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REPRINT

Relating flow, density, speed and travel time models for uninterrupted and interrupted traffic

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NOTE:

This paper is related to the intersection analysis methodology used in the SIDRA INTERSECTION software. Since the publication of this paper, many related aspects of the traffic model have been further developed in later versions of SIDRA INTERSECTION. Though some aspects of this paper may be outdated, this reprint is provided as a record of important aspects of the SIDRA INTERSECTION software, and in order to promote software assessment and further research.

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Relating flow, density, speed and travel time models for uninterrupted and interrupted traffic

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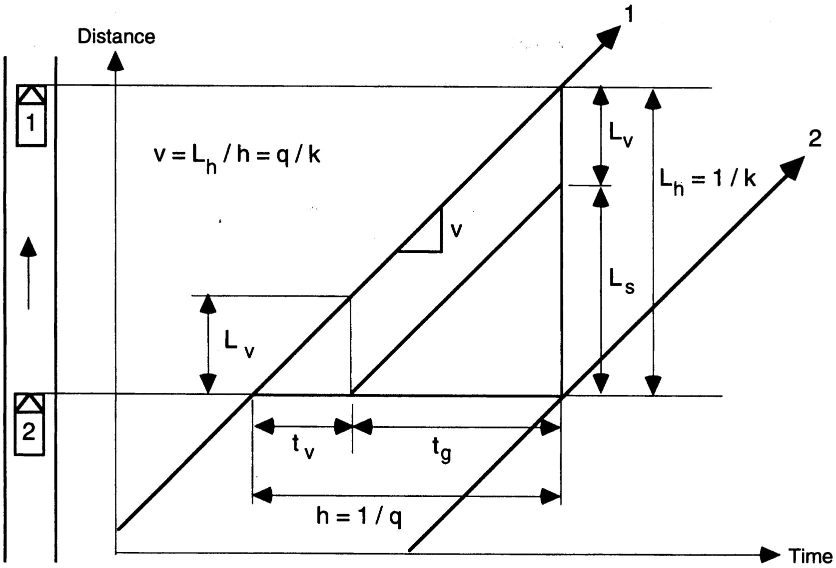

Fig 1. Time-distance diagram showing the relationship between vehicle spacing, headway and speed.

Introduction. A vast amount of literature exists on the fundamental speed-density-flow models of traffic theory (e.g. May[1], TRB[2]) and various travel time (cost)-flow models used for transport planning and demand management purposes (e.g. Akçelik[3]). There is often a confusion about how these two groups of models are related, especially in terms of *congested* traffic conditions. These models have been discussed in some detail and the difficulty of relating the two groups of models has been emphasised in a recent Bureau of Transport and Communications Economics report on traffic congestion and road-user charges[4].

This paper offers an explanation of the relationships between speed, density, flow and travel time models for uninterrupted and interrupted traffic flows. The difference between *arrival* flows measured upstream of a queueing section and *departure* flows measured at a reference point along the road is emphasised. The former is related to *demand* while the latter is related to *capacity*. The difference is of particular importance in oversaturated (congested) conditions where demand exceeds capacity. The speed measured at a reference point along the road under congested conditions can be better understood as a queue discharge, queue formation or moving queue speed. This speed is associated with departure flow which cannot exceed the capacity flow. On the other hand, the average speed based on travel time through a road section including the travel distance upstream of the queueing section is associated with demand flow rate that can exceed the capacity.

Formulae and examples are given for the two types of speed-flow relationships as well as the basic relationships among speed, density, spacing and flow variables. Notations and definitions are given in the Appendix.

Headway, spacing, speed

As a starting-point, three basic variables describing the movement of a vehicle as observed at a reference point along the road are *headway, spacing* and *speed. Headway (h)* is the time between passage of the front ends of two successive vehicles. *Spacing (L_h)* is the distance corresponding to the headway time, i.e. the distance between the front end of the leading vehicle and the front end of the following vehicle. *Speed (v)* is the distance travelled per unit time.

As shown in Fig 1, the relationship between headway, spacing and speed is:

$$v = \frac{L_h}{h} \qquad \text{... (1)}$$

where h = headway (sec.)
L_h = spacing (m/veh)
v = vehicle speed (m/sec.)

Other variables shown in Fig 1 are the vehicle length, space (gap) length, and the corresponding vehicle passage time and gap time. The space (gap) length, L_s, is the distance between two successive vehicles as measured between the back end of the leading vehicle and the front end of the following vehicle, and is equivalent to spacing less vehicle length.

Vehicle passage time, t_v, corresponds to vehicle length, and is the time between the passage of the front and back ends of a vehicle. *Gap time*, t_g, is the time between the passage of the back end of the leading vehicle and the front end of the following vehicle, and is equivalent to headway time less vehicle passage time. Thus:

$$L_s = L_h - L_v \qquad \text{... (2)}$$

$$t_v = \frac{L_v}{v} \qquad \text{... (3)}$$

$$t_g = h - t_v = h - \frac{L_v}{v} = \frac{L_s}{v} \qquad \text{... (4)}$$

where h = headway (sec.)
t_v = vehicle passage time (sec.)
t_g = gap time (sec.)
L_s = vehicle length (m/veh)

In the calculations relating to average traffic conditions, the vehicle length should represent the actual traffic composition. Where the traffic stream is represented as a mixture of light vehicles (LVs) and heavy vehicles (HVs), the average vehicle length can be calculated as:

$$L_v = (1 - p_{HV})\, L_{vm} + p_{HV}\, L_{vHV} \qquad \text{... (5)}$$

where p_{HV} = proportion of heavy vehicles in the traffic stream
L_{vm} = average vehicle length for light vehicles/or passenger car units (m/LV or m/pcu)
L_{vHV} = average vehicle length for heavy vehicles (m/HV)

Typical average vehicle lengths of L_{vm} = 4.0 m and L_{VHV} = 10 m can be used where information is not available. For example, with 5 per cent heavy vehicles (P_{HV} = 0.05), the average vehicle length is found as $L_v = 0.95 \times 4 + 0.05 \times 10 = 4.3$ m.

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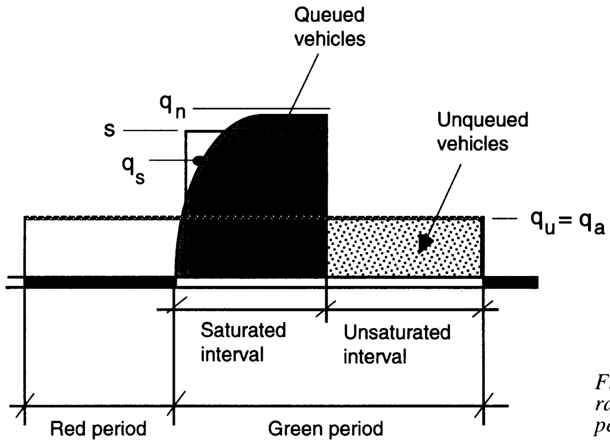


Fig 2. Departure flows during the saturated and unsaturated portions of green period at a signalised intersection.

Flow rate
Flow rate (veh/sec.) is the number of vehicles per unit passing *(arriving* or *departing)* a given reference point, and can be related to *headway* (sec.) through:

$$h = 1/q \qquad \text{... (6)}$$

Thus, Equation (1) is equivalent to $v = L_h q$, and vehicle spacing is given by $L_h = v/q$.

To understand the difference between congested and uncongested traffic operations, it is important to distinguish between the arrival (demand) flow rate and the departure flow rate for a given traffic facility. For example, at a signalised intersection approach lane, the departure flow rate measured at the stop-line is the queue discharge flow rate during the saturated portion of green period, $q=q_s$ (departures from queue), and the arrival flow rate after queue has cleared, $q = q_u$ (unqueued vehicles) as seen in Fig 2. The departure flow rate after queue clearance corresponds to the *arrival* (demand) flow rate measured under uninterrupted conditions at a point upstream of the back of queue, $q_u = q_a$.

Density
Density (concentration) is the number of vehicles per unit distance, and is related to the average spacing through:

$$k = \frac{1}{L_h} \qquad \text{... (7)}$$

where L_h is in metres and k is in veh/m (or $k = 1\,000 / L_h$ in veh/km).

Since $L_h = v/q$, the density is related to flow rate and speed as $k = q/v$.

The average spacing in a stationary queue, L_{hj} *(jam spacing)* is the sum of vehicle length, L_v, and jam space length, L_{sj}:

$$L_{hj} = L_v + L_{sj}$$

L_v = vehicle length (m/veh)
L_{sj} = average space length in a stationary queue *(jam space length)* measured from the back of the leading vehicle to the front of the following vehicle (m/veh).

The *jam density*, i.e. the number of vehicles per unit distance in a stationary queue, can be calculated from the average spacing in queue *(L_{hj})*:

$$k_j = \frac{1\,000}{L_{hj}} \qquad \text{... (8)}$$

where L_{hj} is in m/veh and k_j is in veh/km.

Typical jam space length of 2.0 m, hence jam spacing of $L_{hj} = 6$ m per car and 12 m per heavy vehicle, could be used where information is not available. In the previous example, where the average vehicle length is $L_v = 4.3$ m, the jam spacing is $L_{hj} = 4.3 + 2.0 = 6.3$ m/veh, and the corresponding jam density is $k_j = 1\,000 / 6.3 = 159$ veh/km.

Similarly, density at maximum flow is $k_n = 1\,000 / L_{hn}$ where the spacing at maximum flow $L_{hn} = 1\,000\, v_n / q_n$, therefore $k_n = q_n / v_n$ (L_{hn} in m/veh, q_n in veh/h, v_n in km/h, k_n in veh/km).

Speed-density-flow relationships
As the vehicles speed up from a stationary queue, the space length between vehicles increases gradually, and therefore the spacing increases and the density decreases. The corresponding flow rate increases to a maximum flow (q_n) value and then decreases as the speed increases towards the free-flow speed (v_f). The relationship between speed, density and flow is known as the fundamental relationship in traffic flow theory:

$$q = vk \qquad \text{... (10)}$$

where q is the flow rate (veh/h or veh/sec.), v is the speed (km/h or m/sec.) and k is the density (veh/km or veh/m).

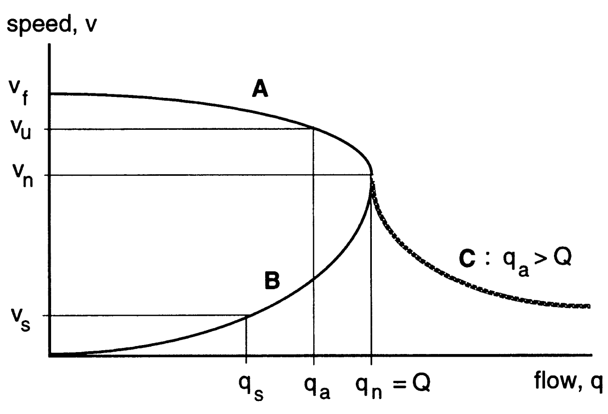
This relationship is often discussed in the context of uninterrupted flows, e.g. as observed on freeways and rural roads. Recent ARRB Transport Research studies of vehicle spacing, headway and speed relationships for vehicles departing from a queue formed at traffic signals[5,6] helped to understand the congested region of the speed-flow-density relationship. Also refer to an earlier paper discussing application of speed-density-flow relationships to traffic observed at the signalised intersection stop-line (Fehon and Moore[7]).

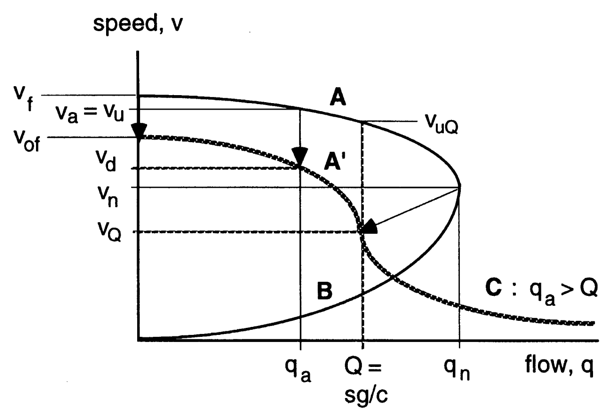
The speed-flow relationships for uninterrupted and uninterrupted conditions are depicted in Figs 3 and 4, respectively.

For uninterrupted traffic (Fig 3), the maximum flow rate, q_n is the capacity *($Q = q_n$)*. Region A represents undersaturated conditions with arrival flows below capacity ($q = q_a \leq Q$) which are associated with uninterrupted speeds ($v_f \geq v_u \geq v_n$, where v_f is the free-flow speed and v_n is the speed at maximum flow). Region B as observed at a reference point along the road represents oversaturated (congested) conditions with flow rates below the maximum flow rate ($q = q_s \leq q_n$) which are associated with reduced speeds ($v_s \leq v_n$).

Changes in conditions from Region A to Region B through the maximum flow point represents *queue formation* (e.g. due to two lanes of traffic merging into one lane, or traffic stopping at traffic signals). On the other hand, changes in conditions from Region B to Region A through the maximum flow point represents *queue discharge* (e.g. one lane of traffic diverging into two lanes, or traffic departing from a queue at traffic signals).

Speed as a function of flow rate: Fig 3 (left) uninterrupted traffic; and Fig 4 (right) interrupted traffic.





Region C for uninterrupted flow represents arrival flows above capacity ($q_a > q_n$) associated with average speeds based on *travel time through the section.* In this case, the flow represents the *demand* flow rate which can exceed the capacity value (as measured upstream of the reference point), and therefore Region C of the speed-flow curve is valid for degrees of saturation in excess of 1.0.

In Fig 4 for interrupted traffic, capacity is given by $Q = sg/c$ where s is the average queue discharge (saturation) flow rate, g is the effective green time and c is the cycle time. The average saturation flow rate is smaller than the maximum queue discharge rate ($s < q_n$) because of lower discharge rate at the start of the green period as seen in Fig 2, and the capacity is the average saturation flow reduced by the available green time ratio, g/c. The use of effective green and cycle time for unsignalised intersection capacity by traffic signal analogy has been described in Akçelik and Chung[8].

Region A for interrupted traffic represents approach (mid-block) travel conditions before the queueing section is entered.

The flow rate for the congested flow region (B), e.g. at a signalised intersection stop-line, is the rate of departure from the queue. This corresponds to the instantaneous queue discharge flow rate during the green period (q_s) that increases from zero to a steady maximum queue discharge flow rate (q_n) while the queue discharge speed increases from zero (stationary queue) to steady speed (v_n) corresponding to the maximum flow. This consideration brings an explanation to the speed-flow models for uninterrupted flows as well: the flow rate measured for forced-flow (congested) conditions can be considered to be a *reduced capacity flow* (not arrival flow). This is a result of vehicles (or moving queues) in the same stream interfering in the same way as the downstream queue interferes with the departure rate at the signal stop-line, resulting in reduced vehicle spacings, speeds and flow rates. Wardrop's speed-flow surveys of traffic in a circular track[9] are relevant in this respect.

Figure 4 shows the average speeds (v_d) based on section travel time (regions A′ and C) to be reduced from the approach (uninterrupted) speeds ($v_a = v_u$) as a result of the delays experienced due to traffic interruptions, e.g. red signal or give-way to major road traffic at a STOP sign ($v_d < v_a$). Average speeds estimated by the SIDRA software package[10] for interrupted movements correspond to the speed-flow relationship consisting of regions A' and C in Fig 4.

The *free-flow speed* for uninterrupted flow (v_f) is the average speed that occurs under zero-flow conditions. The corresponding *zero-flow speed* for interrupted flow (v_{of}) includes the free-flow travel time for uninterrupted flow plus total minimum (zero-flow) delay (d_m) at traffic interruptions.

There is a vast amount of literature on speed-density-flow relationships for uninterrupted traffic offering empirical as well as theoretical models based on car-following, hydrodynamic and kinetic theories of traffic flow. Based on theoretical approaches, various speed-density models (linear, parabolic, exponential) can be derived from $dv/dk = -c\, v^m k^{l-2}$ where parameters m and l are selected to satisfy boundary conditions such as $v = v_f$ when $q = 0$ and $k = 0$, $v = 0$ when $k = k_j$, $dq/dk = 0$ when $k = k_n$, etc. However, it has not been possible to obtain a satisfactory model based on these approaches, that gives realistically high values of $v_f/v_n = 0.6 - 0.9$ (e.g. see TRB[2]) associated with low values of $k_n/k_j = 0.15$-0.40 where v_n and k_n are the speed and density corresponding to the maximum flow rate. For example, $q_n = 2\,000$ veh/h and $v_n = 55$ km/h give $k_n = q_n/v_n = 36.4$ veh/km, and using $k_j = 1\,000/6.3 = 159$ veh/km, $k_n/k_j = 0.23$.

A model based on the use of vehicle spacing as a function of speed (L_h-v) as a starting-point (rather than a speed-density relationship) for the derivation of speed-density -flow models has been discussed in Akçelik[5] and Akçelik and Besley[6].

Alternatively, a speed-flow model can be used as a starting point. For Region B, the following model form derived using exponential queue discharge flow and speed models can be used[6].

$$v_s = v_n\,[1-(1-\frac{q_s}{q_n})\,k_n/k_j] \qquad \ldots (11)$$

where
- v_s, q_s = speed (km/h) and flow rate (veh/h) in Region B of the speed - flow model
- v_n = speed at maximum flow (km/h)
- q_n = maximum flow rate (veh/h)
- k_n = density at maximum flow (veh/km)
- k_j = jam density (veh/km)

If the speed v_s is known, the flow rate in Region **B** (q_s) can be estimated from the following formula derived from Equation (11):

$$q_s = q_n\,[1-(1-\frac{v_s}{v_n})\,k_j/k_n] \qquad \ldots (11a)$$

For six signalised intersection sites in Melbourne (each representing a traffic lane with cars only), $q_n = 1\,840$ to $2\,880$ veh/h, $v_n = 27$ to 56 km/h, $k_n = 50$ to 68 veh/km, and $k_n / k_j = 0.30$ to 0.40 ($L_{hj} = 6.0$ m/veh, $k_j = 167$ veh/km) were observed. For these sites, free-flow speeds were in the range $v_f = 60$ to 80 km/h using the speed limit as the free-flow speed, and the ratios $v_n / v_f = 0.4$ to 0.8 were found.

The following model based on a time-dependent travel time function (Akçelik[3,4] can be used for Regions A and C (or A′ and C) of the speed-flow relationship for uninterrupted or interrupted flow conditions:

$$v = \frac{v_{of}}{1 + 0.25\ \ v_{of} T_p\,[z + \sqrt{z^2 + \frac{m_c x}{Q T_p}}]} \qquad \ldots (12)$$

where
- v = travel speed in km/h ($v = v_u$ for uninterrupted flow, $v = v_d$ for interrupted flow)
- v_{of} = zero-flow travel speed in km/h ($v_{of} = v_f$ for uninterrupted flow)
- T_p = peak flow (analysis) period in hours (e.g. $T_p = 0.25$ to 2.0 h)
- Q = capacity in veh/h ($Q = q_n$ for uninterrupted flow)
- z = $x - 1$
- x = q_a/Q (q_a is the demand flow rate; for uninterrupted flows, $x = q_a / q_n$)
- m_c = a delay parameter

The slope of the speed - flow curve in regions A and C (or A′ and C) is determined by the delay parameter m_c. This slope indicates the rate of change of delay, i.e. the difference between the zero-flow travel time and the travel time at a given flow rate. For uninterrupted conditions, this delay is due to vehicle interactions within the traffic stream. For interrupted facilities, the delay also includes delays at traffic interruptions. The delay parameter m_c is related to parameter J_A used in Akçelik[3] through $m_c = 8\,J_A$. Various values of J_A for different traffic facilities are given in Akçelik[3], indicating delay parameters in the range from $m_c = 0.8$ ($J_A = 0.1$) for a freeway to $m_c = 12.8$ ($J_A = 1.6$) for an interrupted facility with high friction. These values are not based on real-life data. They were constructed using various delay scenarios based on assumptions about the number and characteristics of delay-producing elements (interruptions) with $T_p = 1\ h$.

Where real-life data are not available, m_c can be calculated using estimates or measured values of capacity, zero-flow speed, speed at capacity, and duration of the analysis (peak demand) period:

$$m_c = \frac{16\,Q}{T_p v_{of}^2}\ (\frac{v_{of}}{v_Q} - 1)^2 \qquad \ldots (13)$$

where v_Q is the average travel speed at capacity (for uninterrupted traffic, $v_Q = v_n$, $v_{of} = v_f$, $Q = q_n$).

It should also be noted that uninterrupted travel speeds in Region **A** estimated from Equation (12) are assumed to be the same as the speeds observed at a reference point along the road. The error involved in this assumption is negligible for low degrees of saturation, but may increase as arrival flows approach the capacity value.

A speed-flow function for interrupted traffic flow can be constructed from uninterrupted speed-flow function by calculating the zero-flow and speed at capacity (v_{of}, v_Q) from

$$v_{of} = \frac{v_f}{1 + \frac{d_m v_f}{3\,600}} \qquad \ldots (14)$$

$$v_Q = \frac{v_{uQ}}{1 + \frac{d_Q v_{uQ}}{3\,600}} \qquad \ldots (15)$$

where v_f is the uninterrupted zero-flow speed, v_{uQ} is the uninterrupted traffic speed when the demand flow equals traffic capacity ($q_a = Q$), d_m is the minimum (zero-flow) delay per unit distance, and d_Q is the delay per unit distance at capacity ($q_a = Q$). Minimum delay and capacity for various types of signalised and unsignalised intersections can be calculated using the delay models used in SIDRA[8,10,12,13].

Travel time-flow relationships

The travel time-flow relationships corresponding to Equations (11) and (12) can be obtained using the relationship:

$$t = \frac{3\,600}{v} \qquad \ldots (16)$$

where t is the travel time per unit distance (seconds/km), and the speed is $v = v_s$ for Region B, $v = v_u$ for Regions A and C (uninterrupted traffic), or $v = v_d$ for Regions A′ and C (interrupted traffic). In fact, application of this to Region B observed in the literature is not relevant, but is used here to contrast it with the section travel time under congested conditions (Region C).

The travel time - flow model corresponding to Equation (12) is:

$$t = t_{of} + 0.25\, T_p \left[z + \sqrt{z^2 + \frac{m_c x}{Q T_p}} \right] \qquad \ldots (17)$$

For uninterrupted flow, $t_{of} = t_f = 3\,600/v_f$, for interrupted flow, $t_{of} = 3\,600/v_{of} = t_f + d_m$ (see Equations (14) and (15), and other parameters are as in Equation (12). The travel time per unit distance for uninterrupted traffic under maximum flow conditions is $t_n = 3\,600/v_n$.

The travel time corresponding to the congested flow speed in Region B ($3\,600/v_s$) and the section travel time in Region C for uninterrupted flow conditions ($3\,600/v_u$ for $x > 1$) cannot be equated.

The congested flow speed is measured at a particular reference point along the travel section. Different speed values would be measured at different reference points along the travel section due to the transient queueing characteristics (queue formation and queue discharge). This also explains the difficulty in determining the maximum flow value in real-life surveys.

The travel time predicted by the time-dependent model Equation (17) includes the average delay considering all vehicles that enter the travel section during the analysis period (T_p) including delays until the last vehicle that entered during T_p leaves the section. While the model implies time-dependence for undersaturated conditions, the effect of time-dependence is negligible for low degrees of saturation (the last vehicle is likely to leave the section during the same period). However, the effect of time-dependence increase as arrival flows approach the capacity value. For oversaturated conditions ($x > 1$), time-dependent delays are the main component of travel time.

For uninterrupted traffic operating in undersaturated conditions (Region A), arrival (demand) and departure flows are equal. For demand flows above the maximum flow rate ($q_a > q_n$), traffic operates in Region B for the speed-flow curve, and the demand is not known at the detection point. This corresponds to oversaturated (congested) conditions where the demand can only be measured at the back of the queueing section. An approximate method for estimating the demand (derived using assumptions about density) can be expressed as follows:

$$q'_a = \frac{v_n}{v_s} q_s \qquad \ldots (18)$$

where q_s, v_s represent the conditions on the Region B curve, and v_n is the speed at maximum flow (this expression would probably underestimate the demand).

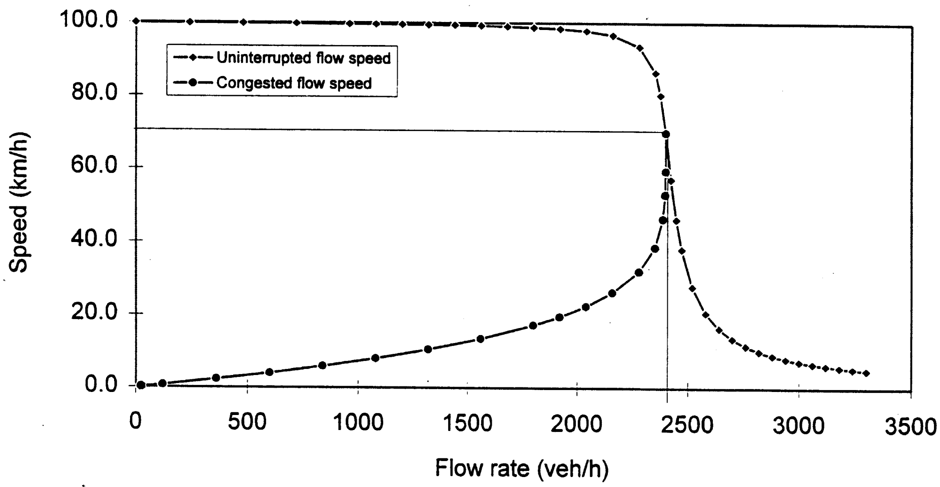


Fig 5. Speed as a function of flow (uninterrupted traffic).

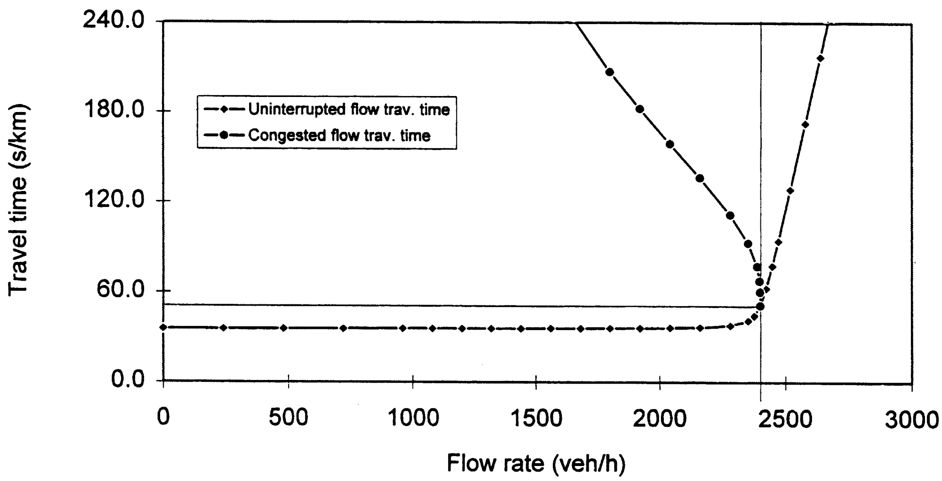


Fig 6. Travel time as a function of flow (uninterrupted traffic).

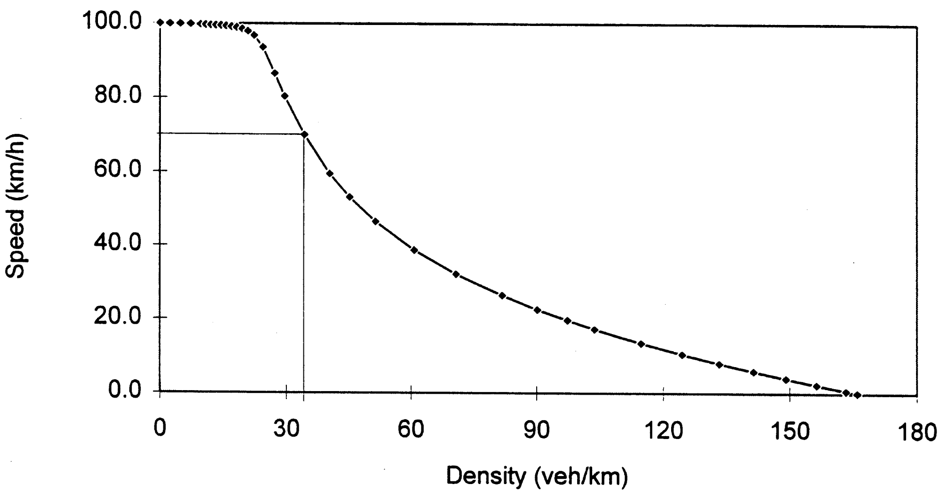


Fig 7. Speed as a function of density (uninterrupted traffic).

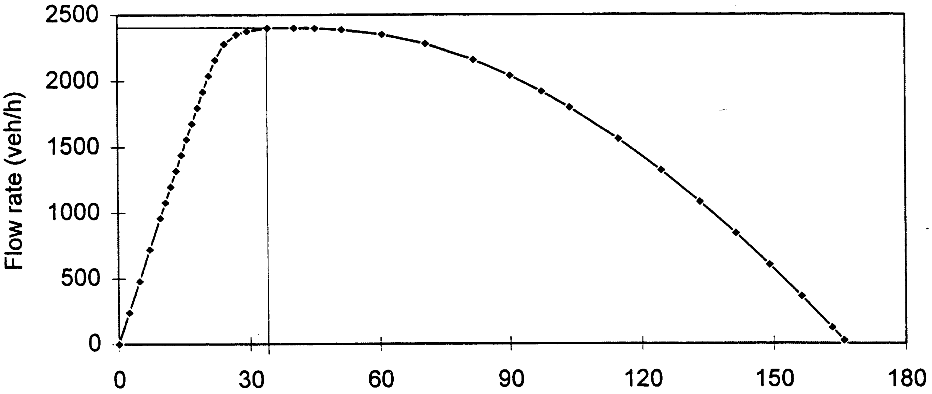


Fig 8. Flow rate as a function of density (uninterrupted traffic).

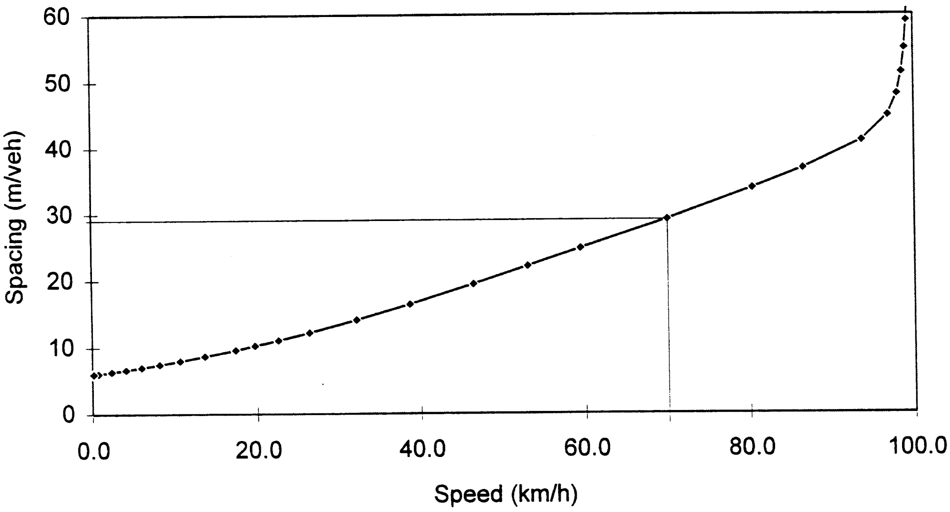


Fig 9. Vehicle spacing as a function of speed (uninterrupted traffic).

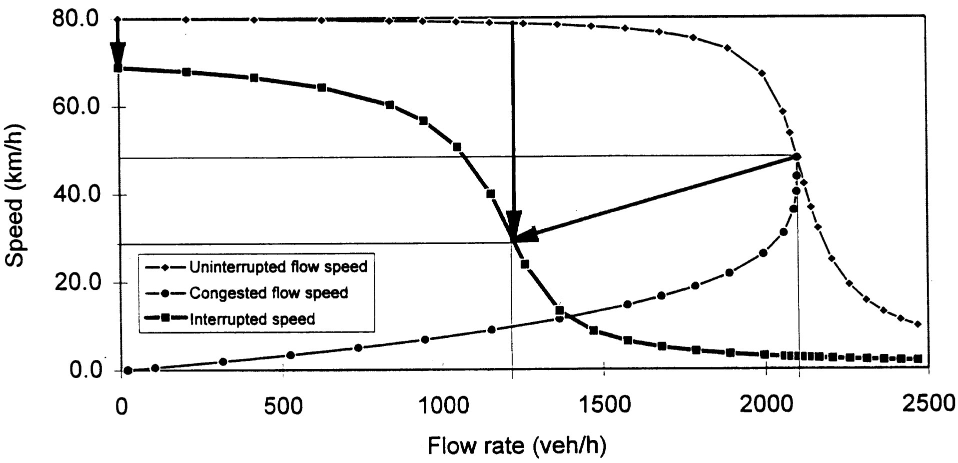


Fig 10. Speed as a function of flow (interrupted traffic).

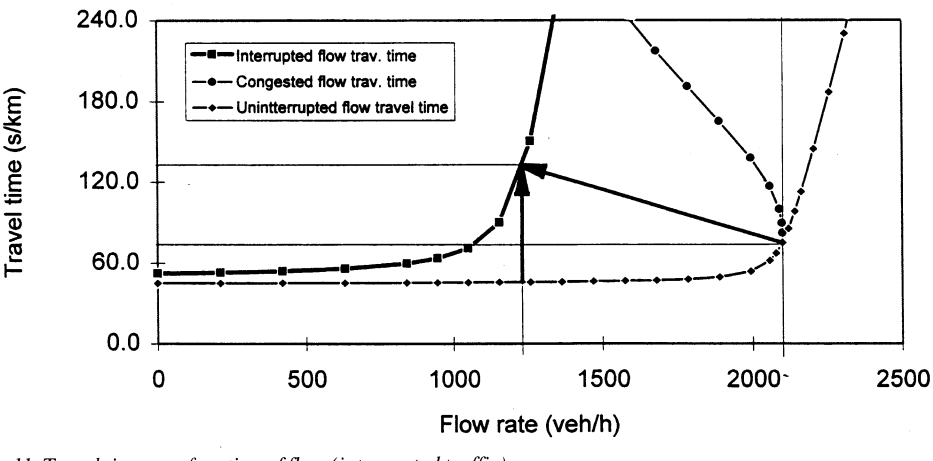


Fig 11. Travel time as a function of flow (interrupted traffic).

Example 1: uninterrupted traffic

Graphs of speed-flow *(v-q)*, travel time-flow *(t-q)*, speed-density *(v-k)*, flow-density *(q-k)*, and spacing-speed $(L_h\text{-}v)$ relationships developed using the formulae given in this paper are shown in Figs 5 to 9 for the following example for an uninterrupted flow case:
$v_{of} = v_f = 100$ km/h, $v_n = 70$ km/h $(v_n/v_f = 0.70)$, $Q = q_n = 2\,400$ veh/h, therefore $k_n = q_n/v_n = 2\,400/70 = 34.3$ veh/km,
spacing at maximum flow $L_{hn} = 1\,000/34.3 = 29.2$ m/veh,
jam spacing, $L_{hj} = 6.0$ m/veh (cars only), jam density, $k_j = 1\,000/6.0 = 166.7$ veh/km, $k_n/k_j = 34.3/166.7 = 0.206$, and for the speed-flow model for Regions A and C, parameter $m_c = 0.71$ is calculated from Equation (13) using $T_p = 1\ h$.

Example 2: interrupted traffic

Graphs of speed-flow *(v-q)* and travel time-flow *(t-q)* relationships are shown in *Figs 10 and 11* for the following example for an interrupted flow case:
For mid-block (approach) section, uninterrupted flow parameters are $v_f = 80$ km/h, $v_n = 48$ km/h $(v_n/v_f = 0.60)$, $q_n = 2\,100$ veh/h, therefore $k_n = q_n/v_n = 2\,100/48 = 43.8$ veh/km, $L_{hn} = 1\,000/43.8 = 22.9$ m/veh, jam spacing, $L_{hj} = 6.0$ m/veh, and jam density, $k_j = 1\,000/6.0 = 166.7$ veh/km, $k_n/k_j = 43.8/166.7 = 0.263$, and parameter $m_c = 2.33$ is calculated from Equation (13) using $T_p = 1\ h$.

A lane of traffic controlled by fixed-time signals along a travel section of 1 km is considered. Signal parameters are $g = 54$ sec., $c = 90$ sec., saturated flow, $s = 2\,066$ veh/h $(< q_n)$, capacity is $Q = sg/c = 1\,239$ veh/h. From Akçelik and Chung[13], minimum delay is calculated as $d_m = 7.2$ sec./km, and delay at capacity is calculated as $d_Q = 87.4$ sec./km. Using these in Equations (14) and (15), $v_{of} = 68.9$ km/h, $v_Q = 27.0$ km/h are found, and using these in Equation (13), the delay parameter for the interrupted speed-flow model is found as $m_c = 10.02$.

Concluding remarks

This paper presented an explanation of the relationships between fundamental traffic variables spacing, headway, speed, density, flow and travel time for uninterrupted and interrupted traffic conditions. An Excel spreadsheet application has been developed to implement the formulae given in the paper.

Further work is recommended to calibrate the models presented here using real-life data from uninterrupted and interrupted traffic facilities with demand flows below and above capacity.

The speed-flow (or travel time-flow) model given in this paper is currently in use in the SIDRA software package[10] for uninterrupted traffic facilities. It is an appropriate model for estimating travel times for such purposes as traffic assignment and congestion pricing as used in recent work by Luk and Hepburn[14] and BTCE[4].

The application of the models given in this paper to the derivation of occupancy time, space time and SCATS DS (degree of saturation) parameters for vehicle-actuated and SCATS Master Isolated signal control purposes is discussed in Akçelik[5], and will be presented in a future paper.

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APPENDIX
Notations and Definitions

h — Headway (seconds/veh); the time between passage of the *front* ends of two successive vehicles
$h = 1/q = L_h/v = t_g + t_v$ (q in veh/sec., v in m/sec.), or $h = 3\,600/q$ (q in veh/h)

k — Density (concentration): number of vehicles per unit distance
$k = q/v = 1/L_h$ in veh/m, or $k = 1\,000/L_h$ in veh/km

k_j — Jam density: number of vehicles per unit distance at zero speed, i.e. for a stationary queue
$k_j = 1/L_{hj}$ in veh/m, or $k_j = 1\,000/L_{hj}$ in veh/km

k_n — Density (veh/km) at maximum flow
$k_n = q_n/v_n$ (q_n in veh/h and v_n in km/h)

L_h — Spacing (m/veh); the headway *distance* as measured between the *front* ends of two successive vehicles (sum of space length and vehicle length)
$L_h = vh = v/q = Lv + L_s$ (q in veh/sec., v in m/sec.)

L_{hj} — Average jam spacing (m/veh), which is the sum of vehicle length and average space length for vehicles in a stationary queue
$L_{hj} = L_v + L_{sj}$

L_{hn} — Average spacing (m/veh) at the maximum queue discharge flow rate of the actual traffic mix, q_n

L_s — Space (gap) length (m/veh): the following distance between two successive vehicles as measured between the *back* end of the leading vehicle and the front end of the following vehicle
$L_s = L_h - L_v = v\,t_g$

L_{sj} — Average space length for vehicles in a stationary queue (m)

L_v — Average vehicle length (m/veh) for the actual traffic mix
$L_v = (1 - p_{HV})\,L_{vm} + p_{HV}\,L_{vHV}$

L_{vHV} — Average vehicle length for heavy vehicles (m/HV)

L_{vm} — Average vehicle length for light vehicles, or passenger car units (m/LV or m/pcu)

p_{HV} — Proportion of heavy vehicles (HVs) in the stream, e.g. $p_{HV} = 0.05$ means 5 per cent HVs and 95 per cent LVs (light vehicles, or pcus)

q — Flow rate (veh/sec. or veh/h): the number of vehicles per unit time passing *(arriving or departing)* a reference point along the road

q_a — Arrival (demand) flow rate (veh/h), i.e. the number of vehicles per unit time as measured at a point upstream of the queueing section

q_n — Maximum flow rate for uninterrupted traffic (veh/h); maximum queue discharge flow rate at a signal stop-line

q_s — Flow rate under congested flow conditions observed at a reference point along the road (veh/h or veh/sec.); departure flow rate during the saturated part of the green period at a signal stop-line

q_u — Flow rate for uninterrupted traffic (veh/h); departure flow rate during the unsaturated part of the green period at a signal stop-line ($q_u = q_a$: departure flow rate = arrival flow rate)

Q — Capacity (veh/h): maximum arrival flow rate that can be serviced under prevailing flow conditions at a traffic facility; for uninterrupted traffic, $Q = q_n$

s — Average saturation (queue discharge) flow rate during the effective green period at a signal stop-line (veh/h)

t — Travel time per unit distance (sec./km)
$t = 3\,600 / v$ (v in km/h)

t_f — Free-flow travel time per unit distance for uninterrupted traffic: average travel time under zero-flow conditions (can be approximated by the speed limit) (sec./km)
$t_f = 3\,600 / v_f$ (v_f in km/h)

t_g — Gap time (sec./veh): the time between the passage of the *back* end of leading vehicle and the *front* end of the following vehicle as measured at a reference point along the road; this is the time taken to travel the space (gap) length, L_s
$t_g = L_s / v = h - t_v$ (v in m/sec.)

t_n — Travel time per unit distance under maximum flow conditions for uninterrupted traffic ($q = q_n$) (sec./km)
$t_n = 3\,600 / v_n$ (v_n in km/h)

t_{of} — Zero-flow travel time per unit distance for interrupted traffic: average travel time under zero-flow conditions including the approach free-flow travel time, t_f, plus total minimum delay at traffic interruptions (sec./km)
$t_{of} = 3\,600 / v_{of}$ (v_{of} in km/h)

t_v — Vehicle passage time (sec./veh): time it takes for the vehicle length (from front end to back end) to pass a reference point at speed v (m/sec.)
$t_v = L_v / v$

v — Speed (m/sec. or km/h): distance travelled per unit time
$v = L_h / h = q / k$

v_a — Average approach speed (km/h) measured at a point upstream of the queueing section: average cruise speed at arrival flow rate q_a

v_d — Average speed for interrupted traffic including the effect of delays at traffic interruptions (km/h)

v_f — Free-flow speed for uninterrupted traffic (km/h)
$v_f = 3\,600 / t_f$ (t_f in sec./km)

v_n — Speed at maximum flow (km/h); speed corresponding to the maximum queue discharge flow rate, q_n at a signal stop-line

v_{of} — Zero-flow speed for interrupted traffic (km/h)
$v_{of} = 3\,600 / t_{of}$ (t_{of} in sec./km)

v_Q — Average travel speed when the demand flow rate equals the capacity for interrupted traffic (km/h)

v_s — Speed under congested (forced-flow) conditions observed at a reference point along the road (km/h): departure speed during the saturated part of the green period at a signal stop-line

v_u — Average travel speed (km/h or m/sec.) for uninterrupted traffic ($v_u = v_a$)

x — Degree of saturation, i.e. the ratio of arrival (demand) flow rate to capacity; for uninterrupted traffic, $x = q_a / q_n$