4.28}$$

Again, the system performance metrics of interest are the analytical total system travel time $\diamond E_C$ and analytical standard deviation of total system travel time $\diamond S_C$, where the subnote C indicates metrics from the StrUEC model. In order to aid with the presentation of metrics, consider the two parts of the travel cost function, where F remains the same. However, the link capacity distribution is link specific and therefore stays inside of summation.

$$D_C = \sum_{(i,j)\in A} \alpha t_{ij}^f p_{ij}^{\beta+1} L_{ij,\beta} \tag{4.29}$$

Using the short hand notation, the analytical total system travel time for the StrUEC model may be calculated as:

$$\diamond E_C = F M_1 + D_C M_{\beta+1} \tag{4.30}$$

The derivation of standard deviation from Equation (14) is more algebraically demanding. In this case, the expected value is with respect to either T or $C_{inv}(ij)$, so most of the travel cost function can be treated as constants and integrated

appropriately. For clarity, we factor out the constant part of each link specific quantity attributed to the delay as D_{ij}:

$$D_{ij} = t_{ij}^{f} \alpha p_{ij}^{\beta+1} \tag{4.31}$$

Finding the expected value of the square of a summation over each link is less straightforward. In the demand case, we assume that T is not link specific and therefore even when it is multiplied together in the sum, it factors out. However in the case of Equation (4.28), when the summation of the travel time over each link is squared, the capacity random variable on each link must be multiplied by the capacity random variable on every other link, after which the expected value is calculated. Equation (4.32) shows this term.

$$D_{C,2} = \int_0^{\infty} \int_0^{\infty} \left(\sum_{(i,j)\in A} D_{ij} T^{\beta} C_{inv}^{\beta}(ij) \right)^2 g(T) h_{ij}(C_{inv}) dC_{inv}(ij) dT \tag{4.32}$$

Using the property that for independent, real value variables, $E(XY) = E(X)E(Y)$ and ordering the links in a "list", it is still relatively simple to calculate this value. Using manipulation to arrange the equation in a form that is easy to compute, the squared part of the system "delay" can be found as:

$$D_{C,2} = \sum_{(i,j)\in A} \left(D_{ij} L_{ij,2\beta} + 2 \sum_{(m,n)=1}^{mn<ij} D_{mn} D_{ij} L_{mn,\beta} L_{ij,\beta} \right) \tag{4.33}$$

Therefore the $\diamond \, STD_C$ is calculated as:

$$\diamond V_C = F^2 M_2 + D_{C,2} M_{2\beta+2} + 2F D_C M_{\beta+2} - \left(F M_1 + D_C M_{\beta+1}\right)^2 \qquad (4.34)$$

$$\diamond STD_C = \sqrt{\diamond V_C} \qquad (4.35)$$

4.4.3 Solving the design problem

The network design problem as formulated in Section 4.4.3 cannot be solved to a guaranteed global optimal value using standard optimization techniques because of the non-convex cost function (Equation (4.15)). Therefore heuristic solution methods are necessary. This work applied a genetic algorithm, an optimization technique inspired by principles of natural evolution. GAs provide a flexible, rigorous framework to solve challenging optimization problems, and are a relatively common research method to solve the bi-level traffic network design problem. Karoonsoontawong and Waller (Karoonsoontawong & Waller, 2006) showed that in terms of heuristic approaches to solve the continuous NDP, GAs perform better than simulated annealing or random search algorithms. A GA will correctly identify local extrema, but as is the case with all heuristics, the solution is not guaranteed to be the global optimal value. In this approach, steps were taken to ensure that the GA had converged on the best solution.

A GA locates an optimal solution by searching for promising regions in which there are a high proportion of "good" solutions. It begins with a randomly generated initial population of individuals that represent potential solutions (called

chromosomes). Over "time", the population evolves according to a natural selection process, in which the best individuals are selected and combined using a crossover technique to form new populations of individuals. Table 4-1 summarizes the terminology used in genetic algorithms. The procedure for a genetic algorithm is described in more detail below.

Table 4-1 Summary of genetic algorithm terms and problem-specific representation

Term	**Explanation in GA context**	**Representation in the Chapter 4 Application**
GA variable	The quantities that the GA changes to find better solutions, which the GA represents as binary numbers	A binary variable representing whether to add n capacity to a link
Chromosome	Possible solution consisting of a set of GA variables	Represents a possible design scenario, where each "1" indicates that capacity is added to that link
Generation	An iteration of the algorithm	A complete cycle of performing each GA procedure a single time, including: solving a Frank Wolfe assignment for each new chromosome, crossover, and mutation
Population	The set of chromosomes at any given generation	The set of all design scenarios that the GA is currently testing
Fitness	The measure of how "good" a chromosome is in terms of minimizing the objective function	The expected total system travel time $\diamond$ E_s associated with a particular design scenario
Crossover	A GA procedure to find new solutions based on the evolutionary equivalent of "breeding"	Using a binary representation
Mutation	A GA procedure that also finds new solutions, based on the evolutionary equivalent	Randomly changes a design plan such that capacity is added or not added to a link
Mutation rate	The probability that each bit of the binary chromosome representation	Mutation rate = 0.001

The fitness evaluation procedure provides a means to judge how "good" each chromosome is in terms of minimizing the specified objective of total travel time or variation of travel time. The GA uses this fitness score to provide a search direction. As a chromosome represents a possible design plan, its fitness is obtained by solving an instance of StrUE, resulting in values for $\diamond E$ and $\diamond S$. The crossover procedure and the mutation procedure are the methods used by the GA to find new, better solutions. The crossover procedure begins by eliminating a percentage of the population as specified by the input culling percentage. The "weakest" members of the population are deleted, here represented by chromosomes with the highest total delay or total emissions value (depending on the objective). To generate new chromosomes, two chromosomes from the remaining population are randomly selected to be "parents". Two "child" chromosomes are created by taking half of the binary representation of each parent to form a new number for each of the GA variables. This procedure requires the culling percentage as an input from the user. Similar to the principles of natural biology, the mutation process maintains diversity in a population and again helps with the search direction of the algorithm. As a general rule, the crossover procedure explores the "nearby" solution space, while mutations help explore further away.

This work utilized a single-objective binary-coded variation of the nondominant sorting genetic algorithm II (NSGA-II) by Deb (2002). NSGA-II is a well-known algorithm that has proven to be the best GA tool for solving multi-

objective optimization problems, and utilizes several techniques that provide superior performance.

An important aspect of using a GA as a solution method is how the problem variables are represented. As in previous applications, this work uses a "binary" approach in order to limit the feasible solution space. Each "chromosome" is specified to have as many bits as there links in the network. Then a "0" represents the decision not to add capacity to a link and a "1" means to add capacity (where the amount of capacity to add is a model input). In general, GAs perform better without constraints. However, a constraint was unavoidable in this application due to the fact that we consider the cost of adding capacity to a link to be related to the length of the link. We avoided the use of a penalty function by initializing each population (set of GA chromosomes) to be feasible. A GA relies on crossover and selection procedures to explore the solution space. However, for binary approaches where the solution contains many more 0's than 1's, there is a much higher probability that crossover or selection will result in infeasible solutions. This issue was addressed by running the GA for more generations to give it more time to explore the solution space. A crossover probability of 0.9 and a mutation probability of 0.001 were used in this work.

Algorithm 4.1 provides a more methodological outline of the GA procedure as utilized in this chapter.

Algorithm 4.1: Genetic Algorithm Pseudocode

INPUT: PS (population size), crossover probability, mutation probability, I (number of generations), RS (random seed)

procedure GA($s_{p,n}^{w}$)

1: $P(\phi)$ = initialize_population(RS,PS);
2: **while** generation $i < I$ **do**
3: **for** $\forall \phi \in P$ **do**
4: $\diamond E_{\phi}, \diamond STD_{\phi} \leftarrow$ solve_strategic($s_{p,n}^{w}$,StrUE,ϕ_s);
5: **end for**
6: rank chromosome fitness by objective w;
7: crossover_procedure(crossover probability);
8: mutation_procedure(mutation probability);
9: increment i;
10: **end while**
11: return ϕ_s^*;

end procedure

OUTPUT: optimal design scenario ϕ_s^*

The genetic algorithm in this chapter implemented an adapted the original code by Deb, which is available online (and written in the C programming language, which can also be complied using C++ compilers). Changes to the code include the function to initialize the population to a feasible solutions, and variations that adapted the crossover and mutation procedures to maintain feasibility. The interface between the strategic code that was also used for Chapters 3 and 4 and the genetic algorithm was programmed using the C++ language. As indicated in Algorithm 5.1, the GA primarily operated as a separate function. However, a small interface between the GA and StrUE was needed in order to test the fitness of each

chromosome, where the set of link capacities are adjusted to match the GA variables of a design plan chromosome.

4.5 Results and Discussion

This section demonstrates the NDP model accounting for strategic user behaviour and discusses the implications for planning for uncertainty in transport networks. The GA is used to solve a variety of design scenarios. Results are presented for a small network in order to demonstrate the model and then on a slightly larger network where more rerouting effects can be captured. Three modelling approaches are compared: StrUE, StrUEC, and a deterministic UE approach.

In the design scenarios, there are three sets of input parameters that can be changed: the total budget, the cost of building on a link, the amount to be added to the link (in the binary relaxation). The user inputs regarding the demand are: the expected value of total trips, and the coefficient of variation of the demand distribution. Networks with a higher degree of fluctuation in the realized demand will have a higher CV_{Str}. The link capacity follows an inverse gamma distribution, where for each link, the primary input is the expected value of the capacity for each link and the coefficient of variation for each link.

In the strategic assignment network design application, the planner seeks to rank and compare different design scenarios, indexed by $s_{B,p}^{w} \in S$. In this experiment, the objective is $w = \diamond E$, and we focus on the case where $p = 1{,}500\ vph$.

Lacking the appropriate data, we assume that the cost to add capacity to each link is equivalent to the length of that link. Essentially, this captures the fact that the cost to add capacity to all links is not equivalent, but links that are longer will cost more to enhance their vehicle capacity. If the cost is $10M/km, then a budget of 10 is a proxy for $100M budget.

4.5.1 Determining link capacity coefficient of variation

Due to a lack of real world data regarding the day-to-day capacity of links in a network, this thesis compares six different methods that may capture different aspects of real networks. These methods are: capacity normalised variability, high variability on congested links, high variability on links with greater capacity, identical variability on all links, random variability on all links, and normally distributed variability on all links. Each method is further detailed below.

Table 4-2 Methods of generating coefficient of variation for all links in the network

Method		**Explanation**
Congested	CON	the links with the highest (or lowest) congestion have higher CV
Capacity	CAP	the links with the highest (or lowest) capacity have the larger CV
Distribution	DIST	link cv is sampled from a specified distribution (i.e., uniform or normal)
Normalized	NORM	link capacity is normalized to a range specified by the user
Identical	ID	all links have the same CV

For a number of methods, it is convenient to divide the links into n bins, where a link coefficient of variation is specified for each bin. Algorithms 4.2, 4.3, and 4.4 show general outlines for the procedures to generate the cov_a for each link. The general method would be a function where the type of variability is specified (from Table 4-2). The function would generate the necessary data to run the function `solve_strategic()` that was introduced in Chapter 2.

Algorithm 4.2 shows the method for giving the links with the highest congestion the highest level of variability. For this, solve_strategic() must be called first (possibly with $CV_s = 0$) so that the congestion on a link can be estimated. The concept is to divide the links into n groups based on their level of congestion and then assign a cov_a to each group.

Algorithm 4.2: Generate link cv data

```
INPUT: set of links ℒ, number of bins n, coefficient of variation
of each bin cv(b);
    procedure link_cv()
1:  if type: CONGESTION
2:        sort links by flow/capacity ratio in ascending order
3:        //create bins
4:        for j = {1, … , n}
5:              B ← B ∪ (Range: (j − 1)/n − j/n; cv(j))
6:        end while;
7:        while i < |ℒ|
8:              if i ∈ Range of b(n)
9:                    cv(i) = cv(b)
10:             increment i;
11:       end while
12: end if
13: return cv;
    end procedure
```

The second method of generating a scheme of link variability is to use link capacity as a measurement tool. In the case of the capacity enhancement network design problem, link capacity is the only indication that the model can control in order to decrease expected total system travel time. According to the link cost function, travel time will decrease as capacity increases in a nonlinear manner. Therefore, it is interesting to investigate the impact of the links with either the highest, or the smallest, capacity when those links also have the most variable travel time, or the least. The method to generate the link cvs is similar to the method according to link congestion. The main difference is to sort the set of links by capacity, instead of by v/c ratio. The procedure in outlined in Pseudo-Algorithm 4.3.

Algorithm 4.3: Generate link cv data

INPUT: set of links $\mathcal{L}$, number of bins n, coefficient of variation of each bin $cv(b)$;

```
    procedure link_cv_continued()
1:  if type: CAPACITY
2:        sort links by expected capacity in ascending order
3:        //create bins
4:        for j = {1, ..., n}
5:              B ← B ∪ (Range: (j − 1)/n − j/n; cv(j))
6:        end while;
7:        while i < |ℒ|
8:              if i ∈ Range of b(n)
9:                    cv(i) = cv(b)
10:             increment i;
11:       end while
12: end if
13: return cv;
    end procedure
```

The third method that is outlined normalises the link capacities over a range of 0 – a specified maximum range (e.g., 0.3). The procedure to generate this data is outlined in Pseudo-Algorithm 4.4.

Algorithm 4.4: Generate link cv data

```
INPUT: set of links ℒ; max cov;
    procedure link_cv_continued()
1:  if type: NORMALIZED
2:        find max(i) and min(i);
3:        if (max == min)
4:              set all cv(i) = max_cov;
5:        else
6:              for each i ∈ ℒ do
7:                    cov(i) ← max_cov*(1-(cap(i)-min/(max-min))
8:              end for
9:        end if;
10: end if
11: return cov(i), ∀i ∈ ℒ;
    end procedure
```

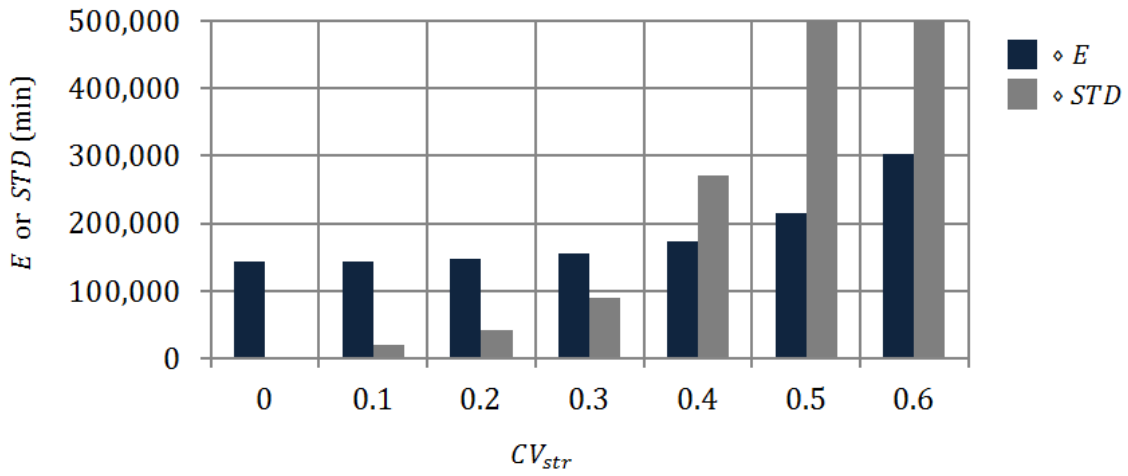
The other cases in Table 4-2 were relatively straightforward and so the pseudo-code is not included here. Note that the distribution case (either normal or uniform) introduces a random element, because each link would just sampled from a specified distribution. While this case may not represent a common network behaviours, it is useful for the purposes of model comparison. However, the element of stochasticity does mean that any specific design scenario could be quite biased. This method may be used to test possible extreme cases for the StrUENDP model.

4.5.2 Small network demonstration

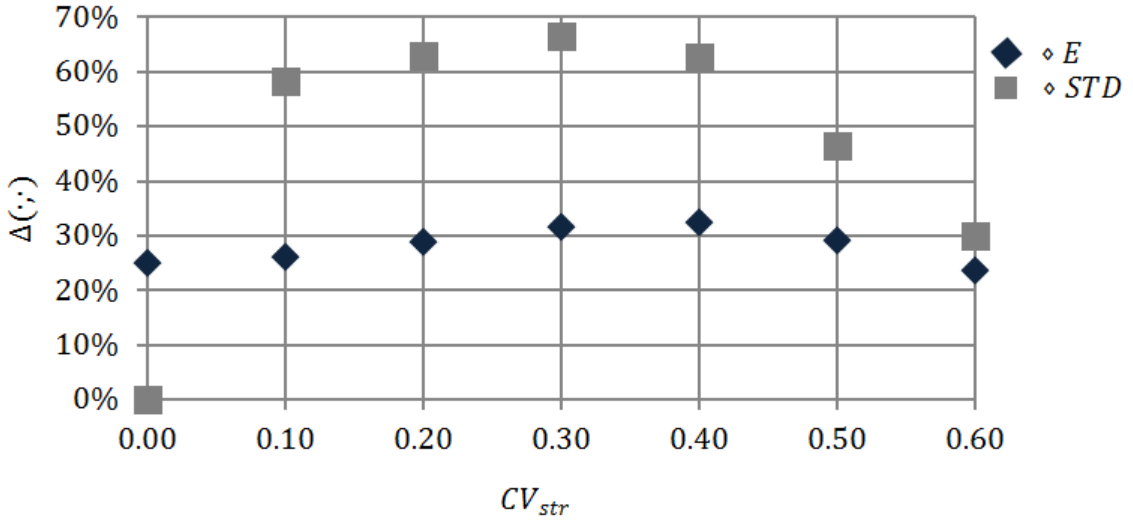
The first demonstration utilizes a network based on the Nguyen-Dupuis network that is popular for small transport test cases. The network data can be found in

Appendix I. There are two origins (1 and 4) and two destinations (2 and 3) with a strategic demand parameter $g(T\text{: }6{,}240, CV_{str})$. Note that the original demand resulted in a network that was highly congested and therefore a deflated demand was utilized in this work.

Figure 4:2 presents results for the StrUE network (where $cov_a = 0, \forall a$) for a specific design scenario $s^{E}_{25,1500}$, meaning that a total of 25 "length units" of 1,500 vph capacity were added to the network. The horizontal axis shows the value of CV_{str}as it varies between 0 and 0.6. Figure 4:2(a) shows the value (absolute not relative) of $\diamond E_s$ and $\diamond STD_s$ (in minutes). Figure 4:2(b) shows the performance metrics ΔE_s and ΔSTD_s for the same cases of CV_{str}.



(a)

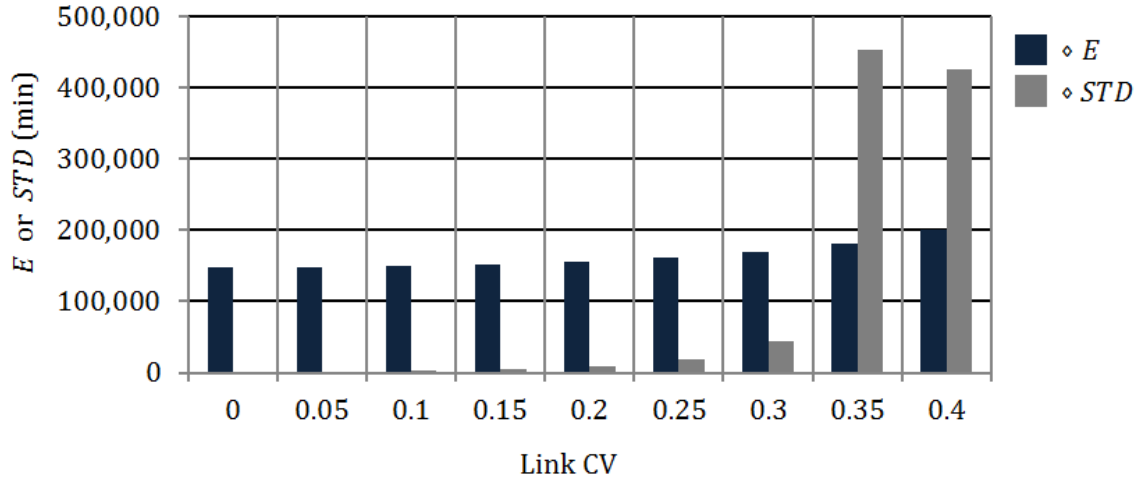


(b)

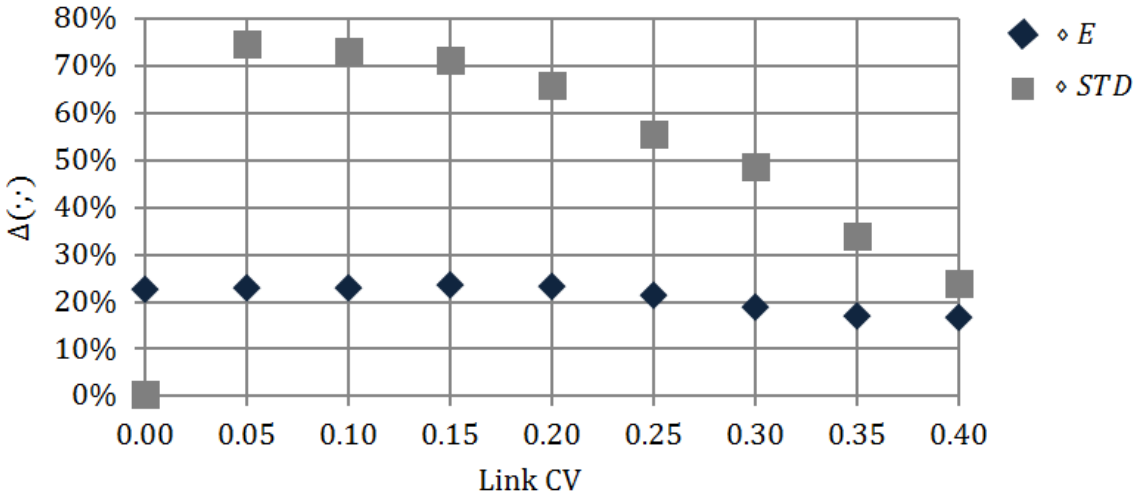
Figure 4:2 Results for Nguyen Dupuis network and StrUE subproblem

Figure 4:2 suggests that for a small network and low levels of volatility, a design scenario will receive similar evaluations of performance. However, once the CV_{str} reaches a certain point, the $\diamond STD$ becomes much larger and the reduction in STD is less. It is also empirically observed that in most, but not all, cases the GA identifies the same set of projects.

Figure 4:3 presents the same demonstration where the demand is treated as a deterministic quantity. It is assumed that all links in the network have the same level of volatility. The horizontal axis of Figure 4:3 shows the values of cov_a as it varies between 0 – 0.4 in increments of 0.05.



(a)



(b)

Figure 4:3 Results for Nguyen Dupuis network and StrUEC subproblem

Increasing levels of link volatility did not have an immense impact on project evaluation. This is likely due to the fact that the links were all treated as uniform, i.e., same C_a and cov_a. Therefore the design projects affects $\diamond\, STD$ more than $\diamond\, E$. In networks where certain links have higher levels of volatility, this might not be the case.

Next we examine the case where there is volatility in both the demand and the capacity. For the results in Figure 4:4, $CV_{str} = 0.3$ and $cov_a = 0.3, \forall a$. In this experiment, we examine the impact of different budgets, which are shown on the horizontal axis. The vertical axis shows the system performance metric, where the blue columns represent $\diamond\, E$ and the grey columns represent the results for $\diamond\, STD$. The red crosses in Figure 4:4 represent the predicted performance of the design scenario in the deterministic case (where $CV_{str} = 0$ and $cov_a = 0, \forall a$).

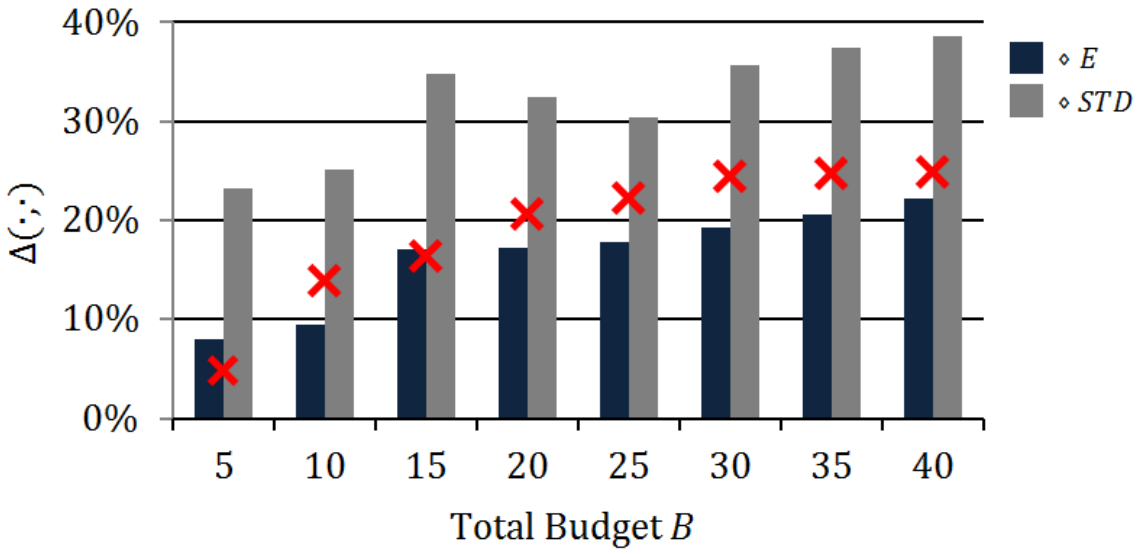


Figure 4:4 Results for Nguyen Dupuis where $CV_{str} = 0.3$ and $cov_a = 0.3 \forall a$

In many cases, the project selection is different when network uncertainty is accounted for. Of course, project evaluation is also different. Network design projects can impact the network by either lowering the travel time on routes and thereby lowering the total system travel time, according to the travel cost function, or by causing people to change routes, which will have unintuitive and

unpredictable impacts on system performance metrics. For the simple case of the Nguyen Dupuis network, a deterministic approach appears to overestimate the impact of design projects.

4.5.3 Medium network demonstration

While the network used in Section 4.5.2 is useful to isolate individual behaviours, it is too small to capture any significant effects of route choice. Therefore, this work presents results from a second experiment on the well known Sioux Falls network, the data for which was obtained from Bar-Gera (2014). Sioux Falls consists of 24 nodes, 76 links and 24 zones. The strategic demand parameter is $g(T: 360{,}000, CV_{str})$.

Figure 4:5 illustrates an example of the genetic algorithm performance. Generally, due to the design of the GA, the population converged relatively quickly. However, this does not mean that an optimal solution has been found. The performance of the GA depends upon the starting population that is randomly generated (although all populations were initialised to include a solution with the best set of design projects based on congestion). In order to control for premature convergence of the GA population, multiple random seeds and multiple scenarios were tested.

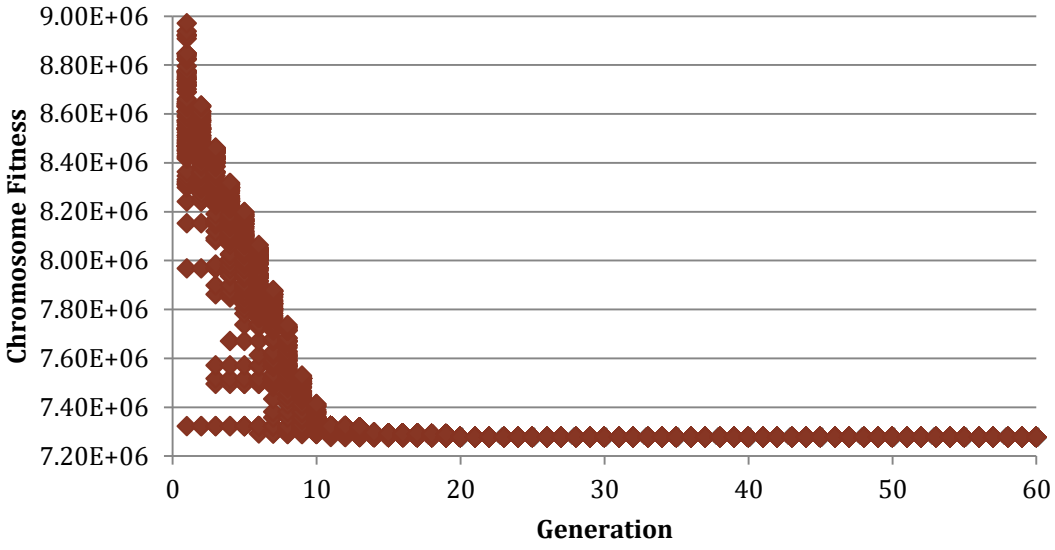
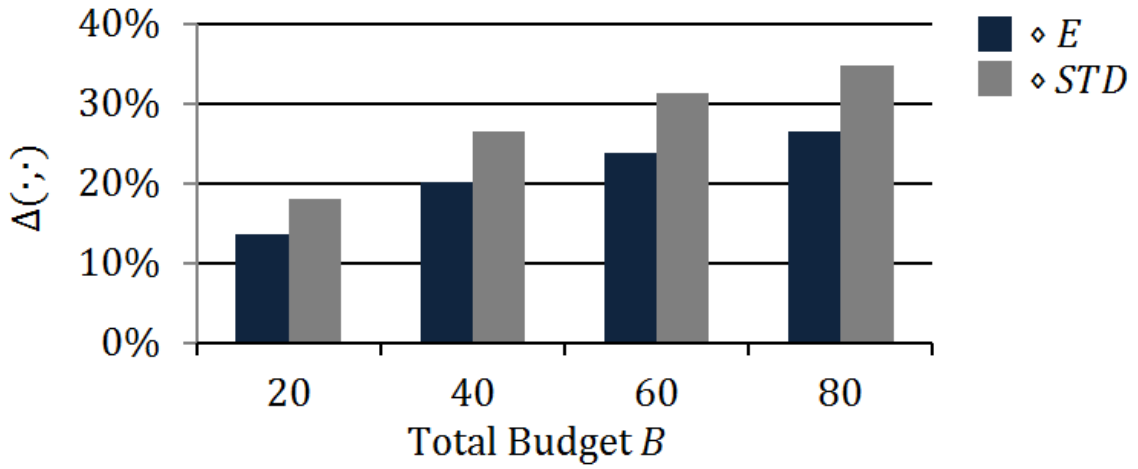
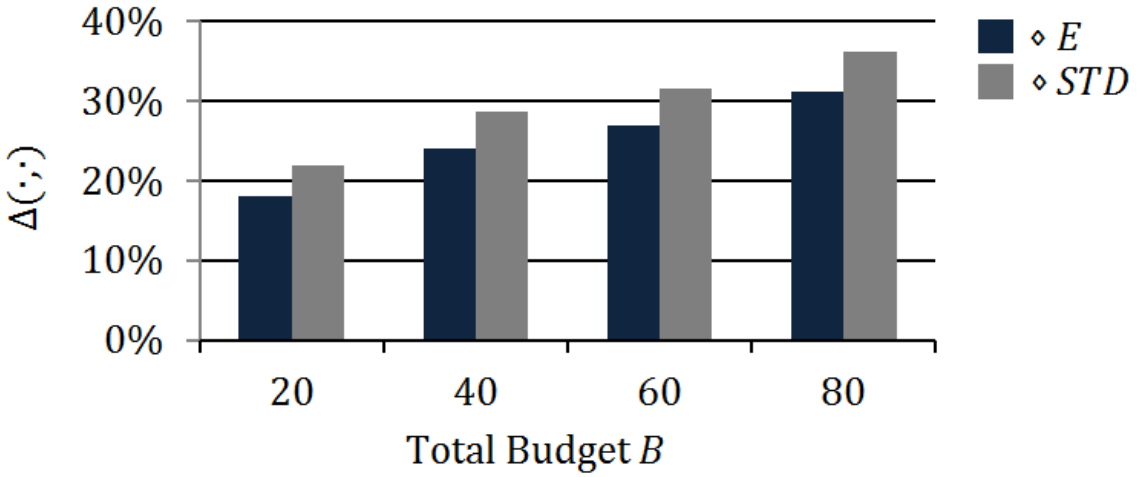


Figure 4:5 Example of genetic algorithm convergence

Figure 4:6 presents the results for a design scenario $s^{E}_{B,1500}$ for the when the budget varies from 20, 40, 60, or 80. The horizontal axis indicates the total budget, while the vertical axis indicates the performance metric, which is the reduction in travel time or standard deviation due to the design scenario. Figure 4:6(a) presents the result for the StrUE model where $CV = 0.3$ (and $cov_a = 0, \forall a$) and Figure 4:6(b) presents the results for StrUEC, where cov_a is based on normalizing the capacity to a range of 0.0-0.2.



(a)



(b)

Figure 4:6 Results for the Sioux Falls network with (a) StrUE subproblem and (b) StrUEC subproblem

The GA found similar solutions, although the reduction in $\diamond E$ and $\diamond STD$ was greater for the results of the StrUEC model where there was also more volatility. In nearly all cases, the GA identifies a different set of links for capacity addition for the deterministic versus the stochastic models. Additionally, the level of volatility (as captured by increasing the coefficient of variation of the probability distributions)

affects the selection of optimal links. In most cases, increasing the budget resulted in links being added to the optimal set for capacity addition. More research is needed to determine the relationship between the volatility on individual links and network performance metrics.

The volatility of capacity on each link could have a significant impact on network design decisions. Lacking the appropriate data, Section 4.5.1 described five methods of generating link capacity data. The results of those methods on the Sioux Falls network, with $CV_{str} = 0.2$ and $B = 50$ are shown in Table 4-3. The upper bound was set as 0.3, and the lower bound as 0.05, with only a small percentage of links being assigned a high cov_{ij}.

Table 4-3 Sensitivity analysis of link capacity volatility types

Type	$\diamond E_{StrUE}$	$\diamond S_{StrUE}$	$\Delta(\diamond E)$	$\Delta(\diamond S)$
congestion	9.58E+06	8.52E+06	19.9%	28.1%
normal	1.06E+07	9.92E+06	20.9%	28.5%
uniform	1.30E+07	1.77E+07	34.6%	21.5%
identical	9.36E+06	8.21E+06	19.8%	28.2%
normalized	1.60E+07	1.90E+07	24.4%	31.0%
capacity	9.61E+06	8.57E+06	20.0%	28.1%

Table 4-3 shows the impact of network characteristics on project selection and evaluation. Of course, it should be noted that the base case $\diamond E$ and $\diamond S$ vary significantly because the capacity volatility in the different network types varies

significantly. Without more investigation, it is difficult to say whether a network that displays a certain characteristic of capacity volatility would be easier or more difficult to minimize with capacity enhancements.

Next, Figure 4:7, Figure 4:8, and Figure 4:9 compare the results from the genetic algorithm for the NDP for the StrUE, StrUEC, and StrSO sub-problems respectively. StrUE represents accounting for day-to-day demand uncertainty, while StrUEC represents accounting for day-to-day demand and capacity uncertainty. The StrSO model does not represent user behaviour and is included for the purposes of comparison.

Each model shows the results for a varying CV_{str} between 0 and 0.5 in increments of 0.1. The CV_{str} is shown on the horizontal axis. The vertical axis shows the performance measure, which is the reduction in expected travel time and standard deviation of total travel time. The budget for all scenarios is $B = 60$. In the StrUEC model, link cov is normalized based on capacity.

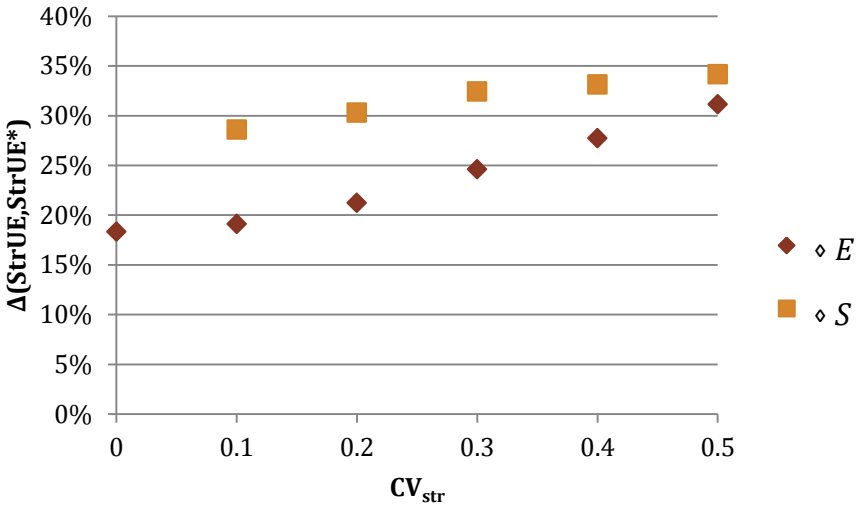


Figure 4:7 Sioux Falls network, NDP results for StrUE

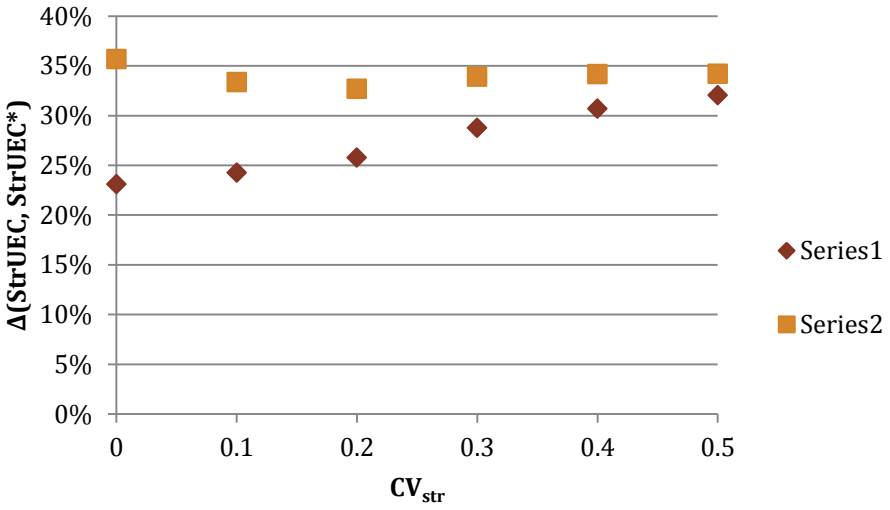


Figure 4:8 Sioux Falls network, NDP results for StrUEC

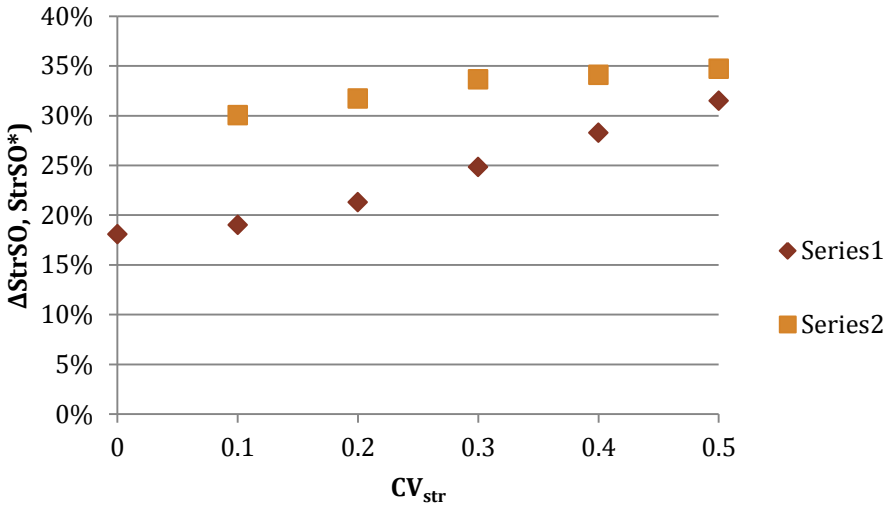


Figure 4:9 Sioux Falls network, NDP results for StrSO

Finally, Table 4-4 illustrates a small sensitivity analysis of the budget, where the effects of the random seed are also shown. The rightmost column indicates the random seed (0.3 and 0.8 were chosen here). Thus, the scenario with each budget is shown in two rows. The percentage difference between the results are shown in the sixth and last columns.

Table 4-4 Comparing the effects of random seeds on the Sioux Falls network

RS	B	$\diamond E^*$	$\diamond S^*$	$\Delta(E, E^*)$		$\Delta(S, S^*)$	
0.3	10	1.43E+07	1.63E+07	10.8%	0.0%	14.5%	0.0%
0.8	10	1.43E+07	1.63E+07	10.8%	-	14.5%	-
0.3	20	1.34E+07	1.49E+07	16.5%	-1.6%	21.6%	-2.5%
0.8	20	1.34E+07	1.50E+07	16.2%	-	21.0%	-
0.3	30	1.27E+07	1.40E+07	20.5%	0.0%	26.4%	0.0%
0.8	30	1.27E+07	1.40E+07	20.5%	-	26.4%	-
0.3	40	1.24E+07	1.36E+07	22.3%	0.0%	28.3%	0.7%
0.8	40	1.24E+07	1.36E+07	22.3%	-	28.5%	-

Table 4-4 makes it clear that there are numerous design projects that result in similar model evaluation, making the uncertain NDP a very difficult problem for the GA to solve. However, the many different objectives are in a very close range. Therefore, it is vital for planners to consider any additional criteria when making a selection of infrastructure design projects.

4.6 Concluding Remarks

The network design problem has a solid foundation in the literature, but remains a challenging topic among researchers. The problem becomes even more complex when uncertainty in real-world parameters is included in the modelling procedure. However, network design project rankings may be impacted by the extreme

behaviours caused by drivers reaction to uncertainty, which could change ranking and evaluation. Additionally, the performance of infrastructure design scenarios will almost certainly be forecasted incorrectly and it is not intuitive whether they will overestimate or underestimate project performance.

This work proposed a network design model that uses the strategic assignment approach to capture the reaction of vehicle travellers to day-to-day demand uncertainty. Additionally, an extension of the strategic approach where day-to-day link capacity is also a random variable, is compared. The model is solved using a tailored genetic algorithm. Results show that at low levels of volatility, project rankings may not be as significantly impacted; however, project evaluations will change. As the volatility in the network increases, not accounting for uncertainties in modelling parameters means that suboptimal projects could be selected. Future work will explore more in-depth implications of link capacity uncertainty on the network design problem and incorporating reliability into the strategic route choice decisions of users.

Chapter 5

Dynamic Strategic Modelling Approaches

5.1 Introduction

Chapters 2, 3, and 4 of this thesis explored static time models to incorporate strategic traveller behaviour and represent daily traffic flow volatility as well as two network management applications. However, as previously discussed, practical applications of transport planning models often employ multiple models of different scales in order to provide a more comprehensive approach to model forecasting. In addition, due to the aggregated nature of the models in previous chapters, there are a number of traffic phenomenon that may be underrepresented.

Chapter 5 and 6 explore strategic modelling approaches that relax one of the key assumptions of the models in the previous chapters: time invariance. Given the overall complexity of the problem and historical limitations in computational processing power, it is not difficult to understand why traffic equilibrium models began with assumptions of static time. Additionally, over the decades of development and implementation, practitioners grew comfortable with the static equilibrium models, like those explored in Part I, and most importantly, the

stability of the solutions obtained. However, there is no denying the fundamental fact: traffic is a time dependent phenomena with important characteristics that cannot be accounted for using static assumptions. Even factors such as departure time choice may significantly change model predictions. Figure 5:1 shows a dynamic profile of travellers in the Sydney network for a 24 hours period for several different modes. A static planning model could not reproduce the situation that is displayed in Figure 5:1. These concerns led to the development of the cutting edge field of *dynamic traffic assignment* (DTA), where the base problem remains the same as static traffic assignment – assigning routes to vehicles trips – but the impact of additional phenomena such as queuing and spillback can be accurately represented.

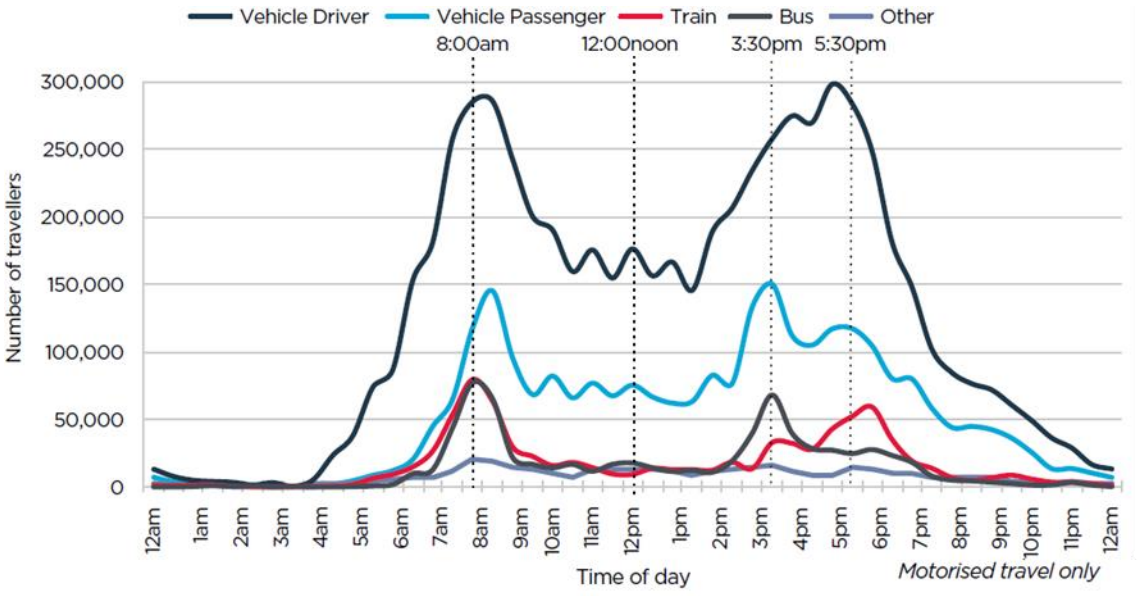


Figure 5:1 Example of dynamic profile of travelers in the Sydney network (Sydney Household Travel Survey)

Since the pioneering work of Merchant and Nemhauser (1978a, 1978b), DTA has been an active field of research (the background of which will be recounted in Section 5.2) The core behavioural assumption within many DTA models is that equilibrium route choice exists. This assumption provides substantial descriptive capabilities but also a potential weakness due to the rarity of observed traffic patterns to be in an equilibrium state. Furthermore, this observation is often employed as the key criticism of DTA approaches to an even greater degree than to traditional static techniques due to the ability of the dynamic models to generally represent traffic in a more realistic and comparable manner to observable conditions. Numerous approaches have emerged to address this criticism, often referred to as disequilibrium, stochastic, day-to-day, or transient modelling.

Chapters 5 and 6 of this thesis explore the concept of strategic assignment in the dynamic setting. As in previous chapters, strategic assignment modelling approaches do not attempt to optimize path (or link) flow directly, but rather to discern strategies that are applied following the realization of some uncertain variable. Often, the goal of the new strategic approach is to equilibrate an expected condition as opposed to a deterministic cost equilibration. The current analysis focuses on a priori approaches, where routes are assigned prior to travel, in order to develop a model that provides superior insights into how traffic volatility emerges in the presence of uncertainty.

The dynamic strategic assignment approach presented in Chapter 5 is based on a DTA formulation by Ziliaskopoulos (2000) that embeds the cell transmission model (Daganzo, 1994, 1995) to realistically propagate traffic according to the hydrodynamic flow equations (Lighthill & Whitham, 1955; Richards, 1956) and captures traffic phenomena (e.g., shockwaves) reasonably well. This formulation is adapted to the strategic traffic assignment problem and the implications are discussed.

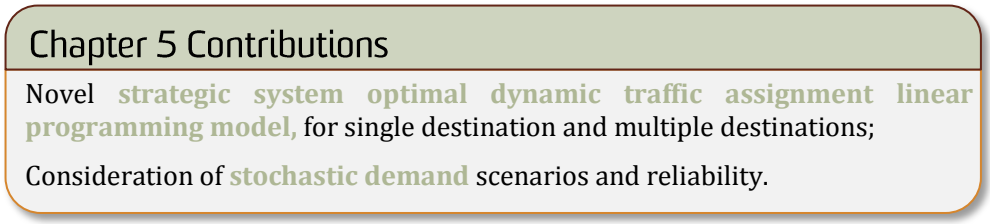


Figure 5:2 Summary of research contribution

Specifically, Chapter 5 of this thesis expands the linear programming System Optimal Dynamic Traffic Assignment (SODTA) modelling framework developed by Ziliaskopoulos (2000), which embeds the cell transmission model (Daganzo, 1994) for traffic propagation. While system optimal conditions are modelled (i.e., marginal cost equilibration) due to the specific capabilities of the LP approach, substantial analytical tools become utilizable, such as stochastic linear programming. The novelty of the presented approach is the development of a two-stage stochastic variation where trip demand is uncertain but represented as a random variable that gives rise to multiple potential future scenarios

(characterizing different days none of which are in equilibrium/optimality when viewed myopically). In the first stage, routing strategies (i.e., flow proportions) are developed to minimize expected system cost and then employed in the second stage after the trip demands are realized to produce scenario-dependent dynamic flows and densities.

5.2 Literature Review

This section provides a literature review of relevant works in the field of dynamic traffic assignment and works related to accounting for uncertainty and the strategic approach. For completeness, some research mentioned in previous sections may be recounted here.

A great number of advancements in the field of dynamic traffic assignment (DTA) have been proposed since the ground-breaking work of Merchant and Nemhauser (1978a, 1978b). The promise of DTA lies in its ability to capture time-varying flows and thereby achieve superior representations of vehicular traffic. However, the addition of temporal phenomena substantially complicates the mathematical representation of traffic assignment.

As with all models, different approaches face their strengths and weaknesses. A comprehensive review of the entire field of DTA is beyond the scope of this thesis, but this section provides a background of selected, relevant works. In a general sense, many DTA models can be classified by two broad approaches. The first uses analytical methods to formulate and solve the DTA problem, usually

based on mathematical programming, optimal control theory, or variational inequality approaches (Peeta & Ziliaskopoulos, 2001). The analytical approaches are able to capture a fine grain of detail, but may be limited by their dependence on link performance functions which are difficult to determine, the holding of traffic, or the inability to scale up to realistic sized problems. The second approach is based on heuristic methods, usually simulation-based. Simulation based DTA has been successfully deployed in a number of metropolitan areas, although static approaches remain far more popular. However, these approaches may lack the capacity to guarantee global optimality or to offer significant insight on the problem. Peeta and Ziliaskopoulos (2001) provide a more comprehensive review of keystone DTA literature prior to the year 2000 and discussion on various proposals for overcoming the aforementioned shortcomings.

Similar to the static models discussed in Section 2.2, DTA can be used in the place of static traffic assignment in the four-step planning process. Additionally, DTA models can use the same behavioural assumptions as static models, except with the added consideration of time: a *dynamic user equilibrium* implies that the travel time on all used paths between each origin-destination pair at each *departure time period* is equal. A general approach to finding the DUE solution for a network involves iterative procedure: finding a time-dependent shortest path for each ODT, a network loading component where the demand is loaded on to the network in order to determine the network conditions that result from the path set, and then adjusting the demand between the current path set.

However, this thesis looks at an alternative approach to DTA that uses a linear programming formulation to find the system optimal flows, instead of the more common simulation based procedures that solve for DUE.

5.2.1 Fundamentals of dynamic traffic modelling

Dynamic traffic modelling seeks to capture the fundamental relationship between speed, density, and flow. The earliest empirical experimentation was conducted in the 1930s by Bruce D. Greenshields, who used a 16mm movie camera to record vehicles at different intervals in space. These experimentations led to the development of Greenshields linear relationship between speed and traffic density and the fundamental relation, where Q is flow, D is density, and v is speed:

$$Q = D \times v \tag{5.1}$$

Building upon these relationships, the (Lighthill & Whitham, 1955) and Richards (1956) applied cinematic wave theory to explain the propagation of shockwaves in a traffic stream. While researchers have studied hundreds of different traffic models over the decades, one which has achieved special attention and success is the cell transmission model (CTM) (Daganzo, 1994, 1995). The CTM is a numerical method that captures the cinematic wave theory in traffic flow. The CTM is relatively straightforward to implement and has proved to be a popular tool in conjunction with DTA.

5.2.2 Overview of the single destination SO-DTA model

While the CTM can be solved using a set of straightforward equations, the problem remains nonlinear and discontinuous, and faces similar drawbacks to other simulation-based methods. However, in Ziliaskopoulos (2000) proposed a linear programming formulation for the system optimal traffic assignment problem that embeds the cell transmission model. Ziliaskopoulos transforms the CTM into a series of linear equations that are powerful in their opportunity to solve using well established methods and commercial solvers such as CPLEX. The original SODTA model is recounted here for completeness.

In order to formulate the SODTA model, consider the set C of all cells and set E of cell connectors between cell i and cell j. Consider C_R as the set of origin cells and C_S as the set of destination cells. Consider T the set of discrete time intervals, where $T = \{\pi, 2\,\pi, 3\,\pi, \ldots, |T|\,\pi\}$, and with no loss of generality, assume that $\pi = 1$. Consider two more sets: $\Gamma^-(i), \forall i \in C$, which defines set of cells preceding cell i and $\Gamma^+(i), \forall i \in C$, which defines the set of cells succeeding cell i. Additionally, let N_i^t be the maximum number of vehicles that can be present in cell i at time t and let Q_i^t be the maximum amount of flow that can enter or exit cell i at time t. Finally, let d_i^t be the demand at cell $i \in C_R$ at time t. This formulation provides for only a single destination and therefore the demand is only indexed by a single cell.

The SO-DTA model is formulated as a linear program. The decision variables are $x_i^t, \forall i \in C, \forall t \in T$, which is the density of each cell during each time period, and

the cell connectors, y_{ij}^t, which represents the flow between cell i and cell j at time t. The objective is to minimize total system travel time, which is simply the sum of all of the density in network over all time periods. This model consistently propagates traffics through a single destination network, with expressions (5.2) – (5.10) as linear constraints (Ziliaskopoulos, 2000).

$$\text{minimize} \sum_{\forall t \in T} \sum_{\forall i \in C \setminus C_s} x_i^t \tag{5.2}$$

subject to

$$x_i^t - x_i^{t-1} - \sum_{k \in \Gamma^-(i)} y_{ki}^{t-1,} + \sum_{j \in \Gamma^+(i)} y_{ij}^{t-1} = 0 \quad \forall i \in C \setminus \{C_R, C_S\}, \forall t \in T, \tag{5.3}$$

$$\sum_{\forall j \in \Gamma^+(i)} y_{ij}^t - x_i^{t,} \leq 0 \quad \forall i \in C, \forall t \in T \tag{5.4}$$

$$\sum_{\forall i \in \Gamma^-(j)} y_{ij}^t + x_j^t \leq N_j^t \quad \forall j \in C \setminus \{C_R, C_S\}, \forall t \in T \tag{5.5}$$

$$\sum_{\forall i \in \Gamma^-(j)} y_{ij}^t \leq Q_j^t \quad \forall j \in C \setminus C_R, \forall t \in T \tag{5.6}$$

$$\sum_{\forall j \in \Gamma^+(i)} y_{ij}^t \leq Q_i^t \quad \forall i \in C \setminus C_S, \forall t \in T \tag{5.7}$$

$$x_i^t - x_i^{t-1} + y_{ij}^{t-1} = d_i^{t-1} \quad \forall j \in \Gamma^+(i), \forall i \in C, \forall t \in T \tag{5.8}$$

$$x_i^0 = 0, y_{ij}^0 = 0 \qquad \forall i \in C, \forall (i,j) \in E, t \in T \quad (5.9)$$

$$x_i^t \geq 0, y_{ij}^t \geq 0, \qquad \forall i \in C, \forall (i,j) \in E, t \in T \quad (5.10)$$

Constraints (5.3) enforces conservation of flow, ensuring that all flow that enters a cell, leaves the cell, except for source and sink cells. Constraint (5.4) ensures that the amount of flow that can move from one cell to the next is restricted by the density of the current cell. Constraint (5.5) addresses the capacity of the cell. Furthermore, note that the capacity of origin and destination cells is assumed to be large enough to allow proper loading of the network demand. Constraints (5.6) and (5.7) limit the total inflow and outflow of a cell, while constraint (5.8) loads demands onto origin cells. Constraints (5.9) and (5.10) represent initial conditions and non-negativity constraints.

The formulation presented above is cell-centric because it involves only a single origin; the path flows are not directly computed or represented in the formulation. The route choice element is not critical in the original formulation and thus, extracting path-based information is not trivial. The model in this thesis needs to explicitly account for path flows and time-based demand departure, which will be discussed more thoroughly in Section 5.3.

The initial model proposed in this work is adapted from Ziliaskopoulos (2000), which proposed a linear programming framework for dynamic traffic assignment based on the cell transmission model (Daganzo 2004a, 2004b). The

CTM propagates traffic in accordance with the hydrodynamic flow equations but also realistically represents flow variability inside the link without the use of a link performance function. Ziliaskopoulos' formulation contributes a single destination model that does not completely overcome the computational difficulties common to the field of DTA, but thanks to the LP formulation it does allow us to gain significant insight into the problem. In addition, a number of works refining the original model and expanding it to solve other problems common to the transportation field have been produced. expand upon this LP formulation of SODTA to allow for multi-origins and multi-destinations, while preserving first in first out (FIFO) requirements. Li et al (2003) propose a decomposition algorithm for the LP SODTA that allows it to be solved on more meaningful sized networks. Waller and Ziliaskopoulos (2006) extend the deterministic LP to account for stochastic demands, using a chance-constrained stochastic program and provide solution techniques. Ukkusuri and Waller (2008) formulates a user-optimal version of this problem. Additionally, Waller et al (2006) use the LP formulation to optimally solve for the continuous network design problem, a result which would not be possible given the usual non-convex formulations for the NDP.

5.2.3 Stochasticity in DTA

Dynamic traffic assignment in a deterministic setting is a challenging problem. When stochasticity in elements such as travel demand, road capacity, and user route choice are accounted for, most approaches become even more complex. This

section recounts other works in the field of DTA that account for uncertainty in a relevant way.

Examining the effects of daily volatility in traffic flow is a problem that can be considered from multiple aspects. For example, one source of daily variation may be a result of supply side reductions in capacity, like that resulting from traffic incidents or adverse weather conditions (Asakura and Kashiwadani, 1991; Clark and Watling, 2005). However, daily variation can also imply an uncertain travel demand which is the focus of this work.

Traditional equilibrium models are, as the name suggests, dependent upon the idea of equilibrium - that is, a consistent state of the network that will remain under some rational set of behaviours. Refer to Watling and Hazelton (2003) for an in-depth discussion on the definition of equilibrium. These authors note a common criticism of traffic assignment models (dynamic included) that questions the existence of an observable equilibrium, a critique that this work seeks to address. Building on this issue, and the field of behavioural dynamics, alternative schools of thought claim that decisions are affected by learning from previous days, leading to potentially unstable conditions; users adapt on a day-to-day basis, and their adapting mechanism may undermine the notions of a stable equilibrium (Hamdouch et al, 2004). Understanding this new concept of dis-equilibrium has become an important area of research, in spite of the mathematical and computational complexity of the problem.

Closely related to the issue of day-to-day uncertainty in traffic flow, is the issue of uncertainty in the very behaviour exhibited by the decision-makers being modelled in the system. This uncertainty goes beyond just that of user perception in travel time (as addressed with stochastic user equilibrium models, see Section 2.2.2). Traditionally, traffic assignment models have built upon the assumption that people seek to minimize their own travel time, but a number of works have explored diffing behavioural assumptions.

Horowitz (1984) examines the stability of stochastic equilibrium in a two-link network by analysing different mechanisms of route choice over time. He shows that even when equilibrium solutions are unique, link flow values may converge to their equilibrium values, oscillate about equilibrium perpetually, or converge to a solution not consistent with equilibrium conditions. Cantarella and Cascetta (1995) undertake research on interperiodic demand modelling from both a deterministic and stochastic process approach. Rather than focusing on the concept of equilibrium, this work focuses on fixed point attractors. Additionally this work discusses the conditions for stability of both equilibrium and dynamic processes, and the relationship between the two. Watling (1999) extends Horowitz's two-link example to a general network setting, further clarifying the distinction between stability in discrete vs. continuous time, and that between deterministic and stochastic processes. A dynamical adjustment process is proposed for analysing the stability of a general asymmetric stochastic equilibrium assignment problem in discrete time. Watling and Hazelton (2003) further extend

the concept of dynamic learning route choice and examine the properties of deterministic dynamic systems under perturbation, and the implications of day to day route choice adjustments on the stability of equilibria. The authors further note the importance of the behavioural mechanism in modelling day-to-day fluctuations, and emphasize the need for more analytical techniques over simulation based techniques for their potential to offer greater insight.

Rather than focusing on *a priori* path choice decision making, an alternative method proposes that people may adapt their route choice based on primary, real-time experience. Referred to in the literature as adaptive routing (or routing with recourse), it is assumed that users may gain information and correspondingly change their behavior *en route*. There is an abundance of research in the literature dealing with travel-time adaptive shortest paths (Andreatta and Romeo, 1988; Psaraftis and Tsitsiklis , 1993; Polychronopoulos and Tsitsiklis, 1996 ; Miller-Hooks and Mahmassani , 2000; Waller and Ziliaskopoulos, 2002; Nie and Fan, 2006; Gao and Chabini, 2006). However, extensions of this type of user-level behavior to a system equilibrium are much scarcer. Unnikrishnan and Waller (2009) extend the online shortest path behaviour described in Waller and Ziliaskopoulos (2002) to a user equilibrium framework using a convex math programming formulation.

5.3 StrSODTA Model Formulation

This work addresses the impact of demand uncertainty on traditional equilibrium planning models. Instead of assuming a deterministic demand value and then determining the optimal paths for travellers, this model identifies the optimal *proportion* of flow on each path (for each possible departure time) to minimize *expected* total system travel time across a range of specified discrete demand scenarios. Hence, instead of three different demand scenarios resulting in three different optimal solutions, the model will output a single optimal value; however, no demand scenario is an optimal solution *in and of itself*. The optimality results from a greater strategy prevailing across all potential demand scenarios and may be interpreted as an explanation for the randomness that can be observed in traffic flow.

Section 5.3 presents the strategic system optimal dynamic traffic assignment linear programming formulation. First, a single destination linear program is introduced, and then the implications and the multi-OD formulation presented. The focus of this section is two-fold: to formulate the scenario-based optimal path proportion problem as a linear program, and to re-interpret the optimal path proportions to represent the probability distribution guiding the path choice of each individual belonging to that OD.

5.3.1 Single Destination StrSO DTA Formulation

The proposed model is a powerful tool due to the linear programming formulation at its foundation. While at the current time, heuristic solution methods appear to offer more in regards to deployability, ultimately most heuristic methods do not offer globally optimal solutions. A linear program, on the other hand, can be solved to a guaranteed globally optimal solution using well known methods and commercial solvers, regardless of the size; however, the LP faces a separate set of challenges with regard to the large number of constraints required.

In the strategic approach, route proportions on paths for all origin-destination pairs are optimized to minimize expected system performance over a range of stochastic demand scenarios. Therefore, the solution to the StrSODTA problem will not constitute an optimal solution to any of the individual demand scenarios, representing a level of volatility that is observed in real traffic networks. In addition, it is a level of volatility that could not be measured or represented using traditional system optimal based approaches. The re-interpretation of the path proportions is natural; the expected number of people to follow each path will be equal to the optimal path flows. However, as the flows are being randomly sampled from this distribution, any specific sample taken from this distribution will in general not represent an optimal solution. The variability in the path flows resulting from this sampling approach is a natural way of representing the day to

day variability in flows. This variability in path flows can be extended to link flows, allowing us to measure the reliability and/or volatility of links in the network.

The formulation of the LP follows that of Ziliaskopoulos (2000) for the SODTA problem, but increases in complexity due to the need to track paths, and the scenario-based demand stochasticity. In order to estimate the optimal path proportions in this problem, it is necessary to explicitly separate flows based on cell, time interval, departure time, path, and demand scenario. The LP formulation presented in this chapter aims to find the single set of path proportions that are optimal over all demand scenarios, for each OD pair and departure time. Given these path proportions, scenario-specific demands are loaded onto the network using a CTM-based LP formulation, and system metrics can be calculated.

Consider the following notation: let C be the set of all cells in the network, where a link can be decomposed into cells based on the procedure detailed in Section 5.2.1. Let C_r represent the set of source cells and let C_s be the set of sink cells. Let E be the set of cell connectors, containing a cell connector for each cell i and cell j that are connected. Let $\Gamma^-(i)$ be the set of cells immediately preceding cell i and let $\Gamma^+(i)$ be the set of cells immediately succeeding cell i. Let $N_{t,i}$ be the maximum density of cell i, corresponding to the CTM parameter of the same index. Let $Q_{t,i}^-$ and $Q_{t,i}^+$ be the maximum flow into and out of cell i, again corresponding to the CTM parameter of the same index, and for the sake of simplicity, let $Q_{t,i}^- = Q_{t,i}^+ = Q_{t,i}$.

Let T be the set of time intervals, indexed by t. In order to explicitly represent path-based flow for the strategic approach, the flow needs to be represented by path and by departure time to ensure that the strategic proportions follow the optimal strategy. Consider $\phi \in \Phi$ to be a path within the set of all paths and let the set of departure times be defined as $\tau \in T_D : T_D \subset T$. The proposed model explicitly accounts for the finite number of demand scenarios, where ξ represents a specific demand scenario from the set Ξ. The total demand for a particular scenario is D^{ξ}, which is divided by departure time, D_{τ}^{ξ}. Each demand scenario ξ occurs with a probability of $p^{\xi} : \sum_{\xi \in \Xi} p^{\xi} = 1$.

There are three sets of decisions variables for the StrSODTA model. The first two are similar to the cell density and cell connector flow variables in the original Ziliaskopolous model, but for the strategic approach cells and cell connectors need to be indexed by path, departure time, and demand scenario. Let $x_{t,\tau,i}^{\xi,\phi}$ represent the density of cell i during time interval t and demand scenario ξ, of departure time τ, and path ϕ. Let $y_{t,\tau,ij}^{\xi,\phi}$ represent the flow from cell i to cell j during time interval t and demand scenario ξ, of departure time τ, and path ϕ. Finally, the model requires the strategic path proportion variables: let π_{τ}^{ϕ} represent the proportion of the demand D that uses path ϕ at departure time $\tau \in T_D$. The strategic path proportions are the only variable whose scope is across all demand scenarios $\xi \in \Xi$.

Conceptually speaking, the LP can be understood as a set of separate single origin, single destination DTA problems, one for each path and departure time

combination, which share cell and flow capacities at each time step. This observation is important, as it serves as a starting point for the potential use of decomposition methods similar to those used by Li et al. (2003).

The objective of the LP is to find the proportion of flow assigned to each feasible path at each departure time period such that the total expected system travel time is minimized. Recall that in the discretization of time and space in the CTM model, each cell represents the distance a vehicle can travel during a time increment Δt. Thus, the total travel time in the network is equivalent to the density of all cells (excluding sink cells) during all time periods. Model StrSODTA provides the formulation for the strategic system optimal dynamic traffic assignment model.

Model SD StrSODTA

$$minimize \sum_{\xi \in \Xi} \sum_{\phi \in \Phi} \sum_{t \in T} \sum_{\tau \in T_D} \sum_{i \in C \setminus C_s} p^{\xi} x_{t,\tau,i}^{\xi,\phi} \tag{5.11}$$

subject to

$$x_{t,\tau,i}^{\xi,\phi} - x_{t-1,\tau,i}^{\xi,\phi} - \sum_{j \in \Gamma^-(i)} \delta_{ij}^{\phi} y_{t-1,\tau,ji}^{\xi,\phi} + \sum_{j \in \Gamma^+(i)} \delta_{ij}^{\phi} y_{t-1,\tau,ij}^{\xi,\phi} = 0 \qquad \begin{array}{l} \forall \xi \in \Xi, \forall \phi \in \Phi, \\ \forall t \in T^*, \forall \tau \in T_D, \\ \forall i \\ \in C \setminus (C_r \cup C_s) : \delta_i^{\phi} \end{array} \tag{5.12}$$

$$x_{t,\tau,i}^{\xi,\phi} - x_{t-1,\tau,i}^{\xi,\phi} - \sum_{j \in \Gamma^-(i)} \delta_{ij}^{\phi} y_{t-1,\tau,ij}^{\xi,\phi} = 0 \qquad \begin{array}{l} \forall \xi \in \Xi, \forall \phi \in \Phi, \\ \forall t \in T^*, \forall \tau \in T_D, \\ \forall i \in C_s : \delta_i^{\phi} \end{array} \tag{5.13}$$

$$x_{t,\tau,i}^{\xi,\phi} - x_{t-1,\tau,i}^{\xi,\phi} + \sum_{j\in\Gamma^+(i)} \delta_{ij}^{\phi}\, y_{t-1,\tau,ij}^{\xi,\phi} = \pi_{\tau}^{\phi} D_{\tau}^{\xi} \qquad \forall\xi \in \Xi, \forall\phi \in \Phi, \forall t \in T^*, \forall\tau \in T_D, \forall i \in C_r: \delta_i^{\phi} \qquad (5.14)$$

$$\sum_{j\in\Gamma^+(i)} \delta_{ij}^{\phi} y_{t,\tau,ij}^{\xi,\phi} + x_{t,\tau,i}^{\xi\phi} \le 0 \qquad \forall\xi \in \Xi, \forall\phi \in \Phi, \forall t \in T^*, \forall\tau \in T_D, \forall i \in C\backslash C_s \qquad (5.15)$$

$$\sum_{\phi\in\Phi} \sum_{\tau\in T_D} \left(\sum_{i\in\Gamma^-(j)} \delta_{ij}^{\phi} y_{t,\tau,ij}^{\xi,\phi} + x_{t,\tau,i}^{\xi\phi} \right) \le N_{t,i} \qquad \forall\xi \in \Xi, \forall t \in T, \forall i \in C\backslash(C_r \cup C_s) \qquad (5.16)$$

$$\sum_{\phi\in\Phi} \sum_{\tau\in T_D} \sum_{j\in\Gamma^-(j)} \delta_{ij}^{\phi} y_{t,\tau,ij}^{\xi,\phi} \le Q_{t,i} \qquad \forall\xi \in \Xi, \forall t \in T, \forall i \in C\backslash C_s \qquad (5.17)$$

$$\sum_{\phi\in\Phi} \sum_{\tau\in T_D} \sum_{j\in\Gamma^+(i)} \delta_{ij}^{\phi} y_{t,\tau,ij}^{\xi,\phi} \le Q_{t,i} \qquad \forall\xi \in \Xi, \forall t \in T, \forall i \in C\backslash C_r \qquad (5.18)$$

$$\sum_{\phi\in\Phi} \pi_{\tau}^{\phi} = 1 \qquad \forall\tau \in T_D \qquad (5.19)$$

$$y_{0,\tau,ij}^{\xi,\phi} = 0 \qquad \forall\xi \in \Xi, \forall\phi \in \Phi, \forall\tau \in T_D, \forall(i,j) \in E \qquad (5.20)$$

$$y_{t,\tau,ij}^{\xi,\phi} \ge 0 \qquad \forall\xi \in \Xi, \forall\phi \in \Phi, \forall t \in T, \forall\tau \in T_D, \forall(i,j) \in E \qquad (5.21)$$

$$x_{t,\tau,i}^{\xi,\phi} \ge 0 \qquad \forall\xi \in \Xi, \forall\phi \in \Phi, \forall t \in T, \forall\tau \in T_D, \forall i \in C \qquad (5.22)$$

$$\pi_{\tau}^{\phi} \ge 0 \qquad \forall\phi \in \Phi, \forall\tau \in T_D \qquad (5.23)$$

Constraint (5.12) defines the conservation of flow for basic, merge, and diverge cells, excluding source and sink cells. Constraint (5.13) defines the conservation of flow for sink cells, where the capacity of $i \in C_s$ is assumed to be infinite and can be considered "outside" of the network. Constraint (5.14) defines the conservation of flow for source cells. Constraint (5.14) is particularly important because it loads the demand into the network. The set of time intervals in Constraints (5.12), (5.13), and (5.14) is denoted as T^*, where $T^* \subset T/(0)$, due to the $t-1$ index used in each of these constraints. Note that the network loading is formulated such that D_0^{ξ} enters the network as density during time interval $t = 1$.

Constraint (5.15) provides the connection between density in a cell and the flow in the cell connector. Constraint (5.16) defines the jam density for all cells other than source cells and sink cells. Constraints (5.17) and (5.18) define the saturation flow rate between cells. The combination of Constraints (5.15), (5.16), and (5.17) create realistic traffic movement, in particular the effects of shockwave propagation.

The strategic path proportions enter the program in Constraints (5.14) and (5.19). Finally, Constraints (5.20) – (5.23) provide non-negativity conditions and ensures that all cells are empty during time interval zero.

The SDStrSODTA Model has an important advantage: due to the LP approach, the model can be solved for globally optimal flows using any commercial solver. Additionally, this approach is advantageous because of the relatively simple

extensions to important transport problems like network design and infrastructure evaluation.

The next section expands the StrSODTA model to multiple destinations.

5.3.2 Multiple-Destination StrSO DTA Formulation

Many applications in DTA are desirable due to their ability to capture the route choice component of user behaviour. For example, under the conditions of dynamic user equilibrium, users are assigned routes such that the travel time on all paths between an origin-destination at a departure time are equal and minimal. The travel time on all routes is in turn influenced by the route choice of users from other ODs. While the current approach is based on system optimal flows instead of user equilibrium, the interaction between the routes from different ODs is a critical element of consideration.

The consideration of multiple destinations introduces a well-known issue with the system optimal DTA linear program: the issues of holding back. Because the objective is to minimize system travel time, not for user equilibrium, the program will delay vehicles from certain origin-destinations pairs in favor of others, if the travel time will be minimized.

In order to expand the SDStrSODTA linear program to account for multiple destinations, all flow and density must be indexed by the origin-destination they belong to. Therefore, consider a set of origin-destination pairs OD, indexed by

$\mu \in OD$. Then consider the set of paths to be indexed by origin-destination pair such that $\phi \in \Phi(\mu)$. The indexing of the three decision variables must also be expanded to include a specific path representation. The cell density becomes $x_{t,\tau,i}^{\xi,\mu,\phi}$, where $\phi \in \Phi(\mu)$ indicates a path in a set of paths for the OD pair μ. Similarly, the flow contained in the cell connector becomes $y_{t,\tau,ji}^{\xi,\mu,\phi}$, where the path is represented in the same way. Finally, the path proportion variable has an addition index specifying the OD pair, becoming $\pi_{\tau}^{\mu,\phi}$, such that the proportion on all paths for a single OD pair must sum to 1.

The mathematical programming model for the multiple destination StrSODTA model is presented in equations (5.24) - (5.36). The objective is to minimize the expected total system travel time, which is the summation of the density in each cell for each time period for each demand scenario, multiplied by the probability of that demand scenario.

Model: StrSODTA

$$minimize \sum_{\xi \in \Xi} \sum_{\mu \in OD} \sum_{\phi \in \Phi(\mu)} \sum_{t \in T} \sum_{\tau \in T_D} \sum_{i \in C \setminus C_s} p^{\xi} x_{t,\tau,i}^{\xi,\mu,\phi} \tag{5.24}$$

subject to

$$x_{t,\tau,i}^{\xi,\mu,\phi} - x_{t-1,\tau,i}^{\xi,\mu,\phi} - \sum_{j \in \Gamma^{-}(i)} \delta_{ij}^{\mu,\phi} y_{t-1,\tau,ji}^{\xi,\mu,\phi} + \sum_{j \in \Gamma^{+}(i)} \delta_{ij}^{\phi,\mu} y_{t-1,\tau,ij}^{\xi,\mu,\phi} = 0 \qquad \begin{array}{l} \forall \xi \in \Xi, \forall \mu \in OD, \\ \forall \phi \in \Phi(\mu), \forall t \\ \in T^{*}, \forall \tau \in T_D, \forall i \\ \in C \setminus (C_r \cup C_s): \delta_i^{\phi} \end{array} \tag{5.25}$$

$$x_{t,\tau,i}^{\xi,\mu,\phi} - x_{t-1,\tau,i}^{\xi,\mu,\phi} - \sum_{j\in\Gamma^-(i)} \delta_{ij}^{\phi,\mu}\, y_{t-1,\tau,ij}^{\xi,\mu,\phi} = 0 \qquad \forall\xi\in\Xi, \forall\mu\in OD, \forall\phi\in\Phi(\mu), \forall t\in T^*, \forall\tau\in T_D, \forall i\in C_s: \delta_i^{\phi} \qquad (5.26)$$

$$x_{t,\tau,i}^{\xi,\mu,\phi} - x_{t-1,\tau,i}^{\xi,\mu,\phi} + \sum_{j\in\Gamma^+(i)} \delta_{ij}^{\phi,\mu}\, y_{t-1,\tau,ij}^{\xi,\mu,\phi} = \pi_{\tau}^{\mu,\phi} D_{\tau}^{\xi,\mu} \qquad \forall\xi\in\Xi, \forall\mu\in OD, \forall\phi\in\Phi(\mu), \forall t\in T^*, \forall\tau\in T_D, \forall i\in C_r: \delta_i^{\phi} \qquad (5.27)$$

$$\sum_{j\in\Gamma^+(i)} \delta_{ij}^{\phi,\mu} y_{t,\tau,ij}^{\xi,\mu,\phi} + x_{t,\tau,i}^{\xi,\mu,\phi} \le 0 \qquad \forall\xi\in\Xi, \forall\mu\in OD, \forall\phi\in\Phi(\mu), \forall t\in T^*, \forall\tau\in T_D, \forall i\in C\backslash C_s \qquad (5.28)$$

$$\sum_{\mu\in OD}\sum_{\phi\in\Phi(\mu)}\sum_{\tau\in T_D}\left(\sum_{i\in\Gamma^-(j)} \delta_{ij}^{\phi} y_{t,\tau,ij}^{\xi,\phi} + x_{t,\tau,i}^{\xi\phi}\right) \le N_{t,i} \qquad \forall\xi\in\Xi, \forall t\in T, \forall i\in C\backslash(C_r\cup C_s) \qquad (5.29)$$

$$\sum_{\mu\in OD}\sum_{\phi\in\Phi(\mu)}\sum_{\tau\in T_D}\sum_{j\in\Gamma^-(j)} \delta_{ij}^{\mu,\phi} y_{t,\tau,ij}^{\xi,\mu,\phi} \le Q_{t,i} \qquad \forall\xi\in\Xi, \forall t\in T, \forall i\in C\backslash C_s \qquad (5.30)$$

$$\sum_{\mu\in OD}\sum_{\phi\in\Phi(\mu)}\sum_{\tau\in T_D}\sum_{j\in\Gamma^+(i)} \delta_{ij}^{\mu,\phi} y_{t,\tau,ij}^{\xi,\mu,\phi} \le Q_{t,i} \qquad \forall\xi\in\Xi, \forall t\in T, \forall i\in C\backslash C_r \qquad (5.31)$$

$$\sum_{\phi\in\Phi(\mu)} \pi_{\tau}^{\mu,\phi} = 1 \qquad \forall\mu\in OD, \forall\tau\in T_D \qquad (5.32)$$

$$y_{0,\tau,ij}^{\xi,\mu,\phi} = 0 \qquad \forall\xi\in\Xi, \forall\mu\in OD, \forall\phi\in\Phi(\mu), \forall\tau\in T_D, \forall(i,j)\in E \qquad (5.33)$$

$$y_{t,\tau,ij}^{\xi,\mu,\phi} \ge 0 \qquad \forall\xi\in\Xi, \forall\mu\in OD, \forall\phi\in\Phi(\mu), \forall t\in T, \forall\tau\in T_D, \forall(i,j)\in E \qquad (5.34)$$

$$x_{t,\tau,i}^{\xi,\mu,\phi} \geq 0 \qquad \forall \xi \in \Xi, \forall \mu \in OD, \forall \phi \in \Phi(\mu), \forall t \in T, \forall \tau \in T_D, \forall i \in C \tag{5.35}$$

$$\pi_{\tau}^{\mu,\phi} \geq 0 \qquad \forall \mu \in OD, \forall \phi \in \Phi(\mu), \forall \tau \in T_D \tag{5.36}$$

The model presented in above equations is similar to the single-destination model, except constraints must be added to account for multiple OD pairs. However, as previously noted, the introduction of multiple ODs adds the problem of non-adherence to first-in first-out behaviour, as will be shown in more detail in the following section.

There are two performance metrics for the StrSODTA model: the expected total system travel time for a set of demand scenarios $E(\Xi)$ and the experienced travel time for each demand scenario in a set, $TT(\xi)$. The expected travel time for all demand scenarios is the same as the objective function, and is shown in (5.37).

$$E(\Xi) = \Delta t * \sum_{\xi \in \Xi} \sum_{\mu \in OD} \sum_{\phi \in \Phi(\mu)} \sum_{t \in T} \sum_{\tau \in T_D} \sum_{i \in C \setminus C_s} p^{\xi} x_{t,\tau,i}^{\xi,\mu,\phi} \tag{5.37}$$

The experienced travel time for an individual demand realization ξ is a sum of the density in each cell for each departure time, path, and OD pair. Equation (5.38) shows the calculation for the experienced travel time of a demand realization.

$$TT(\xi) = \Delta t * \sum_{\mu \in OD} \sum_{\phi \in \Phi(\mu)} \sum_{t \in T} \sum_{\tau \in T_D} \sum_{i \in C \setminus C_s} x_{t,\tau,i}^{\xi,\mu,\phi} \tag{5.38}$$

The following section explores the implications of the StrSODTA model.

5.4 Model Demonstration

This section focuses on the implementation and analysis of the-StrSODTA LP model, the mathematical formulation for which was presented in the previous section. A solution approach to testing static network planning data is presented, followed by a detailed numerical analysis.

5.4.1 1800Solution approach

As discussed previously, one of the benefits the StrSODTA approach lies with the linear programming formulation. Linear programs have a long history and well established approaches for efficient solution methods. Commercial solvers such as CPLEX are a common approach to solving linear programs, and the approach that was utilized here. This section discusses how the model was transformed into the necessary form for CPLEX to solve.

The input data required for dynamic models is similar the data needed for the static approaches discussed in Chapters 3, 4, and 5. However, in addition to road network data, including link lengths, capacity, and free-flow cost, DTA models also require data regarding the time-nature of the demand; specifically, the

departure times for all demand, for all origins and destinations. Given the relative newness of DTA models, dynamic network data is not always available. However, based on series of reasonable assumptions, this section discusses the process to transform static data into data that can be used with dynamic models. In addition to being a useful technique given common constraints in practice, this approach also allows a general comparison between static and dynamic modelling results.

The solution approach for the StrSODTA model consists of six general steps, which are described and further outlined in pseudo-code form. The steps are: decompose the link network to a cell network, create the dynamic demand scenarios, manage the link-path incidence matrices, create the linear programming model using the AMPL programming interface, solve the model using CPLEX, and calculate the system performance metrics. While it is possible to directly create a linear programming model using the C++ CPLEX API, the author deemed using the AMPL modelling language to be the more versatile approach.

The first step to solving the StrSODTA model is to transform the link network, such as the Nguyen Dupius or the Sioux Falls networks from previous chapters, into a cell network that is used with the CTM (Section 5.2.1). This procedure is outlined in the form of pseudo-code in Algorithm 6.1. The CTM recognizes four kinds of cells: ordinary cells, merge/diverge cells, source cells, and sink cells. Due to the use of predecessor and successor cell sets, this approach need only explicitly differentiate between ordinary, source, and sink cells. A single link

will be decomposed into a group of ordinary cells, with possible source, sink, merge, or diverge cells on each end. A link is decomposed into cells based on the free flow travel time of the link, where each cell is the distance a vehicle can travel at free flow during the time step Δt. The number of cells is rounded up, leading to the well-known problem of discretization error of the CTM. In order to identify the "type" of each cell, the succeeding and preceding cells are counted, similar to the procedure to create the appropriate sets $\Gamma^+(i)$ and $\Gamma^-(i)$.

Generally planning data for link capacity is provided in *vehicles per hour*, as was discussed in more detail in Section 0. For a link (i, j), capacity was interpolated to a "per second" capacity, and then multiplied by Δt to obtain the jam density of each cell $N(i)$, for each i contained in a. The issue of saturation flow rate of each cell can be a trickier concept, here is literature that supports an assumption.

Algorithm 6.1: StrSODTA Algorithm Pseudocode

```
INPUT: Network G = (N, A); time step Δt;
    procedure decompose_link_network()
1:      for i ∈ A do
2:          create n = ceiling(freeflowTT(i)/Δt) new cells;
3:          for each j ∈ n do
4:              if (n == source cell)
5:                  C_r ← C_r ∪ j, Q_j ← ∞,
6:                  N_j ← 2Q_j;
7:              else if (n == sink cell)
8:                  C_s ← C_s ∪ j,
9:                  Q_j ← (capacity(i)/3600 * Δt), N_j ← ∞;
10:             else (n == ordinary cell)
11:                 C ← C ∪ j
12:                 Q_j ← (capacity(i)/3600 * Δt),
13:                 N_j ← 2Q_j
14:         end for all
15:     end for
16:     for i ∈ C do
17:         create Γ⁺(i) and Γ⁻(i);
18:     end for
    end procedure
OUTPUT: C; C_r; C_s; N_i Q_i, Γ⁻(i), Γ⁺(i) ∀i ∈ C;
```

The next step is to create the demand-related parameters for input to the StrSODTA model. The required input is a set of demand scenarios, where the demand and departure time between each origin and destination is defined for each scenario. This information is not contained within static demand data and a set of justifiable assumptions are required. Therefore, this work tests two relationships between demand scenarios: perfectly correlated and uncorrelated. The difference between these approaches is the amount that the expected demand is inflated or deflated in the additional scenarios. In the perfectly correlated approach, the demand for all

ODs is inflated or deflated by the same amount. In the independent approach, the demand is inflated or deflated based on a randomly generally number.

The number of departure times and the time step of those departures needs to be based on reasonable assumptions, such as a uniform distribution or a "peak" distribution to represent different network conditions that are observed in reality. Sensitivity analysis regarding the departure times could be also reveal interesting model characteristics. Generally, this demonstration keeps the number of departure times relatively small to control the size of the linear program and corresponding computational time, although in theory, this need not be a limitation.

Algorithm 6.2 outlines the procedure to create the demand scenarios from the static demand data. First, the set of OD pairs is created. Then the total demand for each OD $w_{rs}$is divided between a specified number of departure times (i.e., $|T_D|$). Note that the pseudo-code is intended to be functional, regardless of the number of specified demand scenarios. To achieve this purpose, the algorithm uses the cardinality of the demand scenario within the set, denoted $|\xi|$. The first demand scenario is assumed to be the expected demand. For odd numbered demand scenarios within the set (i.e., $|\xi| \,\%\, 2 \neq 0$), the demand is inflated, and for even numbered demand scenarios, the demand is deflated. This approach assumed a uniform distribution between departure times, although relaxing this assumption would require a small addition to lines 27, 29, 33, and 35.

Algorithm 6.2: StrSODTA Algorithm Pseudocode

```
INPUT: W; Ξ; T_D; demand correlation type (CORRELATED,
INDEPENDENT);
    procedure create_demand_scenarios()
1:      for r ∈ |W| do
2:          for s ∈ |W| do
3:              OD ← OD ∪ μ(r, s)
4:          end for
5:      end for
6:      for μ ∈ OD do
7:          for ξ ∈ Ξ do
8:              for τ ∈ T_D do
9:                  if |ξ| == 0
10:                     D_τ^{ξ,μ} ← w_rs/|T_D|
11:                 else if type == CORRELATED
12:                     if (|ξ| % 2) ≠ 0 //odd number
13:                         D_τ^{ξ,μ} ← w_rs(1 + |ξ|/100)/|T_D|
14:                     else //even number
15:                         D_τ^{ξ,μ} ← w_rs(1 − |ξ|/100)/|T_D|
16:                 else type == INDEPENDENT
17:                     r ← uniform random sample
18:                     if (|ξ| % 2) != 0
19:                         D_τ^{ξ,μ} ← w_rs * (1 + r/100)/ |T_D|
20:                     else
21:                         D_τ^{ξ,μ} ← w_rs * (1 − r/100)/ |T_D|
22:             end for
23:         end for
24:     end for
    end procedure
OUTPUT: OD; D_τ^{ξ,μ}, ∀ξ ∈ Ξ, ∀μ ∈ OD, ∀τ ∈ T_D;
```

One of the biggest changes between the StrSODTA model and previous applications of Ziliaskopoulos' LP is the need to explicitly represent and track paths. As the size of a network increases, the number of paths becomes potentially very large and

may have a significant effect on the computational size of the linear program (a topic that was explored by Rey et al (2014)). Additionally, the addition of paths introduces the data handling issue of explicitly representing the relationship between links, cells, and paths (which was handled using a forward star representation in the static case), and the issue of identifying the paths (which was handled using a shortest path approach in the static case). While innovative approaches that don't explicitly save path information (such as bush-based algorithms by Dial) are possible, the paths proportions play a significant role in the strategic approach, requiring the representation of paths.

Algorithm 6.3 describes the method for creating the path set for each OD pair and the link-path incidence matrix that is required for Constraints (5.25), (5.26), and (5.27). First, a k shortest path algorithm (Yen, 1971) finds each path in the set, where the number of paths k is specified by the user. The paths are based on free-flow costs. Rey et al (2014) found that $k = 5$ is a sufficient value. While the k path algorithm is based on link identification, it is also helpful to save the cell representation of each path. In that way, creating the cell connector-path incidence matrix for each path of each OD pair is a relatively straightforward logical check (or in common programming terms, a search) of whether cell i and cell j are included in the set of cells that comprise each path ϕ.

Algorithm 6.3: StrSODTA Algorithm Pseudocode

```
INPUT: OD; k;
    procedure manage_link_path_incidence()
1:      for μ ∈ OD do
2:          Add k (shortest) paths to Φ(μ);
3:      end for
4:      for μ ∈ OD do
5:          for φ ∈ Φ(μ) do
6:              for i ∈ C do
7:                  for j ∈ C do
8:                      if i == j
9:                          δ_ij^(μ,φ) ← 0
10:                     else if ∃(i ∈ φ) && ∃(j ∈ φ)
11:                         δ_ij^(μ,φ) ← 1
12:                     else
13:                         δ_ij^(μ,φ) ← 0
14:                 end for
15:             end for
16:         end for
17:     end for
    end procedure
OUTPUT: Φ(μ), ∀μ ∈ OD; δ_ij^(μ,φ), ∀μ ∈ OD, ∀φ ∈ Φ(μ), ∀i, j ∈ C;
```

Algorithms 6.1.1, 6.1.2, and 6.1.3 create the input data to solve the model. The next step is to create the model in a form that CPLEX can solve. AMPL is a programming language that provides the interface between mathematical programming and solver such as CPLEX, Gurobi, CONOPT, all which may apply different solution methods and be more appropriate for different forms of problems, generally linear, quadratic, and smooth nonlinear. CPLEX is the best known and most widely used large-scale solver, according to the AMPL website. CPLEX requires a license for

large scale programs, which was provided by the RCITI for use in the numerical results of this thesis. While initially alternative modelling interfaces such as GAMS were considered, AMPL was ultimately selected due to its great versatility and straightforward implementation.

The StrSODTA model file for AMPL is written in such a way that it is not dependent on the network itself. Algorithms 5.1, 5.2, and 5.3 provide the appropriate inputs to create a network-specific data file for the AMPL model. The mathematical program presented in Section 5.3.1 and Section 5.3.2 was coded in AMPL. Then an automated process was written to generate the appropriate data for the StrSODTA model. The AMPL program handled the interface between the solver CPLEX and the StrSODTA model. The full procedure to create and solve the StrSODTA model is presented in Algorithm 6.4.

Algorithm 6.4: StrSODTA Algorithm Pseudocode

INPUT: $C(G), \Xi, T_D, OD, D, P_\xi, \forall \xi \in \Xi;$

```
    procedure Solve_StrSODTA()
1:      decompose_link_network();
2:      create_demand_scenarios();
3:      manage_link_path_incidence();
4:      Create model StrSODTA using AMPL language;
5:      Create StrSODTA AMPL data file;
6:      Use CPLEX to solve StrSODTA;
7:      Calculate performance metrics;
    end procedure
```

OUTPUT: $\pi_\tau^{\mu,\phi}, \forall \mu \in OD, \forall \phi \in \Phi(\mu), \forall \tau \in T_D;\ , x_{t,\tau,i}^{\xi,\mu,\phi}, y_{t,\tau,ij}^{\xi,\mu,\phi}, \forall \xi \in \Xi, \forall \mu \in OD,$
$\forall \phi \in \Phi(\mu), \forall t \in T, \forall \tau \in T_D, \forall i \in C, (i,j) \in E;\ E(\Xi);\ TT(\xi), \forall \xi \in \Xi;$

5.4.2 Description of test network

Using the Solve_StrSODTA procedure described in Section 5.4.1, this chapter performs a numerical analysis on the Nguyen Dupius and Sioux Falls networks. The Nguyen Dupuis is a small test network with no cycles and can be used to form a more disaggregate view of model behaviour. Sioux Falls is a common grid-based test network in transport modelling. In order to decompose each network into a cell-based representation instead of a link based representation, a link is divided into sections based on how far a vehicle can travel during a timestep Δt. Therefore, the size of the network and the corresponding size of the StrSODTA LP depends on the desired resolution.

Table 5-1 compares the timestep Δt and the resulting number of cells and cell connectors in the Nguyen Dupius and Sioux Falls networks. The difference is linear according to the time step, with the difference being due to rounding error and the fact that source and sink links are required to be only one cell. A number of CTM-based DTA implementations utilize a time resolution of six seconds. However, the computational complexity of the StrSODTA model increases significantly with the number of cells in the network (as well as the number of OD pairs, paths, departure times, and demand scenarios). In order to keep the solve time pf the LP reasonable, $\Delta t = 120$ was chosen for many applications presented in this thesis. Section 5.4.3 presents a small sensitivity analysis with regard to the timestep. While such a large time step may not be entirely realistic, it was judged to be appropriate for the size and geometric characteristics of the test networks. Additionally, it allows the StrSODTA problem to be solved on networks that can be compared with other modelling approaches, like those presented in Chapter 2.

Table 5-1 Test Network Characteristics

	Nguyen Dupius		Sioux Falls	
Δt	$\|C\|$	$\|E\|$	$\|C\|$	$\|E\|$
6	1325	1335	3148	3339
30	277	287	636	827
60	144	154	322	513
90	102	112	244	513
120	79	89	192	383

The parameters that can be varied for sensitivity analysis include the inputs to the Algorithm 6.1.4: the timestep, the demand values, the number of departure times, the number of paths, the number of demand scenarios, and the relationship between the demand in each demand scenario.

5.4.3 Single-destination model demonstration

This section demonstrates the SD-StrSODTA model in a relatively simple example in order to illustrate the propagation of flow in the proposed model: two demand scenarios, two departure times ($T_D = \{1,2\}$), two origins, one destination, and two paths per OD ($k = 2$), on the Nguyen Dupuis network. The expected demand is shown in Table 5-2. Due to the simple parameters, the results can be examined in more intricate detail, facilitating understanding of the fundamental model behaviour.

Table 5-2 Expected demand for the single destination Nguyen Dupius case

	2
1	400
4	100

N between 58-67, Q = 73.3

A large time step of $\Delta t = 120$ was chosen for this section to keep the results to a reasonable size. A large number of time steps were included, $T = 40$, or 4,800

seconds of “simulation” to ensure that all vehicles left the network. This timestep led to a jam density of all cells is between 58-67, and all vehicles have the same capacity, and therefore the same $Q = 73.3$. This section presents results as a time-space density diagram. Each cell in a particular path lies across the horizontal axis, with each time step on the vertical axis. Thus, each block represents a cell during a particular time step, with time beginning in the top left corner of the diagram and ending in the bottom right corner. The numbers in the blocks represent the cell density at a particular time period, where cells that are shown in red have zero density. Each path is shown in two diagrams; the first shows just the density of a particular path and OD pair, while the second diagram shows the total density in each cell at each time step. In this way, one can read the progression of “vehicles” through the network.

	Scenario 1,OD (1-2), Path 1															
	76	0	1	2	3	16	17	21	22	29	30	31	40	41	42	9
0																
1	127															
2	61	66														
3	61	5.3	61													
4		66		61												
5			66		61											
6				66	17	44										
7				17	66		44									
8				17	22	44		44								
9					39		44		44							
10						39		44		44						
11	130						39		44		44					
12	64	66					39			44		44				
13	64		66				39				44		44			
14	64			66				39				44		44		
15		64		22	44				39				44		44	
16			64		22	44				39				44		44
17			34	29	22		44				39				44	44
18			34		29	22		44				39				88
19				34	29	15	7.3		44				39			88
20				34		44		7.3		44				39		88
21					34		44	7.3			44				39	88
22						34		51				44				127
23							34		51				44			127
24								34		51				44		127
25									34		51				44	127
26										34		51				171
27											34		51			171
28												34		51		171
29													34		51	171
30														34		222
31															34	222
32																256

Figure 5:3 Cell-density diagram for $\xi = 1, m = (1,2), \phi = 1$: path density

The areas of interest in the cell density diagrams occur when "queuing" is possible, or the origin cell and merge/diverge cells. In the first cell-density diagram presented in Figure 5:3, there are two departure times where vehicles enter the network at $t = 0$ and $t = 11$. It requires between 3 and 4 timesteps for all vehicles to leave the origin cell $i = 76$. There are two cells that succeed cell 76: cell 0 and cell 4. However, an immediate example of holding presents itself: the density of cell

76 is 64 for timesteps 11, 12, and 13, despite the fact that the succeeding cell 0 is empty (which we can verify in

	Scenario 1,OD (1-2), Path 1 (Total Flow)															
	76	0	1	2	3	16	17	21	22	29	30	31	40	41	42	9
0																
1	200															
2	64	66														
3	64	5.3	61													
4		66		61												
5			66		61											
6				66	17	44										
7				17	66		44									
8				17	22	44		44								
9					39		44		44							
10						39		59		44						
11	200						39		59		44					
12	64	66					39	59		59		44				
13	64		66				39		59		59		44			50
14	64			66				39		59		59		44		50
15		64		22	44				39		59		59		44	50
16			64		22	44				39		59		59		94
17			34	29	22		44				39		59		59	94
18			34		29	22		44				39		59		153
19				34	29	15	7.3		44				39		59	153
20				34		44		66		44				39		211
21					34		44	7.3	59		44				39	211
22						34		59		59		44				250
23							34		59		59		44			300
24								39		59		59		44		300
25									39		59		59		44	300
26										39		59		59		344
27											39		59		59	344
28												39		59		403
29													39		59	403
30														39		461
31															39	461
32																500

Figure 5:4 Cell-density diagram for $\xi = 1, m = (1,2), \phi = 1$: total density

	Scenario 1,OD (1-2), Path 1															
	76	0	1	2	3	16	17	21	22	29	30	31	40	41	42	9
0																
1	127															
2	61	66														
3	61	5.3	61													
4		66		61												
5			66		61											
6				66	17	44										
7				17	66		44									
8				17	22	44		44								
9					39		44		44							
10						39		44		44						
11	130						39		44		44					
12	64	66					39			44		44				
13	64		66				39				44		44			
14	64			66				39				44		44		
15		64		22	44				39				44		44	
16			64		22	44				39				44		44
17			34	29	22		44				39				44	44
18			34		29	22		44				39				88
19				34	29	15	7.3		44				39			88
20				34		44		7.3		44				39		88
21					34		44	7.3			44				39	88
22						34		51				44				127
23							34		51				44			127
24								34		51				44		127
25									34		51				44	127
26										34		51				171
27											34		51			171
28												34		51		171
29													34		51	171
30														34		222
31															34	222
32																256

	Scenario 1,OD (1-2), Path 1 (Total Flow)															
	76	0	1	2	3	16	17	21	22	29	30	31	40	41	42	9
0																
1	200															
2	64	66														
3	64	5.3	61													
4		66		61												
5			66		61											
6				66	17	44										
7				17	66		44									
8				17	22	44		44								
9					39		44		44							
10						39		59		44						
11	200						39		59		44					
12	64	66					39	59		59		44				
13	64		66				39		59		59		44			50
14	64			66				39		59		59		44		50
15		64		22	44				39		59		59		44	50
16			64		22	44				39		59		59		94
17			34	29	22		44				39		59		59	94
18			34		29	22		44				39		59		153
19				34	29	15	7.3		44				39		59	153
20				34		44		66		44				39		211
21					34		44	7.3	59		44				39	211
22						34		59		59		44				250
23							34		59		59		44			300
24								39		59		59		44		300
25									39		59		59		44	300
26										39		59		59		344
27											39		59		59	344
28												39		59		403
29													39		59	403
30														39		461
31															39	461
32																500

Figure 5:5 Cell-density diagram for $\xi = 1, m = (1,2), \phi = 1$: path density

	Scenario 1,OD (1-2), Path 2																	
	76	4	5	6	7	8	60	61	62	21	22	29	30	31	40	41	42	9
0																		
1	73																	
2	2.9	70																
3	2.9		70															
4		2.9	56	15														
5			2.9	56	15													
6				2.9	56	15												
7					59		15											
8						59		15										
9							59		15									
10								59		15								
11	70								59		15							
12		70								59		15						
13			70								59		15					
14				70								59		15				
15				12	59								59		15			
16				12		59								59		15		
17					12		59								59		15	
18						12		59								59		15
19							12		59								59	15
20								12		59								73
21									12		59							73
22									4.4	7.3		59						73
23									4.4		7.3		59					73
24										4.4		7.3		59				73
25											4.4		7.3		59			73
26												4.4		7.3		59		73
27													4.4		7.3		59	73
28														4.4		7.3		132
29															4.4		7.3	132
30																4.4		139
31																	4.4	139
32																		144
33																		144

Figure 5:6 Cell-density diagram for $\xi = 1, m = (1,2), \phi = 2$: path density

	Scenario 1,OD (1-2), Path 2 (Total Flow)																	
	76	4	5	6	7	8	60	61	62	21	22	29	30	31	40	41	42	9
0																		
1	200																	
2	64	70																
3	64		70															
4		2.9	56	15														
5			2.9	56	15													
6				2.9	56	15												
7					59		15											
8						59		15		44								
9							59		15		44							
10								59		59		44						
11	200								59		59		44					
12	64	70								59		59		44				
13	64		70								59		59		44			50
14	64			70						39		59		59		44		50
15				12	59						39		59		59		44	50
16				12		59						39		59		59		94
17					12		59						39		59		59	94
18						12		59		44				39		59		153
19							12		59		44				39		59	153
20								12		66		44				39		211
21									12	7.3	59		44				39	211
22									4.4	59		59		44				250
23									4.4		59		59		44			300
24										39		59		59		44		300
25											39		59		59		44	300
26												39		59		59		344
27													39		59		59	344
28														39		59		403
29															39		59	403
30																39		461
31																	39	461
32																		500

Figure 5:7 Cell-density diagram for $\xi = 1, m = (1,2), \phi = 2$: total density

There are two paths between OD pair (4,2), but the total demand is only 100, divided between two departure times. Therefore, the total number of vehicles is less than the saturation flow rate of 73. Of the two paths, one path includes cells that are used for OD pair (1,2), but the path 2 does not. Therefore, the optimal proportions were to route all of the flow onto path 2 and zero of the flow onto path 1. Figure 5:8 and Figure 5:9 show the cell density diagrams for OD pair (4,2).

	Scenario 1,OD (4-2), Path 1													
	77	10	11	16	17	21	22	29	30	31	40	41	42	9
0														
1														
2														
3														
4														
5														
6														
7														
8														
9														
10														
11														
12														
13														
14														
15														
16														
17														
18														
19														
20														
21														
22														
23														
24														
25														
26														
27														
28														
29														
30														
31														
32														

	Scenario 1,OD (4-2), Path 1 (Total Flow)													
	77	10	11	16	17	21	22	29	30	31	40	41	42	9
0														
1	50													
2														
3														
4														
5														
6				44										
7					44									
8				44		44								
9					44		44							
10				39		59		44						
11	50				39		59		44					
12					39	59		59		44				
13					39		59		59		44			50
14						39		59		59		44		50
15							39		59		59		44	50
16				44				39		59		59		94
17					44				39		59		59	94
18				22		44				39		59		153
19				15	7.3		44				39		59	153
20				44		66		44				39		211
21					44	7.3	59		44				39	211
22				34		59		59		44				250
23					34		59		59		44			300
24						39		59		59		44		300
25							39		59		59		44	300
26								39		59		59		344
27									39		59		59	344
28										39		59		403
29											39		59	403
30												39		461
31													39	461
32														500

Figure 5:8 Cell-density diagram $m = (4,2)$, $\phi = 1$: path density

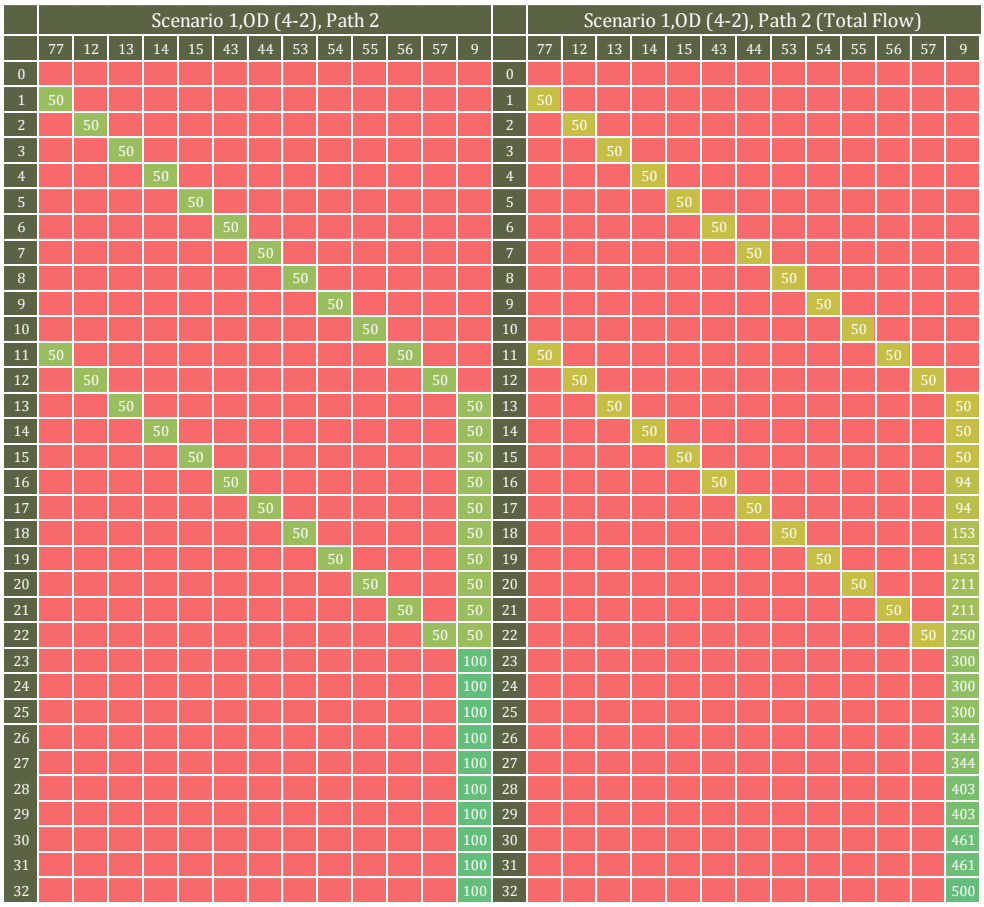


Figure 5:9 Cell-density diagram for $m = (4,2)$, $\phi = 2$: path density

Next, this section performs a sensitivity analysis of the parameters k, T_D, Δt, and Ξ. The sensitivity analysis was performed on the single destination model because the computational time on this model was very small, allowing the sensitivity analysis to be performed almost instantly.

First, a sensitivity analysis on the size of the timestep was performed. Table 5-3 shows the results for 3 demand scenarios (that were determined as in Algorithm 6.2) and $\Delta t = 120,90,60,30$. The demand is the same for all four cases. This sensitivity analysis requires the SDStrSODTA model to be solved four times,

where each column in Table 5-3 represents a model instance, with different optimal proportions.

Table 5-3 Time step sensitivity analysis of performance metrics

Δt	120	90	60	30
$TT(1)$ **[hours]**	321.8	311.2	294.8	300.2
$TT(2)$ **[hours]**	358.8	347.6	332.8	339.9
$TT(3)$ **[hours]**	249.2	240.4	222.5	226.0
$E(\Xi)$ **[hours]**	309.93	299.73	283.37	288.7

At higher levels of time aggregation, the model should show more discretization error, and therefore a higher $E(\Xi)$. For the time steps of 120, 90, and 60, the model shows a decreasing prediction of travel time, as expected. However, when the time step is 30 seconds, the model prediction for all performance measures increases again. This may be due to some balance between cell sizes that are too large to capture the effects of congestion and cell sizes that are computationally feasible.

The next sensitivity analysis compares the number of demand scenarios and the time step. Table 5-4 shows the results for the SDStrSODTA model where $\Xi = 2$.

Table 5-4 Comparison of different time steps and the number of demand scenarios

Δt	**120**	**90**	**60**	**30**
$TT(1)$	321.78	311.20	294.81	300.17

$TT(2)$	358.84	347.60	332.81	339.90
$E(\Xi)$	10209.30	13176.00	18828.70	38404.00

The sensitivity analysis continues with a comparison of the proportions resulting from the SDStrSODTA model. Table 5-5 shows the proportions for OD pair (1,4) for the same scenario that was show in the cell density plots. Although, both the travel time and the expected travel time are the same in all scenarios, the proportions the model predicts are not. Table 5-5 shows for that uncongested models, path proportions are not unique.

Table 5-5 Comparison of $p_{1,4}$ at departure time $\tau = 1$

$m = (1,4)$	$\Xi = 1$	$\Xi = 2$	$\Xi = 3$
Path 1	0.55	0.6	0.433333
Path 2	0.45	0.4	0.566667

The final sensitivity analysis on the small network examines the number of paths k and the number of departure times, $\tau \in T_D$. Increasing the number of paths will lower the total travel time to the point that the extra paths are not used. Adding departure times for the same amount of demand should also lower the total travel time, because the vehicles will be more spread out and there will be fewer congestion issues. Both of these trends are observed in Table 5-6. However, the

additional path from 3 to 4 did not lower total travel time. This may indicate that the additional path was not used, but due to the non-uniqueness of path proportions, the additional path may not have lowered the travel time.

Table 5-6 Sensitivity analysis of k and dt

	$k = 3, dt = 2$	$k = 3, dt = 3$	$k = 4, dt = 3$
$TT(1)$	302.9	296.0	296.0
$TT(2)$	337.3	329.7	329.7
$TT(3)$	238.7	232.3	232.3
$E(\Xi)$	288	286	286

Next, the multiple destination model is examined.

5.4.4 Multiple-destination model demonstration

This section presents the multiple destination StrSODTA model on the Nguyen Dupius and Sioux Falls networks. Although a large time step is necessary to make this approach computationally feasible, the use of the same network data in Parts I and II of this thesis allows for very rough comparison of modelling techniques. This ability to compare was deemed more important than working on a small time step. The parameters in this section were chosen as follows: $k = 4, dt = 3$ ($\Delta t = 0, 10, 20$), $T = 100, \Delta t = 120$. Table 5-7 shows the expected demand for the Nguyen

Dupius network and Table 5-10 shows the expected demand for the Sioux Falls network.

Table 5-7 Full expected demand for Nguyen Dupius

	2	3
1	1400	1100
4	1200	1600

This section explores the system level performance metrics, which are the expected demand for all the demand scenarios $E(\Xi)$, and the total travel time in each individual scenario $TT(\xi)$. Table 5-8 shows the results for five perfectly correlated demand scenarios that were generated using the approach in Pseudo-Algorithm 6.1.4. Table 5-8 shows the $E(\Xi)$ for each of the demand scenarios, where there are 1, 2,3, 4, or 5 total demand scenarios. The MDStrSODTA LP was solved 5 separate times to generate the results in Table 5-8. Note that the individual $TT(\xi)$ do not reflect system optimal system realizations due to the strategic path proportions. It is expected that $E(\Xi)$ will be higher in the 2 and 4 cases because the demand is inflated. It may be noted that the additional deflated demand scenarios appear to lower the $E(\Xi)$ as compared to the expected demand case.

Table 5-8 Multiple-destination, perfectly correlated demand Nguyen Dupius

	$\Xi = 5$	$\Xi = 4$	$\Xi = 3$	$\Xi = 2$	$\Xi = 1$
$\boldsymbol{TT(1)}$	716.6	717.1	716.9	716.9	716.3
$\boldsymbol{TT(2)}$	819.6	819.2	820.2	818.8	-
$\boldsymbol{TT(3)}$	544.4	546.4	543.0	-	-
$\boldsymbol{TT(4)}$	1053.0	1049.8	-	-	-
$\boldsymbol{TT(5)}$	395.4	-	-	-	-
$\boldsymbol{E(\Xi)}$	705.8	783.1	693.4	767.9	716.3

Table 5-9 shows the similar results for the case where the demand is independent. However, there is an important caveat: while the expected demand in each case is the same, the remaining demand cases were all generated independently. In other words, the total demand was generated for when $\Xi = 2$, and then a different demand was generated for when $\Xi = 3$ and so on. To make Table 5-9 the exact same situation as the previous table, the total demand for case in the the $\Xi = 5$ scenario would need to be created, and then the model data files adjusted for each case with fewer demand scenarios. However, in this approach, each model data file was generated separately from all the others. Therefore, the $TT(\xi)$ varies significantly between demand scenarios because the demand itself varied.

Table 5-9 Multiple-destination, uncorrelated demand, Nguyen Dupius

	$\Xi = 5$	$\Xi = 4$	$\Xi = 3$	$\Xi = 2$	$\Xi = 1$
$\boldsymbol{TT(1)}$	721.2	719.9	720.2	718.8	716.3
$\boldsymbol{TT(2)}$	1258.3	840.3	1014.0	1111.1	-
$\boldsymbol{TT(3)}$	360.4	313.4	224.1	-	-
$\boldsymbol{TT(4)}$	1707.3	2091.5	-	-	-
$\boldsymbol{TT(5)}$	456.7	-	-	-	-
$\boldsymbol{E(\Xi)}$	900.8	991.3	652.8	915.0	716.3

Next, this section presents the MDStrSODTA model results on the Sioux Falls network. The same parameters were utilized in this case: $= 4, dt = 3$ ($\Delta t = 0, 10, 20$), $T = 100, \Delta t = 120$. Table 5-10 shows the expected demand for the Sioux Falls network. Note that only 12 OD pairs are considered, and that the demand is lower than it was in the static case of Chapters 2, 3, and 4.

Table 5-10 Expected demand for Sioux Falls

	20	**18**	**7**	**6**
1	0	810	420	900
2	430	0	1160	780
3	730	250	0	270
13	1200	260	590	0

It has previously been noted that the MDStrSODTA model grows in size very quickly. While the computation of the linear program is not of focus in this work due to the fact a commercial solver was utilized, a rough indication of the size of the linear program and the corresponding computational burden is shown below. Figure 5:10 shows the size of the linear program *after* CPLEX has performed the pre-solve treatment to eliminate variables and constraints, for each of the LP models where the $\Xi = 1, 2, 3, 4, 5$. Of course, the size of the linear program grows linearly. The largest linear program had over 14 million variables. To provide an conceptual understanding of what this implies, note that while CPLEX was solving the 5 demand scenario case, it was using about 0.125 TB of memory. Nevertheless, the solve time was only about two hours. CPLEX is capable of solving significantly larger LPs.

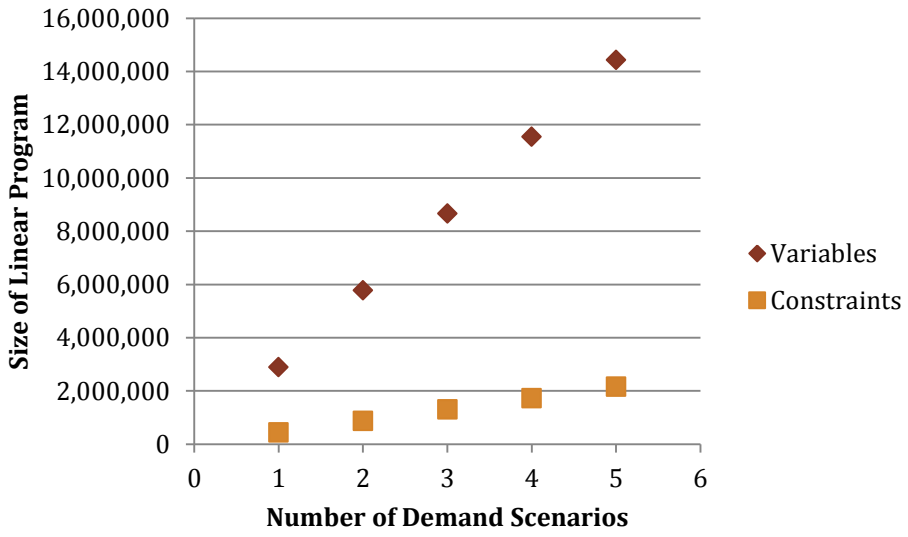


Figure 5:10 Size of LP on Sioux Falls network

Table 5-11 shows the results (in hours) of the perfectly correlated demand case on the Sioux Falls network. The $TT(\xi)$ varies considerably more, indicating that the path proportions are considerably more important. In majority of cases, only one, two, or three paths are used; in only one instance do all four paths have a nonzero value. While the demand was inflated by 20% between $\Xi = 1$ and $\Xi = 2$, the $E(\Xi)$ increased by about 9%, and the difference in the $TT(1)$ and $TT(2)$ was about 12%.

Table 5-11 Sioux Falls, perfectly correlated demand results

	$\Xi = 5$	$\Xi = 4$	$\Xi = 3$	$\Xi = 2$	$\Xi = 1$
$TT(1)$	3473.6	3487.8	3438.3	3449.7	3409.1
$TT(2)$	3932.4	3940.2	3925.7	3912.1	
$TT(3)$	2628.7	2644.3	2588.8		
$TT(4)$	5207.2	5162.5			
$TT(5)$	1869.5				
$E(\Xi)$	3422.2	3808.7	3317.6	3680.9	3409.1

Table 5-12 shows the results for each of 5 demand scenario cases on the Sioux Falls network, for the case where demand is generated independently for each demand scenario. The $TT(1)$ varies more significantly among the five cases in the independent demand case.

Table 5-12 Sioux Falls, independent demand results

	$\Xi = 5$	$\Xi = 4$	$\Xi = 3$	$\Xi = 2$	$\Xi = 1$
$TT(1)$	3558.1	3690.6	3490.9	3531.6	3409.1
$TT(2)$	4802.5	5098.3	4678.8	4975.5	
$TT(3)$	1480.7	1457.8	1492.5		
$TT(4)$	6729.9	11075.2			
$TT(5)$	1995.7				
$E(\Xi)$	3713.4	5330.5	3220.7	4253.5	3409.1

While the linear programming formulation presented here is theoretically sound, the explicit enumeration of paths is concerning in regards to scalability. The number of variables increases in a manner directly proportionally to the number of paths, which in general grows combinatorially with respect to the network size. Therefore, the overall complexity of the problem as stated becomes very challenging when applied to realistically sized networks.

In spite of the drawbacks of the problem complexity, there are several research directions which could aid in the development of scalable solution methods for this problem. Two issues must be addressed in order to manage problem complexity: increased complexity with respect to the number of demand scenarios, and increased complexity due to path enumeration.

In order to manage the problem complexity with respect to the number of demand scenarios considered, decomposition methods, namely Dantzig-Wolfe

decomposition techniques, lend themselves well for implementation on this problem. Because only the path proportion variables link the demand scenario-specific problems, implementations of Dantzig-Wolfe decomposition akin to that of Li et al. (2003) show significant promise.

Managing the enumeration of paths also presents several potential avenues of research. On one hand, column generation methods are well documented as options for generating good solutions to problems in which path enumeration may be otherwise required. Furthermore, other alternatives can be conceived based on the extraction of time-dependent paths from non-path based DTA approaches such as that presented in Ziliaskopoulos (2000). While the problem of extracting such paths is not trivial, the computational advantages of avoiding the solving of a path-based linear program may be significant.

5.5 Concluding Remarks

Chapter 5 introduced a novel strategic dynamic traffic assignment model that examines user responses to uncertain demand. Specifically, demand uncertainty was modelled in a scenario-based framework, and where it was assumed that users react to the uncertainty by generating strategic strategies prior to the observation of the network demand (but with knowledge of the set of demand scenarios). The model is formulated as a linear program, based on a path-centric representation of flow. Substantial formulaic development was presented to enhance previously known DTA modelling techniques.

Numerical results were shown on the Nguyen Dupius and the Sioux Falls networks for demonstrative purposes, where the size of cells was considered quite large to maintain computational feasibility. Cell and link variation across demand scenarios was analysed, and computational complexity discussed. While the problem formulation and solution method are correct and sound, there are limitations regarding scalability due to the size of the linear program solved; both the number of demand scenarios considered and the path-based nature of the formulation severely affect the computational complexity of the problem.

Chapter 6

Dynamic Management Application: Network Design

6.1 Introduction

As noted previously, the traffic network design problem (NDP) is one of the more challenging issues in the transportation research. Robust modelling approaches that provide systematic methods to determine the optimal distribution of a budget over a range of possible projects and quantify the corresponding system impact are vital for the success of transportation planning agencies around the world. However, introducing complexities such as dynamics or inherent network uncertainties, e.g., stochastic demand, results in a challenging mathematical problem to solve.

This chapter expands the strategic system optimal dynamic traffic assignment (StrSODTA) formulation from Chapter 6 to solve the capacity enhancement network design problem. A globally optimal solution to the StrSODTA NDP can be found due to the linear programming model at its foundation. The proposed work incorporates an enhanced version of a system optimal linear programming model proposed by (Ziliaskopoulos, 2000) that embeds the cell

transmission model to realistically propagate traffic through a network. Chapter 6 propose the StrSODTA LP model that incorporates strategic route choice behaviour, demand scenarios based on a discrete distribution, and path based proportions. Strategic behaviour implies that users choose routes in a way to minimize travel time over a range of stochastic demand scenarios, instead of a single deterministic value. This results in a set of flows that are not an optimal solution to any single demand scenario, and thus display a day to day volatility that is commonly observed in traffic. The research in this work proposes simple modifications to apply the StrSO DTA LP to the NDP.

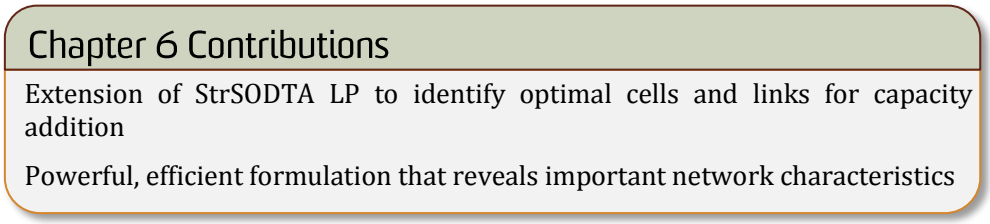


Figure 6:1 Summary of research contribution

This approach overcomes many issues associated with the traditional dynamic traffic assignment approach to the network design problem (i.e., is not simulation based or reliant on a link cost function), although it faces challenges in terms of its computational complexity. This work first presents the model formulation, and then demonstrates results on a sample network. These results are analysed for a range of potential budgetary applications. Furthermore, this work examines the impact of the strategic approach as compared to the results that would be obtained

using a single scenario expected demand approach. Finally, possibilities to address the issue of computational complexity are discussed.

6.2 Background

Chapter 7 focuses on a novel approach to solving the transport road network design problem, in which the optimal links in a network are identified in order to achieve some objective. Traditional approaches to the NDP that utilize a link cost function to represent vehicle movement become challenging nonconvex mathematical problems when a variable is added to the capacity term, as is thoroughly illustrated in Chapter 5. In these approaches, heuristic solution methods such as genetic algorithms are generally necessary. However, in the current application, the network design problem is represented as a linear program and can be solved easily and efficiently by commercial solvers such as CPLEX.

As previously discussed, the static network design problem is a popular and challenge topic in the transport modelling community. Examining network design from a dynamic perspective makes the problem when more complex, and the works on dynamic network design are more rare due to the difficult of formulation and solving the models. Some researchers have tried a genetic algorithm approach for the dynamic network design problem, a popular approach for the static case (Wismans et al, 2012; Wismans et al, 2011b). Lin has produced a number of works related to the dynamic network design problem, including decomposition for the bilevel problem (Lin et al, 2011), a solution method based on the dual

approximation of the similar LP (Lin, 2011), and a genetic algorithm solution approach (Lin et al, 2009).

Dynamic network design approaches accounting for sources of uncertainty are also more rare. Approaches to accounting for demand uncertainty include a chance constraint, two-stage recourse model, or scenario-based simulation method. However, as seen previously in this thesis, simulation based approaches may face disadvantages because they are required to know the probability of each scenario in advance and it becomes computationally expensive when there are a large number of scenarios.

Waller and Ziliaskopoulos (2001) first uses a variation of this approach to examine the network design problem while accounting for stochastic demand. Additionally, Waller et al (2006) use the LP formulation to optimally solve for the continuous network design problem, a result which would not be possible given the usual non-convex formulations for the NDP. Ukkusuri and Waller (2008) formulates a user-optimal version of this problem, which Karoonsoontawong and Waller (2005) use to compare results in the network design application accounting for stochastic demand. Do Chung et al (2011) propose a robust dynamic NDP, demand uncertainty using a set based robust optimization approach to account for demand uncertainty that also uses a linear programming approach at its foundation. All of these approaches in some way incorporate the linear programming SODTA model presented by Ziliaskopoulos (2000), which further

draws on Daganzo's cell transmission model (Daganzo, 1994, 1995) to present a simple formulation of traffic flow that captures flow variability inside the link while avoiding the drawbacks associated with link performance functions.

The problem examined in this research differs from that in the literature in that it incorporates the concept of a strategic approach to equilibrium within a linear programming framework, and further captures the finer grain resolution phenomena observable through the use of CTM, and then uses this approach to investigate the impact on the network design problem. This work differs from previous approaches in the strategic approach to accounting for the impact of multiple demand scenarios, not just expected demand, when identifying optimal path proportions (instead of expected flows).

6.3 Formulation

The model presented in this work is the accumulation of a number of previous works. The StrSODTA model may be conceived as consisting of a two-stage approach; in the first stage, system optimal route *proportions* are determined so as to minimize *expected* total system travel time accounting for a finite range of possible discrete demand scenarios. In the second stage, the actual travel demand is realized, and the model outputs scenario-dependent flows. However, these flows will not represent a system optimal solution for any of the realized demand scenarios, thus representing the changing nature of traffic observed in reality and

additionally, introducing a variance in expected total system travel time for an optimal model that has not previously been possible.

First Section 6.3 briefly recounts the multiple destination StrSODTA LP mode. Next, the enhancements necessary for the NDP are formulated and explained. The model proposed in this work uses a linear program to realistically propagate traffic according to the cell transmission. As a result of the underlying CTM model, the objective function for this model is the expected total system travel time. This becomes simply the aggregate density of each cell $i \in C$ for each time period $t \in T$ for each demand scenario $\xi \in \Xi$, multiplied by the probability of that demand scenario p^{ξ}.

Model: StrSODTA

$$\text{minimize} \sum_{\xi \in \Xi} \sum_{\mu \in OD} \sum_{\phi \in \Phi(\mu)} \sum_{t \in T} \sum_{\tau \in T_D} \sum_{i \in C \setminus C_s} p^{\xi} x_{t,\tau,i}^{\xi,\mu,\phi} \tag{6.1}$$

subject to

$$x_{t,\tau,i}^{\xi,\mu,\phi} - x_{t-1,\tau,i}^{\xi,\mu,\phi} - \sum_{j \in \Gamma^-(i)} \delta_{ij}^{\mu,\phi} y_{t-1,\tau,ji}^{\xi,\mu,\phi} + \sum_{j \in \Gamma^+(i)} \delta_{ij}^{\phi,\mu} y_{t-1,\tau,ij}^{\xi,\mu,\phi} = 0 \qquad \begin{matrix} \forall \xi \in \Xi, \forall \mu \in OD, \\ \forall \phi \in \Phi(\mu), \forall t \\ \in T^*, \forall \tau \in T_D, \forall i \\ \in C \setminus (C_r \cup C_s) : \delta_i^{\phi} \end{matrix} \tag{6.2}$$

$$x_{t,\tau,i}^{\xi,\mu,\phi} - x_{t-1,\tau,i}^{\xi,\mu,\phi} - \sum_{j \in \Gamma^-(i)} \delta_{ij}^{\phi,\mu} y_{t-1,\tau,ij}^{\xi,\mu,\phi} = 0 \qquad \begin{matrix} \forall \xi \in \Xi, \forall \mu \in OD, \\ \forall \phi \in \Phi(\mu), \forall t \\ \in T^*, \forall \tau \in T_D, \forall i \\ \in C_s : \delta_i^{\phi} \end{matrix} \tag{6.3}$$

$$x_{t,\tau,i}^{\xi,\mu,\phi} - x_{t-1,\tau,i}^{\xi,\mu,\phi} + \sum_{j\in\Gamma^+(i)} \delta_{ij}^{\phi,\mu} y_{t-1,\tau,ij}^{\xi,\mu,\phi} = \pi_\tau^{\mu,\phi} D_\tau^{\xi,\mu} \quad \forall \xi \in \Xi, \forall \mu \in OD, \forall \phi \in \Phi(\mu), \forall t \in T^*, \forall \tau \in T_D, \forall i \in C_r: \delta_i^\phi \quad (6.4)$$

$$\sum_{j\in\Gamma^+(i)} \delta_{ij}^{\phi,\mu} y_{t,\tau,ij}^{\xi,\mu,\phi} + x_{t,\tau,i}^{\xi,\mu,\phi} \le 0 \quad \forall \xi \in \Xi, \forall \mu \in OD, \forall \phi \in \Phi(\mu), \forall t \in T^*, \forall \tau \in T_D, \forall i \in C \backslash C_s \quad (6.5)$$

$$\sum_{\mu\in OD} \sum_{\phi\in\Phi(\mu)} \sum_{\tau\in T_D} \left(\sum_{i\in\Gamma^-(j)} \delta_{ij}^{\phi} y_{t,\tau,ij}^{\xi,\phi} + x_{t,\tau,i}^{\xi\phi} \right) \le N_{t,i} + g_i \quad \forall \xi \in \Xi, \forall t \in T, \forall i \in C \backslash (C_r \cup C_s) \quad (6.6)$$

$$\sum_{\mu\in OD} \sum_{\phi\in\Phi(\mu)} \sum_{\tau\in T_D} \sum_{j\in\Gamma^-(j)} \delta_{ij}^{\mu,\phi} y_{t,\tau,ij}^{\xi,\mu,\phi} \le Q_{t,i} \quad \forall \xi \in \Xi, \forall t \in T, \forall i \in C \backslash C_s \quad (6.7)$$

$$\sum_{\mu\in OD} \sum_{\phi\in\Phi(\mu)} \sum_{\tau\in T_D} \sum_{j\in\Gamma^+(i)} \delta_{ij}^{\mu,\phi} y_{t,\tau,ij}^{\xi,\mu,\phi} \le Q_{t,i} \quad \forall \xi \in \Xi, \forall t \in T, \forall i \in C \backslash C_r \quad (6.8)$$

$$\sum_{\phi\in\Phi(\mu)} \pi_\tau^{\mu,\phi} = 1 \quad \forall \mu \in OD, \forall \tau \in T_D \quad (6.9)$$

$$y_{0,\tau,ij}^{\xi,\mu,\phi} = 0 \quad \forall \xi \in \Xi, \forall \mu \in OD, \forall \phi \in \Phi(\mu), \forall \tau \in T_D, \forall (i,j) \in E \quad (6.10)$$

$$y_{t,\tau,ij}^{\xi,\mu,\phi} \ge 0 \quad \forall \xi \in \Xi, \forall \mu \in OD, \forall \phi \in \Phi(\mu), \forall t \in T, \forall \tau \in T_D, \forall (i,j) \in E \quad (6.11)$$

$$x_{t,\tau,i}^{\xi,\mu,\phi} \ge 0 \quad \forall \xi \in \Xi, \forall \mu \in OD, \forall \phi \in \Phi(\mu), \forall t \in T, \forall \tau \in T_D, \forall i \in C \quad (6.12)$$

$$\pi_\tau^{\mu,\phi} \ge 0 \quad \forall \mu \in OD, \forall \phi \in \Phi(\mu), \forall \tau \in T_D \quad (6.13)$$

The continuous NDP application of the LP StrSO model requires the addition of a budgetary constraint, and the modification of the constraints representing the physical characteristics of network links. The budgetary constraint specifies the cost for expanding the capacity of a link. For demonstration purposes, this work assumes a unit cost of β to add z units to the jam density of cell i, and that this amount will proportionally add $\alpha_i = Q_i/N_i$ units to the saturation flow of cell i.

Additionally, the NDP approach in this paper alters constraints (6.6)-(6.8) above to show the amount of capacity and flow that are added to a cell.

$$minimize \sum_{\xi \in \Xi} \sum_{\mu \in OD} \sum_{\phi \in \Phi(\mu)} \sum_{t \in T} \sum_{\tau \in T_D} \sum_{i \in C \setminus C_s} p^{\xi} x_{t,\tau,i}^{\xi,\mu,\phi} \tag{6.14}$$

subject to

$$\sum_{\forall i \in C \setminus C_s} \beta_i z_i \leq G$$

$$x_{t,\tau,i}^{\xi,\mu,\phi} - x_{t-1,\tau,i}^{\xi,\mu,\phi} - \sum_{j \in \Gamma^-(i)} \delta_{ij}^{\mu,\phi} y_{t-1,\tau,ji}^{\xi,\mu,\phi} + \sum_{j \in \Gamma^+(i)} \delta_{ij}^{\phi,\mu} y_{t-1,\tau,ij}^{\xi,\mu,\phi} = 0 \qquad \begin{matrix} \forall \xi \in \Xi, \forall \mu \in OD, \\ \forall \phi \in \Phi(\mu), \forall t \\ \in T^*, \forall \tau \in T_D, \forall i \\ \in C \setminus (C_r \cup C_s) : \delta_i^{\phi} \end{matrix} \tag{6.15}$$

$$x_{t,\tau,i}^{\xi,\mu,\phi} - x_{t-1,\tau,i}^{\xi,\mu,\phi} - \sum_{j \in \Gamma^-(i)} \delta_{ij}^{\phi,\mu} y_{t-1,\tau,ij}^{\xi,\mu,\phi} = 0 \qquad \begin{matrix} \forall \xi \in \Xi, \forall \mu \in OD, \\ \forall \phi \in \Phi(\mu), \forall t \\ \in T^*, \forall \tau \in T_D, \forall i \\ \in C_s : \delta_i^{\phi} \end{matrix} \tag{6.16}$$

$$x_{t,\tau,i}^{\xi,\mu,\phi} - x_{t-1,\tau,i}^{\xi,\mu,\phi} + \sum_{j\in\Gamma^+(i)} \delta_{ij}^{\phi,\mu} y_{t-1,\tau,ij}^{\xi,\mu,\phi} = \pi_\tau^{\mu,\phi} D_\tau^{\xi,\mu} \quad \forall \xi \in \Xi, \forall \mu \in OD, \forall \phi \in \Phi(\mu), \forall t \in T^*, \forall \tau \in T_D, \forall i \in C_r : \delta_i^\phi \quad (6.17)$$

$$\sum_{j\in\Gamma^+(i)} \delta_{ij}^{\phi,\mu} y_{t,\tau,ij}^{\xi,\mu,\phi} + x_{t,\tau,i}^{\xi,\mu,\phi} \le 0 \quad \forall \xi \in \Xi, \forall \mu \in OD, \forall \phi \in \Phi(\mu), \forall t \in T^*, \forall \tau \in T_D, \forall i \in C \backslash C_s \quad (6.18)$$

$$\sum_{\mu\in OD} \sum_{\phi\in\Phi(\mu)} \sum_{\tau\in T_D} \left(\sum_{i\in\Gamma^-(j)} \delta_{ij}^{\phi} y_{t,\tau,ij}^{\xi,\phi} + x_{t,\tau,i}^{\xi\phi} \right) \le N_{t,i} + g_i \quad \forall \xi \in \Xi, \forall t \in T, \forall i \in C \backslash (C_r \cup C_s) \quad (6.19)$$

$$\sum_{\mu\in OD} \sum_{\phi\in\Phi(\mu)} \sum_{\tau\in T_D} \sum_{j\in\Gamma^-(j)} \delta_{ij}^{\mu,\phi} y_{t,\tau,ij}^{\xi,\mu,\phi} \le Q_{t,i} + \alpha_i g_i, \quad \forall \xi \in \Xi, \forall t \in T, \forall i \in C \backslash C_s \quad (6.20)$$

$$\sum_{\mu\in OD} \sum_{\phi\in\Phi(\mu)} \sum_{\tau\in T_D} \sum_{j\in\Gamma^+(i)} \delta_{ij}^{\mu,\phi} y_{t,\tau,ij}^{\xi,\mu,\phi} \le Q_{t,i} + \alpha_i g_i, \quad \forall \xi \in \Xi, \forall t \in T, \forall i \in C \backslash C_r \quad (6.21)$$

$$\sum_{\phi\in\Phi(\mu)} \pi_\tau^{\mu,\phi} = 1 \quad \forall \mu \in OD, \forall \tau \in T_D \quad (6.22)$$

$$y_{0,\tau,ij}^{\xi,\mu,\phi} = 0 \quad \forall \xi \in \Xi, \forall \mu \in OD, \forall \phi \in \Phi(\mu), \forall \tau \in T_D, \forall (i,j) \in E \quad (6.23)$$

$$y_{t,\tau,ij}^{\xi,\mu,\phi} \ge 0 \quad \forall \xi \in \Xi, \forall \mu \in OD, \forall \phi \in \Phi(\mu), \forall t \in T, \forall \tau \in T_D, \forall (i,j) \in E \quad (6.24)$$

$$x_{t,\tau,i}^{\xi,\mu,\phi} \ge 0 \quad \forall \xi \in \Xi, \forall \mu \in OD, \forall \phi \in \Phi(\mu), \forall t \in T, \forall \tau \in T_D, \forall i \in C \quad (6.25)$$

$$\pi_\tau^{\mu,\phi} \ge 0 \quad \forall \mu \in OD, \forall \phi \in \Phi(\mu), \forall \tau \in T_D \quad (6.26)$$

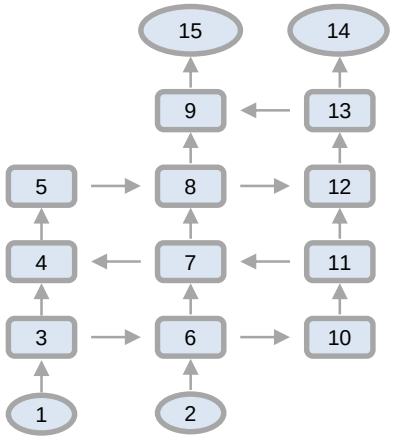
As previously stated, unlike traditional approaches to the NDP, the application presented in this work does not require significant additional computational complexity as compared to the base model. Instead, the addition consists of only a single constraint and one decision variable for each cell.

6.4 Model demonstration

This section demonstrates the LP NDP model described in the previous section. First, results are presented on a small cell network for demonstration purposes. Then results are presented on the Nguyen Dupius and Sioux Falls networks.

6.4.1 Model Demonstration: cell network

A network consisting of 15 cells, 2 origins, and 2 destinations was selected in order to isolate the impact of the strategic approach on infrastructure expansion decisions. The origin cells are 1 and 2, while the destination cells are 14 and 15. This network consists of two arterial corridors and a small "highway" segment with greater flow and capacity.



Cells	N_i	Q_i
3,4,5,9,10,11,12,13	10	8
6,7,8	15	12

Figure 6:2 Demonstration cell network and cell parameters

This network contains three paths for each of the four possible origin-destination pairs. Tables 6-1 and 6-2 contains the demand parameters for this demonstration, including the aggregate demand in each demand scenario, the proportions of the total demand for each OD pair, and then the proportions of the OD demand that leave at each of the four included departure times; for simplicity, the departure time proportions are assumed to be the same for each demand scenario, following a "peak" pattern, but the total demand in each scenario as well as the proportions of the demand for each OD pair for each scenario are changing. This demonstration includes three demand scenarios representing the average congestion case, the lightly congested case, and the heavily congested case respectively. Forty time periods were simulated to ensure all demand was able to exit the network even in the heavily congested case.

Results are demonstrated under a varying budget, where the cost is $\beta_i = \$10{,}000$ add one unit of capacity to a cell. The budget was varied from 0-

200,000, or the equivalent of adding 20 units of capacity. Figure 6:3 displays the results for total system travel time (including all demand scenarios) corresponding with the varying budget for the cases of considering one demand scenario (equivalent to the expected demand case), considering the average and lightly congested demand scenarios, and considering all three demand scenarios.

Table 6-1 Demand parameters for the demonstration network

Demand Scenario	Total Demand	Proportion of demand for OD pair RS			
		(1,14)	(1,15)	(2,14)	(2,15)
Average congestion case	210	0.3	0.2	0.2	0.3
Light congestion case	190	0.4	0.25	0.2	0.15
Heavy congestion case	230	0.43	0.2	0.17	0.2

Table 6-2 Departure time parameters for the demonstration network

OD Pair	Proportion of OD demand at departure time			
	$\tau = 1$	$\tau = 2$	$\tau = 3$	$\tau = 4$
(1,14)	0.35	0.35	0.12	0.18
(1,15)	0.15	0.55	0.2	0.1
(2,14)	0.3	0.5	0.15	0.05
(2,15)	0.25	0.45	0.15	0.15

Figure 6:3 shows a nonlinear decrease in total travel time with increasing budget. Additional analysis shows that including one, or two, or three, demand scenarios in order to solve for the optimal capacity additions in the demonstration network results in selecting generally (but not always) the same set of links. The largest percentage of the budget is added to cell 8 in all cases. However, the exact amount to be added to each cell differs, indicating that accounting for strategic behaviour will result in a different set of project rankings.

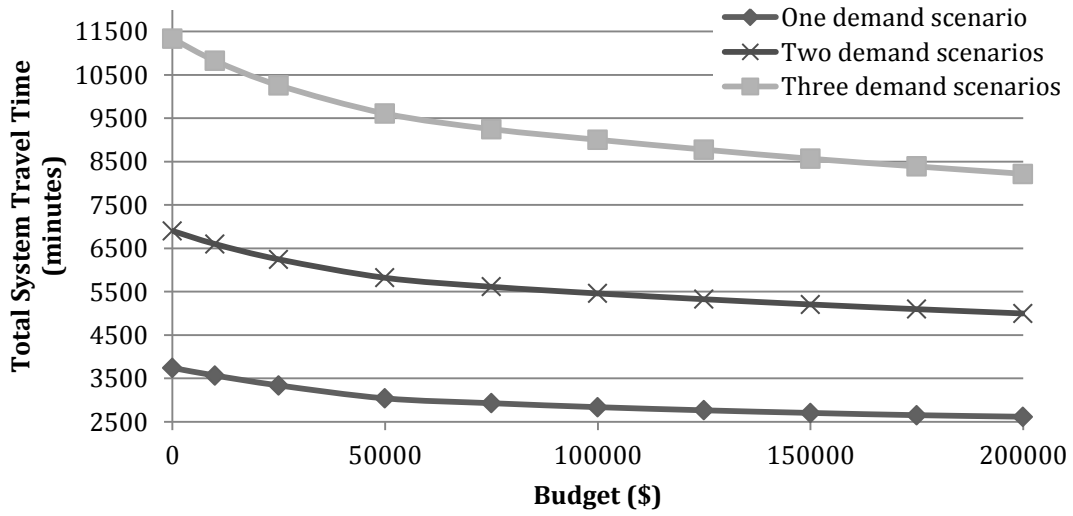


Figure 6:3 Total system travel time for the StrSODTA NDP under a varying budget

6.4.2 Model demonstration: medium networks

This section demonstrates the NDP StrSO model on the Nguyen Dupius and Sioux Falls networks. The networks were decomposed and solved using the same approach that is described in 5.4.1. The only change in the current approach was to the AMPL model file, which was adapted to include the additional budget

constraint, the new decision variable z, and the proportional parameter that represented how additional density also affected additional flow. Table 6-3 shows the expected demand for the Nguyen Dupius network. Note that the same data files can be used for the NDP StrSODTA model as were used in the previous chapter.

Table 6-3 Expected demand for the Nguyen Dupius network

	2	**3**
1	1400	1100
4	1200	1600

Table 6-4 reveals the results for the network design problem on the ND network for the case where $\Xi = 3$. The table shows the cases where the budget adds 0, 25, 50, 75, 100, and 125 "units" of density to the network. In this case, the model always identified the same set of cells to improve, cells 54, 55, and 56. The difference in the budget scenarios is the amount of capacity added to the cells and the corresponding reduction in $E(\Xi)$.

Table 6-4 Results for Nguyen Dupius network, correlated demand, where $\boldsymbol{\Xi = 3}$

$\boldsymbol{B}$	**0**	**25**	**50**	**75**	**100**	**125**
54, 55, 56	-	33.3	16.7	25.0	33.3	41.7
$\boldsymbol{TT(1)}$	5990.9	5769.8	5589.4	5423.8	5271.3	5130.2
$\boldsymbol{TT(2)}$	7167.6	6899.2	6677.1	6473.4	6286.1	6112.8

$TT(3)$	3952.8	3811.7	3701.6	3600.3	3506.6	3420.5
$E(\Xi)$	5703.8	5493.6	5322.7	5165.8	5021.3	4887.8
ΔE		3.7%	6.7%	9.4%	12.0%	14.3%

Table 6-5 shows the identical experiment for the case where demand is independent and $\Xi = 3$. Again, the model identified the cells 54, 55, and 56 in every case, varying how much capacity was added in each case. Capacity was always evenly distributed between the three cells.

Table 6-5 Results for Nguyen Dupius network, independent demand, where $\Xi = 3$

B	0	25	50	75	100	125
$54, 55, 56$	0.0	33.3	16.7	25.0	33.3	41.7
$TT(1)$	5994.9	5776.6	5595.0	5431.1	5276.4	5137.0
$TT(2)$	9429.1	9070.9	8770.1	8493.5	8240.5	8002.4
$TT(3)$	1083.8	1079.3	1074.4	1068.5	1068.3	1072.7
$E(\Xi)$	5502.6	5308.9	5146.5	4997.7	4861.7	4737.4
ΔE		6.9%	9.8%	12.4%	14.8%	16.9%

However, adding unlimited amounts of capacity to the same cells may not be a desirable solution, nor a realistic one. Therefore, the amount of density that was added to each cell was limited to 30. In this case, an increase of 30 vehicles for

every 120 second time step is the equivalent to 1800 vehicles per hour. The expected demand for the Sioux Falls network is replicated in Table 6-6.

Table 6-6 Expected demand for the Sioux Falls network

	20	**18**	**7**	**6**
1	0	810	420	900
2	430	0	1160	780
3	730	250	0	270
13	1200	260	590	0

Table 6-7 shows the system level results for the Sioux Falls network for the cases where $\Xi = 1,2,3,4,5$. The total budget was $B = 200$.

Table 6-7 Results for Sioux Falls network with correlated demand

	$\Xi = 5$	$\Xi = 4$	$\Xi = 3$	$\Xi = 2$	$\Xi = 1$
$TT(1)$	3290.7	3291.9	3266.8	3267.7	3239.2
$TT(2)$	3722.4	3720.6	3700.6	3697.2	
$TT(3)$	2489.3	2492.5	2464.5		
$TT(4)$	4822.6	4816.5			
$TT(5)$	1772.8				
$E(\Xi)$	3219.5	3580.4	3143.9	3482.5	3239.2

Table 6-8 shows the design project selections for each of the five demand scenario cases for the Sioux Falls network, where the budget was $B = 200$. The project selection was not the same for differing number of demand scenarios, indicating

that optimal project selection will change depending on the demand uncertainty accounted for in the modelling approach. The case where $\Xi = 1$ is the "expected demand" case, where the strategic demand scenarios are not accounted for. The project selection is different in this case than it is in any of the strategic demand cases.

Table 6-8 Design project results for the Sioux Falls network: perfectly correlated demand

Cell:	**8**	**9**	**10**	**26**	**27**	**36**	**37**	**100**	**101**	**182**	**183**
$\Xi = 1$	30.0	30.0	30.0	4.4	4.4	-	-	30.0	30.0	20.6	20.6
$\Xi = 2$	30.0	30.0	30.0	-	-	3.8	3.8	30.0	30.0	21.2	21.2
$\Xi = 3$	30.0	30.0	30.0	-	-	1.6	1.6	30.0	30.0	23.4	23.4
$\Xi = 4$	30.0	30.0	30.0	20.3	20.3	4.7	4.7	30.0	30.0	-	-
$\Xi = 5$	30.0	30.0	30.0	19.5	19.5	5.5	5.5	30.0	30.0	-	-

Table 6-9 repeats the Sioux Falls network experiment, where the number of demand scenarios were tested for a fixed budget of $B = 200$ for the *independent* demand case. This is the same demand data that was used in Table 5-12; recall that the demand is not the same between design scenarios, e.g., $TT(2)$ indicates a different demand for each of the columns (and the results are not directly comparable). However, $TT(1)$ is the same expected demand case which does not change; thus, the difference between 3506 seconds in the 4 demand scenario case

compared to 3295 in the 3 demand scenario case resulted from the different optimal path proportions in each case. However, the changes in optimal path proportions result from the model being solved for a different set of values for each demand scenario.

Table 6-9 Results for Sioux Falls network with independent demand

	$\Xi = 5$	$\Xi = 4$	$\Xi = 3$	$\Xi = 2$	$\Xi = 1$
$\boldsymbol{TT(1)}$	3344.3	3506.7	3295.2	3321.4	3239.2
$\boldsymbol{TT(2)}$	4490.3	4753.7	4374.7	4643.3	
$\boldsymbol{TT(3)}$	1398.6	1385.6	1418.7		
$\boldsymbol{TT(4)}$	6215.1	10073.9			
$\boldsymbol{TT(5)}$	1899.5				
$\boldsymbol{E(\Xi)}$	3469.6	4930.0	3029.5	3982.3	3239.2

Finally, the design project results are presented for the independent demand case on the Sioux Falls network in Table 6-10. Across all the demand scenarios (shown in rows), there were no new cells selected for capacity enhancement as compared to the correlated demand case. In other words, it was a similar set of optimal cells to add capacity in all cases, which is line with intuition and results from the static case. Finding the optimal locations to which to add capacity can be narrowed to a smaller set in the context of the entire network. However, the *exact* cells selected in each case (within the set), as well as the amount of capacity added to each cell, changed in the independent demand case. These results suggest that the difficulty in ranking network design projects may be in determining the proper model inputs

(specifically the appropriate demand and demand scenarios) because demand has a greater influence on the optimal capacity expansion in the network.

Table 6-10 Design project results for the Sioux Falls network: independent demand

	8	9	10	26	27	36	37	100	101	182	183
$\Xi = 1$	30.0	30.0	30.0	4.4	4.4	-	-	30.0	30.0	20.6	20.6
$\Xi = 2$	30.0	30.0	30.0	25.4	25.4	12.5	12.5	17.1	17.1	-	-
$\Xi = 3$	30.0	30.0	30.0	19.7	19.7	7.5	7.5	27.7	27.7	-	-
$\Xi = 4$	30.0	30.0	30.0	21.9	21.9	3.1	3.1	30.0	30.0	-	-
$\Xi = 5$	30.0	30.0	30.0	22.2	22.2	2.8	2.8	30.0	30.0		

6.5 Concluding Remarks

This work proposed a system optimal network design model that accounts for stochastic demand using a strategic approach. In the strategic approach, the optimal path *proportions* are assigned so as to minimize the expected total system travel time in all demand scenarios. However, the actual path flows that will be manifested for a given demand realization will not consist of a system optimal solution for that individual demand realization. This work has presented an application of the StrSO DTA LP model to the network design problem, and results demonstrate the differences between accounting for the strategic approach and simply the expected demand case. The system optimal model in this work

represents a lower bound of the user equilibrium problem, but future work will examine a user optimal approach to representing strategic flows.

Future research directions focused on developing scalable versions of the problem were also discussed. Decomposition methods are conceptually promising in reducing the overall size of the LP solved, while path generation and path-flow extraction techniques could curb the effect of the path-based formulation on complexity.

Chapter 7

Conclusion and Future Directions

Transport planning models play a critical role informing policy and decision-making. However, due to the immense complexity in the underlying physical process, advances to the planning models, particularly those that account for sources of uncertainty and stochasticity, continue to be a vital pursuit for researchers.

This thesis proposed a framework that accounted for day-to-day uncertainty in traffic assignment models known as the *strategic* approach. The name of this method stems from one of the core assumptions: users employ a strategy to make a route choice. In this thesis, the day-to-day demand was assumed to be a random variable with a known distribution, which users also have knowledge of (gained through past experience travelling the network). The basic assumption in this framework is that users are homogeneous and risk neutral, and therefore they employ a strategy to minimize their expected travel time. This is an extension over

deterministic approaches, in which users choose a route to minimize a total travel time that does not account for any system variability.

The strategic framework results in a two-stage model. In the first stage, users employ their strategy and choose their expected least cost path, which they do not deviate from regardless of experienced travel conditions. In the second stage, users travel on their chosen route and network conditions manifest. One of the key points of the strategic framework is that the second stage travel realization will result in an equilibrium based on flow proportions. While the route choice of travellers in the second stage remains fixed, the number of people who choose to travel will change and therefore the network conditions and corresponding network performance metrics will vary.

The strategic approach encompasses numerous advantages. Based on the use of a travel demand distribution, the variation in travel time both on the link and the system levels can be quantified and compared. Therefore, the strategic approach could be used to identify areas in the network that are particularly unreliable. One of the most challenging aspects of transport planning models lie in the need to apply the theoretical models on practically-sized problems, which due to the core assumptions of the proposed framework, is relatively straightforward in the strategic approach. Additionally, planning models are relevant and useful because they can be applied to numerous useful applications. Again, the assumptions at the core of the proposed framework make the modelling

approaches ideally suited for numerous applications and future extensions such as tolling and network design.

Chapters 2, 3 and 4 of this thesis explored time invariant strategic modelling approaches. In addition to the user equilibrium model in which the strategy was based on myopic user behaviour, this thesis introduced two variations, in which the strategies were based on minimize expected total system travel time and variation of total system travel time.

Additionally, Chapter 3 explores a first best pricing application of the strategic framework, including the introduction of a marginal social cost based pricing model. While the strategic approach accounts for day-to-day demand uncertainty, uncertainty in the forecasted long term planning scenario presents another challenge for planners and researchers. Therefore, this thesis also proposed a method to compare model predictions under short term uncertainty, and additionally under long term planning scenario uncertainty. Results show that if long term uncertainty is not accounted for, model predictions may overestimate the impact of a pricing scheme. Additionally, long term demand uncertainty is shown to have a significant impact on the robustness of the network.

Finally, Chapter 4 explored a network design application. In addition to day-to-day demand uncertainty, there also exists an important uncertainty in the daily, operational capacity that may have a significant impact on user route choice, and therefore on the project rankings that result from an equilibrium traffic assignment

modelling evaluation. Therefore, Chapter 4 employed an extension to the strategic user equilibrium approach that includes day-to-day capacity uncertainty, where the capacity on each link is a random variable with a known distribution. Users then chose an expected least cost route based on this distribution, and then a day-to-day scenario realization takes place where they decide whether to travel. A network design model including day-to-day demand and capacity uncertainty was proposed, along with a solution method based on a tailored genetic algorithm. Results demonstrated the complexity of the network design problem under uncertainty, as well as the importance of considering multiple sources of uncertainty when ranking network design projects.

In Chapters 5 and 6 of this thesis, the assumption of time invariance was relaxed. Dynamic network modelling approaches are important because they can capture network effects that are fundamentally non-static in nature, such as queueing and backwards wave propagation. However, dynamic traffic assignment models increase significantly in complexity as compared with their static counterparts and are under-utilized in practice.

Chapter 5 introduced the strategic system optimal dynamic traffic assignment model. This model is powerful due to the linear programming formulation at its core, which allows well-established solution methods to be applied. The model proposed in Chapters 5 and 6 is based on an established linear programming based system optimal model that does not account for any sources of

uncertainty. While the system optimal assumptions at the foundation of the model in Chapters 5 and 6 mean that it does not describe user behaviour, it is still a useful model to determine important network characteristics and as a lower bound on the total travel time in a network. Chapter 5 introduces the model and solution approach to transform the static planning data used for testing purposes in Chapter 2 to the dynamic network problem. This will allow the models, evaluations, and performance metrics to be roughly compared which may have interesting implications for both specific networks and for static versus dynamic modelling in general.

As with all models, a number of defendable, yet restrictive, assumptions were required in this thesis in order to make the models tractable and to reduce the problem complexity. In Chapters 2, 3, and 4, these assumptions included a homogeneous, risk neutral user populations, a lognormally distributed demand, a specifically structured travel cost function, a first best tolling structure, and a specific design scenario. The assumptions in Chapters 5 and 6 included a system, not user, optimal user behaviour, a specified stochastic demand based on scenarios, a limited number of paths and departure times, a high discretization time step for the cell transmission model, and network design capacity additions that were specific to a cell, not the entire link.

Each of these assumptions points the way towards an interesting topic for future research. For the time invariant strategic model, the next step may be

relaxing the risk neutral assumption, i.e., incorporating a late arrival penalty in the route choice decision-making. This would incorporate a measure of reliability into user route choice, where a person would be less likely to choose a route if it had a substantial possibility of causing a late arrival. Additionally error in user perception and multiple user classes could be included.

Traffic assignment is only one piece of the transport planning process, in particular the four step planning process. Another avenue of future research would be to integrate a strategic component into the trip generation, trip distribution, and mode choice models, as well as the traffic assignment framework that was proposed in this thesis. An integrated, comprehensive framework would allow for the use of a feedback mechanism, where the estimated costs from the strategic traffic assignment model could lead to an adjusted trip distribution model, which could then adjust the forecasted travel demand distribution as an iterative process.

One of the benefits of the strategic framework is that it could be applied to practically sized problems, such as the Sydney regional network, composed of approximately 60,000 links and an expected demand of 1.3 million, shown in Figure 7:1. The next step to achieving this goal and another avenue for future research may be to adjust the solution approach to account for the tailing effect of the Frank Wolfe method. With an adjusted solution approach, the strategic framework could be solved more quickly on substantially sized networks.

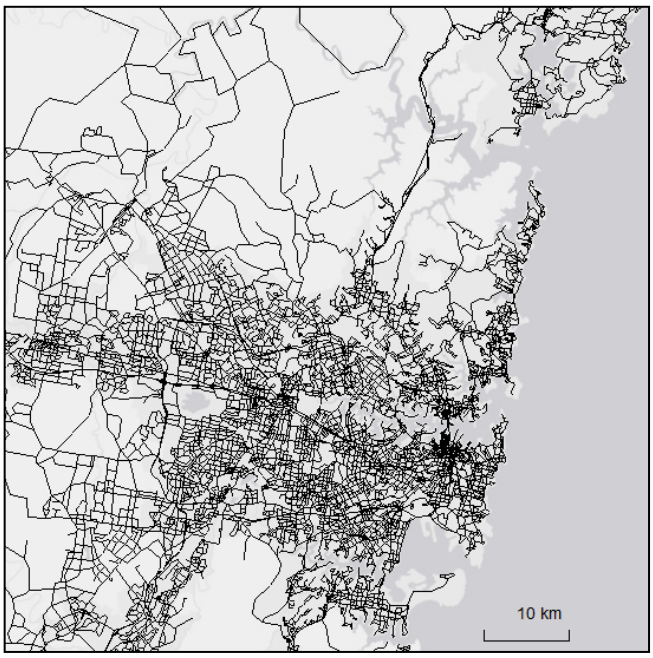


Figure 7:1 Sydney city network, a large-scale network that the StrUE model may be applied on in the future

The dynamic strategic model shows equal promise in terms of future research. As discussed in the concluding remarks of Chapters 5 and 6, issues like a decomposition scheme to reduce the size of the linear program could improve the performance of the model. Additionally, the variance of total system travel time is a quadratic equation that could be used as an objective function that would lead to a quadratic program, which can still be solved by CPLEX. The variance of total travel time would provide an interesting measure of reliability, similar to what was discussed in Chapter 2, that has never been examined on a dynamic network.

The topics discussed in this thesis will continue yielding prolific research possibilities and lead to deployable models for practitioners in the future.

BIBLIOGRAPHY

Ahuja, R. K., Magnanti, T. L., & Orlin, J. B. (1993). Network flows: theory, algorithms, and applications.

Andreatta, G., & Romeo, L. (1988). Stochastic shortest paths with recourse. *Networks, 18*(3), 193-204.

Arnott, R., De Palma, A., & Lindsey, R. (1991). Does providing information to drivers reduce traffic congestion? *Transportation Research Part A: General, 25*(5), 309-18.

Asakura, Y., & Kashiwadani, M. (1991). *Road network reliability caused by daily fluctuation of traffic flow.* Paper presented at the PTRC Summer Annual Meeting, 19th, 1991, University of Sussex, United Kingdom.

Asensio, J., & Matas, A. (2008). Commuters' valuation of travel time variability. *Transportation Research Part E: Logistics and Transportation Review, 44*(6), 1074-85.

Bain, R. (2009). Error and optimism bias in toll road traffic forecasts. *Transportation, 36*(5), 469-82.

Bar-Gera, H. (2002). Origin-based algorithm for the traffic assignment problem. *Transportation Science, 36*(4), 398-417.

Bar-Gera, H. (2010). Traffic assignment by paired alternative segments. *Transportation Research Part B: Methodological, 44*(8), 1022-46.

Bar-Gera, H. (2012). Transportation Test Problems. Retrieved February 29, 2013, from http://www.bgu.ac.il/~bargera/tntp/

Beckmann, M., McGuire, C., & Winsten, C. B. (1956). Studies in the Economics of Transportation.

Bell, M. G. (2000). A game theory approach to measuring the performance reliability of transport networks. *Transportation Research Part B: Methodological, 34*(6), 533-45.

Boyce, D., & Janson, B. (1980). A discrete transportation network design problem with combined trip distribution and assignment. *Transportation Research Part B: Methodological, 14*(1), 147-54.

Boyles, S. D., Kockelman, K. M., & Travis Waller, S. (2010). Congestion pricing under operational, supply-side uncertainty. *Transportation Research Part C: Emerging Technologies, 18*(4), 519-35. doi: http://dx.doi.org/10.1016/j.trc.2009.09.006

Cantarella, G. E., & Cascetta, E. (1995). Dynamic processes and equilibrium in transportation networks: towards a unifying theory. *Transportation Science, 29*(4), 305-29.

Chen, A., Yang, H., Lo, H. K., & Tang, W. H. (2002). Capacity reliability of a road network: an assessment methodology and numerical results. *Transportation Research Part B: Methodological, 36*(3), 225-52.

Chen, A., & Zhou, Z. (2010). The α-reliable mean-excess traffic equilibrium model with stochastic travel times. *Transportation Research Part B: Methodological, 44*(4), 493-513.

Chen, A., Zhou, Z., Chootinan, P., Ryu, S., Yang, C., & Wong, S. C. (2011). Transport Network Design Problem under Uncertainty: A Review and New Developments. *Transport Reviews, 31*(6), 743-68. doi: 10.1080/01441647.2011.589539

Chootinan, P., Wong, S., & Chen, A. (2005). A reliability-based network design problem. *Journal of Advanced Transportation, 39*(3), 247-70.

Chriqui, C., & Robillard, P. (1975). Common bus lines. *Transportation Science, 9*(2), 115-21.

Clark, S., & Watling, D. (2005). Modelling network travel time reliability under stochastic demand. *Transportation Research Part B: Methodological, 39*(2), 119-40.

Connors, R. D., & Sumalee, A. (2009). A network equilibrium model with travellers' perception of stochastic travel times. *Transportation Research Part B: Methodological, 43*(6), 614-24.

Daganzo, C. F. (1994). The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory. *Transportation Research Part B: Methodological, 28*(4), 269-87.

Daganzo, C. F. (1995). The cell transmission model, part II: network traffic. *Transportation Research Part B: Methodological, 29*(2), 79-93.

Daganzo, C. F., & Sheffi, Y. (1977). On stochastic models of traffic assignment. *Transportation Science, 11*(3), 253-74.

Damberg, O., Lundgren, J. T., & Patriksson, M. (1996). An algorithm for the stochastic user equilibrium problem. *Transportation Research Part B: Methodological, 30*(2), 115-31.

Dantzig, G. B., Harvey, R. P., Lansdowne, Z. F., Robinson, D. W., & Maier, S. F. (1979). Formulating and solving the network design problem by decomposition. *Transportation Research Part B: Methodological, 13*(1), 5-17.

Davidson, P. (2011). *Modelling toll roads-where have we gone wrong.* Paper presented at the Australian Transport Research Forum.

Deb, K., Pratap, A., Agarwal, S., & Meyarivan, T. (2002). A fast and elitist multiobjective genetic algorithm: NSGA-II. *Evolutionary Computation, IEEE Transactions on, 6*(2), 182-97.

Dial, R. B. (1971). A probabilistic multipath traffic assignment model which obviates path enumeration. *Transportation research, 5*(2), 83-111.

Dial, R. B. (2006). A path-based user-equilibrium traffic assignment algorithm that obviates path storage and enumeration. *Transportation Research Part B: Methodological, 40*(10), 917-36.

Dixit, V., Gardner, L. M., & Waller, S. T. (2013). *Strategic User Equilibrium Assignment Under Trip Variability.* Paper presented at the Transportation Research Board 92nd Annual Meeting.

Do Chung, B., Yao, T., Xie, C., & Thorsen, A. (2011). Robust optimization model for a dynamic network design problem under demand uncertainty. *Networks and Spatial Economics, 11*(2), 371-89.

Duthie, J., & Waller, S. T. (2008). Incorporating environmental justice measures into equilibrium-based network design. *Transportation Research Record: Journal of the Transportation Research Board, 2089*(-1), 58-65.

Duthie, J. C., Unnikrishnan, A., & Waller, S. T. (2011). Influence of demand uncertainty and correlations on traffic predictions and decisions. *Computer-Aided Civil and Infrastructure Engineering, 26*(1), 16-29.

Fan, Y., & Nie, Y. (2006). Optimal routing for maximizing the travel time reliability. *Networks and Spatial Economics, 6*(3-4), 333-44.

Ferguson, E. M., Duthie, J., & Travis Waller, S. (2012). Comparing Delay Minimization and Emissions Minimization in the Network Design Problem. *Computer-Aided Civil and Infrastructure Engineering*.

Flyvbjerg, B. (2008). Curbing optimism bias and strategic misrepresentation in planning: Reference class forecasting in practice. *European Planning Studies, 16*(1), 3-21.

Frank, M., & Wolfe, P. (1956). An algorithm for quadratic programming. *Naval research logistics quarterly, 3*(1-2), 95-110.

Friesz, T. L., Bernstein, D., Smith, T. E., Tobin, R. L., & Wie, B. (1993). A variational inequality formulation of the dynamic network user equilibrium problem. *Operations Research, 41*(1), 179-91.

Gao, S., & Chabini, I. (2006). Optimal routing policy problems in stochastic time-dependent networks. *Transportation Research Part B: Methodological, 40*(2), 93-122.

Gardner, L., Unnikrishnan, A., & Waller, S. (2008a). Robust Pricing of Transportation Networks Under Uncertain Demand. *Transportation Research Record: Journal of the Transportation Research Board, 2085*(-1), 21-30. doi: 10.3141/2085-03

Gardner, L. M., Boyles, S. D., & Waller, S. T. (2011). Quantifying the benefit of responsive pricing and travel information in the stochastic congestion pricing problem. *Transportation Research Part A: Policy and Practice, 45*(3), 204-18.

Gardner, L. M., Unnikrishnan, A., & Waller, S. T. (2008b). Robust pricing of transportation networks under uncertain demand. *Transportation Research Record: Journal of the Transportation Research Board, 2085*(-1), 21-30.

Gardner, L. M., Unnikrishnan, A., & Waller, S. T. (2010). Solution Methods for Robust Pricing of Transportation Networks under Uncertain Demand. *Transportation Research Part C: Emerging Technologies, 18*(5), 656-67.

Gentile, G. (2014). Local User Cost Equilibrium: a bush-based algorithm for traffic assignment. *Transportmetrica A: Transport Science, 10*(1), 15-54.

Google C++ Style Guide.). Retrieved July 4, 2014, from http://google-styleguide.googlecode.com/svn/trunk/cppguide.html

Hamdouch, Y., & Lawphongpanich, S. (2008). Schedule-based transit assignment model with travel strategies and capacity constraints. *Transportation Research Part B: Methodological, 42*(7), 663-84.

Han, S., & Heydecker, B. (2006). Consistent objectives and solution of dynamic user equilibrium models. *Transportation Research Part B: Methodological, 40*(1), 16-34.

Hazelton, M. L. (1998). Some remarks on stochastic user equilibrium. *Transportation Research Part B: Methodological, 32*(2), 101-08.

Hearn, D. W., & Ramana, M. V. (1997). *Solving congestion toll pricing models*: Citeseer.

Horowitz, J. L. (1984). The stability of stochastic equilibrium in a two-link transportation network. *Transportation Research Part B: Methodological, 18*(1), 13-28.

Jayakrishnan, R., Tsai, W. T., Prashker, J. N., & Rajadhyaksha, S. (1994). A faster path-based algorithm for traffic assignment. *University of California Transportation Center*.

Jiang, Y., & Szeto, W. (2014). Time-Dependent Transportation Network Design that Considers Health Cost. *Transportmetrica A: Transport Science*(just-accepted), 1-48.

Karoonsoontawong, A., & Waller, S. (2005). Comparison of System- and User-Optimal Stochastic Dynamic Network Design Models Using Monte Carlo Bounding Techniques. *Transportation Research Record: Journal of the Transportation Research Board, 1923*(-1), 91-102.

Karoonsoontawong, A., & Waller, S. T. (2006). Dynamic continuous network design problem: linear bilevel programming and metaheuristic approaches.

Transportation Research Record: Journal of the Transportation Research Board, 1964(-1), 104-17.

Lam, W. H. K., Shao, H., & Sumalee, A. (2008). Modeling impacts of adverse weather conditions on a road network with uncertainties in demand and supply. *Transportation Research Part B: Methodological, 42*(10), 890-910.

Lawphongpanich, S., & Hearn, D. W. (2004). An MPEC approach to second-best toll pricing. *Mathematical Programming, 101*(1), 33-55.

Leblanc, L. J. (1975). An algorithm for the discrete network design problem. *Transportation Science, 9*(3), 183-99.

LeBlanc, L. J., Morlok, E. K., & Pierskalla, W. P. (1975). An efficient approach to solving the road network equilibrium traffic assignment problem. *Transportation Research, 9*(5), 309-18.

Lemp, J. D., & Kockelman, K. M. (2009). Understanding and Accommodating Risk and Uncertainty in Toll Road Projects. *Transportation Research Record: Journal of the Transportation Research Board, 2132*(1), 106-12.

Li, H., Bliemer, M. C., & Bovy, P. H. (2008). Network reliability-based optimal toll design. *Journal of Advanced Transportation, 42*(3), 311-32.

Li, Y., Waller, S. T., & Ziliaskopoulos, T. (2003). A decomposition scheme for system optimal dynamic traffic assignment models. *Networks and Spatial Economics, 3*(4), 441-55.

Li, Z.-C., Lam, W. H. K., Wong, S. C., & Sumalee, A. (2012). Environmentally Sustainable Toll Design for Congested Road Networks with Uncertain Demand. *International Journal of Sustainable Transportation, 6*(3), 127-55.

Lighthill, M. J., & Whitham, G. B. (1955). On kinematic waves. II. A theory of traffic flow on long crowded roads. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences, 229*(1178), 317-45.

Lin, D.-Y. (2011). A Dual Variable Approximation-Based Descent Method for a Bi-level Continuous Dynamic Network Design Problem. *Computer-Aided Civil and Infrastructure Engineering, 26*(8), 581-94.

Lin, D.-Y., Karoonsoontawong, A., & Waller, S. T. (2011). A Dantzig-Wolfe Decomposition Based Heuristic Scheme for Bi-level Dynamic Network Design Problem. *Networks and Spatial Economics, 11*(1), 101-26.

Lin, D.-Y., Unnikrishnan, A., & Waller, S. (2009). A genetic algorithm for bi-level linear programming dynamic network design problem. *Transportation letters, 1*(4), 281.

Lo, H. K., Luo, X., & Siu, B. W. (2006). Degradable transport network: travel time budget of travelers with heterogeneous risk aversion. *Transportation Research Part B: Methodological, 40*(9), 792-806.

Lo, H. K., & Tung, Y.-K. (2003). Network with degradable links: capacity analysis and design. *Transportation Research Part B: Methodological, 37*(4), 345-63.

Magnanti, T. L., & Wong, R. T. (1984). Network design and transportation planning: Models and algorithms. *Transportation Science, 18*(1), 1-55.

Marcotte, P., Nguyen, S., & Schoeb, A. (2004). A strategic flow model of traffic assignment in static capacitated networks. *Operations Research, 52*(2), 191-212.

Merchant, D. K., & Nemhauser, G. L. (1978a). A model and an algorithm for the dynamic traffic assignment problems. *Transportation Science, 12*(3), 183-99.

Merchant, D. K., & Nemhauser, G. L. (1978b). Optimality conditions for a dynamic traffic assignment model. *Transportation Science, 12*(3), 200-07.

Miller-Hooks, E. D., & Mahmassani, H. S. (2000). Least expected time paths in stochastic, time-varying transportation networks. *Transportation Science, 34*(2), 198-215.

Mirchandani, P., & Soroush, H. (1987). Generalized traffic equilibrium with probabilistic travel times and perceptions. *Transportation Science, 21*(3), 133-52.

Newbery, D. M. (1990). *Pricing and congestion: economic principles relevant to pricing roads*: Oxford.

Nie, Y. M. (2010). A class of bush-based algorithms for the traffic assignment problem. *Transportation Research Part B: Methodological, 44*(1), 73-89.

Peeta, S., & Ziliaskopoulos, A. K. (2001). Foundations of dynamic traffic assignment: The past, the present and the future. *Networks and Spatial Economics, 1*(3-4), 233-65.

Phibbs, P. (2008). Driving Alone: Sydney's Cross City Tunnel. *Built Environment, 34*(3), 364-74.

Pigou, A. C. (1920). *The economics of welfare* (Vol. 2): Cosimo, Inc.

Polychronopoulos, G. H., & Tsitsiklis, J. N. (1996). Stochastic shortest path problems with recourse. *Networks, 27*(2), 133-43.

Psaraftis, H. N., & Tsitsiklis, J. N. (1993). Dynamic shortest paths in acyclic networks with Markovian arc costs. *Operations Research, 41*(1), 91-101.

Rey, D., Duell, M., Waller, S. T., & Dixit, V. (2014). *A Path Enumeration Algorithm for Strategic System Optimal Dynamic Traffic Assignment*. Paper presented at the The 5th Dynamic Traffic Assignment Symposium, Salerno, Italy.

Richards, P. I. (1956). Shock waves on the highway. *Operations research, 4*(1), 42-51.

Roughan, M., Greenberg, A., Kalmanek, C., Rumsewicz, M., Yates, J., & Zhang, Y. (2002). *Experience in measuring backbone traffic variability: Models, metrics, measurements and meaning.* Paper presented at the Proceedings of the 2nd ACM SIGCOMM Workshop on Internet measurment.

Senna, L. A. (1994). The influence of travel time variability on the value of time. *Transportation, 21*(2), 203-28.

Shao, H., Lam, W. H., Meng, Q., & Tam, M. L. (2006a). Demand driven travel time reliability-based traffic assignment problem. *Transportation Research Record, 1985*, 220-30.

Shao, H., Lam, W. H., & Tam, M. L. (2006b). A reliability-based stochastic traffic assignment model for network with multiple user classes under uncertainty in demand. *Networks and Spatial Economics, 6*(3-4), 173-204.

Sharma, S., & Mathew, T. V. (2011). Multiobjective network design for emission and travel-time trade-off for a sustainable large urban transportation network. *Environment and Planning-Part B, 38*(3), 520.

Sheffi, Y. (1985). Urban transportation networks: equilibrium analysis with mathematical programming methods.

Sheffi, Y., & Powell, W. B. (1982). An algorithm for the equilibrium assignment problem with random link times. *Networks, 12*(2), 191-207.

Siu, B. W., & Lo, H. K. (2008). Doubly uncertain transportation network: degradable capacity and stochastic demand. *European Journal of Operational Research, 191*(1), 166-81.

Sumalee, A., Watling, D. P., & Nakayama, S. (2006). Reliable network design problem: case with uncertain demand and total travel time reliability. *Transportation Research Record: Journal of the Transportation Research Board, 1964*(1), 81-90.

Sumalee, A., & Xu, W. (2011). First-best marginal cost toll for a traffic network with stochastic demand. *Transportation Research Part B: Methodological, 45*(1), 41-59.

Sun, D., Benekohal, R. F., & Waller, S. T. (2003). *Multiobjective traffic signal timing optimization using non-dominated sorting genetic algorithm.* Paper presented at the Intelligent Vehicles Symposium, 2003. Proceedings. IEEE.

Szeto, W. Y., Jaber, X., & O'Mahony, M. (2010). Time-Dependent Discrete Network Design Frameworks Considering Land Use. *Computer-Aided Civil and Infrastructure Engineering, 25*(6), 411-26.

Ukkusuri, S. V., Mathew, T. V., & Waller, S. T. (2007). Robust Transportation Network Design Under Demand Uncertainty. *Computer-Aided Civil and Infrastructure Engineering, 22*(1), 6-18.

Ukkusuri, S. V., & Patil, G. (2009). Multi-period transportation network design under demand uncertainty. *Transportation Research Part B: Methodological, 43*(6), 625-42.

Ukkusuri, S. V., & Waller, S. T. (2008). Linear programming models for the user and system optimal dynamic network design problem: formulations, comparisons and extensions. *Networks and Spatial Economics, 8*(4), 383-406.

Unnikrishnan, A., & Waller, S. T. (2009). User equilibrium with recourse. *Networks and Spatial Economics, 9*(4), 575-93.

Verhoef, E. T. (2002). Second-best congestion pricing in general networks. Heuristic algorithms for finding second-best optimal toll levels and toll points. *Transportation Research Part B: Methodological, 36*(8), 707-29.

Waller, S., & Ziliaskopoulos, A. (2001). Stochastic Dynamic Network Design Problem. *Transportation Research Record: Journal of the Transportation Research Board, 1771*(-1), 106-13.

Waller, S. T., Fajardo, D., Duell, M., & Dixit, V. (2013). Linear Programming Formulation for Strategic Dynamic Traffic Assignment. *Networks and Spatial Economics*, 1-17.

Waller, S. T., Mouskos, K. C., Kamaryiannis, D., & Ziliaskopoulos, A. K. (2006). A Linear Model for the Continuous Network Design Problem. *Computer-Aided Civil and Infrastructure Engineering, 21*(5), 334-45.

Waller, S. T., & Ziliaskopoulos, A. K. (2002). On the online shortest path problem with limited arc cost dependencies. *Networks, 40*(4), 216-27.

Waller, S. T., & Ziliaskopoulos, A. K. (2006). A chance-constrained based stochastic dynamic traffic assignment model: Analysis, formulation and solution algorithms. *Transportation Research Part C: Emerging Technologies, 14*(6), 418-27.

Wardrop, J. G. (1952). *Some theoretical aspects of road traffic research.* Paper presented at the Proceedings, Institution of Civil Engineers.

Watling, D. (1999). Stability of the stochastic equilibrium assignment problem: a dynamical systems approach. *Transportation Research Part B: Methodological, 33*(4), 281-312.

Watling, D. (2006). User equilibrium traffic network assignment with stochastic travel times and late arrival penalty. *European journal of operational research, 175*(3), 1539-56.

Watling, D., & Hazelton, M. L. (2003). The dynamics and equilibria of day-to-day assignment models. *Networks and Spatial Economics, 3*(3), 349-70.

Wen, T., Gardner, L., Dixit, V., Duell, M., & Waller, S. T. (2014). *A Strategic User Equilibrium Model Incorporating Both Demand and Capacity Uncertainty.* Paper presented at the Transportation Research Board 93rd Annual Meeting.

Wismans, L., Van Berkum, E., & Bliemer, M. (2011a). Modelling externalities using dynamic traffic assignment models: A review. *Transport Reviews, 31*(4), 521-45.

Wismans, L. J., Brands, T., Van Berkum, E. C., & Bliemer, M. C. (2012). Pruning and ranking the Pareto optimal set, application for the dynamic multi-objective network design problem. *Journal of advanced transportation*.

Wismans, L. J., Van Berkum, E. C., & Bliemer, M. C. (2011b). *Comparison of evolutionary multi objective algorithms for the dynamic network design problem.* Paper presented at the Networking, Sensing and Control (ICNSC), 2011 IEEE International Conference on.

Xie, C., & Liu, Z. (2014). On the stochastic network equilibrium with heterogeneous choice inertia. *Transportation Research Part B: Methodological*.

Yang, H., & Bell, M. G. H. (1998). Models and algorithms for road network design: a review and some new developments. *Transport Reviews, 18*(3), 257-78.

Yang, H., & Huang, H.-J. (1998). Principle of marginal-cost pricing: how does it work in a general road network? *Transportation Research Part A: Policy and Practice, 32*(1), 45-54.

Yen, J. Y. (1971). Finding the k shortest loopless paths in a network. *management Science, 17*(11), 712-16.

Yin, Y., & Ieda, H. (2001). Assessing performance reliability of road networks under nonrecurrent congestion. *Transportation Research Record: Journal of the Transportation Research Board, 1771*(1), 148-55.

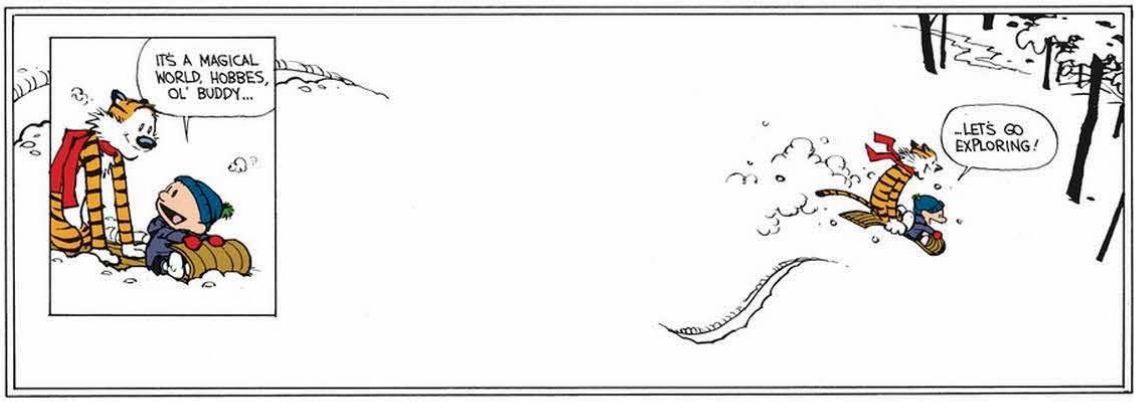
Yin, Y., & Lawphongpanich, S. (2007). A robust approach to continuous network designs with demand uncertainty. *Transportation and Traffic Theory 2007. Papers Selected for Presentation at ISTTT17*.

Zhao, Y., & Kockelman, K. M. (2002). The propagation of uncertainty through travel demand models: an exploratory analysis. *The Annals of regional science, 36*(1), 145-63.

Zheng, H., & Chiu, Y.-C. (2011). A network flow algorithm for the cell-based single-destination system optimal dynamic traffic assignment problem. *Transportation Science, 45*(1), 121-37.

Zhou, Z., & Chen, A. (2008). Comparative analysis of three user equilibrium models under stochastic demand. *Journal of Advanced Transportation, 42*(3), 239-63.

Ziliaskopoulos, A. K. (2000). A linear programming model for the single destination system optimum dynamic traffic assignment problem. *Transportation Science, 34*(1), 37-49.


IT'S A MAGICAL WORLD, HOBBES, OL' BUDDY...
..LET'S GO EXPLORING!