

**TECHNICAL NOTE**

# **HCM 2000 Back of Queue Model for Signalised Intersections**

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# HCM 2000 Back of Queue Model for Signalised Intersections

Rahmi Akçelik

## Introduction

A major improvement in the latest edition of the US Highway Capacity Manual (TRB 2000) is the introduction of a back of queue model for signalised intersections (Chapter 16, Appendix G). This report presents an extended version of the original technical note that described the queue length model developed for HCM 2000 (Akçelik 1998). The model is applicable to both pretimed and actuated signals using different but consistent model parameters.

The model is described fully, and comparisons with microscopic simulation data, real-life data and aaSIDRA (Akcelik & Associates 2000) estimates are presented. Equations are given using the HCM 2000 notation (see the *Notations* list given before the *References* section). Comments on some aspects of the back of queue values predicted by the HCM 2000 model are included. The theoretical background to the development of the HCM 2000 back of queue model is discussed in *Appendix A*, and a method for determining model parameters in the case of unequal lane utilisation is given in *Appendix B*.

A model to predict the queue clearance (service) time, with due allowance for platooned arrivals, is also given for use in the opposed (permitted) turn model and the actuated signal timing method.

All models described in this paper use *progression factors* for platooned arrivals (Akçelik 1995, 1996). The method is consistent between the delay and queue models. Refinements to the application of various conditions on the parameters used in the progression factors in the HCM 2000 queue and delay models are given in Akçelik (2001).

The queue length definition used here is the *back of queue* rather than the cycle-average queue. In addition to a model to predict the average back of queue, a model is given to calculate 70<sup>th</sup>, 85<sup>th</sup>, 90<sup>th</sup>, 95<sup>th</sup> and 98<sup>th</sup> percentile queue values. The back of queue measure is useful for identifying spillback conditions (i.e. the blockage of available queue storage distance). The *queue storage ratio* measure is presented for this purpose.

The queue model allows for an *initial queued demand* at the start of the flow period. Equations with zero initial queued demand are also given.

The queue length model given in this paper is expressed in the form of traditional two-term equations. These were derived by simplification of the more general form used in the aaSIDRA method (Akcelik and Associates 2000) as follows:

- (i) the degree of saturation (demand volume / capacity ratio) for non-zero overflow queue was set to zero ( $x_0 = 0$ ),

- (ii) the variational factor in the first term was set to one, ( $f_{b1} = 1.0$ ) so that the first term represents non-random (uniform) back of queue values and all randomness and oversaturation effects are accounted for in the second term, and
- (iii) a simpler form of the queue parameter ( $k_B$ ) was derived by means of comparison with the aaSIDRA model predictions, extensive ModelC simulation data for pretimed and actuated signals, as well as actuated signal data from a real-life intersection (Akçelik and Chung 1994, 1995a,b, Akçelik, Chung and Besley 1997, Akçelik, Besley and Chung 1998).

The effect of (i) is to allow for the prediction of non-zero overflow queues even at low degrees of saturation. The effect of (ii) is that the first-term queue expression represents non-random back of queue values, and all randomness and oversaturation effects are accounted for in the second term. In the aaSIDRA model, the first term includes some variational effects, and therefore differs between pretimed and actuated signals. In the HCM 2000 model, such variational effects are included in the second term, and therefore the first-term queue is the same for pretimed and actuated signals. The second-term queue is approximately equal to the average overflow queue.

## Queue Models

The models presented here are for use on an individual lane basis. To apply the method to *lane groups*, an “average queue length per lane” is calculated. For this purpose, parameters “average demand flow rate per lane” ( $v_L$ ), “average saturation flow rate per lane” ( $s_L$ ), “average capacity per lane” ( $c_L$ ) and “average initial queue demand per lane” ( $Q_{bL}$ ) are calculated for each lane group by dividing the total lane group values ( $v$ ,  $s$ ,  $c$ ,  $Q_b$ , respectively) by the number of lanes in the lane group. See *Appendix B* for a refinement for determining model parameters in the case of unequal lane utilisation.

### Average Back of Queue

( $Q$ ,  $Q_1$ ,  $Q_2$ ,  $Q_b$ ,  $s_{LG}$  in vehicles,  $C$  in seconds,  $T$  in hours,  $c_L$  in veh/h,  $v_L$  veh/s)

$$Q = Q_1 + Q_2 \quad (1.1)$$

$$Q_1 = PF_2 \frac{v_L C (1-u)}{1 - [\min(1, X_L) u]} \quad (1.1a)$$

$$Q_2 = 0.25 c_L T \left\{ z + \left[ z^2 + \frac{8 k_B X}{c_L T} + \frac{16 k_B Q_{bL}}{(c_L T)^2} \right]^{0.5} \right\} \quad (1.1b)$$

where

parameters  $v_L$ ,  $X_L$ ,  $X$ ,  $z$  are calculated from *Equations (1.6a) to (1.8)*,

$PF_2$  is the queue progression factor (*Equation 1.10*),

$u = g/C$  is the green time ratio,

$Q_{bL}$  = average initial queue demand per lane (veh),

$k_B$  is the queue parameter (incremental queue factor similar to "k" in the HCM delay formula):

$$k_B = 0.12 (s_{LG})^{0.7} I \quad \text{for pretimed signals} \quad (1.1c)$$

$$k_B = 0.10 (s_{LG})^{0.6} I \quad \text{for actuated signals} \quad (1.1d)$$

and  $I$  is the upstream filtering factor for platooned arrivals:

$$I = \begin{cases} 1.0 - 0.91 [\min(1.0, X_u)]^{2.68} & \text{for platooned arrivals} \\ 1.0 & \text{for random arrivals} \end{cases} \quad (1.1e)$$

where

$X_u$  is the degree of saturation (v/c ratio) at the upstream stop line

The queue model presented in this paper adopts the upstream filtering factor ( $I$ ) for platooned arrivals as used in the HCM delay model. This is the variance-to-mean coefficient for number of arrivals. HCM 2000 Chapter 15 qualifies  $X_u$  as the flow weighted average of contributing upstream movement  $X$  values. A more detailed method that allows for metering, splitting and merging of upstream flows has been described by Tarko and Rajaraman (1998). This method could be used to replace the HCM equation in the future.

If no initial queued demand,  $Q_{bL} = 0$ :

$$Q_2 = 0.25 c_L T \left[ z + \left( z^2 + \frac{8 k_B X}{c_L T} \right)^{0.5} \right] \quad (1.1f)$$

Figures 1a and 1b show average back of queue as a function of the degree of saturation for cases with and without initial queued demand using the equations given above.

Equation (1.1b) is the original equation presented in Akçelik (1988). The second-term back of queue equation in HCM 2000 (Equation G16-9 in Chapter 16 Appendix G) differs from the original equation as follows:

- (i)  $(X_L - 1)$  is used instead of parameter  $z = X - 1 + 2 Q_{bL} / (c_L T) = (X_L - 1) + Q_{bL} / (c_L T)$ ,
- (ii)  $k_B X_L$  is used instead of  $k_B X$ .

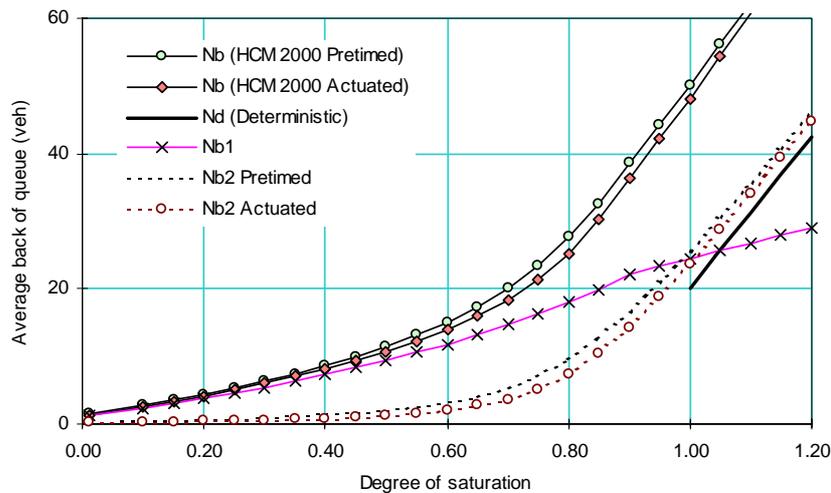
This difference has no effect with no initial queued demand ( $Q_{bL} = 0$ ) since  $X_L = X$  and  $z = X - 1$  in that case. However, the HCM 2000 equation underestimates the back of queue relative to the original equation (Equation 1.1b) when there is initial queued demand

The original formula could be expressed in terms of  $X_L$  (rather than  $z$  and  $X$ ) as follows:

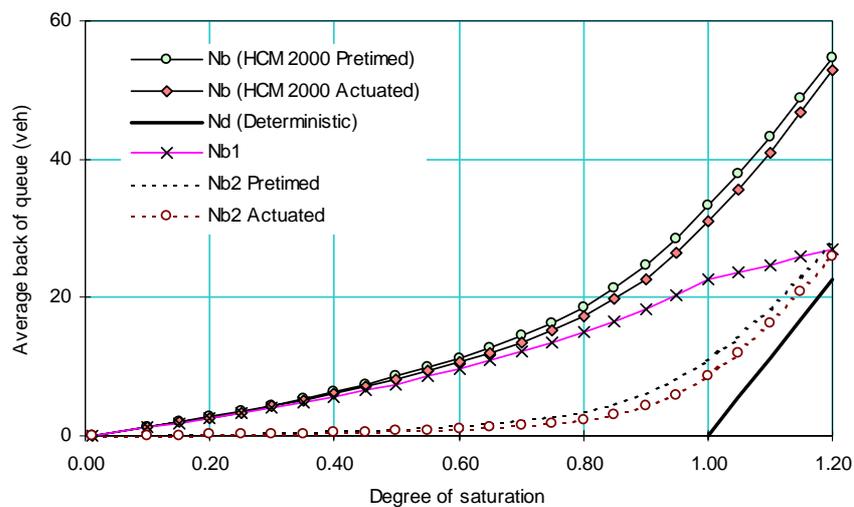
$$Q_2 = 0.25 c_L T \left\{ \left( X_L - 1 + \frac{Q_{bL}}{c_L T} \right) + \left[ \left( X_L - 1 + \frac{Q_{bL}}{c_L T} \right)^2 + 8 k_B \left( \frac{X_L}{c_L T} + \frac{Q_{bL}}{(c_L T)^2} \right) \right]^{0.5} \right\} \quad (1.1g)$$

It is recommended that the HCM 2000 Equation G16-9 is adjusted in line with the above equation.

All discussions in this report are based on the original equations.



**Figure 1a – Average back of queue ( $Q = N_b$ ) as a function of the degree of saturation: the case with initial queued demand,  $Q_{bL} = 20$  veh (various queue components, i.e. deterministic ( $Q_d = N_d$ ), first-term ( $Q_1 = N_{b1}$ ) and second-term ( $Q_2 = N_{b2}$ ) are shown)**



**Figure 1b – Average back of queue ( $Q = N_b$ ) as a function of the degree of saturation: the case without initial queued demand,  $Q_{bL} = 0$  veh (various queue components, i.e. deterministic ( $Q_d = N_d$ ), first-term ( $Q_1 = N_{b1}$ ) and second-term ( $Q_2 = N_{b2}$ ) are shown)**

**Table 1**

**Parameters for the percentile (70<sup>th</sup>, 85<sup>th</sup>, 90<sup>th</sup>, 95<sup>th</sup> and 98<sup>th</sup>) back of queue factors for pretimed and actuated signals**

	Pretimed signals			Actuated signals		
	p <sub>1</sub>	p <sub>2</sub>	p <sub>3</sub>	p <sub>1</sub>	p <sub>2</sub>	p <sub>3</sub>
f <sub>B70%</sub>	1.2	0.1	5	1.1	0.1	40
f <sub>B85%</sub>	1.4	0.3	5	1.3	0.3	30
f <sub>B90%</sub>	1.5	0.5	5	1.4	0.4	20
f <sub>B95%</sub>	1.6	1.0	5	1.5	0.6	18
f <sub>B98%</sub>	1.7	1.5	5	1.7	1.0	13

### **Percentile Back of Queue**

(*Q*, *Q*<sub>%</sub> in vehicles)

$$Q_{\%} = f_{B\%} Q \quad (1.2)$$

where

$$f_{B\%} = p_1 + p_2 e^{-Q/p_3} \quad (1.2a)$$

Parameters p<sub>1</sub>, p<sub>2</sub>, p<sub>3</sub> for the 70<sup>th</sup>, 85<sup>th</sup>, 90<sup>th</sup>, 95<sup>th</sup> and 98<sup>th</sup> percentile back of queue factors for pretimed and actuated signals are given in *Table 1*.

### **Average Overflow Queue**

$$N_o = N_{b2} \quad (1.3)$$

As an approximate method, the average overflow queue can be calculated as the second term of the average back of queue formula using *Equation (1.1b) or (1.1f)*.

### **Queue Storage Ratio**

(*Q*, *Q*<sub>%</sub> in vehicles, *L<sub>a</sub>*, *L<sub>h</sub>* in metres)

*Average Queue Storage Ratio*

$$R_Q = \frac{L_h Q}{L_a} \quad (1.4a)$$

*Percentile Queue Storage Ratio*

$$R_{Q\%} = \frac{L_h Q_{\%}}{L_a} \quad (1.4b)$$

where

$L_h$  is the jam spacing (average spacing in a stationary queue) in metres,

$L_a$  is the available queue storage distance in metres,

$Q$  is the average back of queue (vehicles per lane),

$Q_{\%}$  is the percentile back of queue (vehicles per lane).

The available queue storage distance can be calculated as the blocking queue distance, which is the distance from the downstream stop line to the back of downstream queue that blocks the upstream stop line.

### **Queue Clearance (Service) Time**

( $s_{LG}$  in vehicles,  $g_s$ ,  $r$  in seconds)

$$g_s = \frac{f_q y_L r}{1 - y_L} \quad \text{subject to } g_s \leq g \quad (1.5)$$

$$f_q = PF_2 \quad \text{for pretimed signals} \quad (1.5a)$$

$$= PF_2 [1.08 - 0.1 (G/G_{max})^2] \quad \text{for actuated signals} \quad (1.5b)$$

*subject to  $[1.08 - 0.1 (G/G_{max})^2] \geq 1.0$*

$PF_2$  is the queue progression factor.

### **Model Parameters**

( $s_{LG}$ ,  $Q_{bL}$  in vehicles,  $T$  in hours,  $c_L$  in veh/h,  $v$ ,  $v_L$ ,  $s$ ,  $s_L$  in veh/h)

#### **Adjusted Flow Rate and Flow Ratio**

$$v_L = v_1 / N_{LG} = (v + Q_b / T) / N_{LG} = v / N_{LG} + Q_{bL} / T \quad (1.6a)$$

$$y_L = v_L / s_L = (v / N_{LG} + Q_{bL} / T) / s_L = (v / s) + Q_{bL} / (s_L T) \quad (1.6b)$$

( $v_L = v / N_{LG}$  and  $y_L = v / s$  for zero initial queued demand,  $Q_{bL} = 0$ )

#### **Degree of Saturation**

$$X_L = v_L / c_L = (v / N_{LG} + Q_{bL} / T) / c_L = (v / c) + Q_{bL} / (c_L T) \quad (1.7)$$

$$= X + Q_{bL} / (c_L T)$$

( $X_L = X$  for zero initial queued demand,  $Q_{bL} = 0$ )

$$X = v / c \quad (1.7a)$$

*Second term parameter*

$$z = X - 1 + \frac{2 Q_{bL}}{c_L T} = (X_L - 1) + \frac{Q_{bL}}{c_L T} \quad (1.8)$$

( $z = X - 1 = X_L - 1$  for zero initial queued demand,  $Q_{bL} = 0$ )

*Average Values per Lane*

The demand flow rate ( $v_L$ ), saturation flow rate ( $s_L$ ), capacity per cycle ( $s_{LG}$ ), capacity ( $c_L$ ) and initial queued demand ( $Q_{bL}$ ) in *Equations (1.1) to (1.8)* should be used as *individual lane* values, or when applying the equations on a lane group basis, *average values per lane* should be used. These can be calculated from:

$$v_L = v_1 / N_{LG} = (v + Q_b / T) / N_{LG} \quad (1.9a)$$

$$s_L = s / N_{LG} \quad (1.9b)$$

$$c_L = c / N_{LG} \quad (1.9c)$$

$$Q_{bL} = Q_b / N_{LG} \quad (1.9d)$$

where  $v_1$  and  $v$  are the total flow rates for the lane group with and without the effect of initial queued demand,  $s$ ,  $c$  and  $Q_b$  are the total saturation flow rate, capacity and initial queued demand for the lane group, and  $N_{LG}$  is number of lanes in the lane group.

A refinement for the case of unequal lane utilisation is described in *Appendix B*.

*Queue Progression Factor*

The progression factors apply to the case of platooned arrivals generated by coordinated signals. For more detailed information on progression factors, refer to Akçelik (1995, 1996, 2001). The Queue Progression Factor ( $PF_2$ ) is calculated from:

$$PF_2 = \frac{(1-P)(1-y_L)}{(1-u)(1-R_p y_L)} = \frac{(1-R_p u)(1-y_L)}{(1-u)(1-R_p y_L)} \quad (1.10)$$

*subject to conditions*

- (i)  $PF_2 \geq 1.0$  for Arrival Types 1 and 2,
- (ii)  $PF_2 \leq 1.0$  for Arrival Types 4 to 6,
- (iii)  $P \leq 1.0$  ( $R_p \leq 1/u$ ),
- (iv)  $R_p < 1/y_L$  and
- (v)  $PF_2 = 1.0$  for  $y_L \geq u$  ( $X_L \geq 1$ ).

where  $u$  is the green time ratio ( $g/C$ ),  $y_L$  is the lane group flow ratio per lane, i.e.  $v_L/s_L$  ratio where  $v_L$  is the lane group flow rate per lane (veh/h) and  $s_L$  is the lane group saturation flow rate per lane (veh/h), and  $R_p$  is the platoon ratio:

$$R_p = v_{Lg} / v_L = P / u \quad (1.10a)$$

where  $v_{Lg}$  is the arrival flow rate per lane (veh/h) during the green period ( $v_{Lg} = R_p v_L$ ),  $v_L$  is the average arrival flow rate per lane (veh/h) during the signal cycle, and  $P$  is the proportion of traffic arriving during the green period.

If the user specifies a known value of  $P$  rather than an Arrival Type, then  $R_p = P / u$  can be calculated and used to determine a corresponding arrival type (see Akçelik 2001, Table 1).

For **non-platooned** (*uniform*) arrivals as relevant to isolated intersections, use  $R_p = 1.0$ , therefore:

$$\begin{aligned} PF_2 &= 1.0 \quad \text{and} \\ I_f &= 1.0 \end{aligned} \quad (1.11)$$

### Various cases

#### Two green periods per cycle

In the case of **two green periods per cycle** (e.g. a turning movement that receives a green circle and a green arrow, i.e. permitted and protected turns in HCM terminology):

- (i) in the formula for  $f_q$ ,  $(G/G_{\max})_i$  values are used for the relevant green periods individually ( $i = 1, 2$ ),
- (ii) for the second-term parameter  $k_B$ , the total  $s_{Lg}$  per cycle is used:  $s_{Lg} = (s_{Lg})_1 + (s_{Lg})_2$ , and
- (iii) uniform back of queue and queue clearance time models need to be extended to include any *residual queues from the previous green period* (as in aaSIDRA).

Queue accumulation polygons used in HCM 2000 Chapter 16, Appendix B could be extended for queue prediction in the case of two green periods in future editions of HCM.

#### Permitted (filter) turns in a shared lane

In the formula for  $f_q$ ,  $(G/G_{\max})$  value of the through movement is normally used in the case of permitted (filter) turns in a shared lane. In the case of permitted and protected turns from a shared lane,  $(G/G_{\max})$  value of the through movement is used for the permitted (filter) turn period.

## Model Validation

Comparisons of various model estimates with simulation and real-life data are discussed in this section. All cases are for isolated intersection conditions (random arrivals).

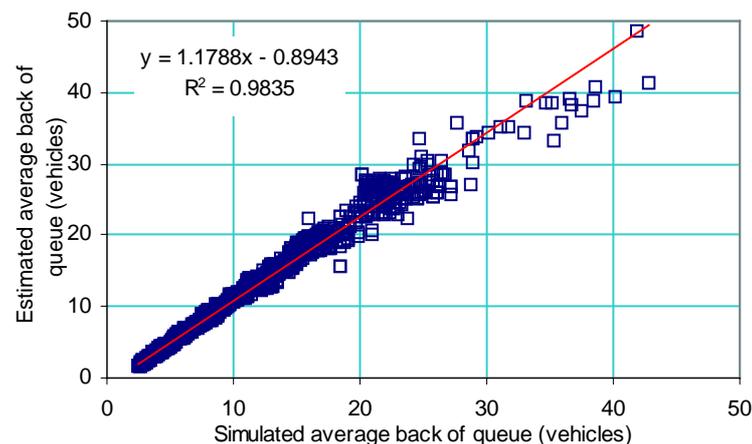
Comparison of average back of queue estimates from the proposed model (*Equation 1.1*) against actuated and fixed-time signal simulation data generated using ModelC (Akçelik and Chung 1994, 1995a,b, Akçelik, Chung and Besley 1997, Akçelik, Besley and Chung 1998) is given in *Figures 2 to 4*. The closeness of results using the estimated and simulated actuated signal timings (*Figures 2 and 3*) indicates the accuracy of the actuated signal timing method introduced in HCM 97 (Courage, et al 1996).

Comparison of average back of queue estimates from the proposed model for actuated signals against back of queue values measured at an intersection in Melbourne, Australia (using measured signal timings) is shown in *Figure 5*.

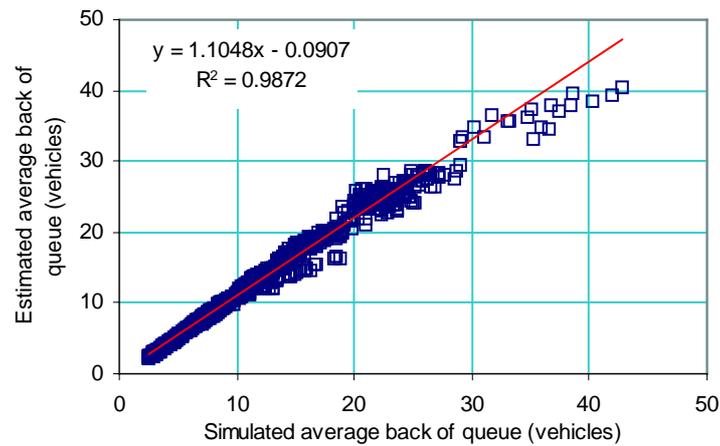
Comparison of average back of queue estimates from the proposed model and the aaSIDRA model for the same site is shown in *Figure 6*.

Slight overestimation of queue length by the proposed and aaSIDRA back of queue models is acceptable because the analytical models include vehicles slowing down at the back of queue without coming to a full stop whereas field surveys are likely to have undercounted such vehicles.

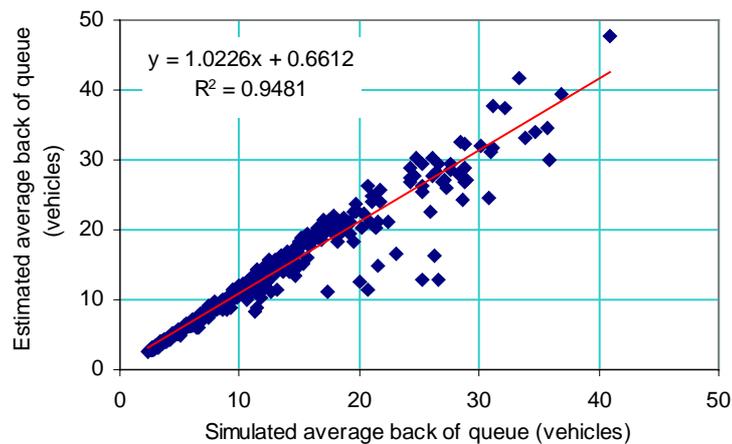
A paper by Vilorio, Courage and Avery (2000) presented a detailed discussion on the HCM 2000 queue model and comparisons with the aaSIDRA and various other queue models for signalised intersections.



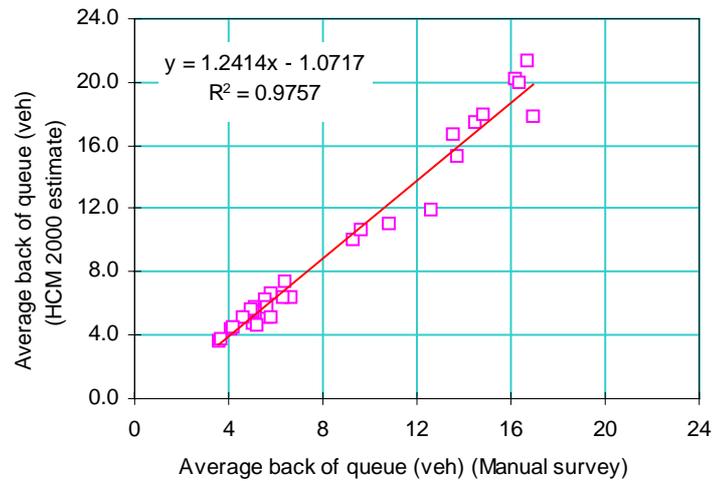
**Figure 2 – Predicted vs simulated average back of queue using estimated actuated signal timings**



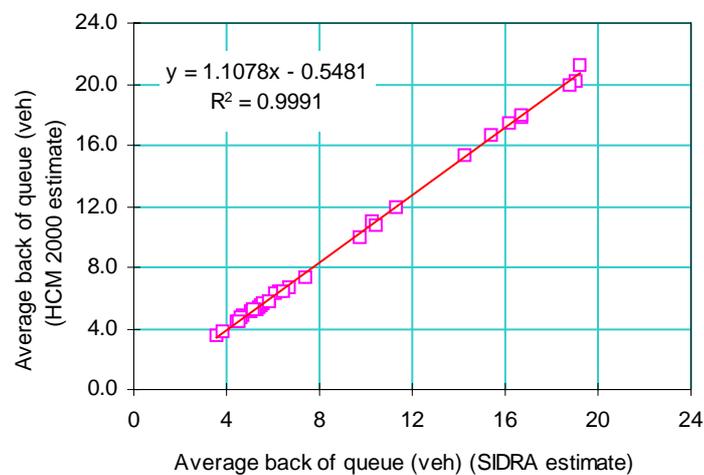
**Figure 3 – Predicted vs simulated average back of queue using simulated actuated signal timings**



**Figure 4 – Predicted vs simulated average back of queue for pretimed signals**



**Figure 5 – Comparison of average back of queue values predicted by the proposed model with those measured at an isolated intersection controlled by actuated signals in Melbourne, Australia (using measured timings)**



**Figure 6 – Comparisons of average back of queue estimates from the proposed model and the SIDRA model for the same site as in Figure 5**

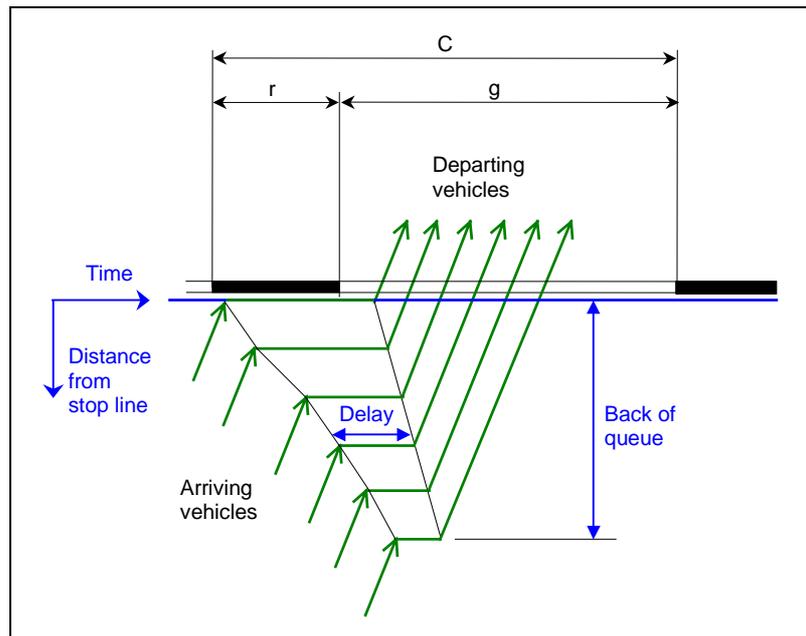
## Large Values of Back of Queue

A frequently asked question by aaSIDRA users who have not been familiar with the back of queue concept (due to the lack of this concept from the HCM editions prior to HCM2000) is related to the large size of queue length that the model can produce. The following issues should be considered in this respect.

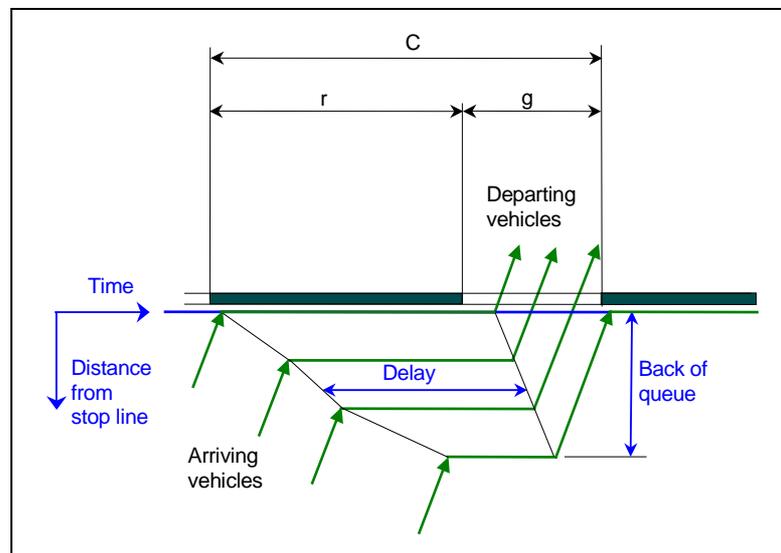
- (i) *Back of queue vs cycle-average queue:* Back of queue is always larger than the more familiar cycle-average queue. The latter is calculated using average delay (e.g. as in the HCM unsignalised intersections chapter), and as such, incorporates all queue states including zero queues, resulting in a smaller value than the back of queue.
- (ii) *Progression effects:* As discussed in Akçelik (2001), *very poor* and *unfavourable* progression conditions result in queue lengths that may be considerably larger than those under random arrival conditions.
- (iii) *Percentile queue length:* If a percentile queue length is used this could be as high as twice or three times the average queue length. However, percentile queues occur only a limited number of times during the analysis period. For example, the 90<sup>th</sup> percentile back of queue is exceeded only in 10 per cent of the signal cycles. With a cycle time of  $C = 100$  s, there are 9 signal cycles in the peak analysis period of 15 minutes used in the HCM. In this case, the 90<sup>th</sup> percentile back of queue is exceeded 0.9 times (say once) during the analysis period.
- (iv) *Arrival (demand) flow rate:* Delay and back of queue are not always consistent in terms of magnitude. *Low average delay* can be associated with a *long back of queue* (as seen in *Figure 7*) as a result of a high arrival flow rate, large green time ratio (relatively short red period) and low degree of saturation. In this case, a large proportion of vehicles may be undelayed, and therefore the cycle-average queue could be small. In contrast, the case of *short back of queue* may be associated with a *large average delay* (as seen in *Figure 8*) as a result of a low arrival flow rate, small green time ratio (relatively long red period) and a high degree of saturation.

In *Figure 7*, vehicles are seen to arrive during the red period and the early part of the green period, which represents platooned arrivals with unfavourable progression. Therefore it corresponds to the case of large progression factors discussed in Akçelik (2001).

A clear understanding of the above issues is needed for effective use of delay and queue length as interrelated performance measures for signalised intersections.



**Figure 7 - The case of low delay associated with a long back of queue (high arrival flow rate, large green time ratio, and low degree of saturation)**



**Figure 8 - The case of short back of queue associated with a large average delay (low arrival flow rate, small green time ratio, and high degree of saturation)**

## Notations and Basic Relationships

HCM 2000 and this document	aaSIDRA and original document	
c		Lane group capacity (veh/h) $c = s g / C$ (where s is in veh/h)
$c_L$	$Q_e$	Lane group capacity per lane (veh/h) $c_L = c / N_{LG} = s_L g / C$ where $s_L = s / N_{LG}$
$c_L T$	$Q_e T_f$	Lane group throughput per lane (maximum number of vehicles that can be discharged during the flow period)
C	c	Average cycle time (seconds) $C = r + g$
$f_{B\%}$	$f_{bp\%}$	Percentile back of queue factor
$f_{LU}$		Saturation flow adjustment factor for unequal lane utilisation
$f_q$	$f_q$	Calibration factor for queue clearance time
g	g	Average effective green time (seconds)
$g/C, u$	u	Green time ratio $u = g / C$
$g_s, g_q$	$g_s$	Average queue clearance (service) time, or saturated portion of the green period (seconds)
$g_u$	$g_u$	Unsaturated portion of the green period (seconds) $g_u = g - g_s$
I	$I_f$	Upstream filtering factor for platooned arrivals
$k_B$	$k_b$	Incremental queue factor (similar to "k" in the HCM delay formula)
$L_a$	$L_a$	Available queue storage distance (lane length in metres)
$L_h$	$L_{hj}$	Average spacing in a stationary queue including the vehicle length and the gap distance in front of the vehicle (jam spacing) (m/veh)
$N_{LG}$	$n_{LG}$	Number of lanes in the lane group (to determine the critical lane flow rate, use $f_{LU} N_{LG}$ as an effective number of lanes instead of $N_{LG}$ )
P	$P_G$	Proportion of traffic arriving during the green period $P = R_p u$

continued >>>

HCM 2000 and this document	aaSIDRA and original document	
PF <sub>2</sub>	PF <sub>2</sub>	Progression factor for back of queue and queue clearance time
Q	N <sub>b</sub>	Average back of queue (vehicles)
Q <sub>1</sub> , Q <sub>2</sub>	N <sub>b1</sub> , N <sub>b2</sub>	First and second terms of the average back of queue formula
Q <sub>%</sub>	N <sub>bd%</sub>	Percentile (70th, 85th, 90th, 95th, or 98th) value of the back of queue
Q <sub>bL</sub>	N <sub>i</sub>	Initial queued demand per lane as observed at the start of a flow period (vehicles)
Q <sub>b</sub>		Initial queued demand for the lane group as observed at the start of a flow period (vehicles)
Q <sub>d</sub> (not in HCM)	N <sub>d</sub>	Deterministic oversaturation queue assuming constant demand and capacity flow rates and ignoring the first-term queue
Q <sub>o</sub> (not in HCM)	N <sub>o</sub>	Average overflow queue  N <sub>o</sub> = N <sub>b2</sub> as an approximate method
r	r	Average effective red time (seconds)  $r = C - g$
R <sub>p</sub>	P <sub>A</sub>	Platoon arrival ratio for coordinated signals: the ratio of the average arrival flow rate during the green period to the average arrival flow rate during the signal cycle  $R_p = P / u$
R <sub>Q</sub>	R <sub>q</sub>	Average queue storage ratio: the ratio of the average back of queue (distance) to the available queue storage distance  $R_Q = L_h Q / L_a$
R <sub>Q%</sub>	R <sub>qp%</sub>	Percentile queue storage ratio: the ratio of the percentile value of the back of queue (distance) to the available queue storage distance  $R_{Q\%} = L_h Q_{\%} / L_a$
s		Lane group saturation flow rate (veh/h)
s <sub>L</sub>	s	Lane group saturation flow rate per lane (veh/h)  $s_L = s / N_{LG}$
s <sub>LG</sub>	sg	Cycle capacity per lane (veh) (s <sub>L</sub> in veh/s, g in seconds)
T	T <sub>f</sub>	Duration of a demand flow (analysis) period (hours)

continued &gt;&gt;&gt;

HCM 2000 and this document	aaSIDRA and original document	
v	$q_a$	Arrival (demand) flow rate (veh/s or veh/h), i.e. the average number of vehicles per unit time as measured at a point upstream of the back of queue (without the effect of the initial queued demand)
$v_1$	$q_{ai}$	Demand flow rate adjusted to take into account the initial queued demand at the start of the flow period $v_1 = v + Q_b / T$
$v_L$		Lane group arrival (demand) flow rate per lane including the effect of initial queued demand (veh/h) $v_L = v_1 / N_{LG} = (v + Q_b / T) / N_{LG} = v / N_{LG} + Q_{bL} / T$ if there is no initial queued demand, $Q_b = 0$ , $v_L = v / N_{LG}$
$v_L C$	$q_{ai}c$	Average demand (vehicles) per lane per cycle corresponding to the total demand including initial queued demand
X	x	Lane group degree of saturation, i.e. the ratio of arrival (demand) flow rate to capacity (without the effect of the initial queued demand) $X = v / c = (v / N_{LG}) / c_L$
$X_L$	$x'$	Lane group degree of saturation per lane, i.e. the ratio of arrival (demand) flow rate to capacity including the effect of the initial queued demand $X_L = v_L / c_L = v_L C / (s_L g) = y_L / u$ $= (v / N_{LG} + Q_{bL} / T) / c_L = X + Q_{bL} / (c_L T)$ if there is no initial queued demand, $Q_{bL} = 0$ , $X_L = X$
$X_u$	$x_u$	Degree of saturation at the upstream intersection (used for upstream filtering factor I). HCM 2000 Chapter 15 qualifies this as the flow weighted average of contributing upstream movement v/c ratios.
$y_L$	y	Lane group flow ratio per lane, i.e. the ratio of arrival (demand) flow rate to the saturation flow rate including the effect of the initial queued demand $y_L = v_L / s_L = u X_L$ if there is no initial queued demand, $Q_b = 0$ , $y_L = v / s$
Z (not in HCM)	z	A performance model parameter used in the second term of the back of queue expression $z = X - 1 + 2 Q_{bL} / (c_L T) = (X_L - 1) + Q_{bL} / (c_L T)$ if there is no initial queued demand, $Q_{bL} = 0$ , $z = X - 1$

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## Appendix A

### Background to the Development of the HCM 2000 Back of Queue Model

The HCM 2000 second-term back of queue model is a *time-dependent* expression. This means that the queue length predicted by this expression depends on the duration of the analysis period, which represents how long the demand flow rate persists (HCM 2000 default is 15 minutes). In contrast, earlier forms of the delay and queue models in the literature are based on *steady-state* conditions, i.e. they assume that flow conditions last indefinitely (Webster 1958, Miller 1968). Time-dependent expressions are derived from steady-state expressions using a well-known coordinate transformation technique (Kimber and Hollis 1979, Akçelik 1980a). The principles of the discussion below are applicable to delay as well (Akçelik 1980a,b, 1981, 1988b, 1990a,b, Akçelik and Roupail 1994).

Time-dependent and steady-state forms of a model predict close values for low degrees of saturation but differ as demand flows approach capacity. A steady-state model is valid only for demand flow rates below capacity, say for degrees of saturation up to about 0.95. A time-dependent model predicts the same values as its steady-state counterpart when a very large value of the analysis period is used.

The origin of the HCM 2000 second-term queue expression can be traced back to the time-dependent model developed by the author (Akçelik 1980a,b, 1981) using an approximation to the steady-state form of the overflow queue model described by Miller (1968). This is explained below using time-dependent expressions without initial queued demand.

Miller's overflow queue model for pretimed signals is given by:

$$Q_{os} = \frac{\exp[-1.33(s_L g / 3600)^{0.5}(1 - X_L) / X_L]}{2(1 - X_L)} \quad (\text{A.1})$$

where  $Q_{os}$  is the average overflow queue in steady-state form (subscript  $s$  is used to denote steady-state),  $s_L g$  is the capacity per cycle in vehicles ( $s_L$  = lane group saturation flow rate per lane in veh/h,  $g$  = effective green time in seconds), and  $X_L$  is the lane group degree of saturation (demand volume/capacity ratio).

In order to facilitate the derivation of a time-dependent function for the average overflow queue, the author developed the following general steady-state model for undersaturated signals using the general structure of Miller's formula that predicts negligible values of overflow queue for low degrees of saturation:

$$\begin{aligned} Q_{os} &= \frac{k_o (X_L - X_o)}{1 - X_L} && \text{for } X_L > X_o \\ &= 0 && \text{otherwise} \end{aligned} \quad (\text{A.2})$$

where  $k_o$  is the overflow queue parameter,  $X_L$  is the degree of saturation, and  $X_o$  is a degree of saturation below which the average overflow queue is zero.

As an approximation to Equation (A.1), the following parameter values were used in Equation (A.2):

$$k_0 = 1.5, \text{ and } X_0 = 0.67 + \frac{s_L g / 3600}{600} \quad (\text{A.2a})$$

where  $s_L g / 3600$  is the cycle capacity (vehicles). These parameters are for pretimed signals.

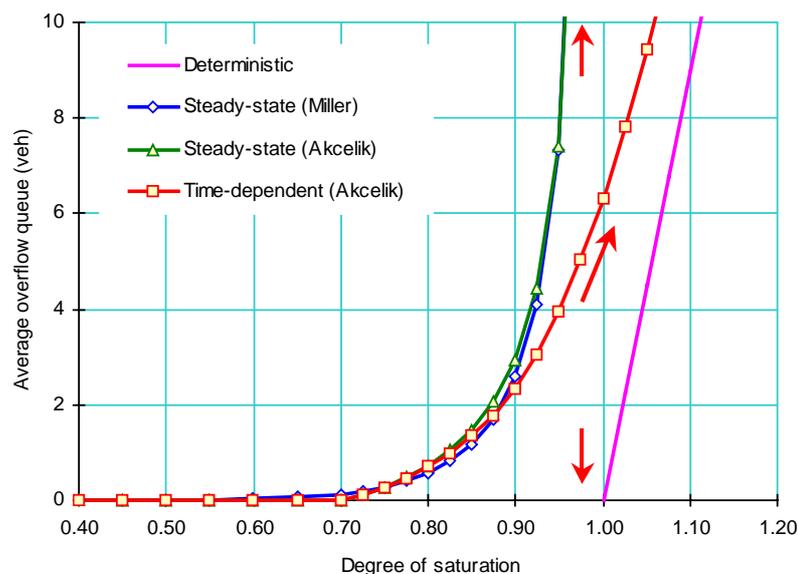
For the derivation of the time-dependent form of the overflow queue expression, a deterministic oversaturation queue ( $Q_d$ ) model is also needed. This is derived by assuming constant arrival flow rate ( $v_L$ ) and constant capacity ( $c_L$ ) over an analysis period ( $T$ ), where  $v_L > c_L$ , therefore  $X_L = v_L / c_L > 1.0$ :

$$Q_d = 0.5 (X_L - 1) T \quad (\text{A.3})$$

Overflow queues estimated by steady-state expressions (*Equations A.1, A.2 and A.2a*) for undersaturated conditions and the deterministic expression (*Equation A.3*) for oversaturated conditions are shown in *Figure A.1*. This figure is based on the following example: no initial queued demand, pretimed signals with no signal coordination effects (Arrival Type = 3), green time,  $g = 40$  s, cycle time,  $C = 100$  s, saturation flow rate,  $s_L = 1800$  veh/h, therefore cycle capacity,  $s_L g = 20$  veh, capacity,  $c_L = 720$  veh/h, and analysis period,  $T = 0.25$  h.

Limitations of steady-state and deterministic oversaturation queue expressions are indicated in *Figure A.1*. While the deterministic expression predicts a zero overflow queue value at capacity (degree of saturation,  $X_L < 1.0$ ), the steady-state expressions predict infinite overflow queue values for flows just below capacity ( $X_L > 1.0$ ). Thus, these equations do not give reasonable results for flows near capacity.

The problem is solved using the coordinate transformation technique to convert the steady-state function to a *transition function* that has the line representing the deterministic function as its asymptote. The result is a *time-dependent* expression that gives realistic finite values of the overflow queue (or delay) for flows around capacity, and adds a random component to the deterministic oversaturation function. The time-dependent function for the average overflow queue obtained using the coordinate transformation technique to relate *Equations (A.2) and (A.2a)* to *Equation (A.3)* is shown in *Figure A.1*.



**Figure 1 - The relationship between the time-dependent, steady-state and deterministic oversaturation models for overflow queue prediction**

Similarly, the steady-state model that is the basis of the HCM 2000 back of queue model (second-term) is given by:

$$Q_{2s} = \frac{k_B X_L}{1 - X_L} \quad (\text{A.4})$$

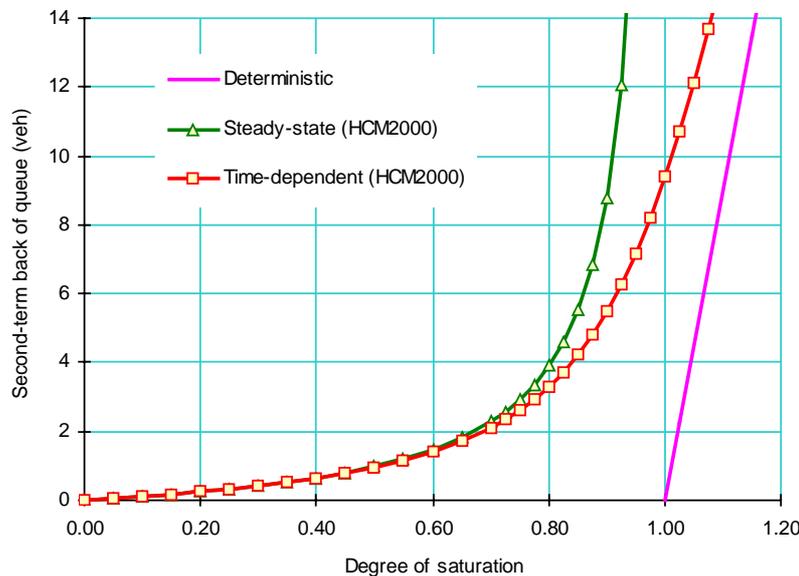
where  $k_B$  is the back of queue model parameter (HCM 2000 Equation G16-10) and  $X_L$  is the degree of saturation. Comparing *Equations (A.2) and (A.4)*, it is seen that and  $X_O = 0$  in *Equation (A.4)*, and  $k_B$  replaces parameter  $k_o$ .

The second-term of the HCM 2000 back of queue model (HCM 2000 Equation G16-9) is a time-dependent model based on *Equation (A.4)*. When there is no initial queued demand, this is given by:

$$Q_2 = 0.25 c_L T \left[ z + \left( z^2 + \frac{8k_b X_L}{c_L T} \right)^{0.5} \right] \quad (\text{A.5})$$

where  $c_L$  = lane group capacity per lane,  $T$  = analysis period,  $z = X_L - 1$ , and  $X_L$  = degree of saturation.

*Figure A.2* shows how the HCM 2000 second-term model (*Equation A.5*) relates to the corresponding steady-state model (*Equation A.4*) and the deterministic oversaturation queue model (*Equation A.3*) for the same example as in *Figure A.1*.



**Figure 2 - The relationship between the HCM2000 second-term back of queue model (time-dependent), and the corresponding steady-state and deterministic oversaturation models**

Figure A.3 shows a comparison of the average back of queue values ( $Q = Q_1 + Q_2$ ) calculated using the HCM 2000 model, its "parent model" used in aaSIDRA (Vitoria, Courage and Avery 2000), and the earlier model based on the use of parameters given in Equation (A.2b) for the same example as in Figures A.1 and A.2. The first-term back of queue results ( $Q_1$ ) for the HCM model are also shown in Figure A.3.

It is seen that the HCM 2000 and aaSIDRA second-term queue models give larger values than the earlier model. This is mainly due to different assumptions in arrival headway distributions (bunched exponential rather than simple negative exponential). For the example shown in Figure A.3, the first-term queue expression makes a larger contribution to the average back of queue for undersaturated conditions, and therefore the differences in the second-term expressions are less important.

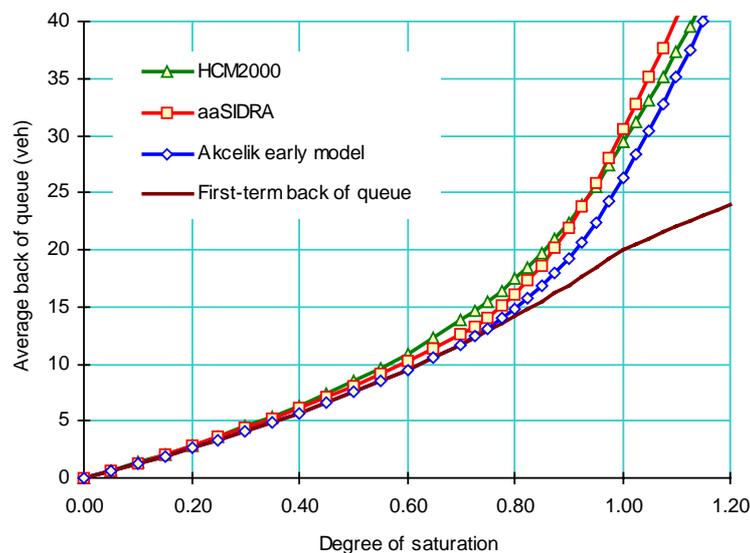


Figure 3 - Comparison of the average back of queue values predicted by the HCM2000, aaSIDRA and an earlier model

## Appendix B

### A Method for Determining Model Parameters in the Case of Unequal Lane Utilisation

The following method is recommended for determining the back of queue in the critical lane of a lane group with unequal lane utilisation. This method has a shortcoming due to the assumption of equal lane saturation flows, which can only be overcome using a lane-by-lane analysis method as in aaSIDRA (Akçelik and Associates 2000; Akçelik 1984, 1988a, 1989, 1997; Akçelik, Chung and Besley 1998).

From *Equations (1.9a) to (1.9d)* in the main text of this report (corresponding to Equations G16-2 to G16-5 of HCM 2000), the lane group arrival flow rate ( $v_L$ ), saturation flow rate ( $s_L$ ), capacity ( $c_L$ ) and initial queued demand ( $Q_{bL}$ ) *per lane* in the case of *equal lane utilisation* are:

$$v_L = v_1 / N_{LG} \quad (B.1)$$

$$s_L = s / N_{LG} \quad (B.2)$$

$$c_L = c / N_{LG} \quad (B.3)$$

$$Q_{bL} = Q_b / N_{LG} \quad (B.4)$$

where  $v_1$  is the total flow rate for the lane group with the effect of initial queued demand,  $s$ ,  $c$  and  $Q_b$  are the total saturation flow rate, capacity and initial queued demand for the lane group, and  $N_{LG}$  is number of lanes in the lane group.

The total flow rate for the lane group with the effect of initial queued demand ( $Q_b$ ) is:

$$v_1 = v + Q_b / T \quad (B.5)$$

where  $v$  is the total flow rate for the lane group without the effect of initial queued demand, and  $T$  is the duration of the demand flow (analysis) period.

The method given here is applicable to the case of no initial queued demand ( $Q_b = 0$  and  $v_1 = v$ ).

On the basis of HCM 2000, Exhibit 16-7, the adjustment factor for lane utilisation can be expressed as:

$$f_{LU} = v_1 / (v_c N_{LG}) \quad (B.6)$$

where  $v_1$  is as in *Equations (B.1) and (B.5)*, and  $v_c$  is the corresponding flow rate for the critical lane in the lane group.

From *Equations (B.1) and (B.6)*, the lane utilisation factor is equivalent to the ratio of the average lane flow rate to the critical lane flow rate:

$$f_{LU} = v_L / v_c \quad (B.6a)$$

Thus, the critical lane flow is given by:

$$v_c = v_1 / (f_{LU} N_{LG}) = v_L / f_{LU} \quad (B.7)$$

Assuming the same lane saturation flow for all lanes in the same group ( $s_i$ ), including the critical lane, the lane group saturation flow is:

$$s = N_{LG} s_i f_{LU} \quad (B.8)$$

In terms of HCM 2000 Equation 16-4 (see HCM 2000 for symbols), the lane saturation  $s_i$  is equivalent to:

$$s_i = s_0 f_w f_{HV} f_g f_p f_{bb} f_a f_{LT} f_{RT} f_{Lpb} f_{Rpb} \quad (B.8a)$$

Since the critical lane saturation flow is  $s_c = s_i$  (the same for all lanes), *Equations (B.8) and (B.2)* imply:

$$s_c = s / (f_{LU} N_{LG}) = s_L / f_{LU} \quad (B.9)$$

The critical lane flow ratio is:

$$y_c = v_c / s_c = (v_L / f_{LU}) / (s_L / f_{LU}) = v_L / s_L = y_L \quad (B.10)$$

It is seen that the flow ratio obtained using the average flow rate and saturation flow rate per lane is the same as the critical lane flow ratio ( $y_L = y_c$ ).

The critical lane capacity is:

$$c_c = s_c g / C = (s / (f_{LU} N_{LG})) g / C = c / (f_{LU} N_{LG}) = c_L / f_{LU} \quad (B.11)$$

The critical lane degree of saturation is:

$$X_c = v_c / c_c = (v_L / f_{LU}) / (c_L / f_{LU}) = v_L / c_L = X_L \quad (B.12)$$

It is seen that the degree of saturation obtained using the average flow rate and capacity per lane is the same as the critical lane degree of saturation ( $X_L = X_c$ ).

Assuming that the initial queued demand in each individual lane is proportional to the lane flow, the initial queued demand for the critical lane is given by:

$$Q_{bc} = Q_b / (f_{LU} N_{LG}) = Q_{bL} / f_{LU} \quad (B.13)$$

It should also be noted that the HCM 2000 method implies equal arrival flow rates for non-critical lanes (for  $N_{LG} > 2$ ):

$$v_n = (v_1 - v_c) / (N_{LG} - 1) \quad (B.14)$$

In summary, parameters for the back of queue equation in the case of *unequal lane utilisation* case can be calculated by replacing  $N_{LG}$  in Equations G16-2 to G16-5 of HCM 2000 by  $(f_{LU} N_{LG})$  as an *effective number of lanes*.

$$v_L = v_1 / (f_{LU} N_{LG}) \quad (B.15)$$

$$s_L = s / (f_{LU} N_{LG}) \quad (B.16)$$

$$c_L = c / (f_{LU} N_{LG}) \quad (B.17)$$

$$Q_{bL} = Q_b / (f_{LU} N_{LG}) \quad (B.18)$$

Equations (B.1) to (B.4) can be considered to be a special case of Equations (B.15) to (B.18), valid only when  $f_{LU} = 1.0$ .

**Example:**

Consider a lane group with three lanes ( $N_{LG} = 3$ ), a total flow rate of  $v = 1095$  veh/h, lane saturation flow rate of  $s_i = 1800$  veh/h, initial queued demand of  $Q_b = 30$  vehicles, demand flow period of  $T = 0.25$  h, and a lane utilisation factor of  $f_{LU} = 0.8333$ . Effective green time and cycle time are  $g = 30$  s and  $C = 100$  s.

The total flow rate for the lane group with the effect of initial queued demand is  $v_1 = 1095 + 30/0.25 = 1215$  veh/h. The saturation flow rate for the lane group is  $s = 3 \times 1800 \times 0.8333 = 4500$  veh/h. The lane group capacity is  $c = 4500 \times 30 / 100 = 1350$  veh/h. The degree of saturation using the flow rate with the effect of the initial queued demand is  $1215 / 1350 = 0.900$ . The flow ratio using the flow rate with the effect of the initial queued demand is  $1215 / 4500 = 0.270$ .

The effective number of lanes to allow for lane underutilisation is  $f_{LU} N_{LG} = 0.8333 \times 3 = 2.5$  lanes. Using this, the parameters required on a per lane basis (reflecting the *critical lane* conditions rather than average lane conditions) are calculated from Equations (B.15) to (B.18):

$$\begin{aligned} v_L &= v_c = 1215 / 2.5 = 486 \text{ veh/h per lane} \\ s_L &= s_c = 4500 / 2.5 = 1800 \text{ veh/h per lane} = s_i \\ c_L &= c_c = 1350 / 2.5 = 540 \text{ veh/h per lane} \\ Q_{bL} &= Q_{bc} = 30 / 2.5 = 12 \text{ veh per lane} \end{aligned}$$

The degree of saturation and flow ratio for the critical lane for use in the back of queue equations are:

$$\begin{aligned} X_L &= X_c = 486 / 540 = 0.900 \text{ (as for the lane group)} \\ Y_L &= y_c = 486 / 1800 = 0.270 \text{ (as for the lane group)} \end{aligned}$$

It is seen that the critical lane and the lane group have the same degree of saturation and the same flow ratio. This is achieved through the use of the lane utilisation adjustment factor ( $f_{LU}$ ) in the saturation flow formula.

For the second-term back of queue equation, we also need the degree of saturation using the flow rate without the effect of initial queued demand. On a lane group basis,  $X = v / c = 1095 / 1350 = 0.811$ . This can also be calculated for the critical lane:

$$\begin{aligned} v_c &= 1095 / 2.5 = 438 \text{ veh/h per lane} \\ X &= 438 / 540 = 0.811 \text{ (as for the lane group)}. \end{aligned}$$

For this example, Equations (1.1a) and (1.1b) give  $Q_1 = 12.95$  veh,  $Q_2 = 6.94$  veh, therefore  $Q = 19.9$  veh. HCM 2000 Equation G16-9 underestimates the second term,  $Q_2 = 4.96$  veh, therefore  $Q = 17.9$  veh.