

REPRINT

Overflow Queues and Delays with Random and Platooned Arrivals at Signalized Intersections

R. AKÇELİK and N. M. ROUPHAIL

REFERENCE:

R. Akçelik and N. M. Roupail (1994). Overflow Queues and Delays with Random and Platooned Arrivals at Signalised Intersections.

Journal of Advanced Transportation, Vol. 28, No. 3, pp. 227-251

NOTE:

This paper is related to the intersection analysis methodology used in the SIDRA INTERSECTION software. Since the publication of this paper, many related aspects of the traffic model have been further developed in later versions of SIDRA INTERSECTION.

Though some aspects of this paper may be outdated, this reprint is provided as a record of important aspects of the SIDRA INTERSECTION software, and in order to promote software assessment and further research.

Overflow Queues and Delays with Random and Platooned Arrivals at Signalized Intersections

*Rahmi Akçelik
Nagui M. Roupail*

The traditional two-term analytical model for predicting delays, queues and stops with random arrivals as found at isolated signalized intersections is extended to the case of platooned arrivals. The work was carried out in the context of modeling traffic performance at signalized paired intersections. A cycle-by-cycle macroscopic simulation model was used to calibrate the overflow terms of the performance formulae for a single stream of platooned arrivals at the downstream approach of a paired intersection system. The steady-state form of the analytical model was used for calibration. The parameters derived for the steady-state model are then used in the time-dependent form of the model. Descriptions of the general analytical model, the cycle-by-cycle simulation model, its validation against several well-known models are presented, and the new models derived from this study are described. Extension of the model to multistream, multiphase applications are discussed and areas of further study are identified.

Introduction

This paper deals with modeling of overflow delays and queues with random and platooned arrivals at signalized intersections. The work was carried out in the context of modeling traffic performance at *signalized paired intersections* (Johnson and Akçelik 1992; Roupail and Akçelik 1991, 1992a). Examples of paired intersections include diamond interchanges, closely-spaced intersections, staggered T junction, large intersections with internal queueing (e.g. due to a wide median). This type of intersection operation has been increasingly more common with the proliferation of signalization in urban street networks.

From a traffic modeling viewpoint, the paired intersection system can be characterized as follows (see *Fig 1*):

- (i) Arrivals at upstream (external) approaches are considered to be random, with traffic encountering no major

Rahmi Akçelik is Chief Research Scientist at the Australian Road Research Board, Ltd. Melbourne, Australia. Nagui M. Roupail is Professor at North Carolina State University, Raleigh, North Carolina, USA.

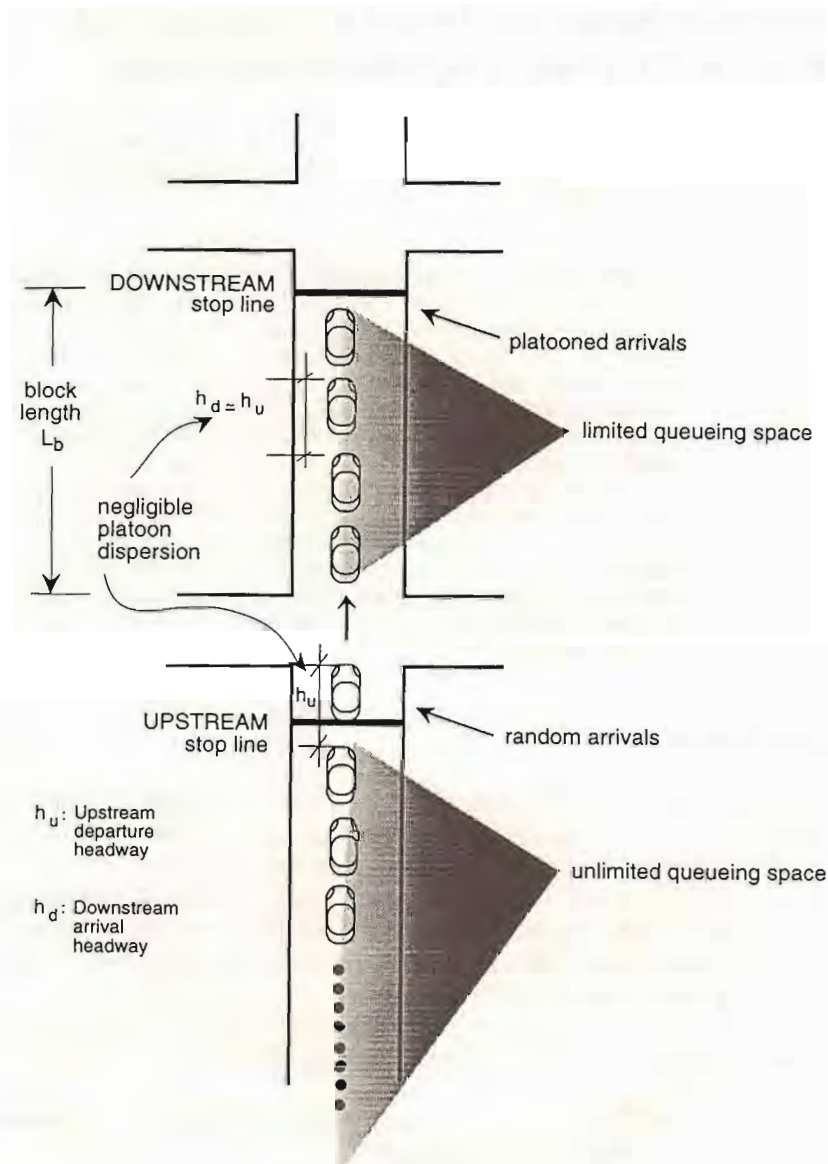


Figure 1. Random and Platooned Arrivals at Signalized Intersection Approaches.

- (ii) interruptions prior to reaching the upstream approach. Arrivals at downstream (internal) approaches occur primarily in platoons which are formed due to queueing at upstream approaches. Thus, downstream operation is governed by the size, density and timing of the arriving platoons.
- (iii) Vehicles in platoons maintain their headways during travel between the intersection pair (negligible platoon dispersion due to the close proximity of stop lines).
- (iv) Midblock traffic generation/absorption is considered negligible, in view of the restrictive geometry between the intersection pair.
- (v) A finite queueing space exists at the downstream intersection. Should demand exceed the queueing capacity, blockage of the upstream stop line may occur. No such restrictions exist on the size of queues at the upstream approaches.
- (vi) Under certain combinations of queueing space, demand levels and signal control parameters, a reduction in the upstream intersection saturation flow rate may occur due to the interference of the downstream queue.

The processes described in items (v) and (vi) are termed *queue interaction*. Description of a preliminary model of queue interaction and an assessment of existing analytical software in describing queue interaction are given elsewhere (Johnson and Akçelik 1992; Ove Arup & Partners 1991; Rouphail and Akçelik 1991, 1992a).

This paper is organized as follows. First, a brief assessment of existing methods is given, and the methodology used in this study is presented. Next, the special simulation program used for calibrating the analytical models is described, and the analytical models for random and platooned arrivals are given. Finally, proposed enhancements and extensions of the model are discussed.

Background

The following criteria may be used in order to evaluate the effectiveness of existing methods in modelling random and platooned arrivals:

- (a) Platoon generation logic: This refers to the ability of the model to construct departing platoons at the upstream stop line which are then projected to the downstream approach.

- (b) Maximum departure flow logic: When demand flows at the upstream intersection exceed capacity, discharge flows occur at capacity.
- (c) Flow variance logic: When the upstream intersection is at or above capacity, departures become virtually uniform from cycle to cycle. This aspect is critical in predicting the size of the downstream random queues and delays.
- (d) Maximum queue length logic: A realistic algorithm should predict queue lengths which can not exceed the physical queueing space between the intersection pair.

A review of a number of well-known network signal evaluation/optimization models in use in Australia and the U.S. revealed that criteria (b), (c) and (d) were not met. The models reviewed were TRANSYT-7F (Federal Highway Administration 1983) and PASSER II-87 (Chang, Messer and Garza 1988) in the U.S. and the SCATES model (RTA-NSW 1991) in Australia. Detailed documentation of this review is given in Ove Arup & Partners (1991) and Johnson and Akçelik (1992). In most cases reviewed, queues are modeled *vertically* at the stop line, with no consideration given to the possibility of queues exceeding the available queueing space. Furthermore, the computation of the effects of random queues and delays are often based on formulae which are insensitive to the arrival flow variance.

In reality, the random nature of arrivals is significantly dampened at the upstream approach when it operates at high levels of saturation. To ignore this phenomenon would result in overestimating queue lengths and delays, and consequently in the assignment of additional green time on the approach and the degradation in performance for other movements. Empirical evidence of the effect of upstream platooning on overflow delays and queues is given by Hillier and Rothery (1967) and more recently by Van As (1991). Theoretical models of stochastic delays and queues which support this evidence have been set forth by Newell (1989, 1990).

Thus, the need for developing analytical tools for predicting traffic performance with specific emphasis on platoon modeling, flow variance-based random queues and delays, and queue interaction models was established.

The analytical models described in this paper can be incorporated into existing traffic analysis software such as SIDRA (Akçelik 1990a, b; Akçelik and Besley 1991), the Highway Capacity Software (1985), and the network analysis packages mentioned earlier. It is important to note that the existing arterial and network analysis methods are based on

modelling traffic streams on a *lane group* or *movement* basis, whereas the lane-by-lane level of detail as used in SIDRA is desirable for paired intersection design.

Methodology

The proposed methodology is to extend the simple two-term analytical performance model (Akçelik 1980, 1981, 1988, 1990b) used for random arrivals (isolated intersections) to the platooned arrivals case. The general analytical model provides an integrated framework for predicting delay, queue length and stop rate. The formula for each of these performance statistics consists of two terms (components), namely a *non-random* term and an *overflow* term.

A cycle-by-cycle (macroscopic) simulation model was developed specifically for this study to assist with the development of analytical models, in particular with the overflow terms of the performance formulae. The need to use simulated data stems from the requirement that cyclic (non-random) and random delays and queues be estimated separately. The intention is to relate the stochastic delays and queues to the arrival type (random vs platooned) as well as to the flow variance. Since microscopic traffic simulation models do not consider the two components independently, e.g. NETSIM (Federal Highway Administration 1980), they could not be used for this purpose.

For the non-random delay and queue components in the case of platooned arrivals, the estimates from the TRANSYT model could have been adequate. For the sake of consistency, however, the simulation model was used as a common tool to assist with the development of both non-random and overflow components of the performance formulae.

The overflow component of the analytical model was derived by calibration against the simulation results for random and platooned arrivals using the steady-state form of the model. The parameters derived for the steady-state model were then used in the time-dependent form of the model.

As an initial step in the development of platooned arrival and queue interaction models, only one traffic stream in a single green phase at the upstream approach was considered. Extension of the model to multistream, multiphase operations is discussed in the section Model Extensions.

A brief description of the simulation model, its output and validation against several well-known models are given in the following sections after the description of the general analytical model.

The analytical and simulation models described in this paper do not include queue interaction considerations. A method to deal with the

effects of queue interaction has been described by Rouphail and Akçelik (1991, 1992a). The models described in this paper should be adopted together with the queue interaction method. Otherwise, the estimated queue lengths may not be realized due to the physical queueing constraints of the system.

General Analytical Model

The general analytical model for the prediction of delays is first described, and then the formulae for queue length and stop rate prediction are given. The discussion on delay models given here is applicable to queue length and stop rate models as well.

The average delay per vehicle, d , can be expressed as sum of two delay terms:

$$d = d_1 + d_2 \quad (1)$$

where

- $d_1 =$ *non-random* delay term (delay due to signal cycle effects calculated assuming non-random arrivals at the average flow rate, either distributed *uniformly* throughout the signal cycle, or in *platoons* arriving at the same time each cycle), and
- $d_2 =$ *overflow* delay term including effects of random arrivals as well as any oversaturation delays experienced by vehicles arriving during the *specified flow period*.

The two components of delay are discussed below.

The Non-Random Delay Component

Non-random delay is estimated by assuming that the number of vehicles which arrive during each signal cycle is fixed and equivalent to the average demand (arrival) rate per cycle. Different expressions are used for the non-random delay term according to the arrival characteristics (uniform, or platooned) and the signal characteristics (one or two green periods).

For *isolated intersections*, the following formula known as the *uniform delay* formula (for the case when arrivals are distributed uniformly throughout the signal cycle) is commonly used as the first term of most delay models (Webster 1958, Miller 1968, Akçelik 1981, Teply 1984, Transportation Research Board 1985):

$$d_1 = \frac{0.5 c (1 - u)^2}{1 - ux} \quad \text{for } x \leq 1.0$$

$$= 0.5 (c - g) \quad \text{for } x > 1.0 \quad (2)$$

where

c = cycle time in seconds,
 u = g/c (ratio of effective green time, g , to cycle time, c), and
 x = degree of saturation (demand/capacity ratio) given by

$$x = q / Q \quad (3)$$

where

q = arrival (demand) flow rate during the specified flow period in vehicles per hour (or per second),
 Q = capacity under the specified flow conditions in vehicles per hour (or per second) given by

$$Q = sg / c \quad (4)$$

where s is the saturation flow rate in vehicles per hour or per second, and g/c is the ratio of effective green time to cycle time, and sg is the capacity per cycle in vehicles.

For **platoon arrivals** (the case when arrivals occur at different rates during different intervals of the signal cycle), various analytical models have been developed (Olszewski 1988, 1990a; Rouphail 1988, 1989; Fambro, Chang, and Messer 1991) for use instead of Eqn. (2). The simplest method which can be adopted is the use of *Progression Factors (PF)* to adjust the uniform delay formula for isolated intersection case (Fambro, et al., 1991):

$$d_1 = \frac{0.5 c (1 - u)^2}{1 - ux} PF \quad (5)$$

where

PF = progression adjustment factor given by

$$PF = \frac{1 - P}{1 - u} f_{at} \quad (6)$$

where P is the proportion of vehicles arriving during the green period (for the isolated intersection case, $P = u$, hence $PF = 1.0$), and f_{at} is an adjustment factor for early or late platoon arrivals.

In this study, a more detailed analytical model was developed and used for estimating the non-random component of the delays and queues for single stream platoon arrivals (see Roupail and Akçelik 1991).

The Overflow Delay Component

The overflow delay term in Eqn. (1) represents the additional delay experienced by vehicles *arriving in a specified flow period*. This delay results from (i) temporary oversaturation due to the random nature of arrivals or (ii) persistent oversaturation when the average flow rate exceeds the capacity.

Two types of models for estimating the overflow delay can be distinguished:

- (i) ***Steady-state models***: These are the older type of models which are based on the assumption that the demand (arrival) flows persist for an infinite period of time. Therefore, delay approaches infinity as the demand flow approaches capacity. These models are able to estimate delays only for below-capacity conditions (for degrees of saturation approaching 0.95). They tend to overpredict delays for high degrees of saturation due to the time-dependence of demand flows.
- (i) ***Time-dependent models***: These are the more recent models which assume that the demand (arrival) flows last for a finite period of time (flow period). As a result, they are able to predict finite delay values at or above capacity. Hence, the time-dependent models are applicable to oversaturated conditions. The related issue of variable-demand modelling is discussed in detail in recent paper by Akçelik and Roupail (1993) and Roupail and Akçelik (1992b).

The well-known *coordinate transformation* method can be used to convert the steady-state form to a time-dependent form. As discussed in Akçelik (1981, 1988, 1990a,b), the following simple *time-dependent* formula provides a general expression for overflow delay:

$$d_2 = 900 T_f \left[(x - 1) + \sqrt{(x - 1)^2 + \frac{8k(x - x_0)}{Q T_f}} \right] \quad (7)$$

for $x > x_0$ (*zero otherwise*)

where

- T_f = duration of the flow period in hours,
- x = degree of saturation in the specified flow period,
- x_0 = the degree of saturation below which the overflow delay is zero ($x_0 \leq 1.0$),
- k = a delay parameter which determines the rate of increase in overflow delay with increasing degree of saturation, and
- Q = capacity in vehicles per hour.

The *steady-state* delay expression which corresponds to Eqn. (7) is:

$$d_s = \frac{k(x - x_0)}{Q(1 - x)} \quad (8)$$

Parameters k and x_0 which determine the shape of the overflow delay curve can be derived by calibrating the steady-state expression using data for undersaturated conditions (for x up to about 0.95).

In the Australian formula (Akçelik 1980, 1981, 1990b), $k = 1.5$ and a variable x_0 parameter are used which were found as an approximation to Miller (1968) model:

$$x_0 = 0.67 + sg / 600 \quad (9)$$

where sg = capacity per cycle in vehicles (s = saturation flow rate in vehicles per second and g = effective green time in seconds).

A well-known formula which constitutes the second term of the Webster's (1958) steady-state model, and forms the basis of the time-dependent delay model used in the Canadian capacity guide (Teply 1984) has $k = 0.5$ and $x_0 = 0$:

$$d_s = \frac{0.5 x}{Q(1 - x)} \quad (10)$$

This formula will be referred to as *Webster's second term* for the purpose of discussion in the following sections.

In this study, the overflow delay term was calibrated for steady-state conditions using simulation results for degrees of saturation approaching 0.95. The steady-state model expressed by Eqn. (8) was used since, once calibrated, its time-dependent form is readily available (Eqn. 7).

The new formulae for k and x_0 derived from the cycle-by-cycle simulation results are given in *Section 4*. For further discussion on the time-dependent delay model for isolated intersections, see Akçelik (1980, 1981, 1990b) and Akçelik and Roupail (1993).

Queue Length and Stop Rate

The formulae for predicting the *maximum back of queue* in an average cycle (N_m) and the *stop rate* (h), which is the average number of stops per vehicle, are similar to the delay formula:

$$N_m = N_{m1} + N_{m2} \quad (11)$$

$$h = h_1 + h_2 \quad (12)$$

where

$N_{m1}, h_1 =$ non-random components of queue length and stop rate, and
 $N_{m2}, h_2 =$ overflow components of queue length and stop rate.

Equations for the non-random componets (N_{m1}, h_1) for isolated intersections can be found in Akçelik (1980, 1981, 1990b). Equations for the case of a single stream with platooned arrivals are given in Roupail and Akçelik (1991).

The *overflow* component of maximum back of queue is equivalent to the *average overflow queue*:

$$N_{m2} = N_0 \quad (13)$$

The average overflow queue can be calculated from:

$$N_0 = d_2 Q = d_2 sg / c \quad (14)$$

where d_2 is the average overflow delay (Eqn. 7) and Q is the capacity in veh / s (Eqn. 4).

The *overflow* component of stop rate can be calculated from:

$$h_2 = N_0 / qc \quad (15)$$

The stop rate predicted by Eqn. (15) does not correct for partial stops (slowdowns at the back of the queue) or multiple stops (move-ups in the queue before clearing the intersection). Akçelik (1981, 1990b) approximates these effects by using only 90% of the stop rate predicted by Eqn. (15).

The Simulation Model

A cycle-by-cycle, macroscopic simulation model was developed

with flows represented in terms of vehicle streams each containing a finite number of vehicles traveling at a constant average headway. At the upstream intersection, the average demand flow rate is used to generate arrivals according to the Poisson distribution. In each cycle, a single stream with an average headway equivalent to the inverse of the simulated arrival flow rate in the cycle is generated.

During the effective upstream green, vehicles waiting in queue depart at the saturation flow rate; subsequent vehicles in the cycle depart at the arrival headway. If a cycle is oversaturated, overflow vehicles are released in subsequent cycles.

Signal system parameters are represented by the system cycle length, effective green splits at each intersection and an offset relationship. When the upstream intersection is close to saturation, the departing streams become nearly identical in each cycle, and, therefore, the variance of the distribution of the departure flows from the upstream signal is much less than that of the arrival flows.

The general concept of the simulation model is illustrated in Fig. 2. The structure of the model allows for the estimation of all delay and queueing parameters for two arrival types: random (Poisson) arrivals at the upstream approach and platooned arrivals downstream. The degree of platooning, is, of course, governed by the prevailing flow rates and signal settings at the upstream approach.

The simulation provides an option to generate regular arrivals (equivalent to the non-random term of the analytical model). This feature was used to estimate the overflow queues and delays as the difference between cases of Poisson and regular arrivals (see Eqn. 1).

The two delay components (non-random and overflow) predicted by the simulation model under a variety of operating conditions were compared with well-established delay models. In all runs, the system cycle length was fixed at 90 seconds, and the saturation flow rate at 1800 veh/h/lane for both intersections. A range of degrees of saturation (x) from 0.40 to 0.93 was examined at cycle capacities (sg) ranging from 4 to 40 vehicles. The analysis period was varied to simulate steady-state queueing conditions. At high x and low sg values, a longer simulation period was used to emulate the long-term delays and queue statistics (Olszewski 1990b). At low x values, steady-state conditions were attained in a relatively small number of cycles. Overall, the simulation time varied from 15 minutes to 5 hours (or from 10 to 200 cycles). No simulation warm-up times were used. The effect of the initial transition period was minimized by setting the demand in the initial signal cycle to the average demand and by extending the simulation period for high degrees of saturation.

A scatter diagram of the average overflow delay from simulation

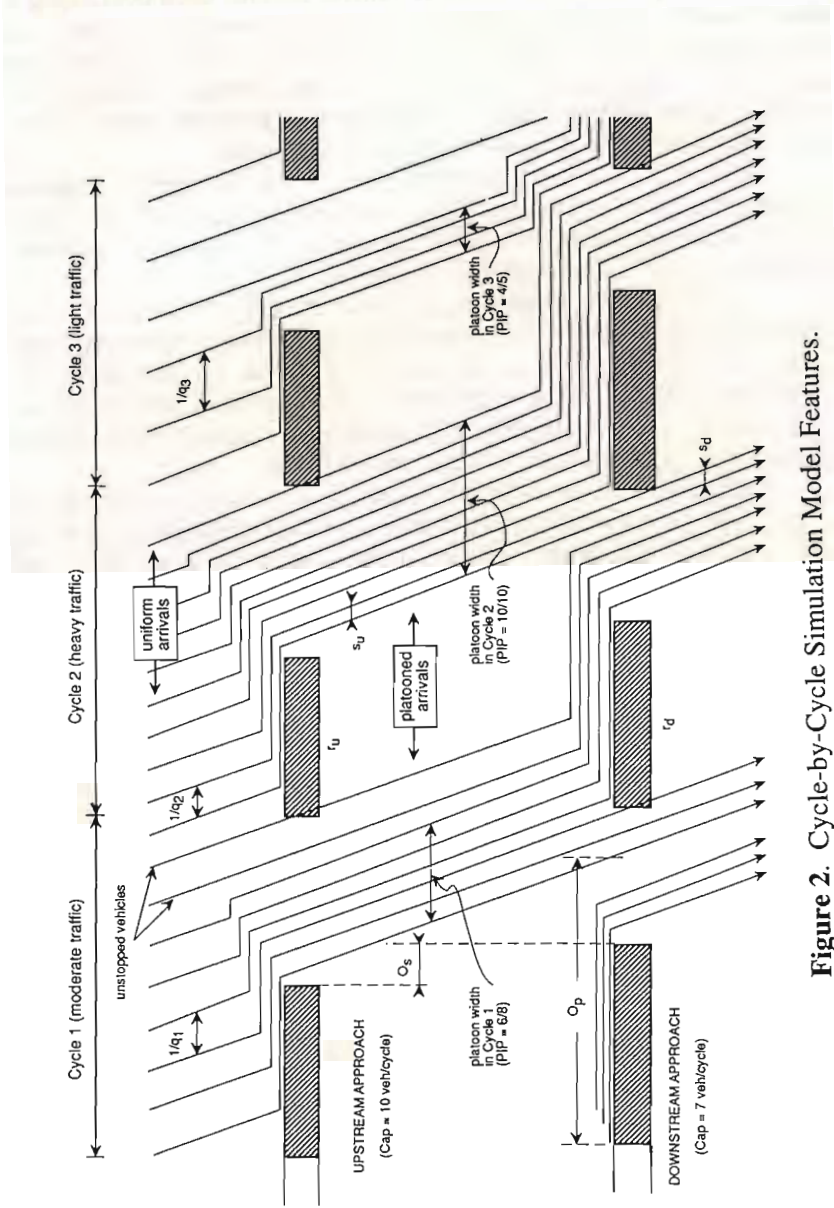


Figure 2. Cycle-by-Cycle Simulation Model Features.

and Webster's second term for random arrivals at an isolated intersection approach (Eqn. 10) against x values is depicted in Fig. 3 (for different cycle capacities, sg). Eqn. (10) tends to overpredict delays compared to the simulation results (because the degree of saturation alone is not a sufficient predictor of the overflow delay). This observation confirms the findings of other theoretical and simulation studies (Miller 1968, Cronje 1983, Newell 1989, Olszewski 1990b, Brilon and Wu 1990).

In the case of platooned arrivals, the simulation experiments were limited to cases where the upstream signal capacity is larger than the downstream capacity. Otherwise, overflow queues at the downstream approach would never develop since the upstream approach cannot discharge flows exceeding its own capacity.

By simulating a number of progression scenarios, it was confirmed that the overflow queues are insensitive to the offset relationship between the intersections (details are given in Rouphail and Akçelik 1991).

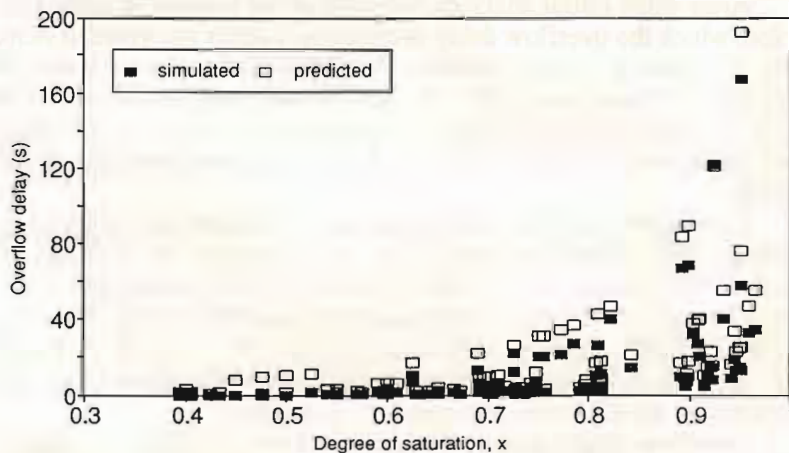


Figure 3. Simulated Overflow Delays and Predictions from Webster's Second Delay Term (Eqn. 10) as a Function of the Degree of Saturation for Random Arrivals and For Different Cycle Capacities.

Calibration Results

Random Arrivals

A new model was calibrated from simulation data to describe the overflow queueing pattern at an isolated intersection approach. Two

delay parameters k and x_0 are used to define the steady-state overflow delay model (Eqn. 8), which can then be used in the time-dependent model (Eqn. 7). The ratio of variance-to-mean of arrivals per cycle at the upstream approach (I_u) derived from simulation is entered as a predictor parameter into the k parameter in a similar manner as in the Hutchinson's model (1972). For this purpose, we define $k = k' I_u$ (for strictly Poisson arrivals, $I_u = 1$ and $k = k'$).

The resulting model for lane-by-lane application of the delay model for isolated signals has a fixed value of x_0 and a variable k parameter given by

$$\begin{aligned} x_0 &= 0.5 \\ k &= k' I_u = 1.22 I_u (sg)^{-0.22} \end{aligned} \quad (16)$$

This equation gives k values in the range 1.0 to 0.5 for sg values in the range 3 to 60 vehicles per cycle.

After some initial analysis, the value of x_0 (degree of saturation below which the overflow delay is effectively zero) was fixed at 0.50, and the value of k was calibrated through a series of runs aimed at maximizing the statistical fit of the steady-state delay model estimates (Eqn. 8) with simulation results. The calibrated x_0 value is consistent with Miller's early model (1963), and confirmed empirically by Sosin (1980).

It is emphasized that the dependence of overflow delay on cycle capacity (sg), as expressed by Eqn. (16) confirms the findings of previous theoretical work (Miller 1968, Cronje 1983) and recent simulation work by Olszewski (1990b) and Brilon and Wu (1990). Note that the US HCM (Transportation Research Board, 1985), the UK (Webster 1958; Burrow 1989) and the Canadian (Teply 1984) methods do not account for the effect of cycle capacity parameter.

Overflow delays and queues predicted by the calibrated steady-state model and simulation were highly correlated (R^2 values of 0.955 for delays and 0.922 for queues) with no apparent bias.

Platoon Arrivals

The general model as described by Eqn. (8) was used for platooned arrivals at the downstream intersection of a paired intersection system with due consideration to the conditions at the upstream intersection. Thus, the delay parameters k and x_0 are used to reflect not only the downstream cycle capacity, but also the magnitude of platooning and cycle to cycle variations in the arriving stream. This approach also

ensures continuity between platooned and random arrival models.

For a random queue to develop at a downstream traffic signal, the capacity of an upstream signal must exceed the capacity of the downstream signal (assuming no flow gains or losses between the two signals). In the case of a single traffic stream through the system, this condition can be stated as:

$$x_d > x_u \quad (17)$$

where x_d and x_u are the downstream and upstream degrees of saturation, respectively.

The delay parameters can be related to a single platooning index, namely, the proportion of upstream departures occurring in platoons, *PIP*. This index is equivalent to the proportion of vehicles stopped at an isolated (upstream) intersection, and is given by:

$$\begin{aligned} \text{PIP} &= \frac{1 - g_u / c}{1 - g_u x_u / c} && \text{for } x_u \leq 1.0 \\ &1.0 && \text{for } x_u > 1.0 \end{aligned} \quad (18)$$

Hence, when the upstream approach is oversaturated, the departure distribution consists of identical platoons whose size is governed by the length of the upstream green, g_u , and the saturation flow rate, s_u . In such cases, the flow variance-to-mean ratio of upstream departures (downstream arrivals) is zero ($I_d = 0$). Thus, for a single stream, it can be stated that the downstream random delays and queues approach zero as the upstream traffic conditions approach saturation. Generally, downstream variations in demand will be smaller than those encountered at the upstream (isolated) approach since variations in the demand flow are partially absorbed at the upstream approach (Van As 1991).

From Eqn. (16), the second delay parameter k can be defined as $k = k' I_d$ where I_d now refers to the variance-to-mean ratio of arrivals at the downstream approach.

Simulation results depicting the ratio of I_d to I_u against *PIP* are shown in *Fig. 4*. At *PIP* values up to 0.75 - 0.85, the ratio I_d / I_u is close to unity. With heavier upstream platooning (i.e. higher x_u), this ratio decreases rapidly, approaching zero as *PIP* approaches 1.0. The data show considerable scatter in the range $0.75 \leq \text{PIP} \leq 0.95$. This reflects a transition region in which the sequence of arrivals from one cycle to the next influences the frequency of overflow queues. In light traffic (low *PIP*), this sequence has no bearing on the mean and variance of vehicles discharged. In very heavy traffic (high *PIP*) with frequent overflow queues, discharge is virtually independent of demand, and

close to capacity, yielding I_d close to zero. The transition zone between the two, however, is very susceptible to arrival flow variations.

The following simple model is adequate for predicting the variance-to-mean ratio of arrivals at the downstream signal for most practical applications:

$$\begin{aligned} I_d &= I_u && \text{for } PIP \leq 0.85 \\ 6.67 I_u (1 - PIP) &&& \text{for } PIP > 0.85 \end{aligned} \quad (19)$$

Predictions from Eqn. (19) are shown in *Fig. 4*. The model satisfies the boundary values for $I_d = 0$ at $PIP = 1.0$.

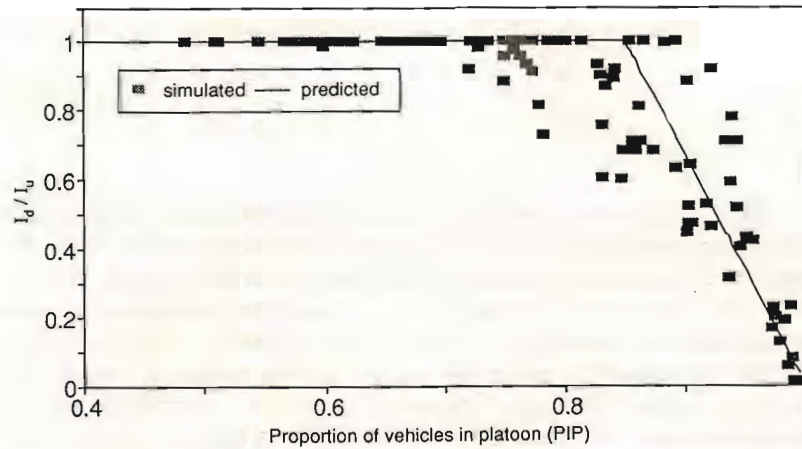


Figure 4. Effect of the Proportion of Vehicles in Platoon (PIP) on the Simulated and Predicted I_d / I_u ratio (I_d, I_u : Variance-to-Mean Ratios of Upstream and Downstream Arrivals).

A lower bound on the value of x_0 is necessary depending on the value of the upstream degree of saturation. No random queues will occur downstream if the downstream capacity is greater than the upstream capacity (see Eqn. 17). As a result, the final x_0 estimate is given by:

$$x_0 = \max [0.5, \min (1, x_u)] \quad (20)$$

Parameter k' for platooned arrivals was derived by adjusting the isolated intersection parameter k' (Eqn. 16) to allow for platooned arrival effects using the PIP parameter. The resulting model is expressed by the following equations:

$$\begin{aligned} k' &= (1.22 - 0.527 \text{ PIP}) (\text{sg})^{-0.22} && \text{for } x_0 = 0.50 \\ &\frac{0.302}{1 - \text{PIP}} (\text{sg})^{-0.22} && \text{for } x_0 > 0.50 \end{aligned} \quad (21)$$

The model requires an upper bound on k' at very high x_d ratios (1.3 and above) to ensure that the estimates of overflow delays for random and platooned arrivals converge to the deterministic oversaturation delay value. This upper bound can be expressed as:

$$k' \leq \frac{0.80 k' (R)}{I_d (1.3 - x_0)} \quad (22)$$

where $k'(R)$ is the corresponding delay parameter for random arrivals given by Eqn. (16).

Restating Eqn. (8) for platooned arrivals as observed at the downstream approach of a paired intersection system gives:

$$d_s = \frac{k' I_d (x_d - x_0)}{Q_d (1 - x_d)} \quad (23)$$

where I_d , x_0 and k' are determined from Eqns. (19) to (22), respectively, and other variables are as defined earlier.

Model estimates of overflow delay are plotted against simulation data in Fig. 5. The model predictions exhibit no apparent bias, and provide a good statistical fit to the simulated delays ($R^2 = 0.866$). Similar results were obtained for the overflow queue estimates from Eqn. (14) ($R^2 = 0.770$).

A sample comparison of the overflow delay estimates for isolated intersection and platooned arrivals are depicted in Fig. 6. It is evident in this case that the overflow delay component is lower than the isolated intersection case throughout, although the differences are small up to an x value of about 0.80. As expected, delay estimates from the two models converge to the deterministic oversaturation delay model at high x values (equivalent to $k = 0$, $x_0 = 1.0$ in Eqn. 7).

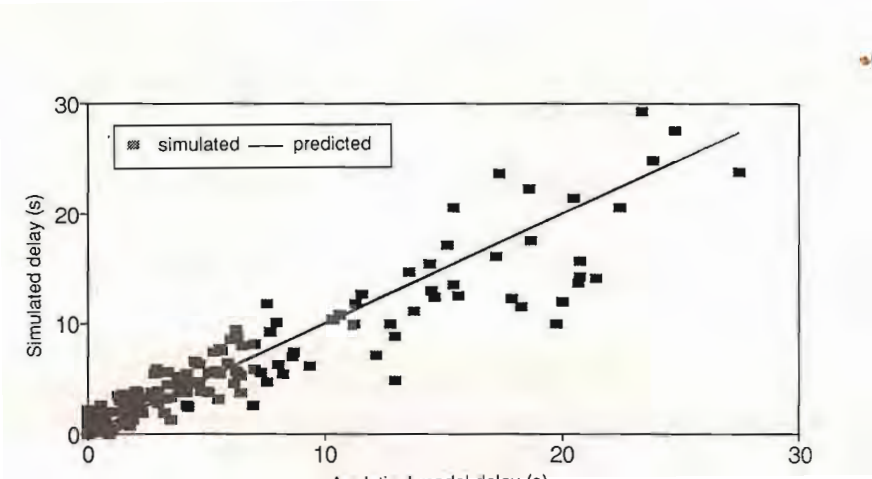


Figure 5. Comparison of Simulated and Predicted Overflow Delays for Platooned Arrivals.

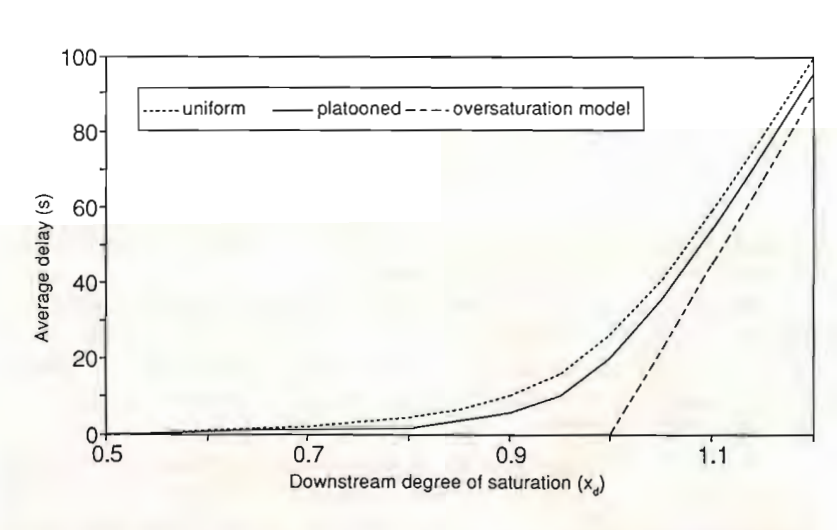


Figure 6. Overflow Delays Predicted for Uniform and Platooned Arrivals as a Function of the Degree of Saturation (Time-Dependent Form).

Model Extensions

The overflow delay model for platooned arrivals described in the previous section applies strictly for a single traffic stream. The extension of the model to multistream, multiphase applications requires the modification of expressions for delay parameters x_0 , I_d , and k' .

Consider that M traffic streams (1, 2, ... m , ... M) enter the downstream approach, each servicing a demand flow rate q_m . Assuming no losses or gains between intersections, the number of vehicles per cycle arriving at the downstream signal is expressed as the sum of all entering streams:

$$q_d c = \sum \min(q_m c, s_m g_m) \quad (24)$$

This equation reflects the fact that the individual stream discharge rate per cycle cannot exceed the stream capacity (i.e. only $s_m g_m$ vehicles can enter the downstream link when $q_m c < s_m g_m$).

Random delays at the downstream approach will not occur so long as the downstream capacity exceeds the sum of the capacities of all entering streams:

$$s_d g_d \geq \sum s_m g_m \quad (25)$$

This condition can be expressed in terms of a lower bound on the downstream degree of saturation ($x_d = q_d c / s_d g_d$):

$$x_0 = \frac{\sum \min(q_m c, s_m g_m)}{\sum s_m g_m} \quad (26)$$

In low demand conditions, a boundary value for $x_0 = 0.50$ is used, as in the isolated intersection case. Thus, a general expression of x_0 for the multistream case is:

$$x_0 = \max \left[0.50, \frac{\sum \min(q_m c, s_m g_m)}{\sum s_m g_m} \right] \quad (27)$$

Note that for a single stream case ($m = 1$), Eqn. (27) reverts to the special case described by Eqn. (20).

Furthermore, Eqn. (25) defines an upper bound on the downstream degree of saturation:

$$x_{d,\max} = \frac{\sum s_m g_m}{s_d g_d} \quad (28)$$

For the multistream case, an approximate value of parameter k' can be calculated using a flow-weighted average PIP in Eqn. (21):

$$PIP = \frac{\sum q_m PIP_m}{\sum q_m} \quad (29)$$

The variance-to-mean ratio for each upstream flow (I_{dm}) can be calculated from Eqn. (19), and then a flow-weighted average can be calculated for use in Eqn. (23):

$$I_d = \frac{\sum q_m I_{dm}}{\sum q_m} \quad (30)$$

Summary and Conclusions

The traditional two-term analytical model for predicting delays, queues and stops at isolated signalized intersections (random arrivals) is extended to the case of platooned arrivals. The work was carried out in the context of modelling traffic performance at signalized paired intersections. The key findings of the study are:

- (i) The characteristics of arrival patterns at signalized intersections have a profound influence on traffic performance. The effects of degree of traffic platooning, cycle by cycle demand fluctuations and the average flow rate must all be incorporated in the analysis if meaningful traffic performance predictions are to be derived.
- (ii) It is generally inadvisable to apply delay or queue length models derived for random arrivals to platooned arrival conditions. The study results indicate that both the non-random and random components of delays and queues are strongly related to the arrival type.
- (iii) Overflow delays and queues are highly dependent on the prevailing congestion levels. For a paired intersection system, highly saturated upstream conditions mean that traffic is released in platoon pulses at a virtually uniform rate each cycle. Fluctuations in demand are therefore absorbed at the entry point (or periphery of the arterial). In this case, the application of the random arrivals model to platooned arrivals may grossly overestimate queue lengths and delays.

Possible extensions of the model to multistream, multiphase conditions have been discussed. The following areas are recommended for further study:

- (a) Lane flow estimation: The effects of lane underutilization and short lanes are important issues. A lane-by-lane analysis method, such as the one adopted by SIDRA (Akçelik 1990b, Akçelik and Besley 1991), is a prerequisite for analyzing this type of effects.
- (b) The case of two green periods (opposed and unopposed turn periods) at the upstream and downstream approaches needs to be considered in modelling platooned arrivals. Formulae for the case of two green periods have been used in SIDRA for isolated intersection cases.
- (c) Modelling of the effect of platooned arrivals on opposed turn capacities would be a useful improvement.
- (d) There is also a need for developing overflow delay models for vehicle-actuated and semi-actuated signal operations.

References

- Akçelik, R. (1980). Time-Dependent Expressions for Delay, Stop Rate and Queue Length at Traffic Signals. Australian Road Research Board. Internal Report AIR 367-1.
- Akçelik, R. (1981). Traffic Signals - Capacity and Timing Analysis. Australian Road Research Board. Research Report ARR 123 (5th reprint 1993).
- Akçelik, R. (1988). The Highway Capacity Manual Delay Formula for Signalized Intersections. *ITE Journal* 58 (3), pp. 23-27.
- Akçelik, R. (1990a). SIDRA for the Highway Capacity Manual. Compendium of Technical Papers, 60th Annual Meeting of the Institute of Transportation Engineers, Orlando, Florida, pp. 210-219.
- Akçelik, R. (1990b). Calibrating SIDRA. Australian Road Research Board. Research Report ARR 180.
- Akçelik, R. and Besley, M. (1991). SIDRA User Guide. Australian Road Research Board. Working Paper WD TE91/012.
- Akçelik, R. and Roupail, N.M. (1993). Estimation of Delays at Traffic Signals for Variable Demand Conditions. *Transportation Research* 27B(2), pp. 109-131.
- Brilon, W. and Wu, N. (1990). Delays at Fixed-Time Traffic Signals Under Time-Dependent Traffic Conditions. *Traffic Engineering*

- and Control* 31 (12), pp. 623-631.
- Burrow, I.J. (1989). A Note on Traffic Delay Formulas. *ITE-Journal* 59 (10), pp. 29-32.
- Chang, E.C., Messer, C.J. and Garza, R.U. (1988). Arterial Signal Timing Optimization Using PASSER II - 87 Microcomputer User's Guide. TTI Research Report 467-1. Texas Transportation Institute, College Station, Texas.
- Cronje, W.B. (1983). Analysis of Existing Formulas for Delay, Overflow and Stops. *Transportation Research Record* 905, pp. 89-93.
- Fambro, D.B., Chang, E.C.P. and Messer, C.J. (1991). Effects of the Quality of Signal Progression on Delay. National Cooperative Highway Research Program Report No. 339. National Research Council, Transportation Research Board, Washington, D.C.
- Federal Highway Administration (1980). Traffic Network Analysis with NETSIM, a User's Guide. FHWA Implementation Package FHWA-IP-80-3, U.S. Department of Transportation, Washington, D.C.
- Federal Highway Administration (1983). TRANSYT-7F User's Manual. Prepared by Transportation Research Center, University of Florida at Gainesville, U.S. Department of Transportation, Washington, D.C.
- Hillier, J.A. and Rothery, R. (1967). The Synchronization of Traffic Signals for Minimum Delays. *Transportation Science* 1(2), pp. 81-94.
- Hutchinson, T.P. (1972). Delay at a Fixed Time Traffic Signal-II, Numerical Comparisons of Some Theoretical Expressions. *Transportation Science* 6 (3), 286-305.
- Johnson, B. and Akçelik, R. (1992). Review of Analytical Software for Applicability to Paired Intersections. Proc. 16th ARRB Conf. 16(5), pp. 347-367.
- Miller, A.J. (1963). Settings for Fixed-Cycle Traffic Signals. *Operations Research Quarterly* 14(4), pp. 373-386.
- Miller, A.J. (1968). Australian Road Capacity Guide - Provisional Introduction and Signalized Intersections. Australian Road Research Board. Bulletin No. 4. (Superseded by ARRB Research Report ARR No. 123, 1981).
- Newell, G.F. (1989). Theory of Highway Traffic Signals. Research Report UCB-ITS-RR-89-7. University of California, Berkeley.
- Newell, G.F. (1990). Stochastic Delays on Signalized Arterial Highways. In: M. Koshi (Ed.), *Proceedings of the Eleventh International Symposium on Transportation and Traffic Theory*, Elsevier, New York, pp. 589-598.
- Olszewski, P.S. (1988). Efficiency of Arterial Signal Coordination.

- Proc. 14th ARRB Conf. 14 (2), pp. 249-257.
- Olszewski, P.S. (1990a). Traffic Signal Delay for Non-Uniform Arrivals. *Transportation Research Record* 1287, pp. 42-53.
- Olszewski, P.S. (1990b). Modelling of Queue Probability Distribution at Traffic Signals. In: M. Koshi (Ed.), *Proceedings of the Eleventh International Symposium on Transportation and Traffic Theory*, Elsevier, New York, pp. 569-588.
- Ove Arup & Partners (1991). Paired Intersections Project. Report for the Australian Road Research Board Ltd. ARRB Working Paper WD TE91/017.
- Roads and Traffic Authority, NSW (1991). *SCATES User Manual*. Haymarket, New South Wales.
- Roughail, N.M. (1988). Delay Models for Mixed Platoon and Secondary Flows. *ASCE Journal of Transportation Engineering* 114 (2), pp. 131-152.
- Roughail, N.M. (1989). Progression Adjustment Factors at Signalized Intersections. *Transportation Research Record* 1225, pp. 8-17.
- Roughail, N.M. and Akçelik, R. (1991). Paired Intersections: Initial Development of Platoon Arrival and Queue Interaction Models. Australian Road Research Board. Working Paper WD TE91/010.
- Roughail, N.M. and Akçelik, R. (1992a). A Preliminary Model of queue Interaction at Signalized Paired Intersections. *Proc. 16th ARRB Conf.* 16 (5), pp. 325-345.
- Roughail, N.M. and Akçelik, R. (1992b). Oversaturation Delay Estimates with Consideration of Peaking. *Transportation Research Record* 1365, pp. 71-81.
- Sosin, J.A. (1980). Delays at Intersections Controlled by Fixed Traffic Signals. *Traffic Engineering and Control* 21 (8/9), pp. 407-413.
- Teply, S. (Ed.) (1984). *Canadian Capacity Guide for Signalized Intersections*. Institute of Transportation Engineers, District 7, and the University of Alberta. Edmonton, Alberta.
- Transportation Research Board (1985). *Highway Capacity Manual*. Special Report 209, Washington, D.C.
- Van As, S.C. (1991). Overflow Delay in Signalized Networks. *Transportation Research* 25A (1), pp. 1-7.
- Webster, F.V. (1958). *Traffic Signal Settings*. Road Research Laboratory Technical Paper No. 39. HMSO, London

Notation

- c Signal cycle time (common for the upstream and downstream intersections)

g_u, g_d	Effective green times at upstream and downstream approaches
q_u, q_d	Average demand (arrival) flow rates at the upstream and downstream approaches
s_u, s_d	Saturation flow rates at the upstream and downstream approaches
sg	Cycle capacity (vehicles per cycle)
Q	Capacity, $Q = sg/c$ (vehicles per hour or vehicles per second)
x_u, x_d	Degrees of saturation (demand/capacity ratio, q/Q) at the upstream and downstream approaches
x_0	Degree of saturation below which the overflow delay is zero
I_u, I_d	Variance-to-mean ratios of the number of arrivals per cycle at the upstream and downstream approaches
k, k'	Delay parameters related to the rate of increase of delay ($k = k'I$).
T_f	Duration of the demand (arrival) flow period
PIP	Proportion in platoon: proportion of vehicles departing from the upstream intersection (arriving at the downstream intersection) which are in a platoon; equivalent to the proportion of vehicles departing with saturation headways
d	Average delay per vehicle, $d = d_1 + d_2$ where d_1 and d_2 are the non-random and overflow components of average delay
N_m	Maximum back of queue in an average signal cycle, $N_m = N_{m1} + N_{m2}$ where N_{m1} and N_{m2} are the non-random and overflow components of the maximum back of queue ($N_{m2} = N_0$ is the average overflow queue)

- h Stop rate (average number of stops per vehicle), $h = h_1 + h_2$ where h_1 and h_2 are the non-random and overflow components of stop rate

Units: Arrival flow and capacity in *veh/h* or *veh/s*, average delay in *seconds*, queue length in *vehicles*.

Acknowledgments

The authors thank D. Ian Johnston, the Executive Director of the Australian Road Research Board, for permission to publish this article. The work reported in the article was carried out during Dr. Nagui Roupail's sabbatical leave at ARRB. The views expressed in the article are those of the authors, and not necessarily those of ARRB.