

## REPRINT

# Gap acceptance modelling by traffic signal analogy

R. AKÇELIK

### REFERENCE:

AKÇELIK, R. (1994). Gap acceptance modelling by traffic signal analogy. *Traffic Engineering and Control*, 35 (9), pp 498-506.

### NOTE:

This paper is related to the intersection analysis methodology used in the SIDRA INTERSECTION software. Since the publication of this paper, many related aspects of the traffic model have been further developed in later versions of SIDRA INTERSECTION. Though some aspects of this paper may be outdated, this reprint is provided as a record of important aspects of the SIDRA INTERSECTION software, and in order to promote software assessment and further research.

# Gap-acceptance modelling by traffic signal analogy

by Rahmi Akçelik, Chief Research Scientist, Australian Road Research Board Ltd

## INTRODUCTION

This paper presents new analytical models of capacity and traffic performance (delay, queue length, proportion queued and queue move-up rate) for approach lanes controlled by road priority signs (stop and give-way/yield).

The derivation of performance models and the calibration of arrival headway distributions are described in more detail, and formulae for fixed-time (pre-timed) isolated signals are given, in recent papers by Akçelik and Chung<sup>1, 2</sup>. Related work on actuated signals is described in Akçelik<sup>3, 4</sup>. This paper discusses the new capacity model for unsignalised intersections in some detail and compares it with existing gap-acceptance based capacity models.

The models for unsignalised intersections were derived by treating *block* and *unblock* periods in a priority (major) stream (as defined in the traditional gap-acceptance modelling) as *red* and *green* periods in a way similar to the modelling of signal-controlled traffic streams. This enables the modelling of the average back-of-queue, proportion queued and queue move-up rate for the entry (minor) stream in a manner consistent with models for traffic signals. This also presents a methodological advantage in that the same conceptual framework is employed in models for different types of intersection.

The models presented here represent a new development to fill the gap in modelling queue length, proportion queued and stop rate (major stops and queue move-ups separately) in the context of gap-acceptance modelling. The traditional gap-acceptance and queueing theory models do not give sufficient information for intersection design purposes since they predict average cycle-based queue lengths rather than the back of queue, and models for predicting stop rates do not exist other than recent work by Troutbeck<sup>5</sup>.

The capacity and performance models presented in this paper were developed using the bunched exponential model of arrival headway distribution for all types of intersection<sup>1</sup>. This model is more realistic than the commonly-used simple exponential and shifted exponential models. However, the models are also applicable to simple negative exponential and shifted negative exponential distributions.

The calibration of performance models was carried out using data generated by the microscopic simulation program MODEL6<sup>6, 7</sup>. The program was modified to incorporate the

New capacity and performance models are presented for unsignalised intersections. The new models were developed by converting the block and unblock periods in traditional gap-acceptance modelling to effective red and green periods by analogy to traffic signal operations. This enabled the modelling of performance statistics (delay, queue length, proportion queued and queue move-up rate) in a manner consistent with models for signalised intersections. The models are based on the bunched exponential model of arrival headway distributions for all traffic streams, and are also applicable to simple negative exponential and shifted negative exponential distributions. The new capacity model is compared with various existing formulae based on gap-acceptance modelling.

calibrated arrival headway distribution model<sup>1</sup> and generate data required for the calibration of the new capacity and performance models.

For capacity and performance modelling, a lane-by-lane method is adopted generally, and therefore the arrival headway distribution in a single lane of the approach road is considered. However, in modelling capacity of entry streams, the headway distribution of

total traffic demand in all lanes of the circulating stream or major stream is adopted with different values of minimum headway and bunching parameters for single-lane and multi-lane cases. When there are several conflicting (higher priority) streams at signal-controlled and signalised intersections, all conflicting streams are combined as one stream and treated using appropriate multi-lane stream parameters.

## ARRIVAL HEADWAY DISTRIBUTIONS

The estimation of arrival headways is fundamental to the modelling of gap acceptance processes for estimating capacities of sign-controlled traffic streams, roundabout entry streams and filter turns at signalised intersections (e.g. Akçelik<sup>8, 9</sup>, Akçelik and Troutbeck<sup>10</sup>, Troutbeck<sup>5, 11-16</sup>).

This paper considers a class of exponential arrival headway distribution models known as negative exponential (M1), *shifted* negative exponential (M2) and *bunched* exponential (M3). The bunched exponential distribution of arrival headways (M3) was proposed by Cowan<sup>17</sup> and used extensively by Troutbeck<sup>5, 11-16</sup> for estimating capacity and performance of roundabouts and other unsignalised intersections. A special case of the model was previously used by Tanner<sup>18, 19</sup> for unsignalised intersection analysis. A detailed discussion of the M3 model and the results of its calibration using real-life data for single-lane traffic streams and simulation data for multi-lane streams are given in Akçelik and Chung<sup>1</sup>. Also see a recent paper by Sullivan and Troutbeck<sup>38</sup>.

The negative and shifted negative exponential distributions are extensively discussed and used in the literature as models of *random arrivals*. On the other hand, the bunched exponential distribution is relatively new, and while more realistic, its use is less common. In particular, the bunched exponential distribution offers improved accuracy in the prediction of small arrival

headways (up to about 12 seconds), which is important for most urban traffic analysis applications.

The cumulative distribution function  $F(t)$ , for the bunched exponential distribution of arrival headways, representing the probability of a headway less than  $t$  seconds, is:

$$F(t) = \begin{cases} 1 - \phi e^{-\lambda(t-\Delta)} & \text{for } t \geq \Delta \\ 0 & \text{for } t < \Delta \end{cases} \dots(1)$$

where:  $\Delta$  = intra-bunch (minimum) headway (sec.),  
 $\phi$  = proportion of free (unbunched) vehicles, and  
 $\lambda$  = a model parameter calculated as:

$$\lambda = \frac{\phi q}{1 - \Delta q} \text{ subject to } q \leq 0.98/\Delta \dots(1a)$$

where  $q$  is the total arrival flow (veh/sec.).

According to the model, the traffic stream consists of:

- (i) *bunched* vehicles with constant intra-bunch headways equal to  $\Delta$  (the proportion of bunched vehicles equals  $1 - \phi$ ); and
- (ii) *free* vehicles with headways greater than the intra-bunch (minimum) headway,  $\Delta$  (thus, the proportion of free vehicles,  $\phi$ , represents the unbunched vehicles with randomly distributed headways).

The M1 and M2 models can be derived as special cases of the M3 model through simplifying assumptions about the bunching characteristics of the arrival stream:

Negative exponential (M1) model:

$$\Delta = 0 \text{ and } \phi = 1 \text{ (therefore } \lambda = q) \dots(2a)$$

Shifted negative exponential (M2) model:

$$\phi = 1 \text{ (therefore } \lambda = q/(1 - \Delta q)) \dots(2b)$$

Thus, models M1 and M2 assume no bunching for all levels of arrival flows. On the other hand, model M3 can be used either with a known (measured) value of  $\phi$ , or more generally, with a value of  $\phi$  estimated as a function of the arrival flow rate. Note that the shifted negative exponential model (M2) is normally used for single-lane traffic only.

The following relationship was derived as a general formula for estimating the proportion of free vehicles in the traffic stream ( $\phi$ ) by generalising the bunching implied by the simple negative exponential model (Akçelik<sup>3,4</sup>):

$$\phi = e^{-b\Delta q} \dots(3)$$

where  $b$  is a bunching factor,  $\Delta$  is the intra-bunch headway, and  $q$  is the arrival flow rate (veh/sec.). The M3 model with estimates of  $\phi$  obtained from Equation (3) will be referred to as the M3A model.

An empirical relationship of a similar form was previously used by Brilon<sup>20</sup> based on previous work by Jacobs<sup>21</sup>:

$$\phi = e^{-b'q} \dots(3a)$$

where  $b' = 6$  to  $9$ . The same empirical relationship has been used by Sullivan and Troutbeck<sup>22</sup>.

The following linear model of the proportion of free vehicles was used by Tanner<sup>18,19</sup>:

$$\phi = 1 - \Delta q \dots(4)$$

The M3 model with estimates of  $\phi$  obtained from Equation (4) will be referred to as the M3T model (in this case,  $\lambda = q$ ). More general forms of the linear  $\phi - q$  model can be considered for calibrating real-life data. The AUSTROADS roundabout guide uses a linear model<sup>10,13,23,24</sup> which has been generalised in SIDRA 4.07 as  $\phi = a(1 - \Delta q)$  where  $a$  is a constant<sup>24,25</sup>.

Both the M3A and M3T models assume that the proportion of free vehicles decreases (the proportion of bunched vehicles increases) with increasing arrival flow rate. They predict zero bunching ( $\phi = 1.0$ ) at very large flows. While the M3T model assumes  $\phi = 0$  at  $q = 1/\Delta$ , the M3A model yields non-zero values of  $\phi$  at high flows.

The parameters for the M3A model calibrated for uninterrupted flow conditions and for roundabout circulating streams<sup>1</sup> are summarised in Table I.

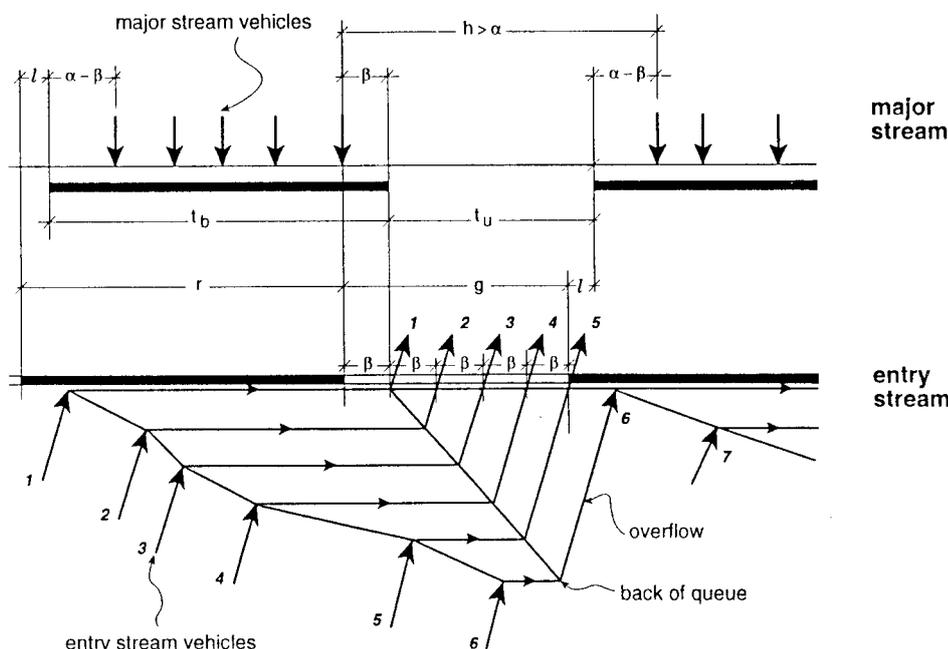


Fig 1. Signal operations analogy for gap-acceptance modelling.

### UNSIGNALLLED INTERSECTION ANALYSIS BY SIGNAL OPERATIONS ANALOGY

A method for treating the traditional gap-acceptance modelling used for roundabouts and sign-controlled intersections by analogy to traffic signal operations was conceived by Akçelik<sup>24</sup>. The underlying assumptions are shown in Fig 1 which depicts an entry (minor) stream at an unsignalled intersection giving way to an uninterrupted major (priority) stream.

The method presented here derives equivalent average red, green and cycle times ( $r$ ,  $g$ ,  $c$ ) for the gap-acceptance process considering average durations of block and unblock periods ( $t_b$ ,  $t_u$ ) in major streams as used in the traditional gap-acceptance modelling<sup>10-16,18,19,26</sup>.

Block periods correspond to continuous periods of no acceptable gap, i.e. consecutive major stream headways less than the mean critical gap ( $\alpha$ ). Unblock periods correspond to headways equal to or greater than the critical gap,  $h_i \geq \alpha$ , where  $h_i$  is the  $i$ th acceptable headway (gap) in the major stream. In accordance with the definition used in the traditional gap-acceptance theory, the duration of the unblock period is  $t_{ui} = h_i - \alpha$  (where  $h_i \geq \alpha$ ). This relationship can be explained by assuming that (a) the first minor stream vehicle departs  $\beta$  seconds after the start of the acceptable

headway, and (b) there cannot be any departures during the last  $(\alpha - \beta)$  seconds of the acceptable headway. Parameter  $\beta$  represents the follow-up (saturation) headway.

The equivalent green time,  $g_p$ , includes the first  $\beta$  seconds of the acceptable headway (or unblock period). However, it is shorter than the unblock period by an amount called lost time ( $l$ ) which cannot be used for any vehicle departures. This is because the number of vehicles ( $n_i$ ) that can depart during an acceptable headway is assumed to be an integer:  $g_i = n_i \beta$ . Therefore,  $g_i = t_{ui} + \beta - l_i = h_i - (\alpha - \beta) - l_i$ . The average value of the lost time is  $l = 0.5\beta$  (this was confirmed by simulation results).

Similarly, the equivalent red time is related to the  $i$ th block period through  $r_i = t_{bi} - \beta + l_i$ . The equivalent cycle time is the sum of the red and green times, and is also equal to the sum of block and unblock periods:  $c_i = r_i + g_i = t_{bi} + t_{ui}$ .

The average capacity per cycle is obtained as  $sg = g/\beta$  where  $g$  is the average equivalent green time and  $\beta$  is considered to be a saturation headway ( $s = 1/\beta$  in veh/sec., or  $s = 3600/\beta$  in veh/h). The entry stream capacity based on the gap-acceptance process can then be expressed as  $Q_q = sg/c$  as in the case of signalised intersections.

The estimates of the average values of block and unblock periods ( $t_b$ ,  $t_u$ ), the equivalent red, green and cycle times ( $r$ ,  $g$ ,  $c$ ), and the corresponding capacity can be calculated using Equations (5) to (9). All capacity and performance calculations are carried out for individual lanes of entry (minor) movements, but traffic in all lanes of the major (conflicting) movement is treated together as one stream. When there are several conflicting (higher priority) streams at sign-controlled and signalised intersections, all conflicting streams are combined as one stream.

Table I. Summary of parameter values for the bunched exponential arrival headway distribution model M3A\*

Number of lanes <sup>†</sup>	Uninterrupted traffic streams <sup>††</sup>			Roundabouts circulating streams		
	$\Delta$	$b$	$\phi$	$\Delta$	$b$	$\phi$
1	1.5	0.6	$e^{-0.9q}$	2	2.5	$e^{-5.0q}$
2	0.5	0.5	$e^{-0.25q}$	1	2.5	$e^{-2.5q}$
>2	0.5	0.8	$e^{-0.4q}$	Same as the 2-lane case		

\* For the M3T model, use  $\phi = 1 - \Delta q$  with same  $\Delta$  values as model M3A

† Total number of lanes available to the traffic stream

†† Use for all traffic at sign-controlled and signalised intersections; and for approach roads (entry streams) only at roundabouts

$$t_b = \frac{e^{\lambda(\alpha-\Delta_m)} - 1}{\phi_m q_m} - \frac{1}{\lambda} \quad \dots(5a)$$

$$t_u = \frac{1}{\lambda} \quad \dots(5b)$$

$$c = t_b + t_u = r + g = \frac{e^{\lambda(\alpha-\Delta_m)} - 1}{\phi_m q_m} \quad \dots(6a)$$

$$g = t_u + \beta - l = \frac{1}{\lambda} + \beta - l \quad \dots(6b)$$

$$r = c - g = t_b - \beta + l = \frac{e^{\lambda(\alpha-\Delta_m)} - 1}{\phi_m q_m} - \frac{1}{\lambda} - \beta + l \quad \dots(6c)$$

$$l = 0.5\beta \quad \dots(7)$$

$$u = g/c = \left(\frac{1}{\lambda} + \beta - l\right) \phi_m q_m e^{-\lambda(\alpha-\Delta_m)} \dots(8a)$$

$$= (1 - \Delta_m q_m + 0.5\beta \phi_m q_m) e^{-\lambda(\alpha-\Delta_m)}$$

$$y = q_e / s = \beta q_e \quad \dots(8b)$$

$$s g = g / \beta \quad \dots(8c)$$

$$Q = \max(Q_s, Q_m) \quad \dots(9)$$

$$Q_s = \frac{s g}{c} = \frac{3600u}{\beta} \quad \dots(9a)$$

$$= \frac{3600\phi_m q_m}{\beta} \left(\frac{1}{\lambda} + \beta - l\right) e^{-\lambda(\alpha-\Delta_m)}$$

$$= \frac{3600}{\beta} (1 - \Delta_m q_m + 0.5\beta \phi_m q_m) e^{-\lambda(\alpha-\Delta_m)}$$

$$Q_m = \min(q_e, 60n_m) \quad \dots(9b)$$

where

$t_b, t_u$  = average durations of the block and unblock periods in the major traffic stream (sec.)

$c$  = equivalent average cycle time corresponding to the block and unblock periods in the major traffic stream ( $c=r+g$ ) (sec.)

$g, r$  = equivalent average green and red times corresponding to the unblock and block periods in the major traffic stream (sec.)

$l$  = equivalent lost time that corresponds to the unused portion of the unblock period (sec.)

$u, y$  = equivalent green time ratio and flow ratio for the entry stream

$s g$  = equivalent capacity per cycle for the entry stream, i.e. the maximum number of vehicles that can discharge during the average unblock period (veh), where  $s$  is in veh/sec.

$s$  = saturation flow ( $s=3600/\beta$ ) (veh/h)

$\alpha, \beta$  = mean critical gap and follow-up (saturation) headway for the entry stream (sec.)

$Q$  = capacity of the entry stream (veh/h)

$Q_s$  = capacity estimate using the gap-acceptance method (veh/h)

$Q_m$  = minimum capacity (veh/h)

$n_m$  = minimum number of entry stream vehicles that can depart under heavy major stream flow conditions (veh/min.)

$q_e$  = arrival flow of the entry lane (veh/h)

$q_m$  = total arrival flow of the major stream (veh/sec. or veh/h; expressed in pcu/sec. or pcu/h if adjusted for heavy vehicle effects using the passenger car equivalents method)

$\lambda, \phi_m, \Delta_m$ , are as in Equations (1) to (4) for the major stream.

When there are several conflicting (higher priority) streams, the total major stream flow ( $q_m$ ) is calculated as the sum of all conflicting stream flows, and parameters  $\Delta_m, \phi_m$  are determined accordingly. Equations (5) to (9) should be used for  $q_m > 0$  (for  $q_m = 0, r = 0, g = c, u = 1.0, Q_s = 3600/\beta$ ).

An example of equivalent red, green and cycle times is given in Fig 2 for the case of a simple gap-acceptance situation with a single-lane major stream ( $\Delta_m = 1.5$  from Table I) with  $\alpha = 4$  sec.,  $\beta = 2$  sec. Figure 3 shows a comparison of the capacities predicted by Equation (9a) with those simulated by MODEL C<sup>2</sup>.

#### COMPARISON WITH EXISTING CAPACITY MODELS

The gap-acceptance based capacity formula given here (Equation 9a) can be compared with various existing capacity formulae. Firstly, Equation (9a) will be rewritten for different arrival headway distributions. For the M3A model (bunched exponential distribution using  $\phi_m$  from Equation (3):

$$Q_s = \frac{3600}{\beta} (1 - \Delta_m q_m + 0.5\beta \phi_m q_m e^{-\lambda \Delta_m q_m}) e^{-\lambda(\alpha - \Delta_m)} \quad \dots(10a)$$

For the M3T model (using  $\phi_m$  from Equation (4):

$$Q_s = \frac{3600}{\beta} (1 - \Delta_m q_m)(1 + 0.5\beta q_m) e^{-q_m(\alpha - \Delta_m)} \quad \dots(10b)$$

For the shifted negative exponential model (M2), setting  $\phi_m = 1$  and  $\lambda = q_m/(1 - \Delta_m q_m)$  as in Equation (2b):

$$Q_s = \frac{3600}{\beta} (1 - \Delta_m q_m + 0.5\beta q_m) e^{-\lambda(\alpha - \Delta_m)} \quad \dots(10c)$$

For the simple negative exponential model (M1), using  $\Delta_m = 0, \phi_m = 1$  and  $\lambda = q_m$  as in Equation (2a):

$$Q_s = \frac{3600}{\beta} (1 + 0.5\beta q_m) e^{-q_m \alpha} \quad \dots(10d)$$

McDonald and Armitage<sup>27</sup> used the concept of *saturation flow* ( $q_s$ ) and *lost time* ( $L$ ) for estimating roundabout capacities with a degree of traffic signal analogy. However, they did not equate the saturation flow with  $3600/\beta$ , and their lost time definition ( $L$ ) is rather different from the lost time ( $l$ ) used in this paper. The practical method they used for measuring the lost time and saturation flow (Fig 1 in McDonald and Armitage<sup>27</sup>) gives a saturation flow close to  $(3600/\beta)$  and the lost time they measure ( $L$ ) is identical to the *zero-gap* ( $t_o$ ) parameter used by Siegloch<sup>28</sup>.

The measurement method by Siegloch

(see Fig 3 in Brilon and Grossman<sup>29</sup>) obtains  $\beta$  as to a saturation headway explicitly, and produces a zero-gap parameter which is stated to be related to the critical gap through  $t_o = \alpha - 0.5\beta$ . Putting  $l = 0.5\beta$  as in Equation (7), the Siegloch/McDonald-Armitage method of measurement can be related to the method described in this paper through:

$$t_o = \alpha - l \quad \dots(11)$$

where  $l$  is the lost time as defined in this paper (Fig 1 and Equation (7)).

The relationship between the critical gap and zero-gap parameters ( $\alpha$  and  $t_o$ ) is depicted in Fig 4.

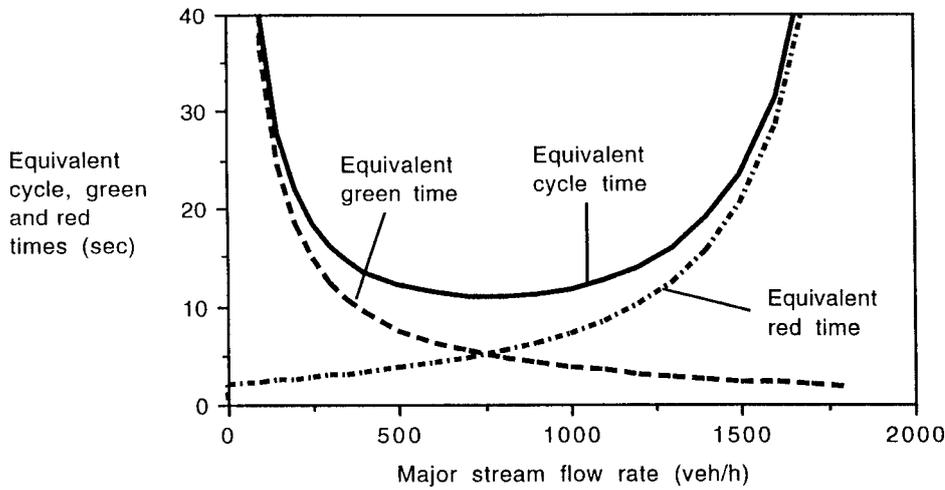


Fig 2. The equivalent red, green and cycle times as a function of the major-stream flow rate for a simple gap acceptance example.

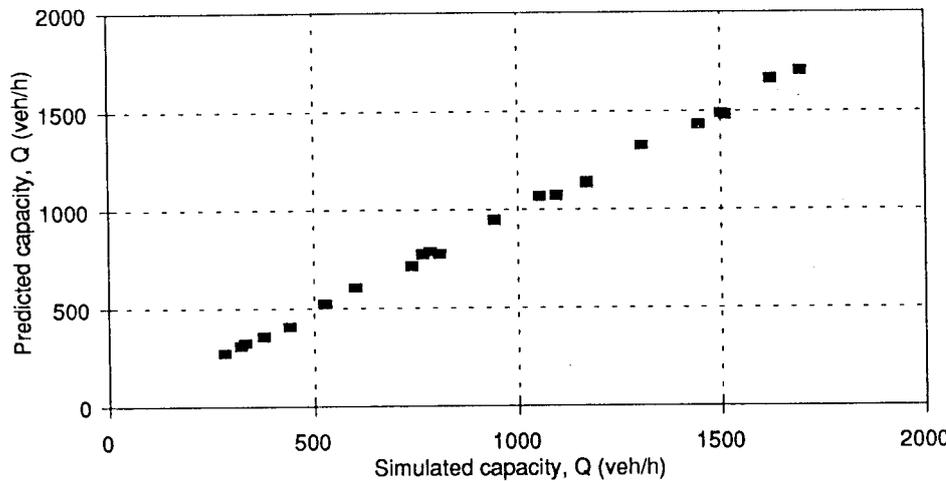


Fig 3. Comparison of the capacities predicted by Equation (9a) with those simulated by MODEL C.

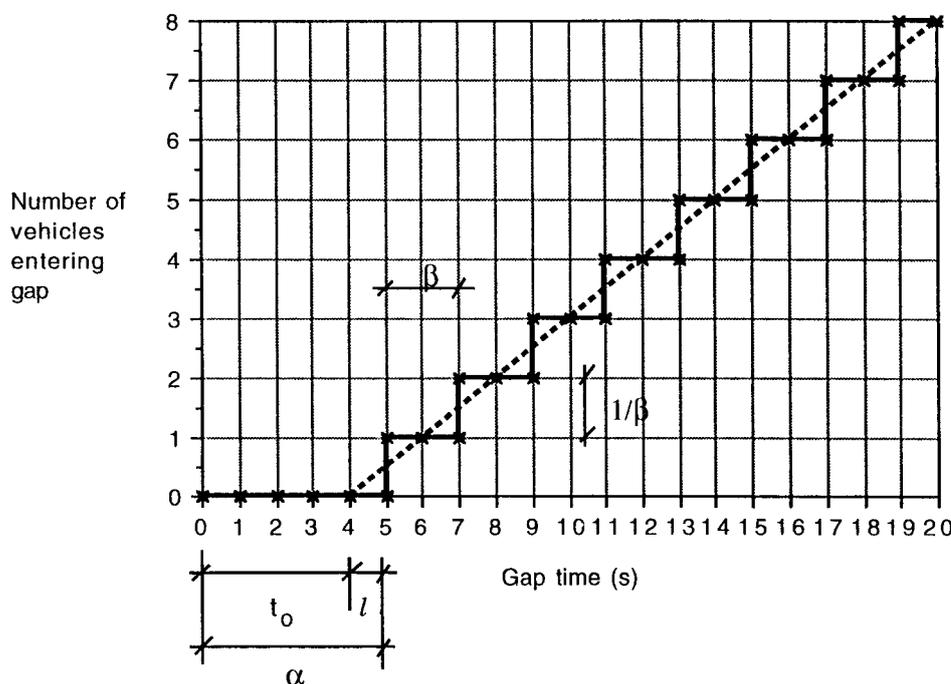


Fig 4. The relationship between the critical gap and zero-gap parameters ( $\alpha$  and  $t_0$ ) constructed for known values of  $\alpha$  and  $\beta$ .

Siegloch's capacity formula, which is used in the German guidelines<sup>20,29</sup>, assumes a negative exponential model of arrival headways (M1), and is given as:

$$Q_g = \frac{3600}{\beta} e^{-q_m t_0} \quad \dots(12a)$$

This is seen to be similar to Equation (10d). The use of  $\alpha-l$  instead of  $\alpha$ , and omission of the factor  $(1 + 0.5\beta q_m)$  tend to compensate, and Equations (10d) and (12a) give close values.

Putting  $q_s = 3600/\beta$  and  $L = t_0$ , the capacity formula given by McDonald and Armitage<sup>27</sup> can be expressed as:

$$Q_g = \frac{3600}{\beta} (1 - \Delta_m q_m) e^{-q_m(t_0 - \Delta_m)} \quad \dots(12b)$$

This is similar to Equation (10b) based on the M3T model, differences being similar to those noted for the Siegloch formula and Equation (10d).

Finally, a formula by Jacobs<sup>21</sup> based on a shifted negative exponential distribution (M2 model) as described by Brilon<sup>20</sup> is:

$$Q_g = \frac{3600}{\beta} (1 - \Delta_m q_m) e^{-\lambda(t_0 - \Delta_m)} \quad \dots(12c)$$

This is seen to be similar to Equation (10c), again, differences being similar to those noted for the Siegloch formula and Equation (10d).

A more traditional capacity formula based on gap-acceptance modelling (Tanner<sup>18,19</sup>, Troutbeck<sup>11,13</sup>) can be expressed in the following general form:

$$Q_g = \frac{3600 \phi_m q_m e^{-\lambda(\alpha - \Delta)}}{1 - e^{-\lambda\beta}} \quad \text{for } q_m > 0 \quad \dots(13a)$$

$$= 3600/\beta \quad \text{for } q_m = 0$$

where  $q_m$  is the total flow for the major stream in veh/sec.

Various capacity formulae found in the literature can be generated from Equation (13a) by applying the special conditions of arrival headway models M1, M2 and M3T (for Tanner's capacity formula). For example, for the simple negative exponential model (M1):

$$Q_g = \frac{3600 q_m e^{-q_m \alpha}}{1 - e^{-q_m \beta}} \quad \text{for } q_m > 0 \quad \dots(13b)$$

$$= 3600/\beta \quad \text{for } q_m = 0$$

For the M3T model, Tanner's formula<sup>19</sup> is obtained:

$$Q_g = \frac{3600 q_m (1 - \Delta_m q_m) e^{-q_m(\alpha - \Delta)}}{1 - e^{-q_m \beta}} \quad \text{for } q_m > 0 \quad \dots(13c)$$

$$= 3600/\beta \quad \text{for } q_m = 0$$

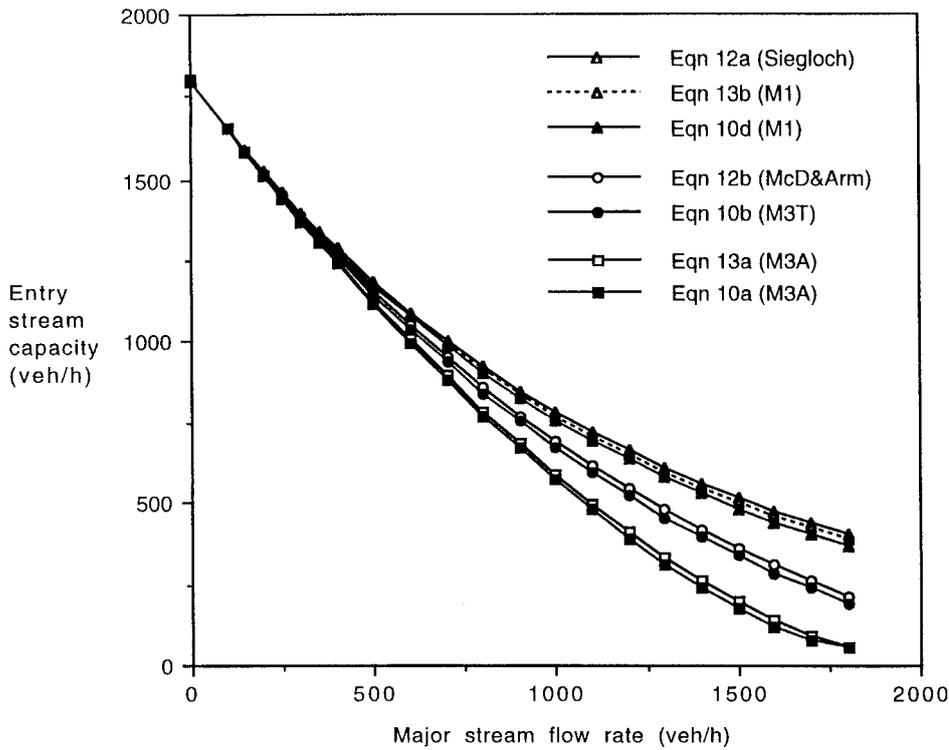


Fig 5. Capacity as a function of the major-stream flow rate estimated from various formulae for the same gap acceptance example as in Fig 2.

Figure 5 shows the capacities estimated from the formulae given above for the same example as in Fig 2 (single-lane major stream with  $\Delta_m = 1.5$  sec.,  $\alpha = 4$  sec.,  $\beta = 2$  sec.,  $l = 1$  sec.,  $t_s = 3$  sec). Figure 5 confirms that (a) generally there is little difference between vari-

ous models for low major-stream flows; (b) the differences among models which use the same arrival headway distribution are negligible; and (c) the impact of the assumption about the arrival headway distribution is significant at high major-stream flow levels.

## PERFORMANCE MODELS

New analytical models for estimating delay, back-of-queue and cycle-average queue length (average, 90th, 95th and 98th percentile values for both queue definitions), proportion queued (major stops) and queue move-up rate for unsignalised intersections are given below. The formula for stop rate involves the use of Equivalent Stop Value (ESV) factors for major stops and queue move-ups. A detailed description of the new performance models is presented, and the ESV factor and stop rate formulae are given in Akçelik and Chung<sup>2</sup>. Similar formulae for fixed-time signals are also given in the same paper.

The new formulae are based on the theoretical framework previously developed for modelling delay, queue length and stop rate in an integrated manner (Akçelik<sup>8,9,30-32</sup>, Akçelik and Rouphail<sup>33,34</sup>). Overflow queue formulation is central to the modelling of delay, queue length and queue move-ups. This provides a convenient link between *steady-state* and *time-dependent* formulations, thus allowing for easy model calibration using field or simulation data.

The performance models for unsignalised intersections are developed by traffic signal analogy (see Fig 1 and Equations (5) to (9)). The traditional two-term model structure is used by introducing a separate calibration factor for each term of each performance statistic. The first-term adjustment factors help to allow for the effect of variations in queue clearance times under low- to medium-flow

conditions (when there are no overflow queues), and any additional delays, etc., due to overflow queues are included in the second (overflow) terms.

Expressions for average delay in seconds per vehicle ( $d$ ), average back-of-queue ( $N_b$ ), cycle-average queue length ( $N_c$ ), proportion queued ( $p_q$ ) and the queue move-up rate ( $h_{qm}$ ) are given in Equations (14) to (21). The formulae for delay, queue length and queue move-up rate are time-dependent expressions. Delays and queue move-up rates are average values for all vehicles queued and unqueued.

$$d = d_1 + d_2 \quad \dots (14)$$

$$d_1 = (1 + 0.3 y^{0.20}) \frac{d_m}{1-y} \text{ for } x \leq 1.0 \quad \dots (14a)$$

$$= d_{l(x=1)} \text{ for } x > 1.0$$

$$d_2 = 900 T_f \left[ (z + \sqrt{z^2 + \frac{8k_d(x-x_o)}{QT_f}}) \right] \text{ for } x > x_o \quad \dots (14b)$$

$$= 0 \text{ otherwise}$$

$$N_b = N_{b1} + N_{b2} \quad \dots (15)$$

$$N_{b1} = 1.2 \varphi_e^{0.8} \frac{q_e r}{1-y} \text{ for } x \leq 1.0 \quad \dots (15a)$$

$$= N_{b1(x=1)} \text{ for } x > 1.0$$

$$N_{b2} = 0.25 QT_f \left[ (z + \sqrt{z^2 + \frac{8k_b(x-x_o)}{QT_f}}) \right] \text{ for } x > x_o \quad \dots (15b)$$

$$= 0 \text{ otherwise}$$

$$p_q = 0.75 \varphi_e (sg)^{0.40} \frac{1-u}{1-y} \text{ subject to } p_q \leq 1.0 \quad \dots (16)$$

$$h_{qm} = \frac{0.25 QT_f}{qc} \left[ (z + \sqrt{z^2 + \frac{8k_{qm}(x-x_o)}{QT_f}}) \right] \text{ for } x > x_o \quad \dots (17)$$

$$= 0 \text{ otherwise}$$

where:

$$x_o = 0.14 (sg)^{0.55} \text{ subject to } x_o \leq 0.95 \quad \dots (18a)$$

$$k_d = 0.17 \varphi_e (sg)^{1.40} y^{-0.40} (d_m Q) \quad \dots (18b)$$

$$k_b = 0.45 \varphi_e (sg)^{1.70} y^{0.40} (d_m Q) \quad \dots (18c)$$

$$k_{qm} = 1.1 \varphi_e (sg)^{1.10} y^{0.50} (d_m Q) \quad \dots (18d)$$

$$d_m = \frac{e^{\lambda(\alpha-\Delta_m)} - \alpha - \frac{1}{\lambda} + \frac{\lambda \Delta_m^2 - 2\Delta_m + 2\Delta_m \varphi_m}{2(\lambda \Delta_m + \varphi_m)}}{\varphi_m q_m} \quad \dots (18e)$$

$$x = q/Q \quad \dots (18f)$$

and:

$r, g, c$  = equivalent average red, green and cycle times in seconds estimated from Equations (6a) to (6c)

$y, u, sg$  = flow ratio, green time ratio and capacity per cycle (vehicles) estimated from Equations (8a) to (8c)

$q_e$  = arrival (demand) flow rate for the entry stream during the specified flow period in vehicles per hour (or per second)

$\varphi_e$  = the proportion of unbunched traffic in the entry stream estimated from Equation (3) and Table I for single-lane conditions

$d_m$  = minimum delay experienced by the entry-stream vehicles (sec.) (see Cowan<sup>26,35</sup>, Troutbeck<sup>3,11,13,15</sup>);

$Q$  = entry stream capacity in vehicles per hour (or per second) estimated from Equation (9)

$x$  = degree of saturation of the entry stream (ratio of arrival flow rate to capacity)

$z = x - 1$

$T_f$  = flow (analysis) period in hours, and  $QT_f$  = throughput, i.e. the maximum number of vehicles that can be discharged during the flow period, and

$d_{l(x=1)}$  and  $N_{b1(x=1)}$  are the values of delay and queue length at  $x=1$  (so that the first terms are constant for oversaturated conditions,  $x > 1$ ).

For  $q_m = 0$  in the above equations, set  $r = 0$ ,  $u = 1.0$  and  $d_m = 0$  (therefore, zero delay, queue length, etc., will result).

The duration of the flow period affects the estimates of performance statistics significantly. Larger delays, queue lengths and queue move-up rates will result from longer flow periods for a given demand level.  $T_f = 0.25$  h is built into the U.S. *Highway Capacity Manual* delay formula for signalised intersections<sup>36</sup> whereas the models given here allow  $T_f$  to be variable.

### Estimation of queue length

The traditional gap-acceptance and queueing theory models do not give sufficient information for intersection design purposes since they predict average cycle-based queue lengths rather than the back-of-queue. The commonly-used *average cycle-based* queue length is the average queue length considering all instances during the cycle including the zero-queue states. The average back-of-queue ( $N_b$ ), estimated from Equation (15), represents the maximum extent of queue in an average cycle as shown in Fig 1. The back-of-queue is a more useful statistic since it is relevant to the design of appropriate queueing space (e.g. for short lane design).

The commonly-used formula to calculate the cycle-average queue ( $N_c$ ) is:

$$N_c = d q_e \quad \dots (19)$$

where  $d$  is the average delay from Equation (14) and  $q_e$  is the average flow rate for the entry stream. Thus, the cycle-average queue is equivalent to the *total delay*, or *delay rate* (strictly speaking, this relationship applies to undersaturated conditions,  $x < 1$ , only).

The 90th, 95th and 98th percentile values of the back-of-queue ( $N_{bp\%}$ ) and the cycle-average queue ( $N_{cp\%}$ ) can be expressed as a function of the average value ( $N_b$  or  $N_c$ ):

$$N_{bp\%} = f_{bp\%} N_b \quad \dots (20a)$$

$$N_{cp\%} = f_{cp\%} N_c \quad \dots (20b)$$

where  $f_{bp\%}$  and  $f_{cp\%}$  are the factors for  $p$ th percentile queue calculated from:

$$f_{b90\%} = 1.9 + 0.7 e^{-N_b/8} \quad \dots (21a)$$

$$f_{b95\%} = 2.5 + 0.7 e^{-N_b/8} \quad \dots (21b)$$

$$f_{b98\%} = 3.0 + 0.7 e^{-N_b/8} \quad \dots (21c)$$

$$f_{c90\%} = 2.0 + 0.6 e^{-N_c/8} \quad \dots (21d)$$

$$f_{c95\%} = 2.5 + 0.7 e^{-N_c/8} \quad \dots (21e)$$

$$f_{c98\%} = 3.2 + 1.0 e^{-N_c/2} \quad \dots (21f)$$

Figure 6 shows the average, 90th, 95th and 98th percentile back-of-queue values as a function of the entry lane degree of saturation for a major stream flow rate of 720 veh/h for the same sample as in Figs 2 and 5 (duration of the flow period is  $T_f = 0.5$  h). For the same sample, the proportion queued as a function of the entry-lane degree of saturation is shown for major-stream flow rates of 360, 720 and 1080 veh/h (to represent low, medium and high flow levels) in Fig 7. A comparison of average back-of-queue and cycle-average queue values simulated using MODEL C with various gap-acceptance parameters is shown in Fig 8.

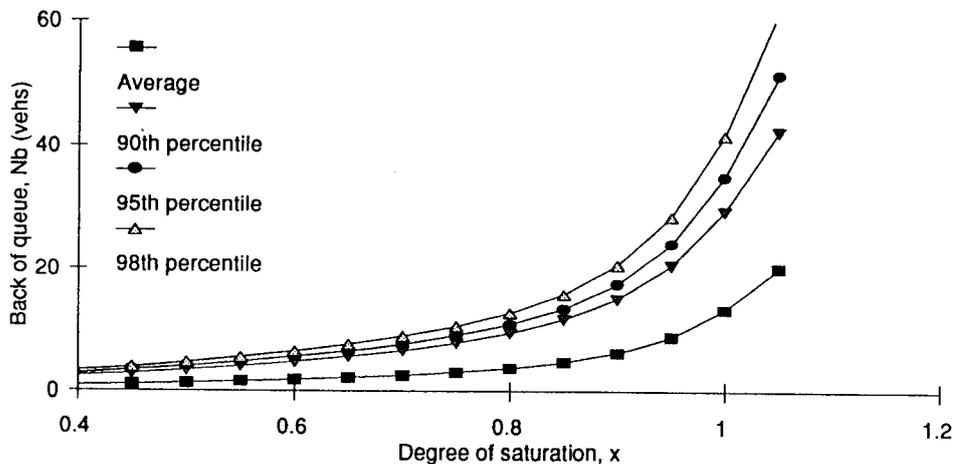


Fig 6. The average, 90th, 95th and 98th percentile back-of-queue functions for a major-stream flow rate of 720 veh/h.

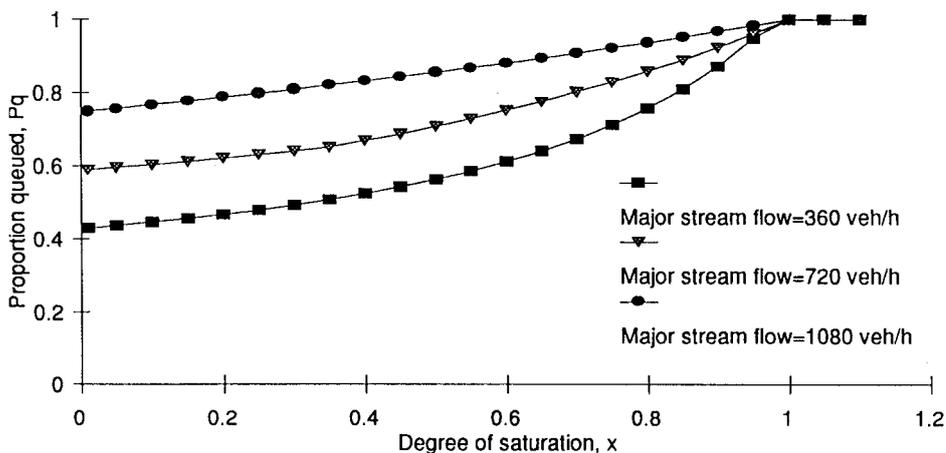


Fig 7. Proportion queued as a function of the entry-lane degree of saturation for three levels of major-stream arrival flow rate.

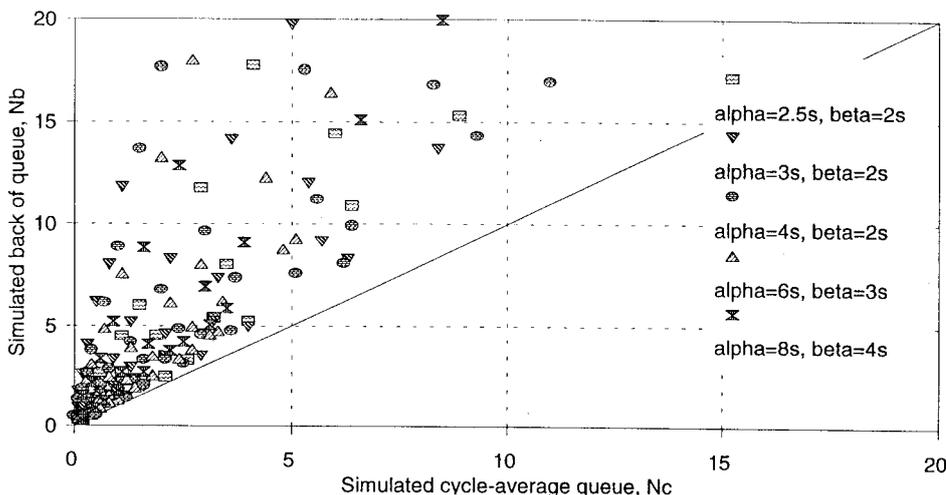


Fig 8. Comparison of average back-of-queue and cycle-average queue values simulated using MODEL C with various gap-acceptance parameters.

## CONCLUDING REMARKS

The modelling of unsignalised intersections by analogy to traffic signal operations enabled the development of a consistent modelling framework for the comparison of different types of intersections. The average back-of-queue, proportion queued and queue move-up rates can now be predicted in a manner consistent with models for signalised intersections. The models have been structured in a form appropriate for developing performance models for vehicle-actuated signals (Akçelik<sup>3,4</sup>). The models were calibrated using a microscopic simulation program (MODEL C). Further work to calibrate the performance models using real-life data would be valuable.

The recommended method for the treatment of conflicting stream flows is to treat traffic in all lanes of all major (conflicting) movements together as one stream. This method is simple and gives results close to the method that treats conflicting movements lane by lane in calculating the parameters necessary for capacity and performance calculations.

On the other hand, the *lane-by-lane* method for the use of performance formulae is recommended, although the formulae could also be used on a *lane group* basis. The lane-by-lane method as used in the SIDRA software package<sup>9,25,32</sup> is preferred due to better accuracies that can be achieved, especially in the prediction of queue length.

Equations to predict the 90th, 95th and 98th percentile queues will provide valuable information to practitioners for the design of queueing space. Effective stop rates predicted in equivalent stop values (ESVs) (see Akçelik and Chung<sup>2</sup>) can be used in simple methods for estimating fuel consumption, pollutant emissions, operating cost and similar statistics (e.g. using excess fuel consumption rate per major stop). Separate prediction of major stops and queue move-up rates is useful for more accurate estimation of such statistics (e.g. using the four-mode elemental model in SIDRA — see Bowyer, Akçelik and Biggs<sup>37</sup>).

Through the use of the bunched exponential model of arrival headways for all traffic streams, the performance models now take into account the effect of bunching in approach (entry) flows as well as major (opposing or circulating) flows.

Comparison of various forms of the new capacity model presented in this paper and those found in the literature confirms that there is little difference between models for low major-stream flows, the differences among models which use the same arrival headway distribution are negligible, and the impact of the assumption about the arrival headway distribution is significant at high major-stream flow levels.

The choice of appropriate gap-acceptance parameter values ( $\alpha$  and  $\beta$ ) is outside the scope of this paper. The Australian method for roundabouts<sup>10,13,23,24</sup> uses a comprehensive method to estimate variable gap-acceptance parameter values. The German and American methods present tables and graphs for the choice of gap-acceptance parameter values for various movements at sign-controlled intersections<sup>29,36</sup>.

The effects of heavy vehicles in the major stream and the entry stream can be taken into account either by adjusting gap-acceptance parameters or using passenger car equivalents (Troutbeck<sup>16</sup>). The use of passenger car equivalents to convert major-stream arrival flow rates and entry-stream capacities as used in the SIDRA software package is described in Akçelik<sup>24</sup>. Further research is recommended on the effects of heavy vehicles on arrival headway distributions and gap-acceptance parameters. Similarly, adjustment of the saturation headway ( $\beta$ ) or the use of an increased lost time ( $l$ ) to allow for the effects of pedestrians at roundabouts and unsignalised intersections could be considered.

The capacity model given in this paper is relevant to a basic gap-acceptance situation where an entry (minor) stream gives way to a single uninterrupted opposing (major) stream. The German and U.S. *Highway Capacity Manual* models<sup>20,29,36</sup> adjust the basic gap-acceptance capacity using *impedance factors* to allow for interactions among various conflicting movements subject to several levels of priority. A critical examination of this method is currently being undertaken.

Traditionally, roundabouts are analysed as a series of T-junctions, i.e. as a basic gap-acceptance process where an entry stream gives way to a circulating stream. This method has been found to overestimate capacities especially under heavy circulating flow conditions. A model developed by the author to adjust basic gap-acceptance capacities at roundabouts to allow for the effects of origin-destination patterns and the amount of queueing of entry streams will be described in a future paper.

The new arrival headway distribution, capacity and performance models described in this paper were being incorporated into the SIDRA software package at the time of the writing of this paper.

#### ACKNOWLEDGMENT

The author thanks Dr Ian Johnston, the Executive Director of the Australian Road Research Board Ltd (ARRB), for permission to publish this article. Any views expressed in the article are those of the author, and not necessarily those of ARRB. The author acknowledges the contribution of Dr Edward Chung to the work reported in this paper.

#### REFERENCES

<sup>1</sup>AKÇELIK, R. and E. CHUNG. Calibration of the bunched exponential distribution of arrival headways. *Road and Transport Research*, **3**(1), 1994, 42-59.

<sup>2</sup>AKÇELIK, R. and E. CHUNG. Traffic performance models for unsignalised intersections and fixed-time signals. In: R. Akçelik (Ed.), *Proceedings of the Second International Symposium on Highway Capacity* (Sydney, August 1994), Vol. 1, pp. 21-50. Australian Road Research Board Ltd, Nunawading, 1994.

<sup>3</sup>AKÇELIK, R. Estimation of green times and cycle time for vehicle-actuated signals. Paper No. 940446 presented at the 73rd Annual Meeting of the Transportation Research Board, Washington DC, January 1994.

<sup>4</sup>AKÇELIK, R. Analysis of Vehicle-Actuated Signal Operations. Working Paper WD TE93/007, Australian Road Research Board, 1994 (in preparation).

<sup>5</sup>TROUTBECK, R. J. The characteristics of the times drivers are stopped at unsignalised intersections. In: C. F. Daganzo (Ed.), *Transportation*

*and Traffic Theory*, Proc., 12th Int. Symp. on the Theory of Traffic Flow and Transportation (Berkeley, California, USA). Elsevier, Amsterdam, 1993, 575-594.

<sup>6</sup>CHUNG, E., W. YOUNG and R. AKÇELIK. MODEL-C: a simulation model for roundabout design. *Proc., 7th REAAA Conference*, Vol. 1, 1992, 66-74.

<sup>7</sup>CHUNG, E., W. YOUNG and R. AKÇELIK. Comparison of roundabout capacity and delay estimates from analytical and simulation models. *Proc., 16th Australian Road Research Board Conf.* **16**(5), 1992, 369-385.

<sup>8</sup>AKÇELIK, R. Traffic Signals: Capacity and Timing Analysis. Research Report ARR No. 123, Australian Road Research Board, 1981 (5th reprint: 1993).

<sup>9</sup>AKÇELIK, R. Calibrating SIDRA. Research Report ARR No. 180, Australian Road Research Board, 1990 (2nd edition, 1st reprint 1993).

<sup>10</sup>AKÇELIK, R. and R. J. TROUTBECK. Implementation of the Australian roundabout analysis method in SIDRA. In: U. Brannolte (Ed.), *Highway Capacity and Level of Service*, Proc. Int. Symp. on Highway Capacity (Karlsruhe). A. A. Balkema, Rotterdam, 1991, 17-34.

<sup>11</sup>TROUTBECK, R. J. Average delay at an unsignalised intersection with two major streams each having a dichotomised headway distribution. *Transpn Sci.*, **20**(4), 1986, 272-286.

<sup>12</sup>TROUTBECK, R. J. Current and future Australian practices for the design of unsignalised intersections. In: W. Brilon (Ed.), *Intersections Without Traffic Signals*, Proc. Int. Workshop (Bochum, West Germany). Springer-Verlag, Berlin, 1988, 1-19.

<sup>13</sup>TROUTBECK, R. J. Evaluating the Performance of a Roundabout. Special Report SR 45, Australian Road Research Board, 1989.

<sup>14</sup>TROUTBECK, R. J. Roundabout capacity and the associated delay. In: M. Koshi (Ed.), *Transportation and Traffic Theory*, Proc. of 11th Int. Symp. on Transportation and Traffic Theory (Yokohama, Japan). Elsevier, New York, 1990, 39-57.

<sup>15</sup>TROUTBECK, R. J. Recent Australian unsignalised intersection research and practices. In: W. Brilon (Ed.), *Intersections Without Traffic Signals II*, Proc. Int. Workshop (Bochum, Germany). Springer-Verlag, Berlin, 1991, 239-257.

<sup>16</sup>TROUTBECK, R. J. Discussion of the effects of heavy vehicles and lane utilisation at roundabouts. ARRB. Working Paper WD TE93/008, Australian Road Research Board, 1991.

<sup>17</sup>COWAN, R. J. Useful headway models. *Transportation Research*, **9**(6), 1975, 371-375.

<sup>18</sup>TANNER, J. C. A theoretical analysis of delays at an uncontrolled intersection. *Biometrika*, **49**(1 and 2), 1962, 163-170.

<sup>19</sup>TANNER, J. C. The capacity of an uncontrolled intersection. *Biometrika*, **54** (3 and 4), 1967, 657-658.

<sup>20</sup>BRILON, W. Recent developments in calculation methods for unsignalised intersections in West Germany. In: W. Brilon (Ed.), *Intersections Without Traffic Signals*, Proc. Int. Workshop (Bochum, Germany). Springer-Verlag, Berlin, 1988, 111-153.

<sup>21</sup>JACOBS, F. Capacity calculations for unsignalised intersections (in German). *Vorlesungsmanuskript*, Stuttgart, 1979.

<sup>22</sup>SULLIVAN, D. P. and R. J. TROUTBECK. Relationship between the proportion of free vehicles and flow rate on arterial roads. Physical Infrastructure Centre Report 92-21, Queensland University of Technology, Brisbane, 1993.

<sup>23</sup>AUSTROADS. Roundabouts. Guide to Traffic Engineering Practice, Part 6. Australian Association of Road and Traffic Authorities, Sydney, 1993.

<sup>24</sup>AKÇELIK, R. Implementing Roundabout and Other Unsignalised Intersection Analysis Methods in SIDRA. ARRB Working Paper WD TE91/002, Australian Road Research Board, 1991.

<sup>25</sup>AKÇELIK, R. and M. BESLEY. SIDRA User Guide. ARRB Working Paper WD TE 91/012 and Quick Reference Guides, Australian Road Research Board, 1993.

<sup>26</sup>COWAN, R. J. An extension of Tanner's results on uncontrolled intersections. *Queueing Systems*, **1**, 1987, 249-263.

<sup>27</sup>MCDONALD, M. and D. J. ARMITAGE. The capacity of roundabouts. *Traff. Engng Control*, **19**(10), 1978, 447-450.

<sup>28</sup>SIEGLOCH, W. Capacity calculations for unsignalised intersections (in German). *Schriftenreihe Strassenbau und Strassenverkehrstechnik*, Heft 154, Bonn 1973.

<sup>29</sup>BRILON, W. and M. GROSSMAN. The new German guideline for capacity of unsignalised intersections. In: W. Brilon (Ed.), *Intersections Without Traffic Signals II*, Proc. Int. Workshop (Bochum, Germany). Springer-Verlag, 1991, 61-82.

<sup>30</sup>AKÇELIK, R. Time-Dependent Expressions for Delay, Stop Rate and Queue Length at Traffic Signals. Internal Report AIR 367-1, Australian Road Research Board, 1980.

<sup>31</sup>AKÇELIK, R. The *Highway Capacity Manual* delay formula for signalised intersections. *ITE J.*, **58**(3), 1988, 23-27.

<sup>32</sup>AKÇELIK, R. SIDRA for the *Highway Capacity Manual*. Compendium of Technical Papers, 60th Annual Meeting of the Institute of Transportation Engineers (Orlando, Florida), 1990, 210-219.

<sup>33</sup>AKÇELIK, R. and N. M. ROUPHAIL. Estimation of delays at traffic signals for variable demand conditions. *Transpn Res.*, **27B**(2), 1993, 109-131.

<sup>34</sup>AKÇELIK, R. and N. M. ROUPHAIL. Overflow queues and delays with random and platooned arrivals at signalised intersections. *J. Advanced Transportation*, **28**(3), 1994, 227-251.

<sup>35</sup>COWAN, R. J. Adam's formula revised. *Traff. Engng Control*, **25**(5), 1984, 272-274.

<sup>36</sup>TRANSPORTATION RESEARCH BOARD. *Highway Capacity Manual*. Special Report 209, Washington, DC, 1985.

<sup>37</sup>BOWYER, D. P., R. AKÇELIK and D. C. BIGGS. Guide to Fuel Consumption Analysis for Urban Traffic Management. Special Report SR No. 32, Australian Road Research Board, 1985.

<sup>38</sup>SULLIVAN, D. P. and TROUTBECK, R. J. The use of Cowan's M3 headway distribution for modelling urban traffic flow. *Traff. Engng Control*, **35**(7/8), July/August 1994, 445-450.

The author's address: Australian Road Research Board Ltd, P.O. Box 156, Nunawading, Victoria 3131, Australia.