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Rahmi Akçelik

ABSTRACT

A travel time function proposed by Davidson for transport planning purposes has been subject to much discussion and efforts of calibration including some controversy over the meaning of its parameters. Modified forms of Davidson's function have been proposed to obtain finite values of travel time for flows near and above capacity. This paper presents a time-dependent form of the original Davidson function, derived using the coordinate transformation technique. The modified form of Davidson's function proposed by Tisato (1991) is shown to overpredict travel times for flows near and above capacity compared with the time-dependent form. The derivation of the original Davidson function is discussed and it is concluded that the function suffers from an inconsistency in its basic parameter definitions. An alternative interpretation of the delay parameter is considered, but this leads to another inconsistency. A new travel time function is proposed as an alternative to Davidson's function to overcome the conceptual and calibration problems. In the proposed function, the delay parameter takes a meaning consistent with the formulae used for estimating intersection delays. Both the steady-state and time-dependent forms of the new function are given.

Correspondence: Rahmi Akçelik
Director, Akcelik & Associates Pty Ltd,
P O Box 1075 G, Greythorn Victoria, Australia 3104
Tel: +61 3 9857 9351, Fax: + 61 3 9857 5397
rahmi@akcelik.com.au

www.aatraffic.com

Introduction

The travel time function proposed by Davidson (1966, 1978) for transport planning purposes has been subject to much discussion and efforts of calibration and improvement including some controversy over the meaning of its parameters. For a general discussion on the usefulness of the Davidson function, see Rose, Taylor and Tisato (1989).

Modified forms of Davidson's function have been proposed to obtain finite values of travel time for flows near and above capacity. Tisato (1991) makes new suggestions for improving Davidson's function based on the author's earlier work (Akçelik 1978, 1981).

This paper presents the time-dependent form of the original Davidson function, derived using the coordinate transformation technique, which was used for intersection delay functions in the past (Akçelik 1980, 1981). Tisato's function is shown to overpredict travel times for flows near and above capacity compared with the time-dependent form.

The derivation of the original Davidson function is discussed and it is concluded that the function suffers from an inconsistency in its basic parameter definitions as pointed out by Golding (1977). An alternative interpretation of the delay parameter is considered, but this leads to another inconsistency.

A new travel time function is proposed as an alternative to Davidson's function to overcome the conceptual and calibration problems. In the proposed function, the delay parameter takes a meaning consistent with the formulae used for estimating intersection delays. Both the steady-state and time-dependent forms of the new function are given (for detailed discussion on the subject of steady-state and time-dependent function forms, refer to Akçelik 1980).

The similarity of the form of time-dependent functions given in this paper to the *conical congestion functions* proposed by Spiess (1990) to overcome the shortcomings of the well-known US Bureau of Public Roads (BPR) function is also interesting. The function proposed here has the advantages of relating to intersection delay modelling and providing explanatory power for the function parameters.

Davidson's function

The following function was proposed by Davidson (1966) as a general-purpose travel-time formula for transport planning purposes:

$$t = t_o \, [1 + J_D \, x \, / \, (1 - x)] \qquad (1)$$

where

t	=	average travel time per unit distance (e.g. in seconds per km),
t_o	=	minimum (zero-flow) travel time per unit distance (e.g. in seconds per km),
J_D	=	a *delay* parameter (or $1 - J_D$ = a *quality of service* parameter),
x	=	q / Q =degree of saturation,
q	=	demand (arrival) flow rate (in veh/h), and
Q	=	capacity (in veh/h).

Davidson (1966, 1978) derived this function from *concepts* of queuing theory but a direct derivation has not been clearly demonstrated.

Davidson modified the well-known steady-state delay equation which is for a single channel queuing system with random arrivals and exponentially distributed service rates:

$$d = (1 / Q) + x / [Q (1 - x)] \qquad (2)$$

where the first term is the service time (reciprocal of the mean service rate) and the second term is the queuing delay.

As the mean service rate, Davidson used saturation flow (s) rather than capacity (Q) in *Equations (1) and (2)* above. These two parameters have the same value for uninterrupted traffic facilities (e.g. freeways), but capacity rather than saturation flow needs to be used for interrupted facilities (e.g. for traffic signals where capacity equals saturation flow multiplied by the ratio of green time to cycle time). This point was discussed in some detail by Tisato (1991).

A distinction between saturation flow and capacity is important for the purpose of calibrating the function and for a clear understanding of the meaning of its parameters.

To obtain *Equation (1)* from *Equation (2)*, Davidson multiplied the queuing delay term by a delay factor (J_D), and equated the service time of the queuing system (1/Q) with the zero travel time of the road section (t_o):

$$t_o = 1 / Q \qquad (3)$$

This equation has an inherent inconsistency that has caused confusion in explaining the meanings of parameters in Davidson's function and frustration in efforts to calibrate the model. The inconsistency of *Equation (3)* is due to its implication that the capacity can be defined as $Q = 1/t_o$ and the degree of saturation as $x = q\, t_o$, which is rather meaningless.

The question of inconsistency in *Equation (3)* was raised by Golding (1977). In his response, Davidson (1978) elaborated on the meaning of the delay parameter (J_D) by considering a number of delay-producing elements (*service facilities*) along the road section, but he still assumed the basic relationship expressed in *Equation (3)*.

Blunden (1978) distinguished between *barrier* and *continuously distributed* traffic elements, and explained that the classical queuing theory models apply to barrier-type elements and these do not provide a rigorous theoretical basis for travel time on a continuously distributed traffic element such as a length of highway. Blunden stated that he originally suggested the type of travel time function expressed by *Equation (1)*, and gave the explanation that *the apparent lack of reciprocity between t_o and s (or Q) is due to the arbitrary definition of the unit length (the mean service time becomes the zero travel time per unit distance).*

The implications of *Equation (3)*, i.e. replacing 1/Q by t_o as a service time, in relation to the definitions of capacity and degree of saturation are not acceptable from the viewpoint of delay formulation. A change in the definition of the delay parameter J_D may be sought to avoid this inconsistency, but this leads to another inconsistency as explained below.

A different interpretation of the delay parameter

Let us consider deriving a travel time function by assuming that the zero-flow (minimum) travel time is simply the sum of the free-flow travel time (t_f) along the section of road (if there were no barrier-type delay elements) and the service time of the delay element (1/Q). Let us consider a single delay element for the sake of simplicity. The generalisation to the case of N delay elements in series each with the same capacity Q (in vehicles per second), and hence the same service time 1/Q (in seconds), as considered by Davidson (1978) does not affect the conclusions.

Thus, the zero-flow travel time is given by:

$$t_o = t_f + (1 / Q) \qquad (4)$$

where the free-flow travel time (t_f) in seconds per km is:

$$t_f = 3600 / v_f \qquad (5)$$

and v_f is the free-flow speed (km/h).

A general form of the steady-state queuing delay at a traffic interruption point (e.g. an intersection) is given by

$$d_q = k\,x / [Q\,(1 - x)] \qquad (6)$$

where k is a *delay parameter* which depends on the level of randomness (or regularity) of the arrival and service processes.

Davidson (1978) considered this form of the queuing delay formula for a queuing system with random arrivals and an Erlang distribution of service times. He quoted a formula equivalent to *Equation (6)* with k = (K+1)/2K where K is the Erlang number.

The special cases of parameter k for the case of random arrivals are k = 1 for exponential service times and k = 0.5 for regular (constant) service times. The former has been used for unsignalised intersections, and the latter for signalised intersections (Webster's well-known second term). A more general form of *Equation (6)* has been discussed by the author (see Akçelik 1990, Section 4, and Akçelik 1988).

The travel time along a road section can be expressed explicitly as the sum of zero-flow (minimum) travel time and total queuing delay along the road section:

$$t = t_o + \Sigma\, d_q \qquad (7)$$

As a rough approximation to the real value of $\Sigma\, d_q$, the delay parameter k in *Equation (6)* can be replaced by k' = p k where p is a parameter which represents the intensity of delay elements along the section of road (e.g. the number of intersections per unit distance). If there are different types of delay elements on the road section, then k' can be considered to be a weighted average of individual delay elements (see the *Discussion* section regarding the important question of capacity changes between delay elements which is ignored at this point!).

Thus, *Equation (7)* can be expressed as:

$$t = t_o + k' \, x / [Q \, (1 - x)] \qquad (8)$$

To derive a function of the same form as *Equation (1)*, the delay parameter must be defined as:

$$J_D = k' / (Q \, t_o) = k' / (Q \, t_f + 1) \qquad (9)$$

so that, from *Equations (8) and (9)*, we have:

$$t = t_o + J_D \, t_o \, x / (1 - x) = t_o \, [1 + J_D \, x / (1 - x)] \qquad (10)$$

Although *Equation (10)* appears to be the same as Davidson's function (*Equation 1*), it has a different definition of the delay parameter (J_D). Unfortunately, while this formulation removes the basic inconsistency resulting from *Equation (3)*, it introduces another since *Equation (9)* implies that the delay parameter decreases as the minimum travel time (t_o) increases. This means that the quality of service improves as the free-flow travel time (t_f) increases. This conflicts with the expectation of lower free-flow travel times on better quality roads (e.g. freeways). Thus, in removing the inconsistency of *Equation (3)*, an inconsistent definition of the delay parameter is obtained in *Equation (9)*.

Alternatively, the only way to retain the form expressed by *Equation (1)* is to accept parameter k' as the true delay parameter and parameter J_D as the delay parameter *normalised* by factor ($Q \, t_o$) as in *Equation (9)*. In this form, it is acknowledged that J_D is not independent of capacity and zero-flow (minimum) travel time, and that a lower J_D does not necessarily correspond to a better quality road.

This definitional inconsistency may explain some of the difficulties encountered in calibrating the delay parameter (J_D) of the original Davidson formula (Taylor 1977a,b,c).

Time-dependent form of Davidson's function

The time-dependent form of Davidson's function (*Equation 1*) derived using the coordinate transformation technique, which was used for intersection delay functions in the past (Akcelik 1981), can be expressed as

$$t = t_o \, \{1 + 0.25 \, r_f \, [z + (z^2 + 8 \, J_D \, x / r_f)^{0.5}]\} \qquad (11)$$

where

t	=	average travel time per unit distance (e.g. in seconds per km),
t_o	=	minimum (zero-flow) travel time per unit distance (e.g. in seconds per km),
J_D	=	a *delay* parameter,
z	=	x - 1,
x	=	q / Q =degree of saturation,
q	=	demand (arrival) flow rate (in veh/h),
Q	=	capacity (in veh/h),
r_f	=	T_f / t_o, i.e. ratio of flow (analysis) period to minimum travel time (T_f and t_o must be in the same units):

The delay parameter J_D in *Equation (11)* can be considered to be either the normalised value ($J_D = k' / (Q\ t_o)$) as given by *Equation (9)*, or a fixed value as in the original Davidson function *(Equation 1)* which assumes $t_o = 1/Q$ *(Equation 3)*.

Flow period is the time interval during which an average demand flow rate, q, persists. Travel time increases as the flow period increases. The corresponding steady-state function (*Equation 1*) assumes infinite flow period. The time-dependent function assumes no initial queue at the start of the flow period. A constant demand pattern (i.e. no peaking) is assumed. Demand (arrival) flows are measured at the back of the queue, not at the intersection stop line or the bottleneck point. Therefore, bottleneck conditions and backward spread of congestion in networks of closely-spaced intersections must be taken into account when applying the time-dependent travel time function.

In *Equation (11)*, the travel time is defined as experienced by all vehicles *arriving* during the specified flow period. Thus, some of these vehicles may leave the road link under consideration after the specified flow period, depending on the level of congestion (degree of saturation). The corresponding delay measurement method is the instrumented car or a vehicle path trace method which measures the difference between arrival and departure times of individual vehicles. This travel time definition is appropriate in the context of transport planning/traffic assignment where route choices of individual vehicles are the main concern.

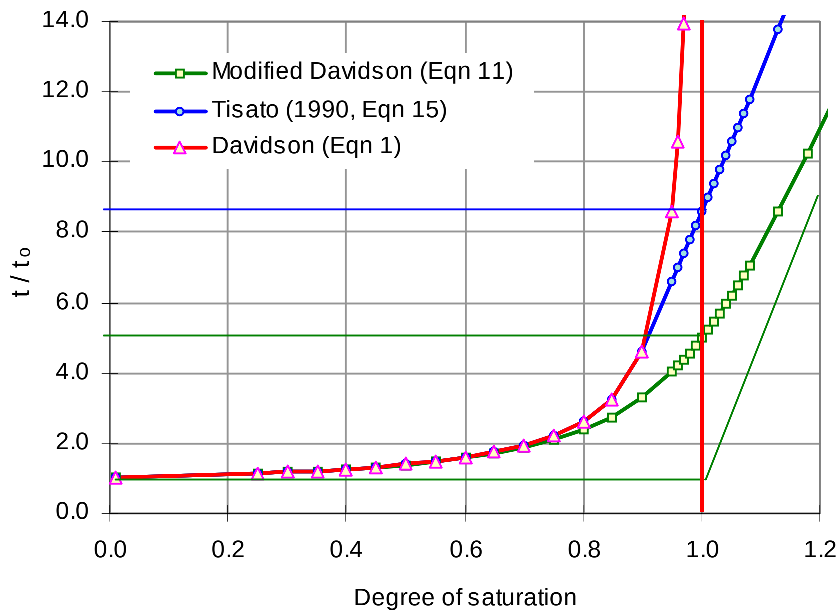

Figure 1 – Travel time graphs representing Davidson's original function, its modified form proposed by Tisato, and its time-dependent form given in this paper (T_f = 1 h, v_o = 80 km/h and J_D =0.4)

Travel time graphs representing Davidson's original function (*Equation 1*), its modified form proposed by Tisato (1990, Equation 15), and its time-dependent form given in this paper (*Equation 11*) are shown in *Figure 1* for T_f = 1 h, v_o =1 / t_o = 80 km/h (t_o = 0.0125 h/km) and J_D =0.4. It is seen that all three functions predict the same travel time for low to medium degrees of saturation (approximately $x < 0.7$), but the differences between the time-dependent function and the other two functions are substantial for near capacity conditions. Tisato's function is seen to overestimate travel times for near and above capacity conditions compared with the estimates obtained from the time-dependent function.

An alternative time-dependent travel time function

The limitations of the steady-state form of Davidson's travel time function (*Equation 1*) apply to its time-dependent form (*Equation 11*). This is due to the definitional problems inherent in the relationships given in *Equations (3) and (9)* which manipulate the meaning of minimum travel time or the delay parameter in order to obtain the convenient functional form expressed in *Equation (1)*.

The problems of Davidson's function can be avoided by developing a different functional form that uses the free-flow travel time and queuing delay terms in an explicit way. For this purpose, the minimum (zero-flow) travel time can be expressed as:

$$t_o = t_f + \Sigma\, d_m \qquad (12)$$

where $\Sigma\, d_m$ is the sum of minimum (zero-flow) delays along the road section. For example, at signalised intersections, minimum delay is the value of uniform delay term for x = 0 (see Akçelik 1981, 1990). The minimum delay formula for roundabouts and other unsignalised intersections can be found in Akçelik and Troutbeck (1991). The mean service time (1/Q) used in *Equation (4)* is the minimum delay in a simple queuing system sense. The geometric delays would be included in the minimum delay values. Using real-life data, the minimum travel time can be estimated from travel times measured under low flow conditions.

Instead of *Equation (9),* the delay parameter can be defined as

$$J_A = k' = p\, k \qquad (13)$$

where p is a parameter which represents the intensity of delay elements along the section of road (e.g. the number of intersections per unit distance).

The steady-state form of the travel time function based on *Equations (12) and (13)* is similar to *Equation (8)*:

$$t = t_o + J_A\, x / [Q\,(1 - x)] = t_o\,[1 + J_A\, x / (Q\, t_o\,(1 - x))] \qquad (14)$$

The corresponding time-dependent travel time function is:

$$t = t_o + 0.25\, T_f\,[z + (z^2 + 8\, J_A\, x / (Q\, T_f))^{0.5}] \qquad (15)$$
$$= t_o\,\{1 + 0.25\, r_f\,[z + (z^2 + 8\, J_A\, x / (Q\, t_o\, r_f))^{0.5}]\}$$

where all parameters are as in *Equation (11)* except the delay parameter, J_A.

As for the time-dependent form of Davidson's function given in the previous section, *Equation (15)* assumes a constant demand pattern (i.e. no peaking) and no initial queue at the start of the flow period, and the travel time is defined as experienced by all vehicles *arriving* during the specified flow period.

Note that, for a signalised intersection, the travel time above the zero-flow value in *Equation (15)* includes that part of the *uniform* delay term (Akçelik 1981, 1990) representing the delay above the minimum delay value (as defined for *Equation 12*). Therefore, an appropriate value of k for isolated signals could be 0.6, and for coordinated signals, k = 0.3 could be used. For roundabouts and other unsignalised intersections, k = 1.0 is appropriate. The effects of any delays during cruise between intersections (the decreased stream speed due to increased flow and mid-block pedestrian crossings, bus stops, etc) would also be included in the delay parameter. For uninterrupted facilities (freeways) where external friction is minimal, $J_A = 0.1$ could be appropriate.

The delay parameter (J_A) in *Equations (14) and (15)* corresponds to the quality of service provided by the road section and is independent of the ($Q\ t_o$) factor in *Equation (9)*. For example, for a road section with one signalised intersection (k = 0.6) along a 4-km road section, p = 0.25 and $J_A = 0.15$. On the other hand, for a road section with four linked signalised intersections along a 1-km road section, p = 4.0, k = 0.3 and $J_A = 1.2$.

The value of travel time at capacity ($t = t_m$ for x = 1.0) can be calculated from *Equation (1)*. For example, using $T_f = 1$ h:

$$t_m = t_o + (0.5\ J_A / Q)^{0.5} \qquad (16)$$

This formula can be used for obtaining rough estimates of the delay parameter. For example, Spiess (1990) adopts the BPR definition of the capacity as the volume at which congested speed is half the free-flow speed, i.e. $t_m / t_o = 2.0$. Using this in *Equation (16)* and putting $v_o = 1 / t_o$, $J_A = 2\ Q / v_o^2$ is obtained. For example, using Q = 2000 veh/h and $v_o = 120$ km/h for uninterrupted flow conditions (freeway), $J_A = 0.28$ is found. On the other hand, $t_m / t_o = 1.5$ for the same data would give $J_A = 0.07$. This shows the sensitivity of the delay parameter to the value of travel time at capacity.

Considering the difficulties of travel time measurement under congested traffic conditions, an appropriate way to calibrate the time-dependent function is to determine the value of the delay parameter J_A in the corresponding steady-state queuing delay function (*Equation 14*) using data points for medium to high flow (but not oversaturated) conditions (e.g. for x in the range 0.4 to 0.95). The resulting delay parameter estimate would be less dependent on the choice of the flow period (T_f) in this case. Direct estimation of minimum travel time and capacity parameters for use in the function before deriving the delay parameter is recommended. Errors in the estimation of these parameters would be taken up by the delay parameter.

The travel time graph representing *Equation (15)* for $T_f = 1$ h, $v_o = 80$ km/h ($t_o = 0.0125$ h/km), Q = 800 veh/h and $J_A = 0.4$ is shown in *Figure 2*. From *Equation (16)*, the travel time at capacity for this case is $t_m / t_o = 2.27$. *Figure 2* also shows the graph representing *Equation (11)* with $J_A = 0.4$ ($T_f = 1$ h and $v_o = 80$ km/h), which is the same as the graph representing the time-dependent function in *Figure 1*. Note that this corresponds to *Equation (15)* with Q = 800 veh/h ($Q\ t_o = 1.0$), and $J_A = 0.4$. The corresponding travel time at capacity is t_m / t_o = 5.0 which is considered to be too large (or the corresponding speed, v_m = 80 / 5.0 = 16 km/h is too small). Also comparing with *Figure 1*, the results in *Figure 2* indicate that Davidson's original function, its time-dependent form (*Equation 11*) and Tisato's function overestimate travel times for near (and above) congested conditions substantially compared with the alternative time-dependent function given by *Equation (15)*.

Travel time-flow graphs from *Equation (15)* representing five road classes (as defined in *Table 1*) are shown in *Figure 3* (T_f = 1 h). The corresponding speed-flow graphs are shown in *Figure 4*. *Table 1* and *Figures 3 and 4* serve to illustrate the range of parameter values that can be expected from real-life data. These graphs were derived by making assumptions about the minimum speed, capacity per lane and delay parameters for some broadly defined road classes.

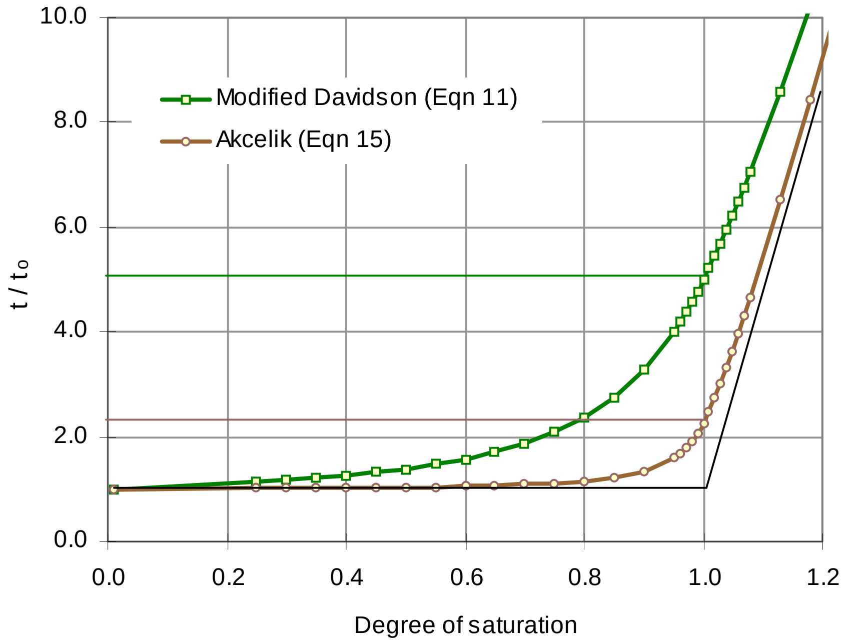

Figure 2 – Travel time graphs representing the time-dependent form of Davidson's function (Equation 11) for T_f = 1 h, v_o = 80 km/h, J_D =0.4, and the alternative function (Equation 15) proposed in this paper for T_f = 1 h, v_o = 80 km/h, Q = 800 veh/h and J_A =0.4

Table 1

Parameters for travel time functions representing various road classes (T_f = 1 h)

Road Class	Description	v_o (km/h)	Q (veh/h/lane)	J_A	v_m / v_o
1	Freeway	120	2000	0.1	0.63
2	Arterial (uninterrupted)	100	1800	0.2	0.57
3	Arterial (interrupted)	80	1200	0.4	0.49
4	Secondary (interrupted)	60	900	0.8	0.44
5	Secondary (high friction)	40	600	1.6	0.41

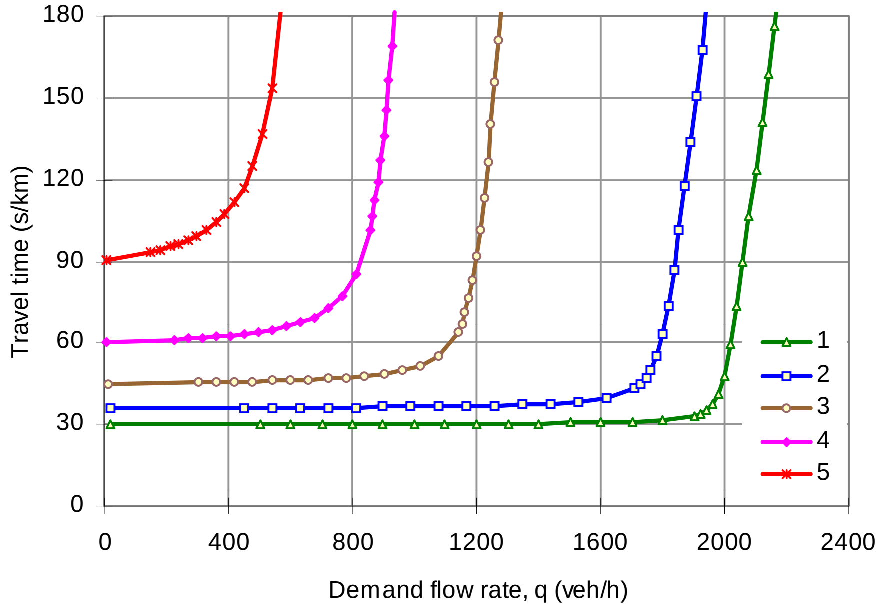

Figure 3 – Travel time-flow graphs (Equation 15) representing five road classes as defined in Table 1

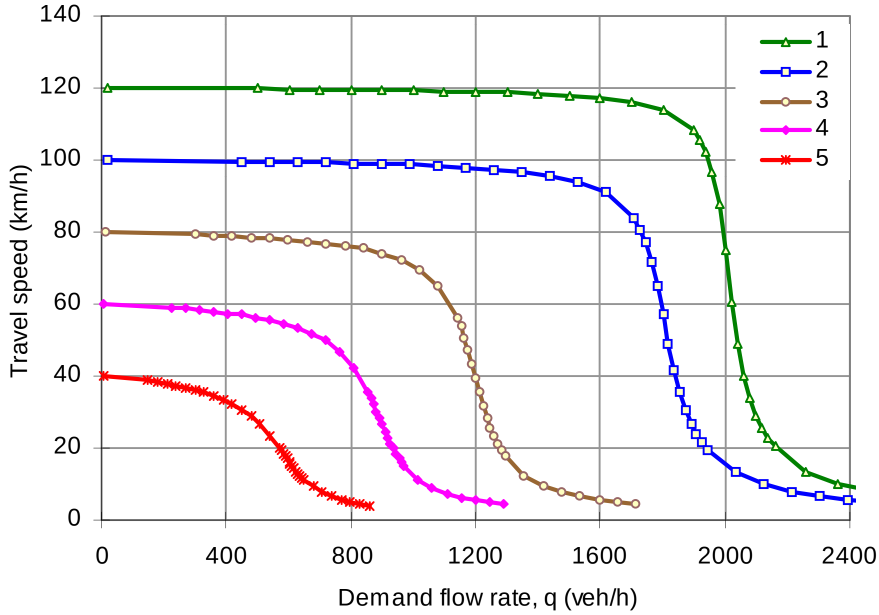

Figure 4 – Speed-flow graphs (corresponding to Figure 3) for five road classes as defined in Table 1

Discussion

The new travel time function given in this paper (*Equation 15*) could overcome various problems with Davidson's function and its modified forms proposed previously. Although this function can be used for individual road links (a link corresponds to one traffic control element, e.g. an intersection), it may be more appropriate to use it as an aggregate function for a series of links described as a *composite link* (see Rose, Taylor and Tisato (1989) for a discussion of the hierarchy of travel time functions).

The loss of accuracy which results from combining a series of links with different traffic characteristics as a single composite link should be kept in mind when interpreting the parameter values obtained by calibrating the travel time function proposed in this paper. In particular, the following points should be remembered (these points also apply to the original Davidson function and its modified forms, and may explain some of the difficulties experienced in efforts to calibrate such functions).

(a) The theoretical basis of the delay term used in the travel time function requires a lane-by-lane application. Therefore, it would be better to apply the function to the *critical lane* on the section of road under study. This requires that the flow and capacity values in the function are single lane values. Using this method, the problem of different numbers of lanes on different links constituting the section of road is avoided. Another reason for this type of analysis is the need to allow for the effect of unequal lane utilisation. For the purpose of simplification, equal lane capacities can be assumed, and the critical lane flow (q_c) can be calculated from

$$q_c = q / \Sigma \rho_i \qquad (17)$$

where q is the total flow and ρ_i is the lane utilisation ratio for lane i. Assuming equal lane capacities, the lane utilisation ratio is defined as $\rho_i = q_i / q_c$ where q_i is the flow in lane i and q_c is the flow in the critical lane. For fully utilised lanes, $\rho_i = 1.0$, and for underutilised lanes $\rho_i < 1.0$. For example, for q = 3000 veh/h, Q_i = 900 veh/h per lane, 4 lanes with one lane underutilised ($\rho_i = 0.50$), the critical lane flow from *Equation (17)* is q_c = 3000/(3x1.0 + 0.5) = 857 veh/h. The capacity and the degree of saturation flow for use in *Equation (15)* are Q = 900 veh/h and x = x_c = 857 / 900 = 0.952, respectively. If all lanes are used equally, q_c = 3000/(4x1.0) = 750 veh/h and x = x_c = 750/900 = 0.833. See Akçelik (1981), for a general discussion of the subject of lane utilisation.

(b) When several links in series are combined and replaced by one aggregate link, the minimum travel times are added, and the delay parameter can be considered to be a weighted average of individual delay elements of all links on the section of road. However, the question arises as to determining the capacity of the composite link. Firstly, note that individual links on the road section can have different number of lanes, but the method of using a critical lane to represent travel conditions on the road section avoids this problem. The capacity of the composite link should be the smallest capacity for any link as this becomes the bottleneck point which is important in the analysis of oversaturated conditions and for the modelling of the backward spread of congestion in the network.

Considering these points, it seems unlikely that a calibration method which simultaneously derives all three parameters of the travel time function (minimum travel time, capacity and delay parameter) by regression analysis would yield reliable values. This is in line with the findings of Taylor (1977a,b,c).

It is recommended that minimum travel time and capacity are determined externally considering the above points, and the delay parameter is derived by regression analysis using the specified minimum travel time and capacity values. The interpretation of the delay parameter which is obtained form this process should consider the type and intensity of delay producing elements on the road section. The effect of traffic composition (relative proportions of light and heavy vehicles) on the calibration parameters should also be considered.

It also appears that a link level analysis would enable better analysis of oversaturated conditions particularly due to better identification of bottleneck points. For the link level analysis, a better form of the travel time function would use explicit (separate) modelling of the free-flow travel time and the delays along the link and at the intersection (node) controlling the link. This would allow for the use of available intersection delay formulae directly, and would have the benefit of making use of any future developments in intersection modelling (for example better modelling of platooned arrivals, queue interaction and variable-demand modelling) without the need to derive new functions for transport planning purposes.

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