

## REPRINT

# The Highway Capacity Manual delay formula for signalised intersections

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### NOTE:

This paper is related to the intersection analysis methodology used in the SIDRA INTERSECTION software. Since the publication of this paper, many related aspects of the traffic model have been further developed in later versions of SIDRA INTERSECTION. Though some aspects of this paper may be outdated, this reprint is provided as a record of important aspects of the SIDRA INTERSECTION software, and in order to promote software assessment and further research.

# The Highway Capacity Manual Delay Formula for Signalized Intersections

BY RAHMI AKCELIK

The purpose of this article is to compare the 1985 U.S. *Highway Capacity Manual* (HCM)<sup>1</sup> delay formula for signalized intersections with the Australian<sup>2</sup> and Canadian<sup>3</sup> formulas and to present a generalized form that embraces them all. The aim is to promote international cooperation in this area of research and development.

Compared with the other delay formulas, the HCM formula predicts higher delays for oversaturated conditions, and the differences between the prediction from the HCM formula and the other formulas increase with increasing degree of saturation. An alternative to the HCM delay formula, which matches the other formulas for oversaturated conditions, is given. The alternative formula, derived from the generalized formula, predicts delays that are very close to those predicted by the original HCM formula for undersaturated conditions, and at the same time predicts delays that are very close to the results from the Australian and Canadian formulas for oversaturated conditions.

The HCM signalized intersection chapter states that its delay formula "yields reasonable results for values of  $x$  between 0.0 and 1.0. . . . The equation may be used with caution for values of  $x$  up to 1.2, but delay estimates for higher values are not recommended." Although traffic engineers do not design for oversaturation, a delay formula that can be expected to give reasonable results for oversaturated, as well as undersaturated conditions, is preferred because the limitations of this type of formula are often

forgotten and the formula misused in practice (e.g., in evaluating alternative designs or in stating benefits from improvements to an existing oversaturated intersection).

## The HCM formula predicts higher delays for oversaturated conditions.

The generalized formula could be calibrated to develop a more suitable formula for U.S. conditions. It might also be possible to calibrate it for vehicle-actuated and fixed-time signals separately. Various other issues related to this discussion and briefly mentioned in this article will be discussed elsewhere.

## A Generalized Formula

The HCM, Australian, and Canadian formulas for delay at traffic signals can be generalized as the following two-term equation:

$$d = \frac{0.5c(1-u)^2}{1-ux} + 900Tx^n \left[ (x-1) + \sqrt{(x-1)^2 + m(x-x_o)/QT} \right] \quad (1)$$

where:

$d$  = average overall delay (including stop-start delays) in seconds per vehicle,

$c$  = signal cycle time in seconds,  
 $u$  =  $g/c$  (ratio of effective green time to cycle time),  
 $x$  = degree of saturation (ratio of arrival flow rate to capacity),  
 $T$  = flow period in hours,  
 $Q$  = capacity in vehicles per hour,  
 $m, n$  = calibration parameters, and  
 $x_o$  = the degree of saturation below which the second term of the delay formula is zero. This can be expressed as  
 $x_o = a + bsg$  (2)  
 where:

$sg$  = capacity per cycle ( $s$  = saturation flow rate in vehicles per second and  $g$  = effective green time in seconds), and  
 $a, b$  = calibration parameters.

The two terms of the delay formula can be referred to as the uniform delay ( $d_u$ ) and the overflow delay ( $d_o$ ) terms:

$$d = d_u + d_o \quad (3)$$

The overflow delay is called the incremental delay because of random arrivals and individual cycle failures in the HCM. The usefulness of this concept is that an overflow queue formulation can be used as a common base for the formulas to predict delay, number of stops, and queue length as in the Australian method.<sup>2,4</sup> The relation between the average overflow queue ( $N_o$ ) in vehicles and the overflow delay ( $d_o$ ) in seconds is

$$3600 d_o = N_o/Q \quad (4)$$

where  $Q$  is the capacity in veh./hr. In this sense, Equation 1 is based on a gener-

alized overflow queue formula. More detailed discussion on the use of the overflow queue concept to predict delay, number of stops, and queue length is given in the Appendix.

Various specific delay formulas can be derived from Equation 1 by setting the calibration parameters  $n$ ,  $m$ ,  $a$ , and  $b$  in the overflow delay term to appropriate values. Therefore, only the second term of Equation 1 will differ between alternative models as no calibration parameters are considered in the first term ( $d_o$ ). The values of the calibration parameters  $n$ ,  $m$ ,  $a$ , and  $b$  for the 1985 HCM,<sup>1</sup> 1981 Australian,<sup>2</sup> and 1984 Canadian,<sup>3</sup> models are given in Table 1. The HCM model differs from the other two models with the  $x^2$  factor ( $n = 2$ ) whereas the Australian model differs from the other two models with a nonzero  $x_o$  parameter ( $x_o = 0.67 + sg/600$ ).

The exact form of the HCM formula differs from Equation 1 because a factor of  $1/1.3 = 0.77$  is applied to convert the overall delay to stopped delay (i.e., it assumes that the stopped delay is always 77% of the overall delay). Whilst the author does not necessarily agree with this simplifying assumption, it is outside the scope of this article.

To facilitate comparisons,  $T = 0.25$  hr., which is fixed in the HCM model, will be used for all models in the following numerical example although it is best to leave this as a variable in the general model. It will also be assumed, by way of example, that  $c = 90$  sec.,  $g = 30$  sec.,  $s = 1500$  veh./hr., and therefore  $g/c = 1/3$ ,  $Q = 500$  veh./hr., and  $sg = 12.5$  veh. The overflow delay,  $d_o$ , from the second term of Equation 1 for this example is given by

$$d_o = 225 x^n [(x-1) + \sqrt{(x-1)^2 + m(x-x_o)/125}] \quad (5)$$

for  $x > x_o$ , (0 otherwise)

The results from Equation 5 for the HCM, Australian, and Canadian models are shown in Figure 1, and tabulated in Table 2. For this example,  $x_o = 0.691$  for the Australian model and  $x_o = 0$  for the other two.

### Time-Dependent Delay Formulation

It is seen from Figure 1 that the Australian and Canadian models produce overflow delay curves that are asymptotic to a deterministic oversaturation delay

**Table 1. Values of the Calibration Parameters in the HCM, Australian, and Canadian Overflow Delay Formulas, Expressed in Terms of Equation 1**

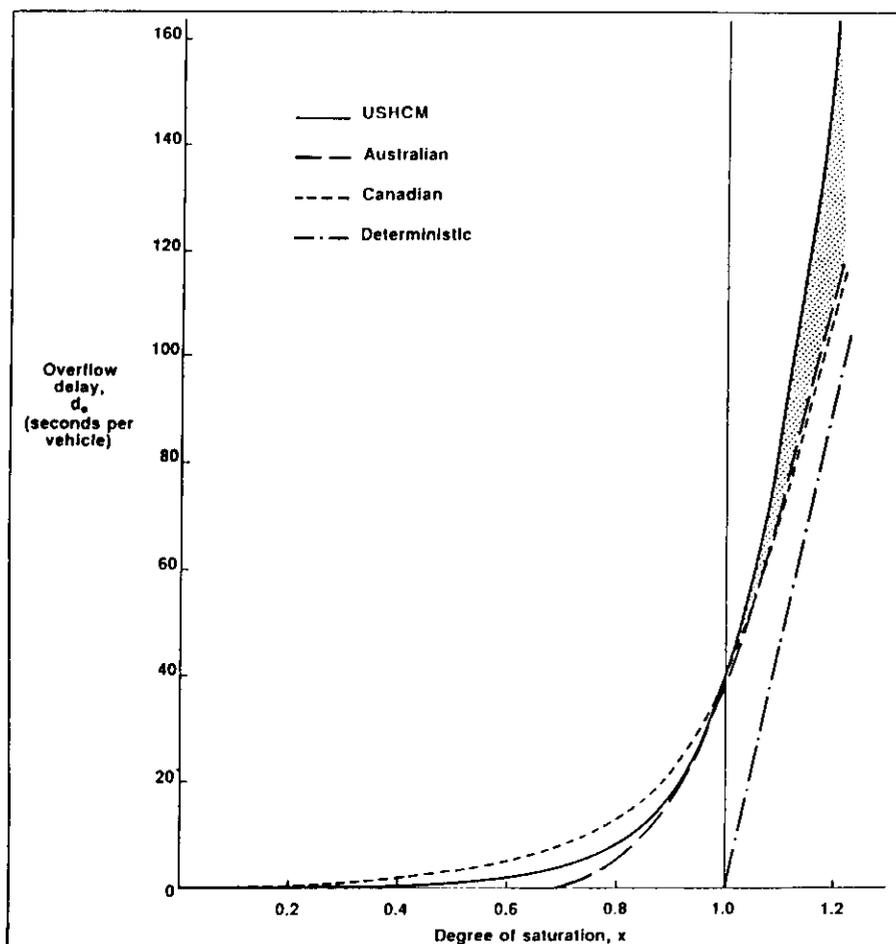
	$n$	$m$	$a$	$b$
HCM	2	4	0	0
Australian	0	12	0.67	1/600
Canadian	0	4	0	0
TRANSYT 8	-1	4	0	0
Alternative to HCM	0	8	0.50	0

**Table 2. Overflow Delays in Seconds per Vehicle from the HCM, Australian, and Canadian Delay Formulas ( $c = 90$  sec.,  $g = 30$  sec., and  $Q = 500$  veh./hr)**

$x$	HCM	Alternative <sup>a</sup>	Australian	Canadian	Deterministic <sup>b</sup>
0	0.0	0.0	0.0	0.0	—
0.4	0.4	0.0	0.0	2.4	—
0.6	1.9	1.8	0.0	5.2	—
0.8	8.1	9.7	5.5	12.6	—
0.9	17.6	19.9	16.5	21.8	—
0.95	26.6	28.5	25.9	29.5	—
1.0	40.2	40.2	38.8	40.2	0.0
1.1	85.1	72.0	72.4	70.3	45.0
1.2	155.5	110.5	112.1	108.0	90.0
1.4	376.0	195.0	197.5	191.8	180.0

<sup>a</sup>Where  $n = 0$ ,  $m = 8$ ,  $a = 0.5$ , and  $b = 0$ .

<sup>b</sup>From Equation 6a.



**Figure 1. Overflow delays predicted by the HCM, Australian, and Canadian formulas ( $c = 90$  sec.,  $g = 30$  sec.,  $Q = 500$  veh./hr., and  $T = 0.25$  hr.).**

line (Akcelik<sup>4</sup>) given by

$$d = 1800T(x-1) \quad \text{for } x \geq 1 \quad (6)$$

For the example above,

$$d = 450(x-1). \quad (6a)$$

This deterministic delay formula is based on a single flow period of length  $T$  with constant flow and capacity and with no initial overflow queue. The delay from Equations 6 or 6a includes delay during  $T$  as well as delay after  $T$  (as experienced by the vehicles that arrive during  $T$  but may depart after  $T$ ). Subsequently, the generalized formula, Equation 1, is based on the same simplifying assumption, which is used as an alternative to the more complicated variable-demand analysis methods that treat the peaking of arrival flows explicitly.

The asymptotic curve is an important characteristic of time-dependent delay formulation and was originally developed by researchers at the U.K. Transport and Road Research Laboratory for the TRANSYT program. In fact, the Canadian delay formula is the same as the formula given by Robertson.<sup>5</sup> However, TRANSYT Version 8 uses a different form of the function<sup>6</sup> equivalent to  $n = -1$ ,  $m = 4$ ,  $a = 0$ , and  $b = 0$  in Equation 1. Different forms of the function were used in earlier versions of TRANSYT.

### Converting the overall delay to stopped delay needs particular attention before any calibration effort.

The time-dependent delay models are derived by converting a steady-state delay function, which is applicable to undersaturated conditions only, to an asymptotic time-dependent function, which becomes applicable to oversaturated conditions also. The steady-state function that corresponds to Equation 1 when  $n = 0$  can be expressed as

$$k(x - x_0)x/(1-x) \quad (7)$$

where parameter  $k$  is related to parameter  $m$  in Equation 1 by  $m = 8k$ . The Canadian<sup>3,5</sup> formula corresponds to Equation 7 where  $k = 0.5$  and  $x_0 = 0$ , which is the random delay term of the

well-known Webster formula.<sup>7</sup> The Australian formula<sup>2</sup> corresponds to  $k = 1.5$  with a variable value of  $x_0$ , which is an approximation<sup>4</sup> to Miller's delay formula<sup>8</sup>. Unlike the Webster formula, Miller's original equation was based on the formulation of overflow queues. This was extended to oversaturated conditions by the author using the TRRL time-dependent delay method and introducing a simplification to the original Miller formula.<sup>4</sup> The reader is also referred to a paper by Hurdle,<sup>9</sup> which discusses the relationship between the steady-state and time-dependent delay formulas.

As seen in Figure 1, the HCM formula appears to produce a curve that does not have the fundamental characteristic of the time-dependent delay formulation. For  $x$  above 1.0, it diverges from the deterministic delay line and predicts very large delay values as indicated by the shaded area. This is due to the  $x^2$  factor ( $n = 2$  in Equation 1). It is not clear if the  $x^2$  factor was introduced in an effort to calibrate the delay formula for undersaturated conditions, or for reasons related to the use of a fixed value of  $T = 15$  minutes (peak flow period). A clarification of this issue would explain a fundamental difference between the HCM formula and the other formulas.

### An Alternative to the HCM Formula

A formula that gives delay values close to the HCM formula for  $x$  values less than 1.0, but remains asymptotic to the deterministic oversaturation line (Equation 6), can be derived from Equation 1

by setting  $n = 0$  (i.e., deleting the  $x^2$  factor and choosing the appropriate values of parameters  $m$ ,  $a$ , and  $b$ ). It has been found by a best-fit analysis of the overflow delay values from the HCM formula for  $x$  values in the range 0.6 to 1.0 that the parameter values of  $m = 8$  and  $a = 0.50$  give a satisfactory solution when parameter  $b$  is set to zero to obtain a formula with constant  $x_0$ . Using  $T = 0.25$  hr., this overflow delay formula is expressed as

$$d_s = 225 [(x-1) + \sqrt{(x-1)^2 + 32 [(x-0.5)/Q]}] \quad (8)$$

for  $x > 0.5$  (zero otherwise).

This equation gives the same delay value as the HCM equation for flow at capacity ( $x = 1.0$ ). Interestingly, it corresponds to an earlier steady-state overflow delay formula given by Miller,<sup>10</sup> which approximates to Equation 7 with  $k = 1.0$  and  $x_0 = 0.5$ . The overflow delay results from Equation 8 for  $Q = 500$  veh./hr. are given in Table 2 as the alternative model.

For direct comparison with the full HCM formula, the stopped delay formula obtained by replacing the second term of Equation 1 by Equation 8 and applying the factor of 0.77 to both terms is as follows:

$$d_s = \frac{0.385c(1-u)^2}{1-ux} + 173 [(x-1) + \sqrt{(x-1)^2 + 32 (x-0.5)/Q}] \quad (9)$$

The results from Equation 9 and the HCM formula for stopped delay for the above example ( $c = 90$  sec.,  $g = 30$  sec., and  $Q = 500$  veh./hr.) are given in Table 3. The values in Table 3 for  $x$  larger than

**Table 3. Comparison of Stopped Delays from the HCM Formula and the Recommended Alternative Formulation Using Factor of 0.77 ( $c = 90$  sec.,  $g = 30$  sec., and  $Q = 500$  veh./hr.)**

$x$	HCM <sup>a</sup> (sec.)	Alternative <sup>b</sup> (sec.)	Difference <sup>c</sup> (sec.)	% Difference <sup>d</sup>
0.2	16.5	16.5	0	0
0.4	18.0	17.8	-0.2	-1.1
0.6	20.7	20.6	-0.1	-0.5
0.8	27.2	28.5	+1.3	+4.8
0.9	35.6	37.3	+1.7	+4.8
0.95	43.0	44.5	+1.5	+3.5
1.0	54.0	54.0	0	0
1.1	88.5	78.5	-10.0	-11.3
1.2	142.7	108.1	-34.6	-24.2
1.3	216.9	140.0	-76.9	-35.5
1.4	312.3	173.0	-139.3	-44.6

<sup>a</sup>Where  $n=2$ ,  $m=4$ ,  $a=0$ , and  $b=0$ .

<sup>b</sup>Where  $n=0$ ,  $m=8$ ,  $a=0.5$ , and  $b=0$ .

<sup>c</sup>(Alternative - HCM).

<sup>d</sup>100 × (Alternative - HCM)/HCM.

1.0 have been calculated using the value of the uniform delay at capacity (first term for  $x = 1$ ) as a fixed value ( $d = 23.1$  sec.) for both models. Explanation of this method used by the SIDRA computer program<sup>11-14</sup> can be found in an earlier article by the author<sup>1</sup>. The maximum difference between the HCM formula and the alternative formula for  $x$  less than 1.0 is about 2 sec. (5%).

## Conclusion

Equation 9 should be considered as an alternative to the HCM delay formula. This formula gives values close to the

HCM formula for degrees of saturation less than 1.0, and at the same time is similar to the Australian, Canadian, and TRANSYT formulas in producing a delay curve asymptotic to the deterministic delay line for degrees of saturation greater than 1.0.

However, the calibration of the generalized formula (Equation 1) directly using actual data for U.S. conditions rather than the results from the original HCM formula would, of course, be a better way of developing an alternative formula. It would also be useful to seek a value of  $x_0$  dependent on capacity per cycle ( $sg$ ) in this process, because this

form of the model has certain advantages.

Furthermore, the question of converting the overall delay to stopped delay needs particular attention before any calibration effort. Researchers could also try deriving separate formulas for fixed-time and vehicle-actuated signals by calibrating the generalized Equation 1 of this article ( $n = 0$  recommended). Again, the important question of applying the delay formulas for individual lanes as against lane groups has not been discussed here.<sup>11</sup> In this respect, any calibration effort should take the particular method of application into account.

## Appendix

The generalized delay formula (Equation 1) brings together the HCM, Australian, Canadian, and the TRANSYT formulas. In fact, a more general approach is to express delay in terms of an average overflow queue, and use this consistently for predicting other primary performance measures such as the number of stops and queue length. This approach has been used in the Australian signal analysis methodology as originally expressed in the Research Report ARR No. 123<sup>2</sup> and subsequently incorporated into the SIDRA computer package<sup>11-14</sup>. The reader is referred to an article by Powell<sup>15</sup> who demonstrated the value of this approach by using a mixture of the Australian (ARR No. 123) and HCM methods as "HCM procedure extension."

Consistent with Equation 1, the following expression can be used to predict an average overflow queue for a movement at an isolated signalized intersection:

$$N_o = 0.25 QT x^m [(x-1) + \sqrt{(x-1)^2 + m(x-x_0)/QT}] \quad (\text{A.1})$$

for  $x > x_0$  (0 otherwise).

where  $N_o$  is the average overflow queue in vehicles allowing for randomness and time-dependent oversaturation effects<sup>2,4</sup>, and all

other parameters and variables are as defined for Equation 1.

Using the overflow queue parameter, the average delay per vehicle is expressed as

$$d = [0.5c(1-u)^2/(1-ux)] + 3600 N_o/Q \quad (\text{A.2})$$

where  $N_o$  is calculated from Equation A.1 and all other the parameters and variables are as in Equation 1.

For the purposes of the HCM method, the average stopped delay can be calculated from

$$d_s = 0.77 d \quad (\text{A.2a})$$

where  $d$  is the average overall delay from Equation A.2.

Similarly, the stop rate (average number of stops per vehicle including multiple stops in queue) can be calculated from

$$h = 0.9 \{[(1-u)/(1-ux)] + 3600 N_o/qc\} \quad (\text{A.3})$$

where  $q$  is the arrival flow rate in vehicles per hour,  $c$  is the cycle time, and  $u$  is the green time ratio as in Equation 1, and 0.9 is a simple reduction factor to allow for partial stops<sup>2</sup>.

The total number of stops in vehicles per hour can be calculated using  $h$  from Equation A.3 as

$$H = qh \quad (\text{A.3a})$$

As  $N_o$  represents the average overflow queue,  $N_o/qc$  represents the multiple stops in queue in ov-

ersaturated cycles. On the other hand, the first term in Equation A.3 represents the major stops. In SIDRA, the fuel consumption values of these two types of stops are different because they represent different stop-start cycles. In other words, a single value of excess fuel consumption per stop should not be applied to the results from Equation A.3.

Another important performance parameter in signal design is the queue length, which is particularly useful in determining the capacities of short lanes (e.g., turn bays). While ARR No. 123 is a little confusing in giving several formulas for queue length prediction, the SIDRA program uses only the *maximum back of the queue* (in an average cycle). This is the performance measure relevant to short lane storage capacities.

The maximum back of the queue,  $N_m$ , in vehicles is given by

$$N_m = \{qr/[3600(1-ux)] + N_o\} \quad (\text{A.4})$$

where  $r$  is the effective red time in seconds,  $qr$  represents the average stop-line queue at the start of green time (based on uniform arrivals), and  $N_o$  is the average overflow queue given by Equation A.1.

The formulas given in this Appendix apply only to the case of a

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single green period per cycle. Extended formulas that apply to the case of two green periods per cycle (e.g., protected and permissive turns) are incorporated in the SIDRA program and are to be documented soon. SIDRA Version 3 allows the choice of HCM delay model, the alternative given in this article, or any other calibration of the generalized formula. A new version of SIDRA with full HCM option taking advantage of the advanced models and computation methods of the SIDRA program is expected to be available soon.

### Example:

For the example in the text,  $c =$

90 sec.,  $g = 30$  sec.,  $s = 1500$  veh./hr., and  $T = 0.25$  hr. (15 minutes) are given, and  $u = 1/3$ ,  $Q = 500$  veh./hr.,  $QT = 125$  veh., and  $r = 60$  sec. are calculated. Let us assume an arrival flow rate of  $q = 480$  veh./hr. to give  $x = q/Q = 0.96$  and  $ux = 0.32$ . The performance measures predicted for this example using the appropriate values of parameters  $n$ ,  $m$ , and  $x_0$  in equation A.1 for the alternative HCM formula, the original HCM formula, and the original Australian (ARR No. 123) formula are given in Table A.1. The parameter values for different formulas are given in Table 1 (note that parameters  $a$  and  $b$  determine  $x_0$ ).

Table A.1. Performance Measures<sup>a</sup>

	Alternative to HCM Formula	Original HCM Formula	Australian Formula	Canadian Formula
Average overflow queue $N_0$ (veh.)	4.26	4.03	3.93	4.37
Average overall delay $d$ (sec./veh.)	60.1	58.4	57.7	60.9
Average stopped delay $d_s$ (sec./veh.)	46.2	44.9	44.4	46.8
Number of stops $H$ (stops/h.)	577	568	565	581
Maximum back of the queue $N_m$ (veh.)	16.0	15.8	15.7	16.1

<sup>a</sup>Predicted using the generalized formula with parameters corresponding to one new and three existing formulas described in Table 1. Numerical example:  $c = 90$  sec.,  $g = 30$  sec.,  $q = 480$  veh./hr.,  $s = 1500$  veh./hr., and  $t = 15$  min.



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fic signal and fuel consumption areas of research and development. He is a member of ITE Technical Committee 5B-27, Left-Turn Lane Storage Length Design Criteria. He graduated from Istanbul Technical University, Turkey, in 1968 and received his Ph.D. degree in transportation engineering from the University of Leeds, England, in 1974. He is a Member of ITE.