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Speed-Flow and Bunching Relationships for Uninterrupted Flows

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1 INTRODUCTION

The author (Akçelik 2002a,b) discussed the speed-flow models given in the US Highway Capacity Manual (TRB 2000; Reilly, et al 1990; Schoen, et al 1995) for basic freeway segments, multilane highways and urban streets, and suggested that the HCM speed-flow models have some features not consistent with expected traffic flow characteristics related to in-stream vehicle interaction and queuing considerations. The HCM models imply that the rate of reduction in speed with increased flow is greater, in other words, traffic delays are larger and increase at a faster rate, for higher-quality facilities. This characteristic of the HCM speed-flow models is in contrast with travel-time - flow models for different road classes used for transport planning purposes (Akçelik 1991, 1996). Higher traffic delays for higher-quality facilities do not appear to be consistent with *queuing* mechanisms inherent to in-stream vehicle interactions (Blunden 1971, 1978; Davidson 1966). It is expected that such physical - environmental characteristics as wider lanes, a larger number of lanes, more lateral clearance and lower interchange or access point density represent higher-quality facilities with lower frequency (intensity) of delay-producing elements and situations.

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A time-dependent speed-flow model developed by the author has been used in various applications successfully, and has been referred to as *Akçelik's function* in the literature (Akçelik 1991, 1996; Akçelik, Roper and Besley 1999; Akcelik and Associates 2002; Dowling and Alexiadis (1997); Dowling, Singh and Cheng 1998; Singh 1999; Sinclair Knight Merz 1998; Nakamura and Kockelman 2000). This function is based on queuing theory concepts, providing a smooth transition between a steady-state queuing delay function for undersaturated conditions and a deterministic delay function for oversaturated conditions. Thus, it allows for estimation of *travel speed*, *travel time* and *travel delay* for both undersaturated and oversaturated conditions. The author used this function to develop alternative versions of the HCM speed-flow models for basic freeway segments, multilane highways and urban streets that are consistent with expected relationships between traffic delay and physical - environmental characteristics of uninterrupted traffic facilities (Akçelik 2002a,b). In the context of uninterrupted flows, *travel delay* will be referred to as *traffic delay*.

This paper introduces an explicit model that describes in-stream vehicle interactions and resulting *queuing* in terms of traffic *bunching* characteristics. For this purpose, the bunched exponential model of the distribution of vehicle headways is used (Akçelik and Associates 2002; Akçelik 1994; Akçelik and Chung 1994; Cowan 1975; Luttinen 1999, 2003; Sullivan and Troutbeck 1993; Troutbeck 1989). A new model of the proportion of bunched vehicles is proposed. The model uses the delay parameter of Akçelik's speed - flow model as a bunching parameter, thus linking the bunching and speed - flow models towards a more integrated framework for modelling uninterrupted traffic streams. This bunching model has been implemented in the latest aaSIDRA version 2.1.

The paper also discusses the *driver response time* parameter at capacity flow (when the average headway equals the intrabunch headway). A model for *forced flow* conditions is then developed. Unsaturated and forced flow conditions are contrasted for the purpose of determining headway distributions.

The new bunching model as well as the associated speed-flow model is applied to roundabout circulating streams although the treatment of roundabout circulating streams as uninterrupted flows, especially based on the assumption of unsaturated conditions, requires some additional modelling considerations. This is further discussed in the Conclusion section.

2 UNINTERRUPTED TRAVEL SPEED CONCEPT

The *average uninterrupted travel speed* can be expressed as:

$$v_u = 3600 / t_u = 3600 / (t_f + d_{tu}) \qquad (2.1)$$

where

v_u = uninterrupted travel speed at a given flow rate (km/h),

t_u = uninterrupted travel time per unit distance, $t_u = t_f + d_{tu}$ (seconds/km),

d_{tu} = traffic delay (uninterrupted travel delay) per unit distance (seconds/km),

t_f = free-flow travel time per unit distance (seconds/km):

$$t_f = 3600 / v_f \qquad (2.2)$$

v_f = free-flow speed (km/h).

Definitions of free-flow speed (v_f) and uninterrupted travel speed (v_u) are shown in a time-distance diagram in *Figure 2.1*.

Speed-flow relationships for uninterrupted movements can be explained with the help of *Figure 2.2* which also shows the associated travel time - flow and traffic delay - flow relationships.

In *Figure 2.2*, Region A represents unsaturated (undersaturated) conditions with arrival flows below capacity ($q_a \le Q$) that are associated with uninterrupted travel speeds, v_u between v_f and v_n ($v_f \ge v_u \ge v_n$) where v_f is the free-flow speed and v_n is the speed at capacity. With increasing flow rate in Region A, speeds are reduced below the free-flow speed due to traffic delays resulting from interactions between vehicles.

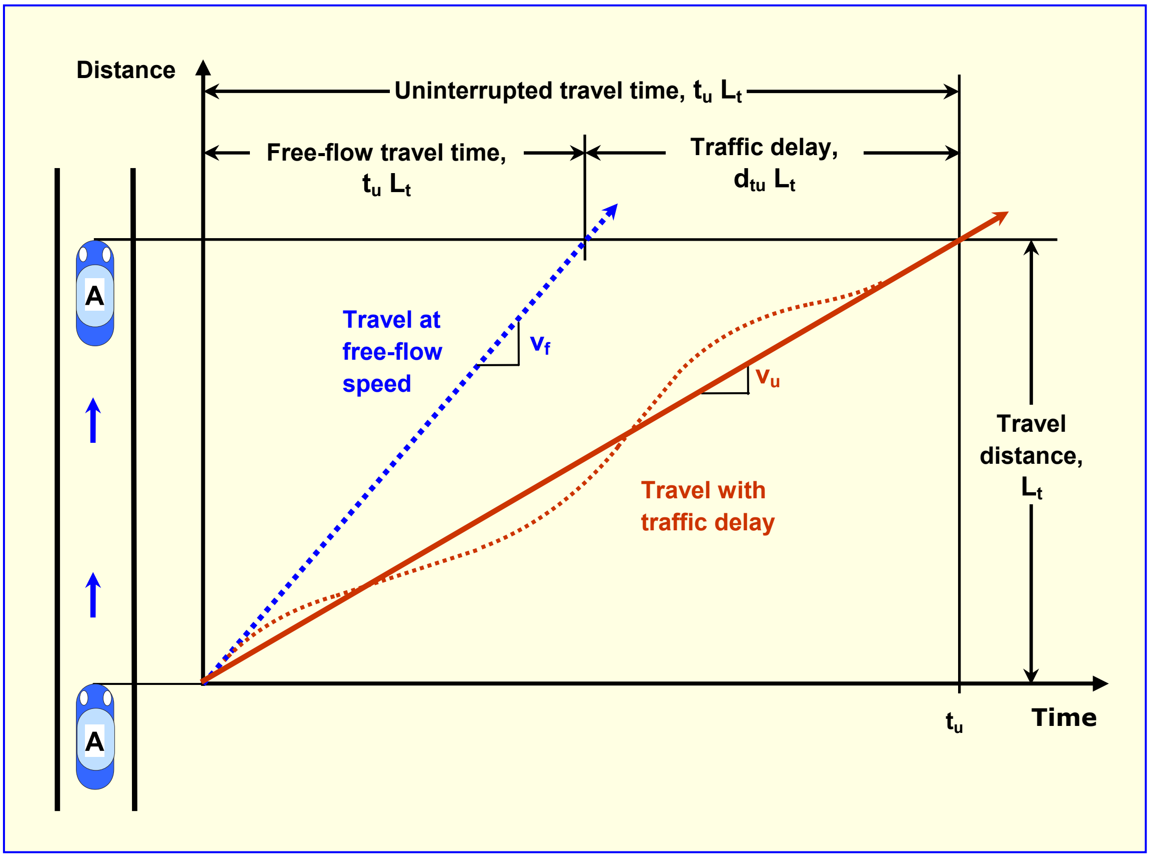

Figure 2.1 - Definition of free-flow and uninterrupted travel speed

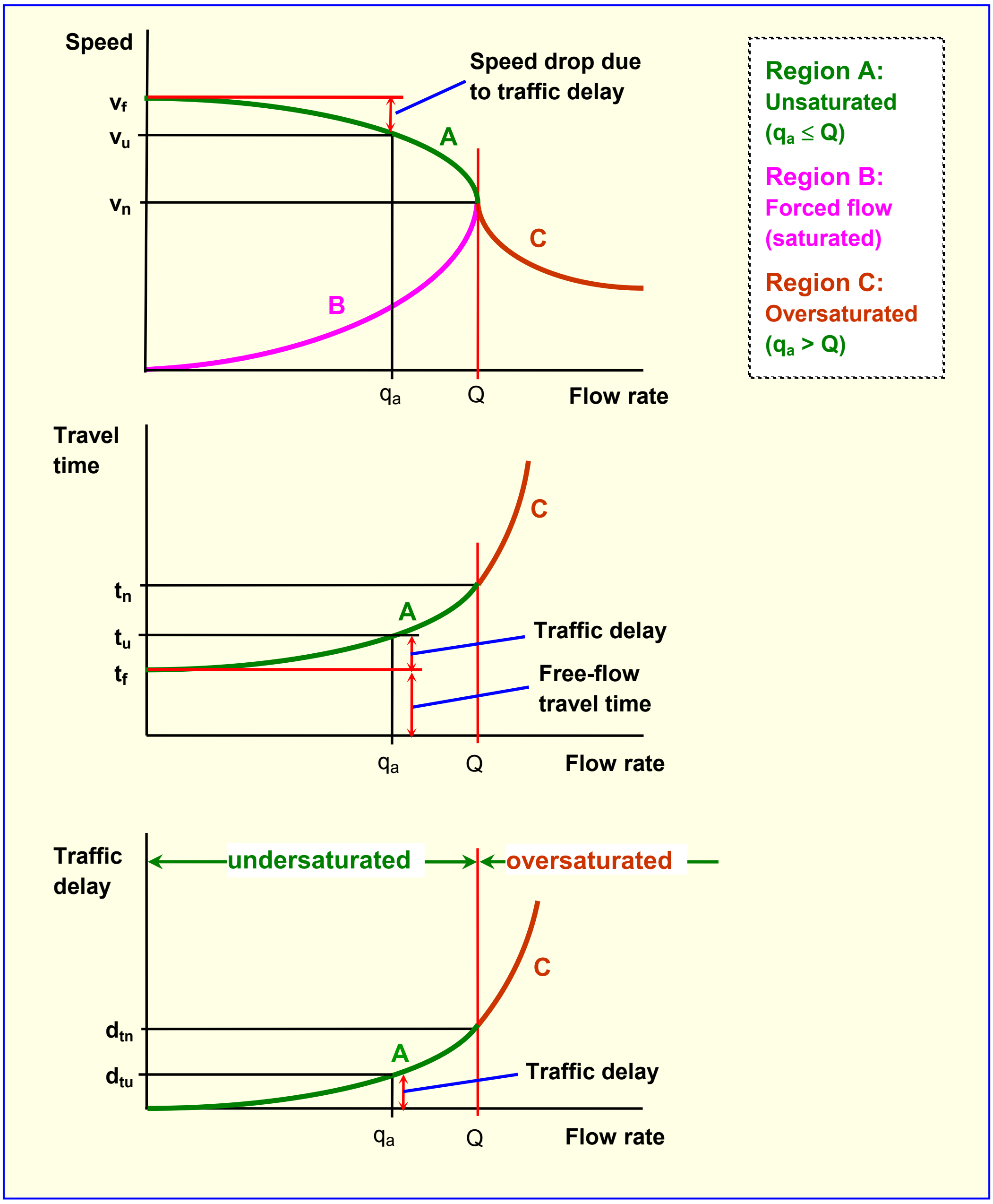

Figure 2.2 - Speed, travel time and delay as a function of flow rate for uninterrupted traffic streams

Region B in *Figure 2.2* represents the forced (saturated) flow conditions with flow rates reduced below capacity ($q < Q$) which are associated with further reduced speeds ($v < v_n$) as observed at a reference point along the road. In this region, flow rates (q) are reduced flow rates due to forced flow conditions, not demand flow rates (q_a), in other words the flow rate measured at a point along the road cannot exceed the capacity flow.

Region C represents oversaturated conditions, i.e. arrival (demand) flow rates above capacity ($q_a > Q$) cause large reductions in travel speeds ($v < v_n$) due to large queuing delays. These speeds can be observed by travel through the total section (along distance L_t), e.g. by an instrumented car. In this case, the flow represents the demand flow rate which can exceed the capacity value as measured at a point upstream of the queuing section.

3 PROPOSED BUNCHING MODEL

The following exponential model was used in aaSIDRA 2.0 and earlier versions for the prediction of proportion free (unbunched) vehicles in a traffic stream (Akcelik and Associates 2002; Akçelik and Chung 1994):

$$\varphi = e^{-b \Delta q} \qquad (3.1)$$

where b is a constant, Δ is the average intrabunch headway (s) and q is the flow rate (veh/s).

The following model has been introduced in aaSIDRA 2.1 to replace the exponential model:

$$\varphi = (1 - \Delta q) / [1 - (1 - k_d) \Delta q] \qquad \textit{subject to } \varphi \le 0.001 \qquad (3.2)$$

where k_d is a constant (*traffic delay / bunching* parameter), Δ is the average intrabunch headway (s), q is the flow rate (veh/s).

The intrabunch headway is treated as the average headway at capacity ($\Delta = 3600 / Q$ where Q is the capacity in veh/h). Previously, it was recommended that the intrabunch headway should be selected on the basis of the best headway distribution prediction (Akcelik and Chung 1994). Although this is still an important objective, the intrabunch headway is treated as the average headway at capacity flow by definition.

Values of parameters b, k_d and Δ for use in *Equations (3.1) and (3.2)* are given in *Table 3.1*. The minimum value of the proportion unbunched (0.001) in *Equation (3.2)* is used for computational reasons.

Extra bunching to allow for the effect of upstream signals, which is used in aaSIDRA for roundabout approach streams, could be used for all uninterrupted streams.

Tanner's (1962, 1967) equation is a special case of *Equation (3.2)* which is obtained when $k_d = 1.0$:

$$\varphi = 1 - \Delta q \qquad (3.3)$$

AUSTROADS (1993) roundabout guide uses the following linear model:

$$\varphi = 0.75 (1 - \Delta q) \qquad (3.4)$$

The bunching model and the bunched exponential model of headway distribution apply for *unsaturated flow* conditions (flow rate below capacity), i.e. for Region A in *Figure 2.2*. Under *forced flow* conditions for (Region B in *Figure 2.2*) all vehicles are bunched with intrabunch headways larger than the minimum intrabunch headway due to lower speeds and spacings of vehicles. This is discussed in *Section 6*, and implications of the forced flow conditions on headway distributions are discussed in the Conclusion section.

The values of the traffic delay / bunching parameter k_d given in *Table 3.1* were determined on the basis of exponential models used previously for uninterrupted streams (Akçelik and Chung 1994) and using data given in SR 45 (Troutbeck 1989) for roundabout circulating streams. Resulting speed-flow relationships were also considered in selecting appropriate values of the parameter.

Figure 3.1 shows the proportion unbunched for one-lane, two-lane and three-lane uninterrupted streams using the bunching model based on the traffic delay parameter (*Equation 3.2*). *Figures 3.2 and 3.3* show the proportion unbunched (measured and estimated by alternative models given by *Equations 3.1, 3.2 and 3.4*) for single-lane and two-lane circulating streams at roundabouts. *Figure 3.4* shows the proportion unbunched for one-lane, two-lane and three-lane roundabouts using the bunching model based on traffic delay parameter together with SR 45 data for single-lane multi-lane roundabouts.

Table 3.1

Parameter values for estimating the proportion of free (unbunched) vehicles in a traffic stream

Total number of lanes	Uninterrupted traffic streams				Roundabout circulating streams			
	Δ	$3600/\Delta$	b	k_d	Δ	$3600/\Delta$	b	k_d
1	1.8	2000	0.5	0.20	2.0	1800	2.5	2.2
2	0.9	4000	0.3	0.20	1.0	3600	2.5	2.2
> 2	0.6	6000	0.7	0.30	0.8	4500	2.5	2.2

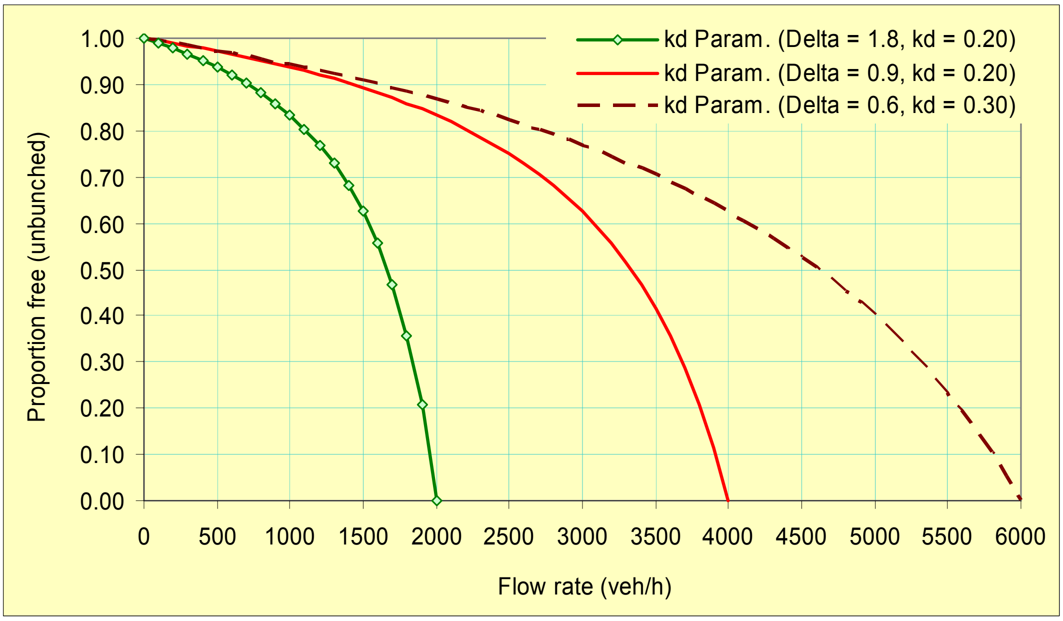

Figure 3.1 - Proportion unbunched for one-lane (Δ = 1.8 s), two-lane (Δ = 0.9 s) and three-lane (Δ = 0.6 s) uninterrupted streams using the bunching model based on traffic delay parameter

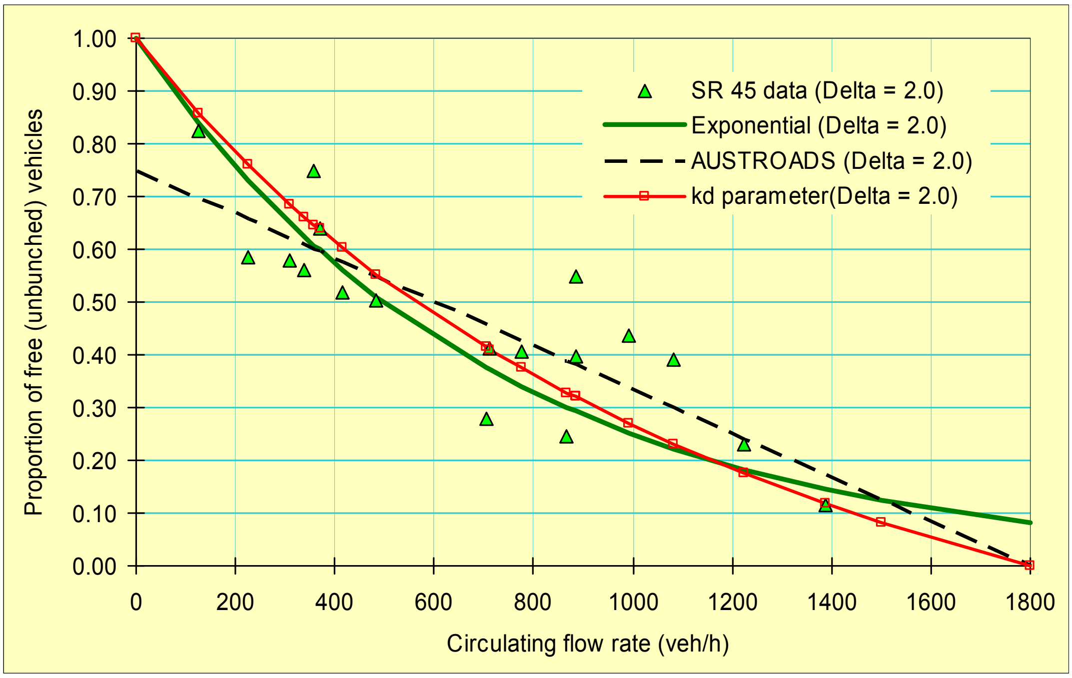

Figure 3.2 - Proportion unbunched for single-lane circulating streams at roundabouts as a function of the circulating flow rate (measured and estimated by alternative bunching models)

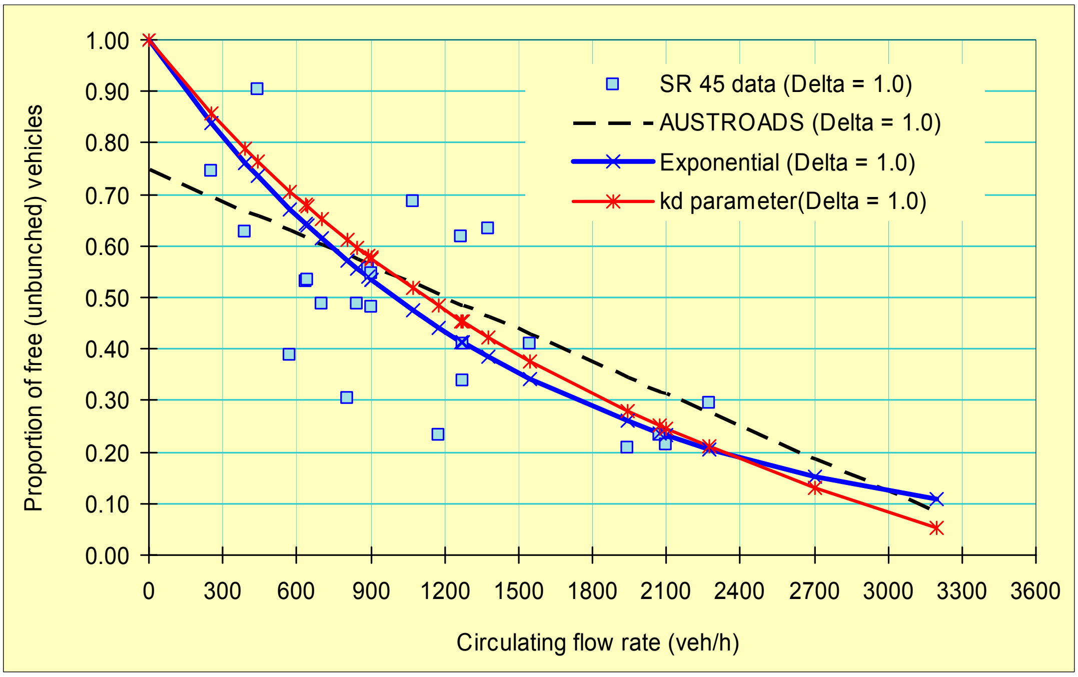

Figure 3.3 - Proportion unbunched for two-lane circulating streams at roundabouts as a function of the circulating flow rate (measured and estimated by alternative bunching models)

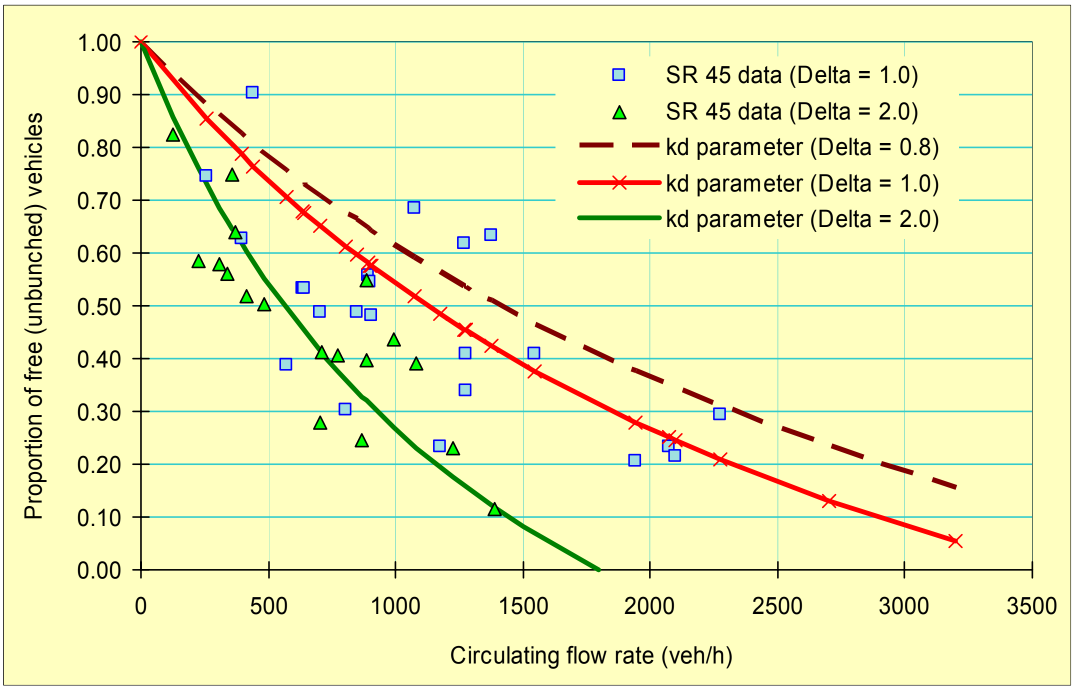

Figure 3.4 - Proportion unbunched for one-lane (Δ = 2.0 s), two-lane (Δ = 1.0 s) and three-lane (Δ = 0.8 s) roundabouts using the bunching model based on traffic delay parameter

4 TRAVEL DELAY, TRAVEL TIME and TRAVEL SPEED

The steady-state travel delay model for an uninterrupted stream corresponding to the bunching model given by *Equation (3.2)* is:

$$d_{tu} = 3600\ k_d\ x\ /\ [Q\ (1 - x)] \tag{4.1}$$

where d_{tu} is the average travel delay for an uninterrupted stream (s/km), k_d is the *traffic delay / bunching* parameter as in *Equation (3.2)*, Q is the capacity in veh/h ($Q = 3600 / \Delta$) and x is the degree of saturation ($x = q / Q = \Delta\ q / 3600$ where q is the arrival flow rate in veh/h).

The corresponding bunch size (n_b) and the corresponding queue size ($n_q = n_b - 1$, considering that the leader of a bunch of vehicles is not queued) are given by:

$$n_b = [1 - (1 - k_d)\ x]\ /\ (1 - x) \tag{4.2}$$

$$n_q = k_d\ x\ /\ (1 - x) \tag{4.3}$$

The time-dependent form of the travel delay model given by *Equation (4.1)* for an uninterrupted stream with no initial queued demand, and the corresponding speed-flow function are given by the following equations (Akcelik 2002a,b):

$$t_u = t_f + 900\ T_f\ \{\ x - 1) + [(x - 1)^2 + 8\ k_d\ x\ /\ (Q\ T_f)]^{0.5}\} \tag{4.4}$$

$$v_u = v_f\ /\ \{1 + 0.25\ v_f\ T_f\ [(x - 1) + ((x - 1)^2 + 8\ k_d\ x\ /\ (Q\ T_f)\ )^{0.5}]\} \tag{4.5}$$

where

t_u = uninterrupted travel time per unit distance at a given degree of saturation x (s/km),

v_u = $3600 / t_u$ = uninterrupted travel speed (s/km),

t_f = free-flow travel time per unit distance (travel time at x = 0) (s/km),

T_f = duration of the analysis period (h), $T_f = 0.25$ h is specified in the HCM,

Q = capacity (veh/h), $Q = 3600 / \Delta$.

The speed-flow model should normally be used for single-lane streams although they could be used for multi-lane streams (lane groups) as a rough approximation.

Figure 4.1 shows travel speed - flow graphs for single-lane uninterrupted and roundabout circulating streams corresponding to the models defined by k_d and Δ parameters given in *Table 3.1* (further parameters are summarised in *Table 4.1)*. For these examples, free-flow speeds are chosen as 70 km/h for the uninterrupted stream and 35 km/h for the roundabout circulating stream, and $T_f = 0.25$ h is used. Safe negotiation speed as used for geometric delay purposes is considered to be appropriate as the free-flow speed in speed-flow relationships for roundabout circulating streams.

Speed- flow and bunching model parameters for the uninterrupted stream models proposed by (Akçelik 2002a,b) for the HCM *basic freeway segment*, *multilane highway* and *urban street* classes are given in *Tables 4.2 to 4.4*. Speed-flow graphs for these models can be found in (Akçelik 2002a,b). The bunching models corresponding to Class 3 facility in each group is shown in *Figure 4.2*.

Table 4.1

Parameters for speed - flow and bunching models for single-lane uninterrupted and single-lane roundabout circulating streams

	Uninterrupted stream (single-lane)	Roundabout circulating stream (single-lane)
Free-flow speed, v_f (km/h)	70	35
Traffic delay / bunching parameter, k_d	**0.20**	**2.2**
Intrabunch headway, Δ (s)	**1.80**	**2.00**
Capacity, Q (veh/h)	2000	1800
Speed at capacity (when q = Q), v_n (km/h)	56	24.4
Speed ratio, v_n / v_f	0.80	0.70
Average spacing at capacity, L_{hn} (m)	28.1	13.6
Response time to stop from speed at capacity, t_{rf}	1.35	0.97

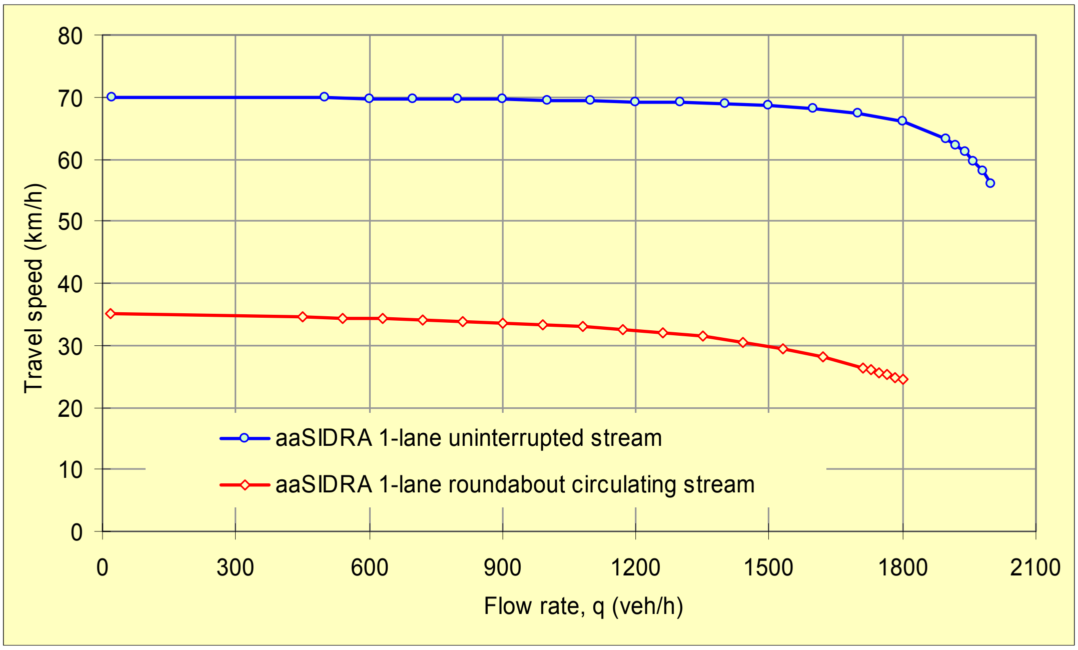

Figure 4.1 - Speed - flow relationships for one-lane uninterrupted and roundabout circulating streams using the bunching model parameters given in Table 3.1 (also see Table 3.1)

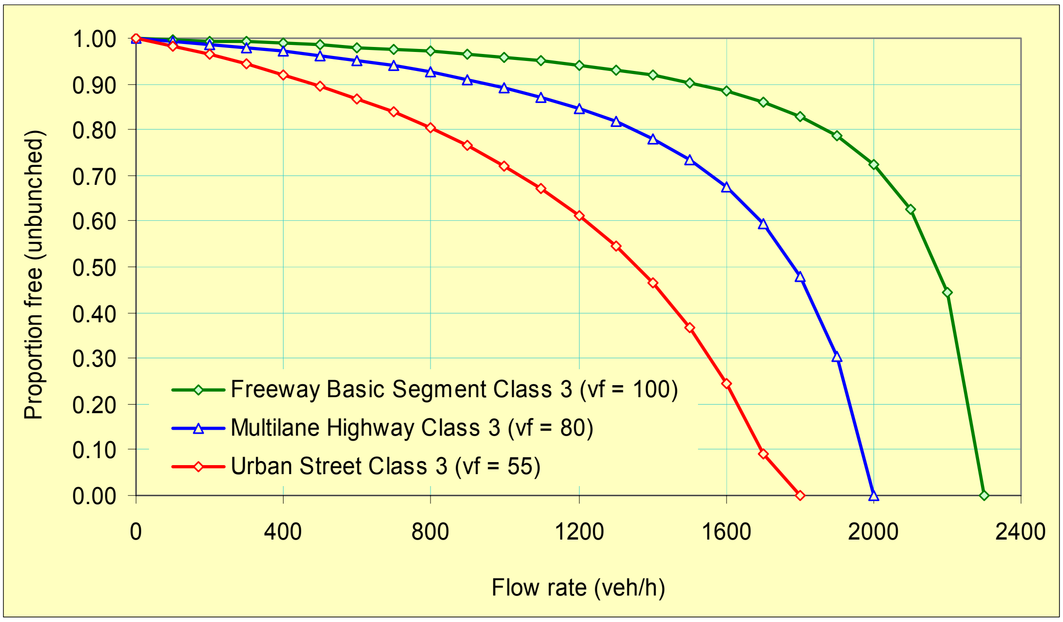

Figure 4.2 - Proportion unbunched for one-lane uninterrupted streams (basic freeway segment, multilane highway and urban street class 3 facilities) (see Tables 4.2 to 4.4)

Table 4.2

Parameters for speed - flow and bunching models for single-lane uninterrupted streams: HCM basic freeway segment classes

Facility class	**1**	**2**	**3**	**4**
Free-flow speed, v_f (km/h)	120	110	100	90
Traffic delay / bunching parameter, k_d	**0.04**	**0.05**	**0.06**	**0.07**
Intrabunch headway, Δ (s)	**1.500**	**1.532**	**1.565**	**1.600**
Capacity, Q (veh/h)	2400	2350	2300	2250
Speed at capacity (when q = Q), v_n (km/h)	102.0	93.5	85.0	76.5
Speed ratio, v_n / v_f	0.85	0.85	0.85	0.85
Average spacing at capacity, L_{hn} (m)	42.5	39.8	37.0	34.0
Response time to stop from speed at capacity, t_{rf}	1.25	1.26	1.27	1.27

Table 4.3

Parameters for speed - flow and bunching models for single-lane uninterrupted streams: HCM multilane highway classes

Facility class	1	2	3	4
Free-flow speed, v_f (km/h)	100	90	80	70
Traffic delay / bunching parameter, k_d	**0.08**	**0.10**	**0.12**	**0.15**
Intrabunch headway, Δ (s)	**1.636**	**1.714**	**1.800**	**1.895**
Capacity, Q (veh/h)	2200	2100	2000	1900
Speed at capacity (when q = Q), v_n (km/h)	82.0	73.8	65.6	57.4
Speed ratio, v_n / v_f	0.82	0.82	0.82	0.82
Average spacing at capacity, L_{hn} (m)	37.3	35.1	32.8	30.2
Response time to stop from speed at capacity, t_{rf}	1.33	1.37	1.42	1.46

Table 4.4

Parameters for speed - flow and bunching models for single-lane uninterrupted streams: HCM urban street classes

Facility class	1	2	3	4
Free-flow speed, v_f (km/h)	80	65	55	45
Traffic delay / bunching parameter, k_d	**0.14**	**0.21**	**0.29**	**0.42**
Intrabunch headway, Δ (s)	1.946	2.000	2.057	2.118
Capacity, Q (veh/h)	1850	1800	1750	1700
Speed at capacity (when q = Q), v_n (km/h)	64.0	52.0	44.0	36.0
Speed ratio, v_n / v_f	0.80	0.80	0.80	0.80
Average spacing at capacity, L_{hn} (m)	28.9	34.6	39.8	47.2
Response time to stop from speed at capacity, t_{rf}	1.55	1.52	1.48	1.42

Sullivan and Troutbeck (1993) presented exponential bunching model results for arterial roads in Brisbane. They found that proportion bunched could be related to lane characteristics, and grouped their results for median lanes and other lanes grouped by lane width (< 3.0 m, 3.0 to 3.5 m and > 3.0 m). The bunching model based on the traffic delay parameter (*Equation 3.2*) was compared with the exponential model results given by Sullivan and Troutbeck using Δ = 2.0 s for all cases. The results are summarised in *Table 4.5* and *Figure 4.3,* which indicate that Sullivan and Troutbeck data represent highly restricted conditions (high k_d parameter values) compared with the HCM multilane highway or urban street conditions, or the aaSIDRA uninterrupted stream model which is based an data collected in Melbourne (Akçelik and Chung 1994). Free-flow speed of 70 km/h was selected only as a representative value to indicate the model characteristics. The data in *Table 4.5* shows that the speed-flow curves have high slopes as indicated by low speeds at capacity.

Table 4.5

Parameters for speed - flow and bunching models for single-lane uninterrupted streams: delay parameter model results based on exponential models developed by Sullivan and Troutbeck (1993) using data collected on arterial roads in Brisbane

	Median lane	Kerb lane (< 3.0 m)	Kerb lane (3.0 to 3.5 m)	Kerb lane (> 3.5 m)
Free-flow speed, v_f (km/h)	70	70	70	70
Traffic delay / bunching parameter, k_d	**4.80**	**3.90**	**2.60**	**1.60**
Intrabunch headway, Δ (s)	**2.0**	**2.0**	**2.0**	**2.0**
Capacity, Q (veh/h)	1800	1800	1800	1800
Speed at capacity (when q = Q), v_n (km/h)	30.7	32.5	36.1	40.3
Speed ratio, v_n / v_f	0.44	0.47	0.52	0.58
Average spacing at capacity, L_{hn} (m)	17.1	18.1	20.0	22.4
Response time to stop from speed at capacity, t_{rf}	1.18	1.23	1.30	1.37

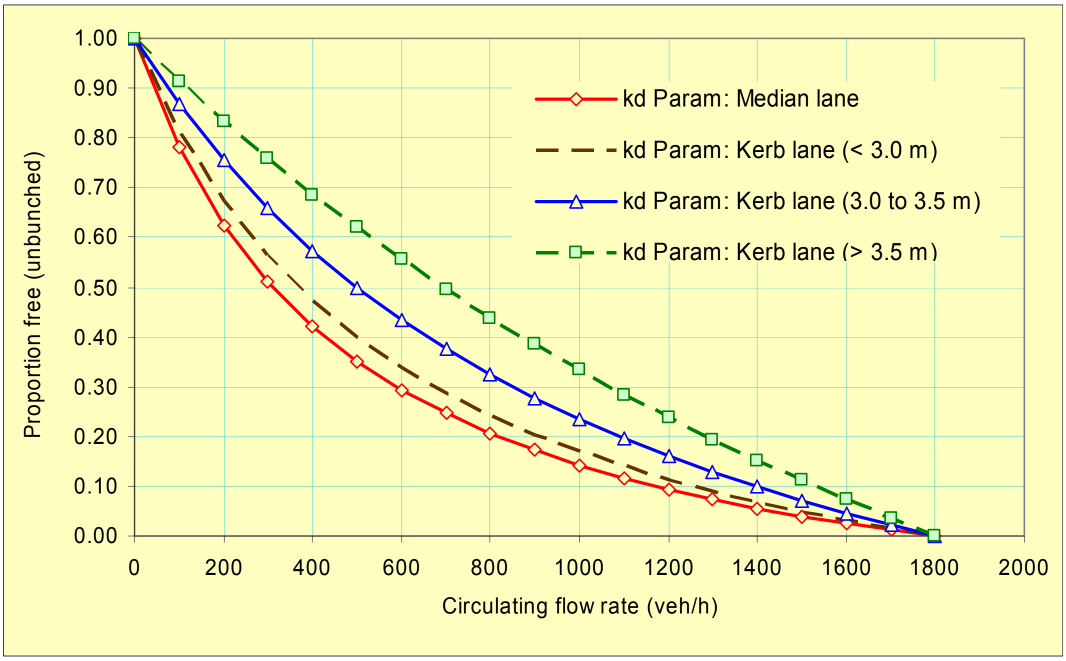

Figure 4.3 - Proportion unbunched for one-lane uninterrupted streams: estimates using the delay parameter model based on exponential models developed by Sullivan and Troutbeck (1993) using data collected on arterial roads in Brisbane

5 RESPONSE TIME

The *driver response time at capacity* (as it applies to bunched vehicles) shown in *Tables 4.1 to 4.5* is calculated from

$$t_{rn} = (3.6 / v_n) (L_{hn} - L_{hj}) = h_n - 3.6 L_{hj} / v_n \quad (5.1)$$

where

t_{rn} = driver response time to stop from speed at capacity (s),

v_n = speed at capacity (km/h),

L_{hn} = $\Delta\ v_n / 3.6$ = spacing at capacity (m), and

L_{hj} = jam spacing (m).

The basis of *Equation (5.1)* is shown in *Figure 5.1*. This assumes that the leading and following vehicles (A and B) have the same braking distance ($L_{bA} = L_{bB}$) and the spacing at capacity (L_{hn}) is sufficient for the following vehicle to stop leaving a gap of $L_{hj} - L_v$ behind the leading vehicle when they both stop (L_v = vehicle length). Therefore, the response time represents a comfortable stopping condition. Note that higher speed at capacity means a lower response time, and a larger jam spacing means a lower response time. The values in *Tables 4.1 to 4.5* were calculated using L_{hj} = 7.0 m (aaSIDRA default value).

The response times in the range 0.97 to 1.55 s, as values relevant to traffic operations (assuming low perception times of alert drivers in heavy urban traffic conditions), indicate that the speed-flow and bunching models are reasonable. These are similar to response times for saturation headways at signalised intersections which were determined to be in the range 0.84 to 1.39 s (Akçelik, Besley and Roper 1999, Akçelik and Besley 2002). *Equation (5.1)* can be applied to saturation headways at signals as a stopping condition as well, giving driver response times equivalent to those obtained from queue discharge models.

In ARR 341 (Akçelik, Roper and Besley 1999), calibration of "Model 4+5" for a basic freeway segment gave v_f = 101 km/h, v_n = 90 km/h (v_n / v_f = 0.89), k_d = 0.09, Δ = 1.44 s (Q = 2500 veh/h), L_{hn} = 36.0 m and L_{hj} = 15.0 m ("Model 4" corresponds to *Equation 4.5*). The corresponding response time from *Equation (5.1)* is t_r = 0.84 s. Application of *Equation (5.1)* to other calibrated freeway models reported in ARR 341 gave response times in the range 0.75 to 1.12 s.

Consistent with *Equation (5.1)*, the spacing, headway and speed at capacity are related as follows:

$$L_{hn} = L_{hj} + t_{rn}\, v_n / 3.6 \quad (5.2)$$

$$h_n = \Delta = t_{rn} + 3.6 L_{hj} / v_n \quad (5.3)$$

$$v_n = 3.6 L_{hj} / (h_n - t_{rn}) \quad (5.4)$$

Equation (5.3) for headway at capacity (or intrabunch headway) is essentially the same as the formula for saturation headway at signalised intersection derived from queue discharge characteristics (Akçelik, Besley and Roper 1999, Akçelik and Besley 2002).

The *stopping wave speed* shown in *Figure 5.1* is given by:

$$v_y = 3.6 L_{hj} / t_{rn} = 3.6 L_{hj} / (h_n - 3.6 L_{hj} / v_n) \quad (5.5)$$

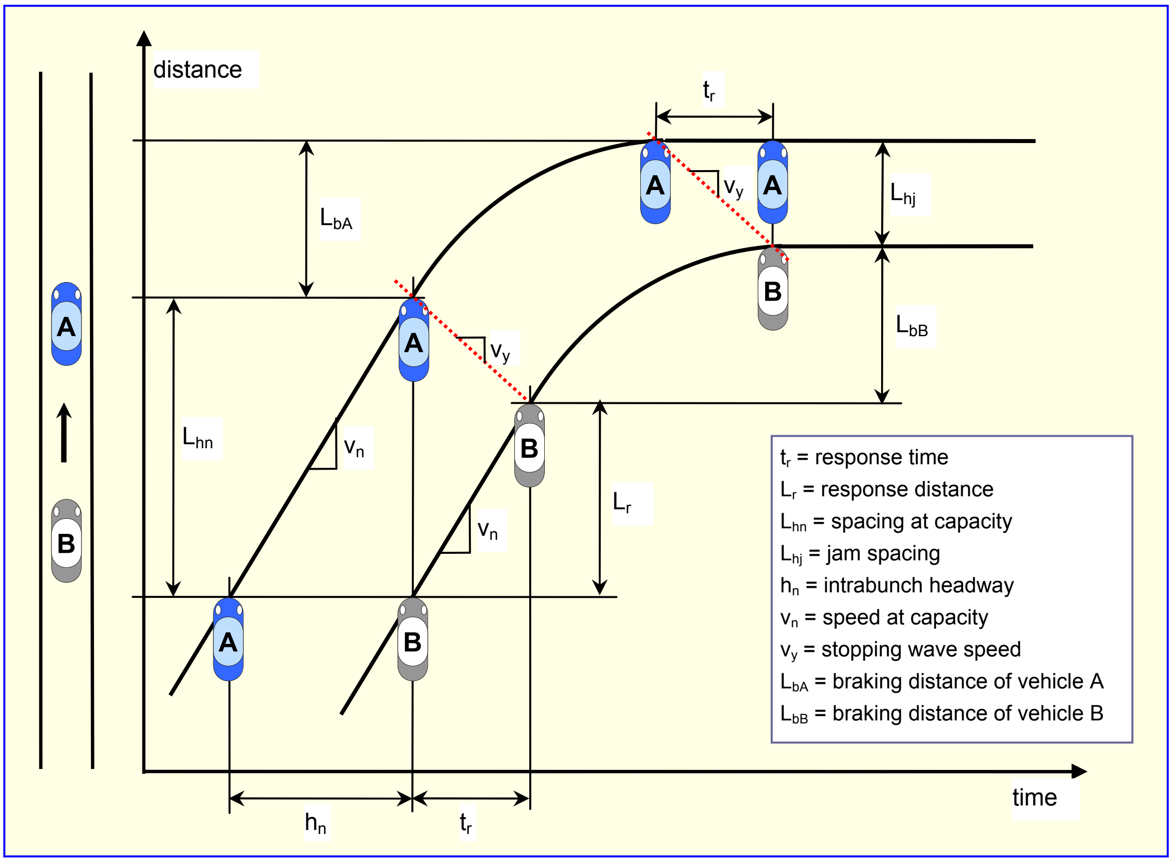

Figure 5.1 - Derivation of the driver response time for vehicles driving with intrabunch headway considering safe stopping conditions

6 FUNDAMENTAL RELATIONSHIP FOR FORCED FLOW CONDITIONS

The speed - flow model given in *Section 4* can be used for regions A and C in *Figure 2.2* (see *Section 2*). A speed-flow model, and associated models for other fundamental traffic flow relationships, for region B in *Figure 2.2*, i.e. for ***forced flow*** conditions can be derived using the driver response time parameter.

Under forced flow conditions when speed drops below the value at capacity ($v \le v_n$) because vehicle spacing is reduced below the value at capacity ($L_h \le L_{hn}$), the bunching model given in *Section 3* no longer applies. The spacing under these conditions can be expressed as:

$$L_h = L_{hj} + t_r \, v / 3.6 \qquad \textit{for } L_h \le L_{hn} \; (v \le v_n) \tag{6.1}$$

Speed and headway are found from:

$$v = 3.6 \, (L_h - L_{hj}) / t_r \qquad \textit{for } L_h \le L_{hn} \; (v \le v_n) \tag{6.2}$$

$$h = 3.6 \, L_h / v = L_h \, t_r / (L_h - L_{hj}) \qquad \textit{for } L_h \le L_{hn} \; (v \le v_n) \tag{6.3}$$

where v is in km/h and h is in seconds.

This can be used to develop fundamental traffic flow relationships for forced flow conditions by making assumptions about the driver response time. For example, a linear driver response - spacing model can be used:

$$t_r = p_1 + p_2 \, L_h \qquad \textit{subject to } 0.5 \text{ s} \le t_r \le 2.5 \text{ s} \tag{6.4}$$

where parameters p_1 and p_2 are derived to achieve a given t_{rn} at capacity ($t_r = t_{rn}$ when $L_h = L_{hn}$) and minimum headway at capacity ($dh/ dL_h = 0$ for $h = h_n$ when $L_h = L_{hn}$):

$$p_1 = t_{rn} \, [1 - L_{hn} \, L_{hj} / (L_{hn}^2 - L_{hn} \, L_{hj})] \tag{6.5a}$$

$$p_2 = t_{rn} \, L_{hj} / (L_{hn}^2 - L_{hn} \, L_{hj}) \tag{6.5b}$$

The linear response time model (*Equation 6.4*) assumes that drivers become more alert as spacing decreases.

Using t_r from *Equation (6.4)* in *Equations (6.2) and (6.3)*:

$$v = 3.6 \, (L_h - L_{hj}) / (p_1 + p_2 \, L_h) \tag{6.6}$$

$$h = 3.6 \, L_h / v = L_h \, (p_1 + p_2 \, L_h) / (L_h - L_{hj}) \tag{6.7}$$

The spacing as a function of speed is:

$$L_h = (L_{hj} + p_1 \, v / 3.6) / (1 - p_2 \, v / 3.6) \tag{6.8}$$

Thus, the linear response time - spacing model implies a hyperbolic spacing - speed function.

Density in veh/km and flow rate in veh/h are given by:

$$k = 1000 / L_h \tag{6.9}$$

$$q = 3600 / h \tag{6.10}$$

The forced-flow model given by *Equations (6.6) to (6.8)* can also be expressed in terms of density and flow:

$$v = 3.6 \, (1 / k - 1 / k_j) / (p_1 / 1000 + p_2 / k) \qquad \textit{for } k \ge k_n \; (v \le v_n) \tag{6.11}$$

$$q = 3600 \, (1 - k / k_j) / (p_1 + 1000 \, p_2 / k) \qquad \textit{for } k \ge k_n \tag{6.12}$$

Other parameters such as occupancy time, space time, etc described in previous publications (Akçelik, Besley and Roper 1999, Akçelik, Roper and Besley 1999) can also be calculated as a result.

For the roundabout circulating stream model given in *Table 4.1*, p_1 = -0.059 and p_2 = 0.079 were found.

For the freeway case described in ARR 341 (Akçelik, Roper and Besley 1999) (t_{rn} = 0.84 s, L_{hn} = 36.0 m, L_{hj} = 15.0 m, v_n = 90 km/h as calibrated for "Model 4+5"), p_1 = 0.240 and p_2 = 0.0167 were found. *Figure 6.1* shows the proportion unbunched for this case, which is consistent with "Model 4" as given by *Equation (4.5)*. Using the new model for forced flow conditions (replacing "Model 5" of ARR 341), the spacing - speed and speed - flow relationships together with measured values for the ARR 341 case (both unsaturated and forced flow conditions) are shown in *Figures 6.2 and 6.3*.

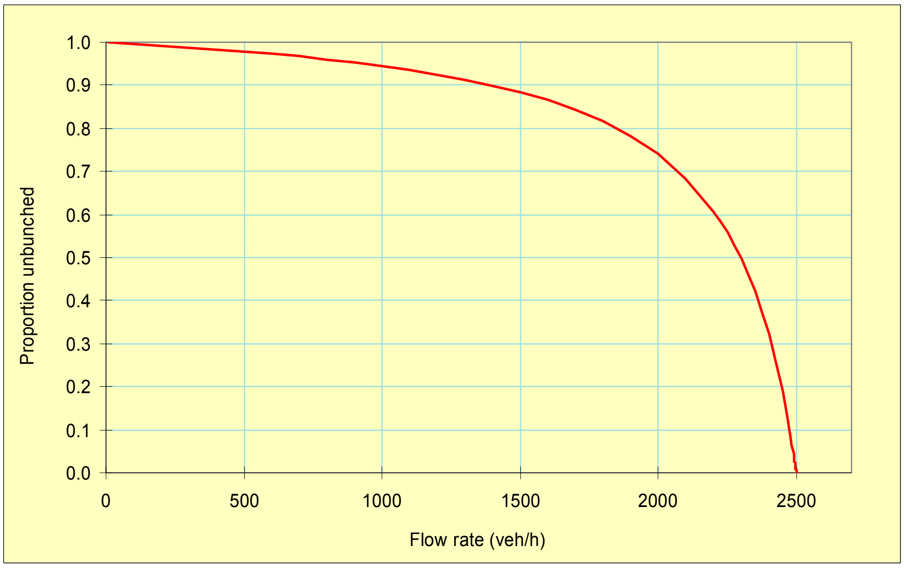

Figure 6.1 - Estimated proportion unbunched for the freeway basic segment data collected in Melbourne (described in ARR 341)

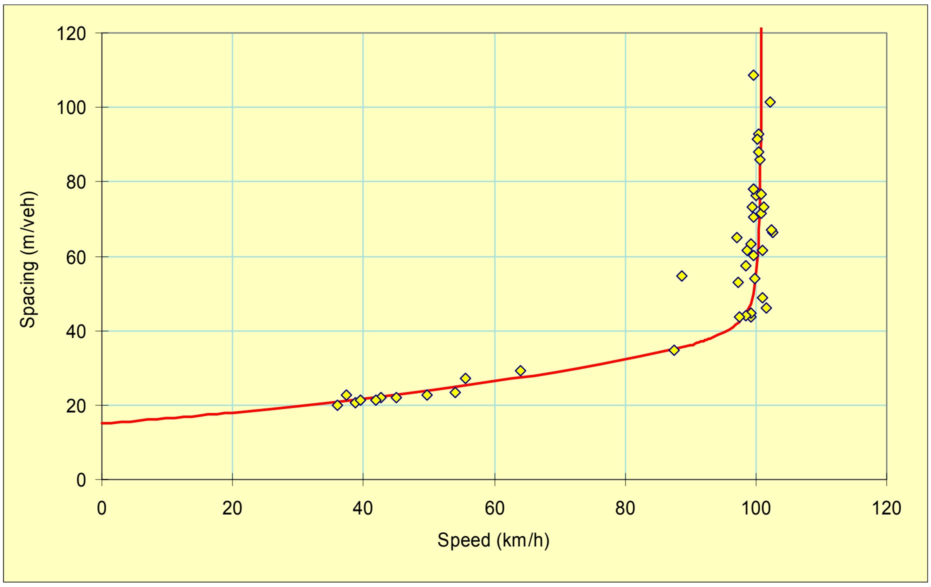

Figure 6.2 - Estimated and measured spacing - speed values for the freeway basic segment data collected in Melbourne (described in ARR 341)

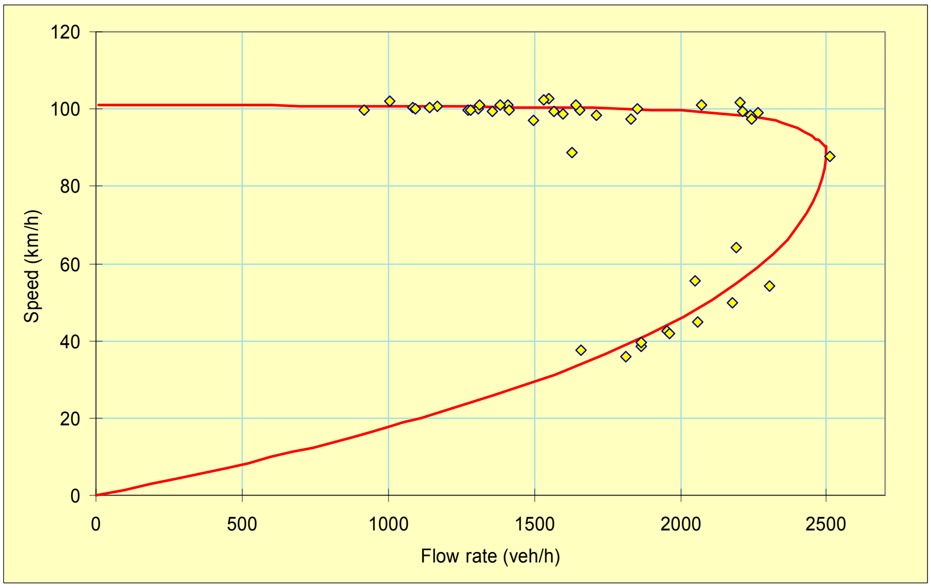

Figure 6.3 - Estimated and measured speed - flow values for the freeway basic segment data collected in Melbourne (described in ARR 341)

7 CONCLUSION

It is strongly recommended that the unsaturated and forced flow conditions should be distinguished in the calibration and application of the bunching model for modelling headway distributions. The bunching model and the bunched exponential headway distribution model should be applied for *unsaturated* flow conditions when the average flow rate is below capacity. Under these circumstances, all headways larger than the intrabunch headway are considered to be unbunched in the headway distribution model. According to the model, when the flow rate reaches capacity, all vehicles are bunched as they travel at the capacity headway (minimum intrabunch headway) at a capacity speed. The traffic stream is considered to be under *forced (saturated) flow* conditions when the average speed drops below the capacity speed and the average vehicle spacing drops below the spacing at capacity. Under these conditions, all vehicles should be considered to be bunched although headways are *larger* than the minimum intrabunch headway due to lower speeds and spacings of vehicles. The bunched exponential model is no longer valid under forced flow conditions, and using headway data collected under these circumstances would result in a biased bunching model.

Of particular concern is the application of the bunched exponential model of headway distribution to roundabout circulating streams without due attention to the headways of vehicles entering from approach queues, i.e. entering with follow-up (saturation) headway, as this may cause problems in capacity estimation. In this paper, the new bunching model as well as the associated speed-flow model is applied to roundabout circulating streams although the treatment of roundabout circulating streams as uninterrupted flows, especially based on the assumption of unsaturated conditions, is not entirely correct. This is partly because circulating streams are relevant to short road segments on the circulating road (between *entry - circulating road junctions*), and partly because they contain *queued vehicles* entering from approach lanes. Vehicles departing from a queue with follow-up headways are in forced flow conditions, and should be considered to be bunched when negotiating the roundabout even though the follow-up headway is longer than the intrabunch headway specified considering unsaturated uninterrupted flow conditions. An additional factor is *priority reversal* which can cause changes to circulating stream headways (Troutbeck 1999, 2002, Troutbeck and Kako 1997).

Thus, roundabout circulating streams may display characteristics of forced flow conditions. Especially under heavy demand conditions, the proportion of queued vehicles in the circulating stream increases, and the effect of priority reversal may be significant. The critical acceptance headways can be very small under heavy circulating flows, and consideration of all headways above the intrabunch (capacity) headway as unbunched headways (although these are between vehicles entering from upstream approach queues with follow-up headways) can cause overestimation of capacity at the downstream entry.

This explains the *unbalanced flow* conditions when a heavy stream can enter the roundabout with little interruption due to a low circulating flow rate against it. This heavy stream represents mainly forced flow conditions (with follow-up headways that can be larger than the intrabunch headway), and cause reduced capacity at downstream entry. The *origin-destination factor* in aaSIDRA takes into account the flow balance as well as the amount of queuing in the circulating stream, in effect modifying the circulating stream headway distribution to allow for these factors. An alternative method would be to determine an intrabunch headway for the circulating stream using the follow-up headway values of contributing streams for queued vehicles to calculate an average intrabunch headway. If the queued vehicles are treated as bunched and their headway used to increase the overall intrabunch headway of the circulating stream, the capacity of the downstream entry would be reduced.

Similar considerations apply to vehicle platoons departing from queues at signalised intersection approaches. These vehicles cross the signal stop line at saturation headway and speed, and then accelerate towards the speed limit (or a desired speed) after clearing the intersection area, and overtaking vehicles ahead when opportunities arise downstream (Akçelik and Besley 2002). Headways of vehicles in such platoons may be larger than the capacity (intrabunch headway) as they travel downstream, but these vehicles are still under forced flow conditions (at least partly) and the application of the bunched exponential model of headway distribution may become problematic.

While the proposed bunching, speed - flow and other associated models described in this paper appear to give reasonable results, they are recommended for further investigation.

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