

REPRINT

Stops at traffic signals

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NOTE:

This paper is related to the intersection analysis methodology used in the SIDRA INTERSECTION software. Since the publication of this paper, many related aspects of the traffic model have been further developed in later versions of SIDRA INTERSECTION. Though some aspects of this paper may be outdated, this reprint is provided as a record of important aspects of the SIDRA INTERSECTION software, and in order to promote software assessment and further research.

STOPS AT TRAFFIC SIGNALS

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ABSTRACT

The queue length, delay and stop rate at isolated undersaturated traffic signals are discussed. Each of these three performance measures is considered to consist of a uniform and a random component. The component which allows for randomness is related to the average overflow queue which is calculated explicitly by the Miller method. An equivalent overflow queue formula is given based on Webster's method for calculating the average queue length at the start of the green period. A formula is given for predicting stop rates at isolated undersaturated signals, which is considered to be satisfactory for most practical purposes. The formula may predict stop rates higher than one for flows approaching capacity because of multiple stops in oversaturated cycles. The formula also allows for a reduction due to partial stops, i.e. vehicles which slow down without coming to a complete stop. Formulae for pedestrian stops, delays and queues are given. The discussions include the consideration of fuel consumption calculations and coordinated signal cases. It is suggested that simulation and other methods, and existing control practices, which ignore multiple stops are severely restricted in their prediction of, or effectiveness in reducing, vehicle fuel consumption and pollutant emissions at traffic signals. A numerical example and a method for the calculation of a stop reduction factor to allow for partial stops are given in the Appendices.

NOTATIONS AND DEFINITIONS

c	=	Cycle time in seconds.	N	=	Average queue length at the beginning of the green period, i.e. the maximum queue length during an average cycle.
d	=	Average delay per vehicle in seconds.	q	=	Flow - average number of arrivals per unit time.
D	=	Total delay per unit time in vehicle-hours per hour or vehicle-seconds per second (= qd).	r	=	Effective red time in seconds (= $c-g$).
g	=	Effective green time in seconds.	s	=	Saturation flow - maximum steady rate of departure from the queue during the green period (vehicles per unit time).
h	=	Stop rate - average number of stops per vehicle.	u	=	Green time ratio - the proportion of the cycle which is effectively green, i.e. the ratio of effective green to cycle time (= g/c).
H	=	Total number of stops per unit time (= qh , in vehicles per hour if q is in vehicles per hour).	x	=	Degree of saturation - the ratio of flow to capacity (= qc/sg).
N_0	=	Average overflow queue - average number of vehicles left in the queue at the end of the green period.			

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y = Flow ratio - the ratio of flow to saturation flow (q/s).

INTRODUCTION

1. The traditional method of calculating optimum signal timings for an isolated intersection is based on the use of delay as a measure of performance (Webster 1958; Webster and Cobbe 1966; Miller 1963, 1964, 1968; Allsop 1971; Sims 1979). A performance index which combines delays and stops is used in the TRANSYT computer program for finding optimum settings for co-ordinated traffic signals (Robertson 1969). It would be desirable to use the same approach for isolated traffic signals because vehicle stops are important when factors such as vehicle operating costs (fuel consumption, wear and tear), air pollution, annoyance to drivers and safety are considered. In particular, the contribution of vehicles stops to total fuel consumption and pollutant emission is significant (Bauer 1975; Courage and Parapar 1975; Dart and Mann 1978; OECD Road Research Group 1977; Patterson 1975, 1976). Similarly, a large proportion of accidents at traffic signals could be attributable to the need to stop vehicles, as suggested by Huddart (1969). These factors make it important to find signal settings which reduce vehicle stops as well as delays.

2. A satisfactory method for the prediction of vehicle stops is therefore a necessary prerequisite. Several formulae for the calculation of stops were given by Webster (1958). In this paper, an attempt is made to clarify the conceptual basis of these formulae. The term stop rate is introduced, which is the average number of stops per vehicle. It is shown that it consists of a uniform stop rate component and a random stop rate component. The uniform stop rate component is equivalent to what was called 'the proportion of vehicles which stop at least once' by Webster (1958). The random component of the stop rate becomes significant for high degrees of saturation (i.e. as flows approach capacity), much the same as delay.

3. It is shown by considering the Webster (1958) and Miller (1963, 1964, 1968) formulae that delay, queue length and stop rate are interrelated, and that each can be considered as having a uniform and a random component. The uniform component is related to the red time and the random component is related to the overflow queue. The Miller method calculates the average overflow queue in an explicit manner. It is shown that the Webster method for the calculation of the maximum queue length in an average signal cycle can be extended to calculate an equivalent overflow queue.

4. By comparing the average overflow queues alone, it can be shown that the predicted values of delays, queue lengths and stop rates from the Miller and Webster methods are very close. Discussions of the relationship between the delay and the overflow queue are given by Miller (1964) and Allsop (1972). A method of measuring delay in the field based on the measurement of overflow queues is described by Sagi and Campbell (1969).

5. Random variations in arrival flow rates result in some signal cycles being oversaturated. In such cycles, some vehicles will be stopped more than once, and this may lead to an average stop rate greater than one. This effect becomes significant for degrees of saturation greater than 0.8, i.e. as flows approach capacity. Since most signals operate near (or at) capacity conditions during peak periods, it is highly desirable that the analytical or simulation models and the field survey methods allow for this effect. Otherwise, the number of stops may be severely underestimated, in which case the relevant fuel consumption, pollutant emission and similar calculation results will reflect these errors.

6. The effect of randomness on the number of stops has usually been neglected in practice. It appears that most simulation models reported in the literature (even those which model individual vehicle acceleration-deceleration manoeuvres) measure the proportion of stopped vehicles, or an equivalent statistic. In other words, they do not measure multiple stops, hence the maximum stop rate which can be measured is one. For example, the macroscopic simulation model of the widely used TRANSYT program (Robertson 1969) measures delays, queue lengths and stops based on uniform arrivals. It makes a correction to delays to allow for randomness, but uses uniform stops in the calculation of a performance index.

7. The field survey method described by Reilly, Gardner and Keil (1976) also ignores multiple stops ('each vehicle is counted only once regardless of the number of stops it may have made'). On the other hand, a recent survey method described by Richardson (1979) allows for extra stops.

8. In this paper, a formula is given for the calculation of stop rates at undersaturated isolated signals. The formula allows for randomness in arrival rates through the use of an average overflow queue. It also allows for partial stops, i.e. vehicles which slow down without coming to a complete stop. The stop rates and the total number of vehicle stops calculated from this formula are expected to be sufficiently accurate for most practical purposes. Its main use would be in the comparison of relative effectiveness of alternative signal designs. A formula for the

number of pedestrians stopped at traffic signals is also given.

9. A numerical example is given in Appendix A to illustrate the use of the formulae presented in this paper and to indicate the closeness of results from the Webster and Miller formulae.

QUEUE LENGTH AT THE START OF GREEN

10. As the formulae for the prediction of stop rates make direct use of the average queue length at the start of green as a parameter, this is discussed first.

11. Following Miller (1964), the expression to give the average number of vehicles in the queue at the beginning of the green period (i.e. the maximum queue during an average signal cycle) can be written as:

$$N = N_u + N_o \tag{1}$$

where N_u = uniform queue, and

N_o = average overflow queue.

The uniform queue based on the assumption of regular arrivals (constant headways) is given by $N_u = qr$, where q = arrival flow rate (veh/s) and r = effective red time (s). Therefore,

$$N = qr + N_o \tag{2}$$

The overflow queue, N_o , i.e. the average number of vehicles left in the queue at the end of the green period is due to the random fluctuations in vehicle arrival rates causing some cycles to be oversaturated. Thus, N_o is the random queue term in eqn (2).

12. Webster (1958) gave the following formula for the average queue at the beginning of green:

$$N = \left(\frac{qr}{2} + qd\right) \text{ or } qr, \tag{3}$$

whichever is the larger

where d = average delay per vehicle (s).

From eqns (2) and (3), the overflow queue is:

$$N_o = \left(D - \frac{qr}{2}\right) \text{ or } 0, \tag{4}$$

whichever is the larger

where $D = qd$ is total delay.

13. The total delay based on Webster's formula is:

$$D = \frac{qc(1-u)^2}{2(1-y)} + \frac{x^2}{2(1-x)} - 0.65 (qc)^{1/3} x (2+5u) \tag{5}$$

where c = cycle time (s),
 qc = average number of arrival per cycle (veh),

u = green time ratio (g/c),
 y = flow ratio (flow/saturation, flow, q/s), and
 x = degree of saturation (flow/capacity, qc/sg).

14. The first term of eqn (5) is the uniform delay term which is the expression for delay when arrival headways are constant. Webster (1958) refers to the second and third terms of his formula as the random delay and the empirical correction terms, respectively. However, as suggested by Allsop (1972), these two terms together can be regarded as the random delay term which estimates the extra delay resulting from overflow queues. Therefore the total delay can be expressed as:

$$D = D_u + D_r \tag{6}$$

15. The Miller (1968) expression for delay is based on an explicit formulation of the overflow queue. In its original form, Miller's formula for average delay per vehicle is:

$$d = \frac{c-u}{2c(1-y)} \left(\frac{2}{q} N_o + (c-u)\right) \tag{7}$$

where N_o is the average overflow queue.

The total delay based on eqn (7) and in the form of eqn (6) is:

$$D = \frac{qc(1-u)^2}{2(1-y)} + \frac{1-u}{1-y} N_o \tag{8}$$

It is seen that the uniform delay term in eqn (8) is the same as Webster's formula (eqn (5)) and the difference between the Miller and Webster formulae is due to the random delay term only. From eqn (8), the random delay term of the Miller formula is:

$$D_r = \frac{1-u}{1-y} N_o \tag{9}$$

16. Miller's formula for the average overflow queue is:

$$N_o = \frac{\exp(-1.33\theta)}{2(1-x)} \tag{10}$$

where $\theta = \frac{1-x}{x} (sg)^{1/2}$ or

$$\frac{1-x}{x^{3/2}} (qc)^{1/2}$$

The values of the average overflow queue calculated from eqn (10) are given in Table I for various degrees of saturation, x , and maximum number of departures per cycle, sg . It is seen that, for low degrees of saturation (about $x < 0.6$), the overflow queue, N_o , is zero, and hence the random delay term D_r , is zero (eqn (9)).

17. Because the Miller formula (eqn (10)) presents an explicit and direct method for the calculation of the average overflow queue, it is conceptually more appealing than the conditional and indirect formulation based on Webster's method (eqns (3) and (4)).

TABLE I

AVERAGE OVERFLOW QUEUE, N_0 FOR ISOLATED SIGNALS

x	sg = 10	sg = 20	sg = 40	sg = 60	sg = 80
0.10	0	0	0	0	0
0.20	0	0	0	0	0
0.30	0	0	0	0	0
0.40	0	0	0	0	0
0.50	0	0	0	0	0
0.60	0.1	0	0	0	0
0.65	0.2	0.1	0	0	0
0.70	0.3	0.1	0.1	0	0
0.75	0.5	0.3	0.1	0.1	0
0.80	0.9	0.6	0.3	0.2	0.1
0.85	1.6	1.2	0.8	0.5	0.4
0.90	3.1	2.6	2.0	1.6	1.3
0.92	4.3	3.7	3.0	2.6	2.2
0.94	6.4	5.7	4.9	4.3	3.9
0.95	8.0	7.3	6.4	5.8	5.4
0.96	10.5	9.8	8.7	8.2	7.6

18. The explicit formulation of the average queue length at start green (eqn (2)) is used below for the derivation of a general formula for stop rates at undersaturated isolated traffic signals.

STOP RATE

DERIVATION OF A GENERAL FORMULA

19. The average number of stops per vehicle will be called the stop rate and will be denoted by h . The total number of stops per unit time experienced by a movement with an arrival flow rate of q (vehicles per unit time) is given by:

$$H = qh \tag{11}$$

20. Webster (1958) derived a formula for 'the proportion of vehicles which stop at least once' (i.e. the proportion of stopped vehicles irrespective of how many times they are stopped). It is shown below that this formula corresponds to the 'uniform stop rate' term given by:

$$h_u = \frac{1 - u}{1 - y} \tag{12}$$

where $u = g/c$ (the green time ratio) and $y = q/s$ (the flow ratio).

21. Webster (1958) also gave formulae for the 'average number of starts and stops per vehicle' and a diagram which compares this with the proportion of stopped vehicles for a particular case. Webster's formulae were given separately for two cases depending on whether the average queue at the beginning of the green period can be cleared during one green time, or not. However, a general method which unifies the two cases was not described. It is also not clear how the curve representing the average number of stops per vehicle relates to the two formulae given by Webster (1958, Fig 9). These formulae and the diagram are not included in the subsequent publication by Webster and Cobbe (1966). Only the formula corresponding to eqn (12) is included.

22. A general formula for stop rates at undersaturated signals is derived below based on Webster's (1958) statement that the total number of stops during a signal cycle is equal to the number of vehicles in the queue at the beginning of the green period plus those vehicles which arrive while the queue is clearing during the green period. Let n be the queue length at the start of green. The queue discharge rate is $(s-q)$, where s is the saturation flow rate and q is the arrival flow rate. The discharge time for n vehicles is $n/(s-q)$, and the number of vehicles which arrive (hence stop) during this time is $qn/(s-q)$. Therefore the total number of stops during

a signal cycle is:

$$n + \frac{qn}{s-q} = \frac{ns}{s-q} \quad (13)$$

This corresponds to the formula given by Webster (1958) for the case where $n/(s-q) < g$, i.e. the queue at the start of green can be cleared during the green period. A generalised formula is derived below using a method similar to that employed by Miller (1963) to derive his delay formula (first put forward by Winsten 1956).

23. Let us first assume that the green period is of unlimited length. The number of stopped vehicles up to the time at which the queue is cleared is given by eqn (13). Let n_1 be the number of vehicles left in the queue at the end of the previous green time. The queue at the start of green is $n = n_1 + qr$, and hence, the number of stopped vehicles is $(n_1 + qr) / (s - q)$. The number of vehicles stopped during the actual (limited) green period is found by subtracting the number of vehicles which would have arrived after the end of the actual green period (i.e. the following cycle). Let n_2 be the number of vehicles left at the end of the limited green period. The time between the end of the limited (actual) green period and the 'unlimited' green period is $n_2 / (s - q)$. The number of vehicles which arrive during this time is $n_2 q / (s - q)$. Hence, the number of stopped vehicles in a cycle is:

$$\frac{(n_1 + qr)s}{s - q} - \frac{n_2 q}{s - q} = \frac{n_1 + qr - n_2 y}{1 - y} \quad (14)$$

The average of this over n_1 and n_2 is the number of stopped vehicles in an average cycle. Under steady-state (undersaturated) conditions, the averages of n_1 and n_2 are both N_0 . Therefore, the average number of stops per cycle is:

$$\frac{qr}{1 - y} + N_0 \quad (15)$$

Dividing this by the average number of vehicle arrivals per cycle, qc , the number of stops per vehicle, i.e. the stop rate, is obtained as:

$$h = \frac{1 - u}{1 - y} + \frac{N_0}{qc} \quad (16)$$

24. Eqn (16) can be expressed as:

$$h = h_u + h_r \quad (17)$$

where h_u and h_r are the uniform and random stop rate terms, respectively. The uniform stop term, h_u , is the number of stops per vehicle assuming regular arrivals. It also corresponds to Webster's (1958) formula for the proportion of stopped vehicles. The second term, h_r , estimates extra stops due to random variations in arrival rates from cycle to cycle. Its effect is negligible for low degrees of saturation (zero for the degrees of saturation, x , less than 0.6

because $N_0 = 0$ as seen in Table 1). However, as the value of x increases, the effect of the random stop term becomes increasingly significant.

25. This can be observed from Fig 1, which shows the stop rate (curve A) as a function of the degree of saturation for the data used by Webster (1958, Fig 9). Curve B in Fig 1 represents the uniform stop rate term, h_u . The difference between curves A and B is the random stop rate term, h_r , calculated using N_0 values from Miller's formula (eqn (10)). The stop rate curve given in Fig 1 was found to be close to the corresponding curve in Fig 9 of Webster (1958).

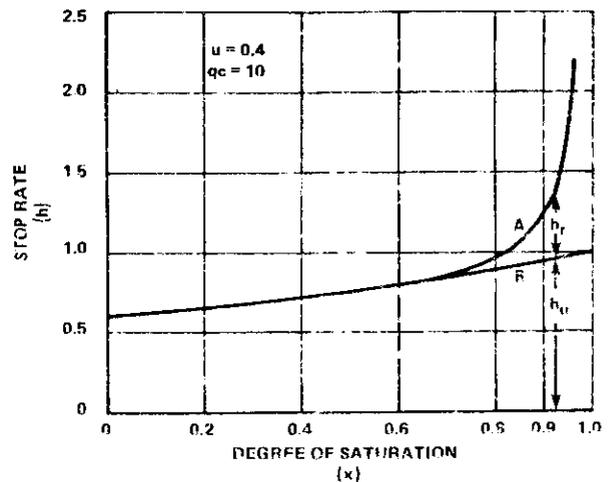


Fig 1 - Stop rate as a function of the degree of saturation

26. The two formulae given by Webster for stops in undersaturated and oversaturated cycles were combined for the purpose of comparing the results with those obtained from eqn (16). A complicated formula was obtained which expressed the random stop rate term as a function of the proportion of oversaturated cycles as well as the average overflow queue. The values of these parameters were calculated from Miller's formulae (1968), and the stop rates were calculated from the complicated formula for the data in Fig 1. It was found that, for large degrees of saturation, the stop rates from the complicated formula were somewhat larger than, but for most practical purposes sufficiently close to, those given by eqn (16). However, the comparison is not considered to be very useful, as there are doubts concerning the basis of the complicated formula.

27. It is thought that a comparison of small differences in stop rates from various formulae may not be very meaningful for the sensitive region of near-capacity conditions (the same applies to delays and queue lengths). Firstly, the analytical formulae are only approximate expressions. Secondly, the accuracy of field measurements under heavy traffic conditions will probably

be limited. Even when microscopic techniques are utilised in which individual vehicles are traced in the traffic stream, it is likely that an error level up to 10 per cent will remain in measured values (Sagi and Campbell 1969). The method described by Richardson (1979) requires a judgement about the definition of a stopped vehicle because 'it was found that many vehicles were effectively stopped although still creeping'. The drivers usually adjust their positions in a long queue, and this would cause similar measurement difficulties.

28. For these reasons, eqn (16) is considered to be a satisfactory expression for stop rates at undersaturated signals (degrees of saturation up to 0.96). It has the advantages of simplicity and consistency with the queue length and delay formulations. However, there is a need to correct for partial stops (vehicles which are delayed without coming to a full stop). This is discussed below.

CORRECTION FOR PARTIAL STOPS

29. The analytical expressions of delay at traffic signals (eqns (5) and (8)) measure the delay at the stop line by assuming infinite deceleration and acceleration rates. Discussions of this subject are given by Webster (1958), Allsop (1972) and Richardson (1979). The time-distance trajectory of vehicle 2 in Fig 2 shows that the stop line delay (BB') differs from the actual stopped delay (t_s) by an amount equal to half the sum of deceleration and acceleration times, ($t_d + t_a$)/2.

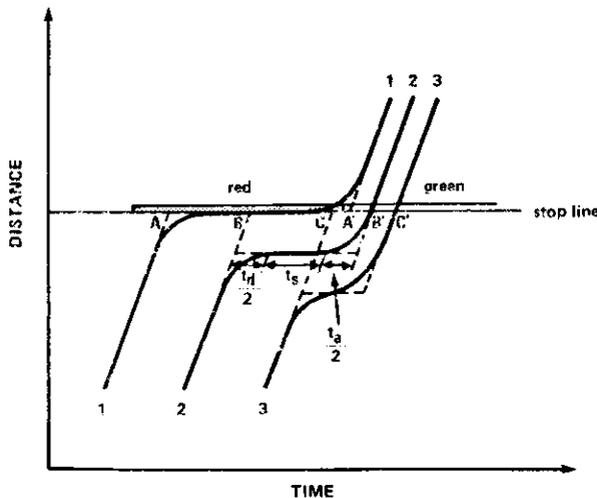


Fig 2 - Time-distance diagram to illustrate the relationship between delays and stops

30. As a result of this method of measurement, a vehicle which is delayed only for a short time without coming to a full stop is counted as a stopped vehicle. The term partial stop may be used to describe such a case (e.g. vehicle 3 in Fig 2). A vehicle whose stop-line delay is less than the time t is subject to a partial stop.

31. A correction is necessary for partial stops because the rates of fuel consumption, pollutant emission, etc. are different for a complete stop and a partial stop. A simple reduction factor can be applied to the stop rate given by eqn (16), i.e.

$$h = f \left(\frac{1-u}{1-y} + \frac{N_0}{qc} \right) \tag{18}$$

32. A method for calculating the value of the reduction factor, f , is given in Appendix 5. The method is somewhat tedious, and the use of a constant value of $f = 0.9$ is considered to be satisfactory for most practical purposes. The resulting formula which is given below may slightly overestimate stop rates under light traffic conditions and may underestimate stop rates under heavy traffic conditions (see Table II in Appendix B).

THE RECOMMENDED FORMULA

33. In summary, the recommended formula for estimating the stop rates, i.e. the average number of complete stops per vehicle is:

$$h = 0.9 \left(\frac{1-u}{1-y} + \frac{N_0}{qc} \right) \tag{19}$$

where q = arrival flow rate (veh/s),
 c = cycle time (s),
 u = effective green time/cycle time ratio (g/c),
 y = flow/saturation flow ratio (q/s), and
 N_0 = the average overflow queue (eqn (4), or eqn (10)).

The total number of (complete) stops per hour is calculated from $H = qh$. A convenient formula for calculating the number of stopped vehicles directly is:

$$H = \frac{3240}{c} \left(\frac{qr}{1-y} + N_0 \right) \tag{20}$$

A numerical example is given in Appendix A.

34. Fig 3 illustrates stop rates as a function of the degree of saturation for various values of the green time ratio, u , and the maximum number of departures per cycle, sg , in order to indicate the differences between the stop rates experienced by minor road traffic ($u = 0.1, sg = 10$) and major road traffic ($u = 0.5, sg = 60$). Fig 4 is given to illustrate the differences between random effects in minor and major road traffic cases. The difference between curve C (uniform stop rate) and curve A for $sg = 10$ (or curve B for $sg = 60$) is the random stop rate. It is seen that random effects in stop rates are higher for minor movements. It is also seen from Figs 3 and 4 that random effects are negligible for degrees of saturation less than 0.6 (because the overflow queue, N_0 , is zero as seen in Table I). Graphs in Figs 3 and 4 are based on the use of eqn (19) with the average overflow queue, N_0 calculated from the Miller formula (eqn (10)).

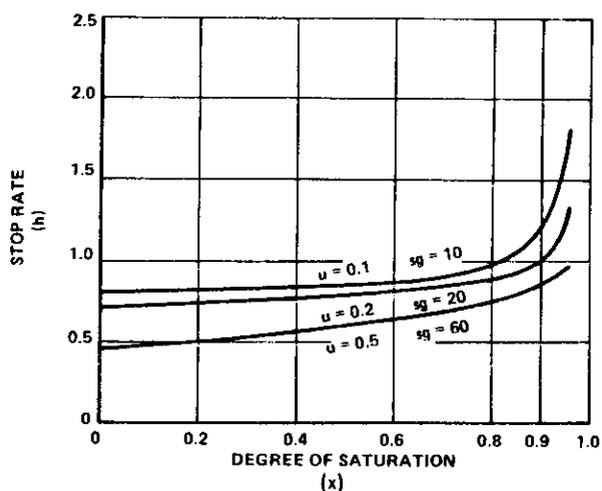


Fig 3 - Stop rate as a function of the degree of saturation for various (u, sg) values

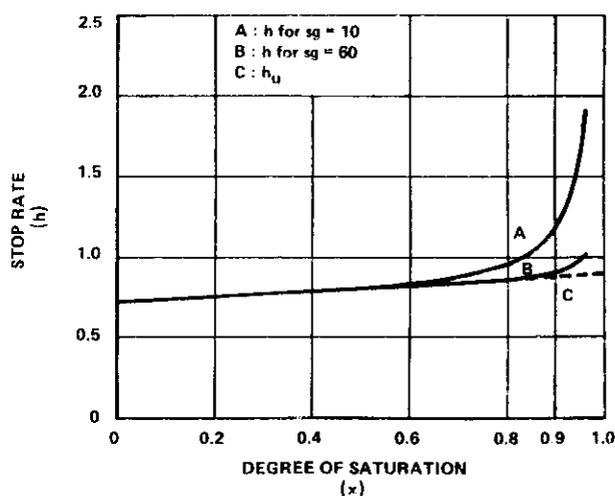


Fig 4 - Random effects on stop rate for various (sg) values, u = 0.2

PEDESTRIAN STOPS

35. The number of pedestrians stopped at traffic signals can be calculated from:

$$H = \frac{qr}{c} \tag{21}$$

- where q = pedestrian arrival flow rate (ped/h, or ped/s),
- r = effective red time to pedestrian(s), including the flashing don't walk period,
- c = cycle time (s), and
- H = is in the same unit as q.

This formula has been derived from eqn (16) by putting y = 0 and N₀ = 0, resulting in a stop rate of h = 1 - u = r/c. This is justified on the basis of very high pedestrian saturation flows (hence small values of y and x). The formula may underestimate the number of stopped pedestrians when pedestrian flows are very high, in which case eqn (16) can be used for better estimates.

36. It should be noted that the number of pedestrians in queue at the start of green is:

$$N = qr \tag{22}$$

where q is in ped/s.

Therefore, eqn (21) assumes that no pedestrians are stopped during the green (walk) period, i.e. assumes an instant discharge of the pedestrian queue. The corresponding formula for average delay in seconds per pedestrian is:

$$d = \frac{r^2}{2c} \tag{23}$$

DISCUSSION

37. Equation (19) is recommended for use as a general-purpose formula for predicting vehicle stop rates at isolated traffic signals. It allows for randomness effects as well as partial stops. The formula is considered to be satisfactory for most practical purposes.

38. It should be noted that the proposed formula is valid for traffic conditions where demand is less than the capacity, i.e. the undersaturated case. Miller (1968) qualified his formula for the average overflow queue, N₀, that it is a good approximation for the degree of saturation, x between 0.4 and 0.96. Because N₀ determines the queue length and delay as well as the stop rate, it is considered to be appropriate to state that the formulae presented in this paper are valid for x up to 0.96.

39. Strictly speaking, the proposed stop rate formula is for fixed-time signals. However, as with the delay and queue length formulae, it can be used for vehicle-actuated signals under heavy flow conditions in order to evaluate relative merits of alternative signal designs.

40. The proposed stop rate formula is a general one which can be used with any queue length prediction method for isolated traffic signals. The difference between various methods would be in the calculation of an average overflow queue. The differences between the overflow queue values calculated from the Webster and Miller formulae (eqns (1) and (10)) have been found to be very small (less than ± 2 per cent). Because the overflow queue determines the random components of delay, queue length and stop rate, and the uniform components are the same, predictions of these statistics using the Webster and Miller formulae would be very close in most cases. The numerical example given in

Appendix A illustrates this for a particular case. However, the Miller formula for calculating an average overflow queue is preferred because it is explicit and direct.

41. In the case of opposed turners, a higher value of the correction factor, f , could be used in the stop rate formula in order to allow for the extra number of 'partial stops' made by vehicles in the queue during the gap acceptance process.

42. The total number of stops, H , given by eqn (19), and the total delay, D , given by eqn (5) or eqn (8) can be used for fuel consumption (similarly, for pollutant emission) calculations of the type $(k_1 D + k_2 H)$, where k_1 and k_2 are the fuel consumption rates for idling and a complete stop, respectively. However, if k_1 is strictly an idling rate, then there is a need for correction because the model delay includes the stopped time (t_s) as well as the deceleration-acceleration time $t = \frac{1}{2}(t_d + t_a)$ as shown in Fig 2. A feasible method is to adjust the fuel consumption rate for stops to $k_2 = k_2 - k_1 t$, i.e. reduce by $k_1 t$ for each stopped vehicle. For example, if $k_1 = 2.4$ L/h, $k_2 = 0.05$ L/stop, and $t = 15$ s, the adjusted rate per stop is $k_2 = 0.04$ L/stop.

43. An interesting extension of the methodology proposed in this paper might be its application to the co-ordinated signal case, making use of the relationship between the overflow queue and the random delay. For example, TRANSYT traffic model (Robertson 1969) uses the random delay term $x^2/4(1-x)$ which is half the value of the second term of Webster's formula (eqn 5). If this is valid, and the relationship between overflow queue and random delay in the case of linked signals is similar to that for isolated signals (i.e. as in eqn 9), then it could be said that the average overflow queue at linked signals would be half the value of the average overflow queue at isolated signals with similar signal and flow characteristics. Once a relationship of this nature is established, then randomness effects could be allowed for in co-ordinated signal cases as well. Such a model improvement would significantly affect the optimisation results if included in a program like TRANSYT (it should be noted that the TRANSYT performance index combines delays and stops using delays corrected for randomness but stops including uniform values only).

44. It is interesting to see that the MITROP computer program (Gartner et al. 1976) for optimising co-ordinated signal timings uses a random delay term (saturation deterrence function) which is calculated as a function of the overflow queue. However, the overflow queue values given by Gartner, et al. as the basis of MITROP are practically the same as the overflow queues given by the Miller formula (eqn (10)) for isolated intersections. This indicates that the random delay term of TRANSYT and MITROP

models are significantly different. The random delay term used in MITROP is equal to the overflow queue values given by Wormleighton (1965). In his paper, Wormleighton discusses the problem in the context of co-ordinated signals, but his overflow queue values appear to be for isolated signals (Poisson arrivals and constant service time), rather than closely spaced signals (platoon behaviour).

FURTHER WORK

45. The following work is recommended for future research and development.

- (a) Testing of the proposed stop rate formula for undersaturated isolated signals by means of field surveys and experiments with microscopic simulation models.
- (b) Formulation of relationships between the overflow queue and the random delay term and between the overflow queue and the random stop rate term for co-ordinated signals and for vehicle-actuated signals.
- (c) Development of a stop rate formula for temporarily oversaturated conditions. Robertson (1979) suggests that, for delays (d) very much larger than a cycle time (c), the number of stops should be related to the ratio (d/c) but modified for the tendency for vehicles to limit their speed changes where it is obvious they will have to stop again before clearing the stop line (i.e. partial stops rather than complete stops). Multiple stops become much more important in oversaturated cases giving rise to stop rates much higher than 1.0. Simulation and other methods, and existing traffic control practices, which ignore multiple stops are severely restricted in their prediction of, or effectiveness in reducing, vehicle fuel consumption and pollutant emissions at traffic signals.
- (d) Determination of the value of the correction factor, f , for opposed turners to allow for extra stops (partial) due to the gap acceptance process.

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APPENDIX A

NUMERICAL EXAMPLE

46. Let us consider a movement with the following flow and signal timing characteristics:

flow, $q = 1310$ veh/h,
 saturation flow, $s = 4800$ veh/h,
 green time, $g = 30$ s, and
 cycle time, $c = 100$ s.

The following parameters are calculated from the above data:

flow, $q = 1310/3600 = 0.364$ veh/s,
 red time, $r = c - g = 70$ s,
 average number of arrivals per cycle,
 $qc = 0.364 \times 100 = 36.4$ veh,
 maximum number of departures per cycle,
 $sg = (4800/3600) \times 30 = 40$ veh
 degree of saturation, $x = qc/sg = 0.91$,
 green time ratio, $u = g/c = 0.30$, and
 flow ratio, $y = q/s = 1310/4800 = 0.27$.

The delays, queue lengths and stops are calculated below allowing for random effects using the method described in the paper. Both Miller and the Webster formulae are used to indicate the closeness of results.

47. Let us first calculate the 'uniform' components of delay, queue length and stop rate which are the same for both formulae. The uniform queue length from eqn (2) is $N_u = 0.364 \times 70 = 25.5$ veh, the uniform delay from the first term of eqn (5) or eqn (8) is $D_u = 36.4 (1-0.3)^2 / 2(1-0.27) = 12.22$ veh-h/h, and the uniform stop rate from the first term of eqn (16) is $h_u = (1-0.3)/(1-0.27) = 0.96$.

48. Let us now calculate the overflow queue, the average queue at start green and the total delay.

(a) The Webster method: The calculation of total delay is required first. From the second and third terms of eqn (5), which are together considered to be the random delay term, $D_r = 3.05$ veh-h/h, hence the total delay is $D = 12.22 + 3.05 = 15.27$ veh-h/h. The overflow queue from eqn (4) is $N_o = 15.27 - 0.364 \times 70/2 = 2.5$ veh and the average queue at start green in $N = N_u + N_o = 25.5 + 2.5 = 28.0$ veh.

(b) The Miller method: From eqn (10), $N_o = 2.4$ veh (using $\theta = 0.628$). Therefore $N = 25.5 + 2.4 = 27.9$ which is practically the same as the queue length from the Webster method (28 vehicles). The random delay from eqn (9) is $D_r = (1-0.3) \times 2.40 / (1-0.27) = 2.30$ veh-h/h, hence the total delay from Miller's formula is $D = 12.22 + 2.30 = 14.52$ veh-h/h (5 per cent less than that given by the Webster formula).

49. Finally, let us calculate the stop rate and the total number of stops. Let us use $N_o = 2.4$ calculated from Miller's formula. From eqn (19), the stop rate is found $h = 0.9 \times (0.96 + 2.4/36.4) = 0.9 \times 1.026 = 0.92$ stops per vehicle ($0.9 h_u = 0.9 \times 0.96 = 0.86$ stops per vehicle is due to uniform stops). The total number of stops from eqn (11) is $H = 1310 \times 0.92 = 1210$ vehicle stops per hour (or from eqn 20, $H = (3240/100) \times (25.5/0.73 + 2.4) \approx 1210$).

APPENDIX B

STOP REDUCTION FACTOR

50. The time for a vehicle to decelerate from speed v down to speed v' , and to accelerate back to speed v is:

$$t_d + t_a = (v-v') \left(\frac{1}{a_1} + \frac{1}{a_2} \right) \quad (24)$$

where the deceleration and acceleration rates, a_1 and a_2 are both positive and assumed to be constant. The partial stop time which corresponds to the stop line delay is $t' = (t_d + t_a) / 2$ (see Fig 2).

51. Therefore, the time for a complete stop ($v'=0$) is:

$$t = \frac{v}{2} \left(\frac{1}{a_1} + \frac{1}{a_2} \right) \quad (25)$$

Assuming, $a_1 = a_2 = a$, the simpler formulae, $t' = (v-v')/a$ and $t = v/a$ are found. For example, $v = 60$ kph = 16.7 m/s, $v'=20$ kph = 5.6 m/s, and $a = 1.1$ m/s² give $t' = 10$ s and $t = 15$ s.

52. The stop reduction values, β , to be associated with the individual delay values of less than t seconds can be calculated as follows. Assume that the fuel consumption (or a particular pollutant emission) rates (or cost, etc.) associated with speed changes ($v-v'$) and ($v-0-v$) are k' and k , respectively. Then, the proportion of stop to be associated with a partial stop time, t' , is $\beta = k'/k$. The value of β will vary between one for time $t' = t$ (full stop) and zero for time $t' = 0$ (undelayed vehicle).

53. Using the fuel consumption data for complete and partial stops given by Claffey (1971), the relationship between the stop reduction value, β , and the partial stop time, t' , has been found to be approximately linear for a cruising speed, v , of about 60 km/h. On the other hand, the data recommended by Robertson and Gower (1977) for use with the stop reduction facility of the TRANSYT/6 computer program indicates an exponential relationship (see Fig 5).

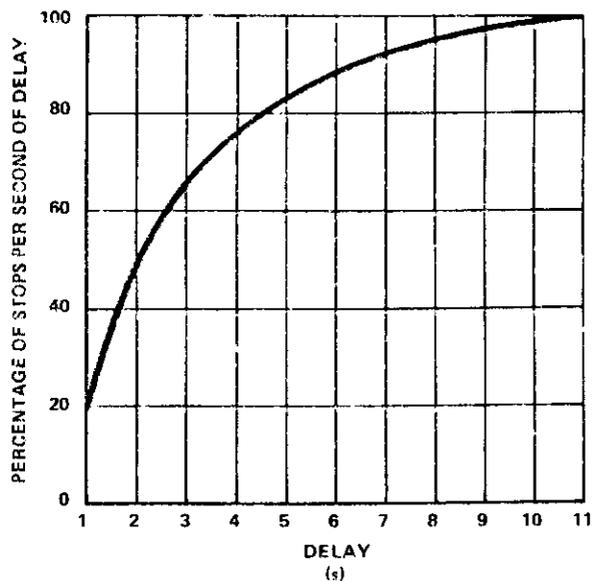


Fig 5 - Data for partial stops (Robertson and Gower 1977)

54. Under the assumption of regular arrival headways, the delay as a function of the vehicle arrival order is linear, and varies from r (effective red period) for the first vehicle in the queue to zero for the last vehicle in the queue (see Richardson 1979). Under this assumption, and when the relationship between the stop reduction value, β , and the partial stop time, t' ,

is linear, the number of complete stops per vehicle, i.e. the uniform stop rate, is:

$$h_u = f_u \frac{1-u}{1-y} \quad (26)$$

where f_u is a stop reduction factor which is given by:

$$f_u = 1 - \frac{t}{2r} (1-y) \quad (27)$$

where t = time for a complete stop (eqn 25),
 r = effective red time, and
 y = flow/saturation flow ratio.

55. If the (β, t') relationship is exponential as shown in Fig 5, the value of the stop reduction factor, f_u , will be higher (i.e. less reduction) than that given by eqn (27). This has been tested using the uniform stop reduction facility of the TRANSYT/6 program.

56. The stop reduction factor given by eqn (27) does not allow for the effect of oversaturated cycles in which most vehicles make at least one complete stop. The following method can be used for the calculation of f to allow for this (for use in eqn (18)). Firstly, calculate the saturation time, $g_s = N/(s-q)$, where $N = qr + N_0$ as in eqn (2). If g_s is larger than the green time, g , calculate the stop reduction time $t_f = t - (g_s - g)$, where t is the time for a complete stop. Then, calculate the stop reduction factor using the appropriate formula given below:

$$f = 1 - \frac{t}{2(r+g_s)} \quad \text{for } g_s \leq g \quad (28)$$

$$f = 1 - \frac{t_f}{2ct} \quad \text{for } g_s > g \text{ and } t_f > 0 \quad (29)$$

$$f = 1.0 \quad \text{for } g_s > g \text{ and } t_f \leq 0 \quad (30)$$

where t , t_f , g_s are as described above, c is the cycle time, and r is the red time.

For the example given in Appendix A, $g_s = 28/(1.333-0.364) = 29$ s, and since $g_s < g$, $f = 1-15/2(70+29) = 0.92$ is found.

57. Various values of f calculated from eqn (28) to (30) using $t = 15$ s are given in Table II for various (x, sg, u, c) values. It is seen that the reductions due to partial stops are higher for light traffic conditions. For desirable operating conditions during peak traffic periods (degrees of saturation of 0.70 to 0.90 and cycle times of 80 to 120 s), a stop reduction factor of approximately 0.9 is a typical value. This figure has been chosen as a general reduction factor in the recommended formula for calculating stop rates at isolated traffic signals (eqn 19).

TABLE II

STOP REDUCTION FACTORS, f

x	The value of f		
	(a)	(b)	(c)
0	0.63	0.81	0.88
0.5	0.69	0.86	0.91
0.7	0.73	0.88	0.93
0.9	0.96	0.93	0.94
0.95	1.0	1.0	1.0

(a) $sg = 5, u = 0.33, c = 30$

(b) $sg = 20, u = 0.50, c = 80$

(c) $sg = 40, u = 0.57, c = 140$

58. The calculation of a precise value of the stop reduction factor, f , should allow for the fact that the drivers may adjust their deceleration rates according to traffic conditions, that the deceleration-acceleration rates vary according to the vehicle type (traffic composition), and that the deceleration and acceleration rates are not constant. The assumption of a linear relationship between the stop reduction value, β , and the partial stop time, t' , can also be relaxed, and other relationships can be accommodated. However, the resulting complications may not be justified for a small improvement in the accuracy of the predicted stop rate.