Progress in Fuel Consumption Modelling for Urban Traffic Management

R. AKÇELIK (Ed.)

REFERENCE:

NOTE:
This report is related to the intersection analysis methodology used in the SIDRA INTERSECTION software. Since the publication of this report, many related aspects of the traffic model have been further developed in later versions of SIDRA INTERSECTION. Though some aspects of this report may be outdated, this reprint is provided as a record of important aspects of the SIDRA INTERSECTION software, and in order to promote software assessment and further research. This report was originally published by the Australian Road Research Board.
Progress in Fuel Consumption Modelling for Urban Traffic Management

R. Akcelik (Ed.)

with contributions from

R. Akcelik  A.J. Richardson
H.C. Watson  C. Bayley

\[
 f_c = f_i - 0.50 \quad d + 20 \quad h + 124 \text{ ml/km}
\]

where

\[
 f_i = 25 + \frac{1800}{v_c} + 0.00743 v_c - 160
\]

OR

\[
 f_c = 25 + \frac{1800}{v_c} + 0.00743 v_c + 114 \text{ PKE}
\]

\[
 = 119 \text{ ml/km}
\]

AUSTRALIAN ROAD RESEARCH BOARD
RESEARCH REPORT
THE PURPOSE OF THIS REPORT

is to present edited versions of the papers presented at the ARRB Seminar on Fuel Consumption Modelling for Urban Traffic Management held on 9 October 1981, as well as two subsequent papers. These papers represent an evaluation of ideas and findings during the period from September 1981 to July 1982.

THIS REPORT SHOULD INTEREST

traffic engineers, automotive engineers and researchers concerned with fuel consumption modelling and urban traffic management.

THE MAJOR CONCLUSIONS OF THE REPORT ARE

1. Substantial progress has been achieved in specifying fuel consumption models for the design and analysis of urban traffic management schemes.
2. The relation between the models proposed by different traffic engineering and vehicle design engineering groups has been established.
3. Fuel consumption models of different level of detail are available, which can be used for different purposes.

AS A CONSEQUENCE OF THE WORK REPORTED, THE FOLLOWING ACTION IS RECOMMENDED

Further work is necessary on several unresolved questions, in particular on fuel consumption during acceleration, using good quality on-road data representing a wide range of speeds and acceleration rates and realistic acceleration profiles. The work should be extended to include road gradient as a parameter. Papers presented at the 2nd SAE-A/ARRB Conference 'Traffic Energy and Emissions', 19-21 May 1982 should be considered in this respect.

RELATED ARRB RESEARCH

P388 – Appraisal of Small Area Traffic Management Study Tools
P390 – Energy Savings from Traffic Management Actions
P395 – Integrating Evaluation into Area Traffic Control
P397 – Emissions and Fuel Consumption Models
PROGRESS IN
FUEL CONSUMPTION MODELLING
FOR URBAN TRAFFIC MANAGEMENT

by

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Principal Research Scientist
Australian Road Research Board

P367 — Traffic Signal Control Techniques
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FOREWORD

Synergy in research is an often discussed but not so often observed event. It is my view that this report represents a real example of the fruits of such synergy. When the need to develop a better understanding of urban fuel consumption became paramount, we were fortunate in having in the one city the automotive engineering skills and major fuel consumption technology contributions of Harry Watson, the wide analytical skills of Chris Bayley, the transport and traffic modelling skills of Tony Richardson and last, but not least, Rahmi Akcelik with his extensive knowledge of traffic flow and particularly of signalised intersection behaviour. Rahmi and Harry in particular also brought to the group a dogged determination to produce a scientifically credible and practically useful end product. I believe this report, although not yet that final end product, is sufficiently close to it to both demonstrate the value of synergy in research and to provide a powerful and practical tool for the achievement of energy conservation in traffic management.

The report itself is built around events at and the consequences of a seminar on fuel consumption modelling held at ARRB in late 1981 with a view to bringing together the various local researchers in fuel consumption modelling and urban traffic management. The seminar itself revealed a convergence of views towards a common goal and an understanding of outstanding research needs. One key point to emerge was the realisation that fuel consumption models serve a variety of different purposes — vehicle design, traffic engineering, transport planning and strategic planning — and we would be naive to believe that one model would satisfy all needs. Too often in the past, modellers had attempted to force one particular model down a whole range of somewhat unwilling throats. Even traffic engineering models, it was realised, needed to be subdivided into different levels of system aggregation — intersections, links, routes, networks. Rarely could data collected in the field for some aggregate purpose be disaggregated sufficiently to produce data of use to others. Beware of multi-collinearity. But that is the bad news ... the good news is in this report. Read it and use it.

As well as thanking Watson, Bayley, Richardson and Akcelik for their technical work and Akcelik for his persistent editorship, I should also thank Peter Lowrie of DMR-N.S.W. for his practical contributions as the reporter at the seminar.

M.G. LAY
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ABSTRACT

The report collects together edited versions of four papers presented at an ARRB Seminar on Fuel Consumption Modelling on 9 October 1981, as well as two subsequent papers. The objective of the Seminar was to focus on the issue of fuel consumption modelling and data needs for urban traffic management purposes. The first paper (Part 1) specifies the general requirements of a fuel consumption model which is compatible with other elements of the traffic system analysis process. It then discusses an elemental model of fuel consumption as the most appropriate 'simple model' for traffic design and evaluation purposes. In Parts 2 and 3, more detailed discussions on the elemental model are presented, and its relation to the Positive Kinetic Energy (PKE) model is explored. In Part 4, problems associated with fuel consumption measurement are described. The elemental and the PKE models are then discussed in detail and criticism of the elemental model is provided. In Part 5, the authors of the four previous papers present a joint statement of the resolution of differences in the approaches adopted by them for developing simple fuel consumption models. It is shown that, subject to various simplifications and an unexplained term, the elemental and PKE models are very similar. In the last paper (Part 6), results of further studies are reported which answer some of the questions raised in previous parts of the report.
NOTATION AND DEFINITIONS

\( a \) Instantaneous acceleration rate \( (= \frac{dv}{dt}) \) 
\( \dot{a} \) Mean acceleration rate 
\( d \) Delay per vehicle (average) — the difference between interrupted and uninterrupted travel times \( (= t_1 - t_2) \), which consists of stopped delay and deceleration-acceleration delays due to stops \( (d = d_s + hd_n) \) 
\( d_s \) Stopped (idling) delay per vehicle along total section distance 
\( d_s', d_s'' \) Delays per unit distance \( (= \frac{d}{x_s}, \frac{d_s}{x_s}, \text{respectively}) \) 
\( d_n \) Average deceleration-acceleration delay per stop (for \( h \) stops along total section distance, total deceleration-acceleration delay per vehicle is \( hd_n \) ) 
\( F \) Fuel consumption (total) 
\( f \) Instantaneous fuel consumption rate 
\( f_c \) Constant-speed fuel consumption per unit distance 
\( f_t \) Average fuel consumption per unit time (\( = \frac{F}{t_s} \) ) 
\( f_s \) Average fuel consumption per unit distance (\( = \frac{F}{t_s} \) ) 
\( f_c \) Fuel consumption per unit distance while cruising \( (> t_c) \) 
\( f_i \) Fuel consumption per unit time while idling 
\( f_i' \) Excess fuel consumption per stop (see the definition of 'stop' below) 
\( h \) Average number of 'stops' per vehicle — a 'stop' is defined as a complete stop-start manoeuvre which involves a deceleration from an initial (cruise) speed to zero speed and an acceleration back to a final (cruise) speed. A speed-change manoeuvre which involves a non-zero intermediate speed (i.e. slow-down) can be converted to an equivalent number of 'effective stops' 
\( \bar{h} \) Average number of stops per unit distance \( (= \frac{h}{x_s}) \) 
\( \text{PKE} \) 'Positive Kinetic Energy' — sum of positive kinetic energy changes during a trip divided by total section distance \( (= - \frac{\Delta v^2}{2} \cdot 12960 \cdot x_s) \) where \( \Delta v^2 = v_f^2 - v_i^2 \), where \( v_f, v_i \): final and initial speeds (km/h) in a positive acceleration. 
\( t \) Time 
\( t_c \) Cruise time — part of section travel time spent while travelling uninterrupted by traffic control devices 
\( t_u \) Uninterrupted travel time — time to travel along the total section distance without incurring any delay by traffic control devices \( (t_u \geq t_c) \) 
\( t_r \) Running time — time to travel along the total section distance, including deceleration-acceleration delays due to traffic control devices, but excluding stopped delay time, i.e. time while vehicle is in motion \( (= t_u + hd_n = t_s - d_s) \) 

Units

\( \text{km/h/s} \) (or \( \text{m/s}^2 \)) 
\( \text{km/h/s} \) 
\( \text{s (or h)} \) 
\( \text{s (or h)} \) 
\( \text{s} \) 
\( \text{s} \) (or \( \text{h} \)) 
\( \text{mL} \) (or \( \text{L} \)) 
\( \text{mL/s} \) 
\( \text{mL/km} \) 
\( \text{mL} \) (or \( \text{ml/h} \)) 
\( \text{stops} \) 
\( \text{stops/km} \) 
\( \text{m/s}^2 \) 
\( \text{s (or h)} \) 
\( \text{s (or h)} \) 
\( \text{s (or h)} \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_s$</td>
<td>Interrupted travel time — time to travel along the total section distance, including all delays due to interruptions by traffic control devices ($= t_u + d = t_u + d_a + d_h$)</td>
<td>s (or h)</td>
</tr>
<tr>
<td>$t_u$, $t_r$, $t_s$</td>
<td>Travel times per unit distance ($t_u/x_s$, $t_r/x_s$, $t_s/x_s$, respectively)</td>
<td>s/km (or h/km)</td>
</tr>
<tr>
<td>$t_a$</td>
<td>Acceleration time</td>
<td>s</td>
</tr>
<tr>
<td>$t_d$</td>
<td>Deceleration time</td>
<td>s</td>
</tr>
<tr>
<td>$t_h$</td>
<td>Deceleration-acceleration time per stop ($= t_d + t_a$)</td>
<td>s</td>
</tr>
<tr>
<td>$v$</td>
<td>Instantaneous speed ($= dx/dt$)</td>
<td>km/h (or m/s)</td>
</tr>
<tr>
<td>$v_c$</td>
<td>Cruise speed — average speed while cruising uninterrupted by traffic control devices, not necessarily constant ($= x_c/t_c$)</td>
<td>km/h</td>
</tr>
<tr>
<td>$v_r$</td>
<td>Running speed ($= x_r/t_r$)</td>
<td>km/h</td>
</tr>
<tr>
<td>$v_s$</td>
<td>Interrupted travel speed (or 'section' speed) — average speed along the total section distance, including the effects of all delays ($= x_s/t_s$) (used as $\bar{v}$ in previous publications)</td>
<td>km/h</td>
</tr>
<tr>
<td>$x$</td>
<td>Distance</td>
<td>km (or m)</td>
</tr>
<tr>
<td>$x_c$</td>
<td>Part of the total section distance travelled at a speed ($v_c$) uninterrupted by traffic control devices</td>
<td>km</td>
</tr>
<tr>
<td>$x_a$</td>
<td>Acceleration distance</td>
<td>km (or m)</td>
</tr>
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<td>$x_d$</td>
<td>Deceleration distance</td>
<td>km (or m)</td>
</tr>
<tr>
<td>$d_h$</td>
<td>Deceleration-acceleration distance per stop ($= x_d + x_a$)</td>
<td>km (or m)</td>
</tr>
<tr>
<td>$x_s$</td>
<td>Total section distance</td>
<td>km</td>
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</tbody>
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Part 1

FUEL CONSUMPTION MODELS AND DATA NEEDS FOR THE DESIGN AND EVALUATION OF URBAN TRAFFIC SYSTEMS

by

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(First written in September 1981)
1. INTRODUCTION

In cases where there are many factors which have a notable bearing on a problem, we find that for research to be tolerable at all we have to restrict our investigation to the observation of relatively few of the factors. We shut our eyes to the rest, either deliberately because we just cannot cope with everything, or unconsciously because we just cannot name all the factors anyway. But the fact that we shut our eyes to factors does not mean that they cease to exist and to exert an influence. When we can name a factor which we are going deliberately to ignore, we can often do something to minimise the disturbing effect of its existence on our results by experimental design before the experiment is put under way. We can arrange for the factor to be held constant during the course of the experiment, or failing this, we take steps to ensure that such a factor shall not introduce bias into our data which would lead to misleading conclusions. When we are ignorant of the nature of disturbing factors we just have to let them do their worst and hope that they will not introduce such confusion into our data that we can never find anything significant in them.

(Moroney 1951)

The above quotation illustrates one of the problems encountered when attempting to estimate fuel consumption in urban traffic systems. Whilst there are undoubtedly very many factors which may affect fuel consumption, the choice of which factors to consider will be determined largely by the circumstances under which such estimations are to be made. For example, the requirements of vehicle designers, traffic engineers and urban planners are substantially different with respect to the input data and output required from such fuel consumption models.

This paper is written in an attempt to summarise the requirements and data needs of traffic engineers when attempting to incorporate fuel consumption as an explicit design and/or evaluation parameter in urban traffic management/control studies. The range of such traffic engineering work is considerable and includes schemes such as:

(a) isolated traffic signals;
(b) traffic signal co-ordination/area traffic control;
(c) Give-Way/Stop signs;
(d) roundabouts;
(e) public transport priority lanes and signals;
(f) clearways;
(g) turn prohibitions.

It is important to note that fuel consumption is only one of several performance measures which can be used to assess the effect of each, or any combination of these traffic management schemes. Other measures which might be used include various traffic performance measures (travel time or speed, delay, number of stops, queue length, etc.), safety, air pollution, noise, and the elements of vehicle operating cost other than fuel consumption (e.g. tyre wear). The effects on different road user groups such as pedestrians, buses, commercial vehicles as well as cars, and effects on different elements of the road system such as major roads and side roads need also to be considered. Detailed discussions on this aspect of the problem can be found in earlier papers by the authors (Akcelik 1981a; Richardson and Graham 1980). Whilst recognising the importance of these other measures, the present paper concentrates on the prediction of changes in fuel consumption which would result from implementing various traffic management schemes.

2. TRAFFIC SYSTEM ANALYSIS PROCESS

Before proceeding to discuss fuel consumption modelling in detail, the role of fuel consumption models in traffic system analysis should be specified. As shown in Fig. 1, the traffic system analysis process can be considered to utilise three distinct types of model: traffic system model, traffic demand model and traffic impact model. The process starts with an initial description of the traffic system in terms of the following components.

(a) Physical characteristics: intersection layouts, lane configurations, site factors related to interferences from commercial activities, property access, bus stops and pedestrians, distances between intersections, mid-block road characteristics, etc.

(b) Control characteristics: the type of control (signals, roundabout, Give-Way/Stop signs), and the details of control, e.g. signal phasing and settings, any turn prohibitions, etc.

(c) Traffic flow characteristics: total flow rates and turning volumes (ideally specified separately for successive time intervals) and the composition of traffic, i.e. relative proportions of cars, public transport vehicles and commercial vehicles.

These data are then used by the traffic system model to obtain predictions of the traffic system performance characteristics in terms of, for example, average travel time (or delay) per vehicle, number of vehicle stops, queue length, etc. (e.g. see Akcelik (1981b) for the formulae used for isolated traffic signals).

Depending on the complexity required of the traffic analysis process, and considering the nature of the performance characteristics, some studies then examine the effect of the traffic system performance characteristics on the traffic demand at the site, or in the area, in question. If scope exists for changes in travel demand due to generation, suppression or diversion of trips to or from other routes, modes or destinations then the extent of such changes should be assessed at this time by means of a traffic demand model. Such a model predicts the response of the traveller population to the performance characteristics of the traffic system to produce new estimates of traffic flows for the system in question. This new estimate of traffic flows should then be input into the traffic system model to produce a new set of traffic system performance characteristics. The process is then repeated until a stable set of traffic flow estimates is obtained. Because of the relatively minor effects of many traffic management schemes, the traffic demand model is often omitted from the traffic system analysis process on the grounds that no demand changes are to be expected. This conclusion, however, should not be
drawn in all circumstances, especially when long-
term demand changes are considered or when short-
term route changes are likely, e.g. attraction due to a
free right turn phase at traffic signals. Some
familiarity with the concepts of travel demand modell-
ing is therefore recommended (e.g. Stopher and
Meyburg 1976).

Once stable estimates of traffic flow and perfo-
rance characteristics have been obtained (either
with or without demand model iterations or directly
from simple field survey methods), these may be input
into various traffic impact models to assess the over-
all impacts of the traffic system. It is at this stage in
the traffic analysis process that a fuel consumption
model is needed. Other traffic impact models might
include air pollution and noise models, vehicle
operating cost models, travel time evaluation models
and possibly traffic safety models such as models of
accident exposure. The term ‘traffic impact’ corres-
ponds to the term ‘secondary measure of perfor-
mance’ used in Akcelik (1981b).

The next step is to examine the traffic system im-
(physical performance characteristics
to determine whether they are satisfactory or not (the
evaluation phase). If it is expected that they can be
improved upon, then modifications can be affected to
the physical characteristics (e.g. additional lanes at
the intersections) or control characteristics (e.g. sig-
nal co-ordination to replace isolated operation of
signals) of the system. The relevant data (system
description) can be changed and the analysis pro-
cess repeated until an optimum set of impacts, or
simply the best possible solution given all practical
constraints, is achieved. Normally, separate analyses
are carried out for different times of the day, e.g.
morning peak, evening peak and off-peak periods,
and an overall evaluation is made in terms of the per-
formance and impact measures obtained for different
times of the day.

One feature which emerges from this traffic
system analysis process is that traffic system
models, traffic demand models and many of the traffic
impact models are already well developed. What is
needed in terms of a fuel consumption model is one
which can be readily incorporated into this overall
process.

3. GENERAL MODEL
SPECIFICATION

Given that it is desirable that a fuel consumption
model be compatible with other elements of the traffic
system analysis process, the general requirements of
such a model can be specified by reference to five
major factors: the range of options to be evaluated,
the method of collecting (or generating) input data,
the type of input data available, the output required
from the model and the statistical considerations of
model building.

3.1 RANGE OF OPTIONS

It is perhaps obvious that as well as the range of
traffic management schemes listed earlier, there is
also a wide range of design/control options available
for each, or any combination, of these schemes. For example, the following are among the design/control variables which can be manipulated for improved operation of traffic at signal-controlled intersections:

(a) alternative phasing arrangements, including considerations of opposed (filter) turns against unopposed (free) turns, pedestrian-only against concurrent vehicle-pedestrian phases, etc.;
(b) alternative criteria for signal settings;
(c) alternative vehicle-actuated control algorithms for isolated signals;
(d) various fixed-time plan selection and dynamic control strategies for co-ordinated signals;
(e) the width and number of lanes;
(f) alternative arrangements to allocate the available lanes to various movements, e.g. exclusive or shared lanes.

3.2 DATA COLLECTION METHODS

The form of fuel consumption models is influenced by the method in which data are collected on traffic system performance. Two distinctly different methods of traffic observation (both in models and real-life) exist and these have led researchers to develop different types of fuel consumption model.

The first method is based on the observation of traffic flows as they pass certain critical points in the system, e.g. stop-lines at intersections. Traffic signal capacity calculations (e.g. Akcelik 1981b) and a signalised intersection survey method to obtain traffic performance statistics (Richards and Read 1979 and 1980a) are related to this approach. Similarly, area traffic control systems collect traffic data from detector loops buried in the road pavement. From such information, a picture of the overall system state can be drawn and the relevant performance measures calculated. These methods of observation are most common in traffic engineering practice.

The second method is based on observing the performance of individual vehicles as they travel through the system. In real life, this corresponds to the 'moving observer' method often used to measure travel time along a route. In modelling, this requires a detailed vehicle-by-vehicle simulation model which can produce an output of individual vehicle time-distance trajectories (e.g. Lieberman et al. 1979, Gipps and Wilson 1980). This method has naturally been adopted by researchers in the field of vehicle design, since they are concerned with the performance of individual vehicles. The 'driving cycle' methods have been used to measure individual vehicle performances under standardised conditions. However, it has been recognised that this method is not of particular use for traffic management purposes (Watson 1978, Kent 1981). In contrast with the 'bird's eye view' of the overall system preferred by traffic engineers, the moving observer method gives a 'worm's eye view' from a single car in a traffic stream. This method suffers from the fact that it is difficult, or impossible, to relate the observed data to the causes of delays, stops, etc. as affected by traffic control, physical and flow characteristics of the system. For example, a vehicle may stop several times in a long queue before it can clear an intersection, and it may be impossible to know if this is caused by inadequate capacity at the next intersection, or say the third intersection downstream, or some mid-block interference, e.g. by a turning vehicle.

Given these different methods of data collection, it is essential that models are developed which are compatible with the type of data which have been collected, or are available.
3.3 INPUT DATA DETAIL

It is important to realise that whilst the traffic engineer would normally possess information regarding traffic performance at a site, he would not have complete information about that traffic stream. In particular, he would not have details about:

(a) vehicle fleet characteristics such as engine size, vehicle weight, transmission type, etc.;
(b) vehicle performance characteristics such as engine speed or torque, current gear, engine tune-up condition, tyre inflation, engine temperature, etc.; and
(c) driver population characteristics such as age, sex, degree of aggressiveness (as reflected in speed and rates of acceleration/deceleration).

Whilst these factors may have a direct influence on fuel consumption, and may be the subject of research studies, practical design studies must account for them by means of aggregation.

Thus vehicle performance and driver population characteristics must be accounted for by typical values which are appropriate in the given circumstances. With respect to vehicle fleet characteristics, some aggregation of data is necessary, whereby vehicles may be classified into a limited number of classes, the simplest classification being "light and heavy vehicles" (e.g. see Akcelik 1981b, p. 15).

3.4 REQUIRED MODEL OUTPUT

In determining the output required of a fuel consumption model, one should consider the two major applications of such a model in traffic management: design and evaluation. In a design context, the objective is to find a set of traffic system parameters which result in an optimal set of traffic system impacts. In such an optimisation problem, the actual value of the objective function (e.g. fuel consumption) is not as important as the changes in the value of the objective function for changing values of traffic system parameters. In an evaluation context, the problem is again one of comparing alternative sets of impacts (usually before and after implementing a traffic management scheme). It therefore appears that, for some traffic management purposes, relative measures of fuel consumption may be adequate. Given this, it is realised that other applications may well require accurate absolute measures and that the development of a fuel consumption model which predicts absolute fuel consumption will, inter alia, produce relative measures of fuel consumption if adequate data are available. However, particular applications of fuel consumption models to traffic system design and evaluation may omit elements of the system which are not expected to experience changes. For this reason, and because data are normally only collected on those aspects of system operation which are expected to experience changes, it is desirable to use some form of model which treats different components of traffic system operation separately.

3.5 STATISTICAL CONSIDERATIONS

A note is appropriate at this stage about the use of formulae based on regression analyses of observed data for predicting fuel consumption from measures of average speed and, perhaps, physical descriptions of the system such as number of intersections per kilometre of road (e.g. Evans and Herman 1976; Watson, Wilkins and Marshall 1980). The authors believe that such formulae do not satisfy various requirements to enable them to be useful for traffic engineers in designing and evaluating urban traffic management schemes (however, this is not to say that they are not useful in another context, e.g. for general transport planning purposes where more macroscopic models are needed, or in cases where prediction only is sufficient, i.e. no need for optimisation).

The reasons for this reservation about regression models are three-fold. Firstly, those equations which attempt to use physical descriptions as independent variables cannot hope to account for the enormous range of options described earlier and hence such models will always be mis-specified for traffic engineering applications. The variables in a regression equation are subject to upper and lower limits representing the range of observations on which the equation is based. In practice, the existence of limitations on the range of applicability of variables is often neglected resulting in mis-use of regression equations. For the examples of traffic work which give due emphasis on this point, the reader is referred to Beard and McLean (1974) and Freeman Fox and Associates (1972) on speed-flow relationships and Kimber (1980) on roundabout capacities.

Secondly, those equations which use average speed as the only independent variable describing traffic performance do now allow for the different fuel consumption rates whilst cruising, stopping and idling. As shown by Akcelik (1981a), trips with identical average speeds can have very different fuel consumption performance, depending on the pattern of stops. Since many traffic management options may change the correlation between average speed and stops it is necessary to have both variables in the equation.

Thirdly, if both average speed (or delay) and stops are included as predictor variables in the regression equation, then the coefficients obtained from field data are likely to be unreliable due to multicollinearity of the predictor variables. That is, there is correlation between these two supposedly predictor variables, because average speed and number of stops will always be related to each other in uncontrolled field data. Under such conditions, it is difficult, if not impossible, to identify the individual contribution of each variable to overall fuel consumption.

It is therefore argued that a model of fuel consumption for traffic management design and evaluation should be constructed using data derived from carefully designed and controlled experiments to relate various fuel consumption rates to their causes directly and explicitly.

4. THE ELEMENTAL MODEL

Given the above requirements and limitations, the authors consider that an elemental model of fuel consumption is the most appropriate 'simple model' for traffic management studies. Such a model, which expresses fuel consumption as a function of the three
The side friction encountered along a route is a function of such factors as the number and width of lanes, quality of the road geometry, adjacent land use (affecting property access, commercial activities, kerb parking conditions, bus stops, pedestrian activities), traffic turning in from side streets, etc. The internal friction is a function of the flow level and traffic composition, and it is considered to include the effects of lane changes and overtakings due to different vehicle speeds in a traffic stream. The effects of interference by vehicles turning into side streets at mid-block locations could also be considered to be part of internal friction.

The coefficient \( f_s \) therefore accounts for both fuel consumed whilst travelling at a steady (constant) cruise speed \( (f_c) \) and fuel consumed as a result of speed fluctuations due to side and internal frictions \( (\Delta f_c) \):

\[
f_s = f_c + \Delta f_c
\]

Fuel consumption at steady speeds has been investigated by several researchers (Claffey 1971 and 1976; Kent 1981; Kent et al. 1981; Vincent et al. 1980; Watson et al. 1980). Fig. 2 summarises the following data:

- U.S. composite car data (Claffey 1971);
- Data for the Melbourne University test car (Watson et al. 1980); and
- Data for 11 cars (Kent et al. 1981).

It is seen that all data show similar trend, and significantly, minimum fuel consumption occurs at a steady cruise speed in the range of 40-60 km/h.

In real driving conditions, however, it is impossible to maintain a steady cruise speed because of side and internal friction and, as a result, fuel consumption under real driving conditions is higher than under steady speed conditions. This increase in fuel consumption \( (\Delta f_c) \) should be related to the magnitude and frequency of speed fluctuations. However, in the absence of detailed information on actual speed fluctuations, it appears that these fluctuations may need to be accounted for by a factor which is dependent on the side and internal friction conditions, e.g. using \( \Delta f_c = \alpha f_c \) and \( f_s = (1 + \alpha) f_c \).

It may not be feasible to quantify all side friction factors explicitly (see e.g. Beard and McLean 1974; Freeman Fox and Associates 1972). For this reason, and for the purpose of deriving an easy-to-use method, it may be useful to define several 'types of environment' \( (E) \) representing aggregate, and to some extent subjective, values of side friction factors. For example:

**Type 1:** *Ideal* conditions of uninterrupted travel (no side friction, typically in freeway-type environment)

**Type 2:** *Good* conditions of uninterrupted travel (major arterial roads with negligible side friction).

**Type 3:** Average conditions of uninterrupted travel (arterial and other roads with a moderate degree of side friction).

**Type 4:** *Poor* conditions of uninterrupted travel (roads with a high degree of side friction, typically in city centre areas).
The internal friction can be measured in terms of varying flow levels, e.g. as a function of the flow ratio, \( y \) (the ratio of flow to saturation flow, where saturation flow is the uninterrupted road capacity). The speed fluctuation adjustment factor \( (1 + \omega) \) can then be calculated as a function of both \( E \) and \( y \), perhaps by using \( E \) to modify the saturation flow as is done with signalised intersection design (Akcelik 1981b).

As an alternative to this method, free speed (average speed under very low flow conditions) can be used as a surrogate measure of various environment types, and the ratio of the cruise speed (i.e. the achieved speed) to the free speed can be used as a combined measure of side and internal frictions. This speed ratio may then be related to the speed fluctuation adjustment factor. Already such a speed ratio has been shown to be related to travel time variability (Richardson and Taylor 1978) and hence a similar relationship with speed fluctuations (or variability) may also be quite viable.

In summary, the cruise component of the elemental model could be expressed in terms of a steady-speed fuel consumption term modified by a speed fluctuation adjustment factor which is related to the side and internal friction experienced on that road segment. Similarly, if required, data in the form of adjustment factors could be given to account for the effects of geometric features of the road such as grade and curvature. Such geometric and physical characteristics may, however, be of more relevance in rural road fuel consumption than in urban fuel consumption calculations.

4.2 IDLE

The second component of the elemental model, \( f_2 \), is the fuel consumed whilst idling when a vehicle is stopped by a control element of the traffic system, e.g. a Stop sign, or traffic signals. With respect to the measurement of idling time, it should be emphasised that stopped delay \( (d_s) \) in eqn (1) is different to the average delay normally used in many traffic models. Stopped delay, as the name implies, is only that delay incurred when the vehicle is actually stopped. Delay incurred in decelerating and accelerating is not included in stopped delay (see Richardson 1980a; Akcelik 1981a, and Parts 3 and 5 of this report).

The fuel consumed whilst idling appears to be dependent essentially on the engine idling speed, although Martin (1974) states that idling consumption is also dependent on ignition timing, engine temperature and combustion efficiency. The elemental model coefficient for idling \( (f_2) \), however, simply expresses the fuel consumption rate per second (or hour) of idling time independent of other factors. This is because engine idling rate is a vehicle design parameter and not a traffic management parameter. However, the effect of vehicle design changes with respect to idling fuel consumption should be recognised in terms of the effect on traffic management strategies. Thus, a change in fuel consumption whilst idling will change the trade-off between the stopped delay (idling) time and the number of stops in traffic design and evaluation.
4.3 STOP-START MANOEUVRES

The third component of the elemental model, \( f \cdot h \), is the excess fuel consumption associated with stop-start manoeuvres caused by traffic control devices. The measurement of the average number of effective stops per vehicle, \( h \), needs to take account of two factors:

(a) vehicles which do not stop but which change speed significantly as a result of the control system should be counted as partial stops; and

(b) vehicles which stop more than once as a result of the control system should contribute multiple (partial) stops to the total count.

The effects of partial stops have been considered by several authors (Richardson 1979; Vincent et al. 1980; Akcelik 1980) whilst consideration of the partial effects of multiple stops has more recently been addressed (Richardson 1980a and b; Ferreira 1981). In both situations, the critical factors are the maximum and minimum speeds experienced during the manoeuvre and the rates of acceleration and deceleration utilised by the driver.

Fuel consumption rates for partial stops (slowdown cycles), which involve a change in speed from the cruising speed to a lower speed and then back to the cruising speed, could be presented as fractions of complete stops in proportion to the fuel consumed in the slowdown manoeuvre. In the case of multiple stops, all stops except the first one correspond to a reverse cycle, i.e. acceleration from zero speed to a higher speed (maximum value \( v_c \)) and back to zero speed again. This may occur at Give-Way/Stop signs and roundabouts where all vehicles in the queue move forward and stop again as one or more vehicles accept a gap in the opposing stream and depart. Similarly, at over-saturated signals, several vehicles at the end of the queue speed up but get stopped again before they can clear the intersection because of insufficient capacity. This type of fuel consumption rate could also be presented as 'effective stop' figures as in the case of slowdown manoeuvres.

The coefficient \( f \cdot \) is related to the excess amount of fuel consumed in a stop-start manoeuvre (with no idling) from an initial cruising speed to zero speed and back up to the same cruising speed. The coefficient is obviously a function of the cruising speed adopted. This is discussed in detail in Parts 3 and 5 of this report.

It is important to note that \( f \cdot \) in eqn (1) is an excess consumption figure. It is calculated as the consumption during a complete stop-and-go cycle (with no idling time) less the consumption when the stop-start manoeuvre distance is covered at cruise speed. By using \( f \cdot \) as an excess consumption figure in eqn (1), the first component of the model is conveniently expressed in terms of the total length of the road section under study. The separation of the first and third components of the model is most useful when estimating changes in fuel consumption as a result of a change in traffic management strategies at an intersection. In such circumstances, where average cruise speed and cruise speed fluctuations are unlikely to be affected, the change in fuel consumption may be estimated as:

\[
\Delta f_s = f \cdot \left( \Delta d_s \right) + f \cdot \left( \Delta h \right)
\]

where \( \Delta f_s \) = change in fuel consumption due to stops and delays, \( \Delta d_s \) = change in stopped delay, and \( \Delta h \) = change in effective stops.

It is interesting to note that Tarnoff and Parsonson (1981) abandoned the use of a detailed simulation model for fuel consumption (NETSIM by Lieberman et al. 1979) in favour of the use of the elemental model approach based on eqn (3) for individual signalised intersections.

In practice, it is difficult to separate the cruise and stop-start components of fuel consumption from a continuous record of fuel consumption in a single vehicle trip. Away from intersections, the distinction between partial stops and speed fluctuations, as described earlier in connection with cruise fuel consumption, becomes rather blurred and may be difficult to identify in the field by a moving observer. Similarly, it may be difficult in the field to identify the cause of multiple stops in a long queue, i.e. if they are due to the control system (related to the third component of the elemental model) or due to mid-block interferences (related to the first component of the elemental model). Therefore, careful experimental design is required for this purpose. It is relatively easier to allow for the differences between different speed change manoeuvres in a model. However, it is important that the relevant fuel consumption data are presented in a form which matches the requirements of the traffic model.

5. FORMAT OF FUEL CONSUMPTION DATA

Given the form of the elemental model of fuel consumption specified in eqn (1), the question remains as to the most convenient format for the fuel consumption data. For routine traffic engineering work, where information and time is at a premium, the most convenient format would appear to be look-up tables or equations approximating those tables. Data could be given as 'correction factors' rather than actual consumption figures in some cases.

The values of all fuel consumption parameters depend on the type of vehicle. A major requirement of traffic engineers is therefore to have data for different types of vehicle. Too much detail in terms of vehicle and driver characteristics should, however, be avoided as discussed in Section 3. Data should be aggregated and presented for a set of 'representative vehicles', e.g. for cars, light trucks, heavy trucks and buses, or simply for 'light vehicles' and 'heavy vehicles'. Whilst greater detail is obviously required for vehicle design purposes, and perhaps in traffic engineering research studies, further detail for general traffic engineering practice would tend to be more confusing than illuminating. Data for present day vehicle populations should be used as a basis of data aggregation (e.g. Fehon 1980).

For each representative vehicle, the cruise fuel consumption rate could be shown as in Table I. As discussed in Section 4.1, the choice of 'environment types' for use in Table I is arbitrary and could also be
represented by the free speed of the section of road under consideration. It should be noted that within each environment type, the effect of flow (internal friction) is implicitly accounted for by reduced average cruise speed.

The idling fuel consumption rate will be a single value in millilitres per second (or hours) of stopped delay for each representative vehicle. The excess fuel consumption per effective stop might be tabulated as shown in Table II. Such a Table would be constructed with an implicit assumption about the patterns and rates of acceleration and deceleration used in the manoeuvre. The top row of the matrix gives the excess fuel consumption during a multiple stop manoeuvre (i.e. from zero speed to finite speed and back to zero), the left column gives values for a complete stop manoeuvre, the lower left triangle of the matrix gives values for partial stops (slowdown manoeuvres), while the upper right triangle gives values for speed-up manoeuvres (if needed). In each case, the speed at the start and end of the manoeuvre is the same.

To conclude, reference should be made to the way in which data might be obtained to enable construction of the Tables described in the paper. Already it has been argued that regression analysis of uncontrolled field survey data should not be used to obtain fuel consumption rates for each of the three components of the elemental model. Rather, a carefully controlled experimental program should be undertaken to obtain the rates in one of two ways. Firstly, the rates could be obtained directly by performing test runs which conform to strictly defined manoeuvres, e.g. a series of partial stop cycles from an initial speed to an intermediate speed and back up to the final speed (with specified rates of acceleration and deceleration).

An alternative, and more general, method of obtaining the data is by the generation of vehicle maps as described by Kent (1981). These maps show the rate of fuel consumption (in mL/min) as a function of speed and rate of acceleration and would need to be generated for a set of representative vehicles. It would be desirable if such maps were also obtained under controlled conditions whereby the sample size for each point in the matrix (within feasible boundaries) was approximately equal. This would ensure that equal statistical reliability could be attached to each point in the matrix. Given this matrix, the fuel consumption for any manoeuvre could be calculated by tracing the speed/acceleration trajectory over time on the matrix and integrating the resultant fuel consumption rates. This would enable Tables I and II to be generated and would also be more useful in traffic research work where, occasionally, the researcher has access to detailed vehicle trajectories (e.g. Gipps and Wilson 1980).

6. CONCLUSION

This paper has attempted to summarise the requirements of traffic engineers with respect to fuel consumption models. As a result of this summary, a number of conclusions can be reached.

(a) Fuel consumption models are but one part of the traffic system modelling process. They should therefore be designed to complement existing models for other parts of the process.

(b) Information on changes in fuel consumption rather than total fuel consumption values may be adequate for some traffic management studies.

(c) The data available to traffic engineers are usually limited and hence fuel consumption models must be able to be used with such limited data.

(d) Traffic engineers are often concerned with individual elements of the traffic system (e.g. an intersection) and hence fuel consumption models must be applicable at this level.

(e) Because of the large range of traffic options to be evaluated, and bearing in mind the statistical problems with regression analysis on field data, it is unlikely that a reliable set of regression equations can be developed to cover all combinations of the many possible traffic management (design/control) options. In view of the large extant body of knowledge covering traffic

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>CRUISE FUEL CONSUMPTION RATES (mL/km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Cruise Speed (km/h)</td>
<td>Side Friction</td>
</tr>
<tr>
<td>Environment Type</td>
<td>1 (Ideal)</td>
</tr>
<tr>
<td>Free Speed (km/h)</td>
<td>120</td>
</tr>
<tr>
<td>20</td>
<td>—</td>
</tr>
<tr>
<td>40</td>
<td>—</td>
</tr>
<tr>
<td>60</td>
<td>—</td>
</tr>
<tr>
<td>80</td>
<td>—</td>
</tr>
<tr>
<td>100</td>
<td>—</td>
</tr>
<tr>
<td>120</td>
<td>—</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>EFFECTIVE STOP FUEL CONSUMPTION RATES (mL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial (or final) Speed (km/h)</td>
<td>Intermediate Speed (km/h)</td>
</tr>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td>20</td>
<td>—</td>
</tr>
<tr>
<td>40</td>
<td>—</td>
</tr>
<tr>
<td>60</td>
<td>—</td>
</tr>
<tr>
<td>80</td>
<td>—</td>
</tr>
<tr>
<td>100</td>
<td>—</td>
</tr>
</tbody>
</table>
system models, the development of such regression equations would also appear to be unproductive.

(f) In view of these requirements, some form of elemental fuel consumption model would seem most appropriate for traffic engineering purposes. Such a model would account for the cruise, idle and stop-start components of driving.

(g) The most appropriate format for fuel consumption data would appear to be in the form of tables or equations for practising traffic engineers. Data to allow for the effects of different road grades, various friction factors, different deceleration-acceleration patterns and rates, etc. are needed.

(h) Data enabling the calculation of fuel consumption rates should be obtained under strictly controlled experimental conditions, with the construction of vehicle maps being a useful intermediate step.

(i) The fuel consumption data should be available for a range of 'representative vehicles' such as cars, light trucks, heavy trucks and buses, or simply light vehicles and heavy vehicles.
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FREEMAN FOX AND ASSOCIATES (1972). Speed-flow relationships on suburban main roads (a report for the Road Research Laboratory, Crowthorne), London.


INSTITUTE OF TRANSPORTATION ENGINEERS (1980). Selection of traffic signal control and timing at individual intersections. ITE Educational Foundation, Washington, D.C.


APPENDIX

FURTHER READING


Part 2

PREDICTION OF CHANGES IN FUEL CONSUMPTION: TWO EXAMPLES

by

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(First written in October 1981)
1. INTRODUCTION

This paper presents two simple examples in order to compare the prediction abilities of the elemental model (Parts 1 and 3 of this report) and the PKE model (Watson, Milkins and Marshall 1980; Watson 1980 and Part 4 of this report). Both the overall and the incremental fuel consumption predictions from:

(a) the elemental model for three different cars; and
(b) the elemental model and three differently calibrated forms of the PKE model for the same car are considered.

The first example is related to the effects of changes in signal timings allocated to a movement at an isolated intersection. The traffic performance variables (delay, number of stops, speed and PKE) are predicted using the simple analytical models given in Akcelik (1981a) as a basis. These are then used as input for the fuel consumption models. Thus, the example shows the combined use of traffic and fuel consumption models to analyse the effects of small changes in traffic operating conditions in a specific situation. The example is simplistic in the treatment of traffic signal timings. The reader is referred to Akcelik (1981b) for a full intersection example.

The second example is related to the prediction of incremental fuel consumption due to extra idling (stopped) time. It is given to complement the discussion of the prediction of incremental fuel consumption due to stops in the first example.

2. EFFECTS OF CHANGES IN SIGNAL TIMINGS

2.1 BASIC DATA

This example considers a through traffic movement in two lanes of an approach road to an isolated signalised intersection under three different signal timing, and hence capacity, conditions described as Cases A, B and C in Table I. Hypothetical cruise (uninterrupted travel) conditions are also indicated in Table I. For basic definitions and formulae related to traffic movement variables, the reader is referred to Akcelik (1981a). Data common to all cases are as follows.

<table>
<thead>
<tr>
<th>BASIC DATA</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Cruise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green time, ( g ) (s)</td>
<td>60</td>
<td>60</td>
<td>64</td>
<td>N.A.</td>
</tr>
<tr>
<td>Cycle time, ( c ) (s)</td>
<td>120</td>
<td>100</td>
<td>100</td>
<td>N.A.</td>
</tr>
<tr>
<td>Green time ratio, ( g/c )</td>
<td>0.50</td>
<td>0.60</td>
<td>0.64</td>
<td>N.A.</td>
</tr>
<tr>
<td>Capacity, ( Q ) (veh/h)</td>
<td>1800</td>
<td>2160</td>
<td>2304</td>
<td>3600 (( =s ))</td>
</tr>
<tr>
<td>Degree of saturation, ( q/Q )</td>
<td>1.00</td>
<td>0.63</td>
<td>0.78</td>
<td>0.50 (( =y ))</td>
</tr>
</tbody>
</table>

2.2 TRAFFIC PERFORMANCE CALCULATIONS

Using the basic data given above and in Table I, the average delay \( d \) and the average number of stops \( h \) are calculated for each case from eqns (6.1) to (6.5) of Akcelik (1981a, Section 6). From these, the results given in Table II are calculated as follows (see Part 3 for the basis of these calculations):

(a) Delay and number of stops per unit distance (s/km, stops/km):

\[
\frac{d}{x_s}, \frac{h}{x_s}
\]

(b) Average speed (km/h):

\[
\frac{3600}{3600 + \frac{d}{x_s}} = \frac{3600}{66.7 + \frac{d}{x_s}}
\]

(c) Total positive kinetic energy (m/s²):

\[
PKE = \frac{h v_c^2}{12960} = 0.225 h
\]

It is assumed that there are no speed fluctuations during cruise, and hence, PKE is only due to stops at the intersection. For this example, it is estimated that all vehicles will reach cruise speed during multiple stops.

Absolute and percentage changes in traffic performance variables are given in Table III. It should be noted that a change from Case A to Case B represents a substantial change (from saturated at-capacity conditions in Case A to an acceptable level of service in Case B). A change from Case B to Case C represents a relatively minor change in that both cases have similar degrees of saturation, and hence similar levels of service.

2.3 FUEL CONSUMPTION CALCULATIONS

The traffic performance data given in Table II are used as input for fuel consumption calculations using the elemental model (\( E \)) and the PKE model (\( P \)) with different data as given below.

E1. Elemental model with data from Claffey (1976) for 1972 Chevrolet sedan (V-8, 6.5 L):

\[ f_e = 116 + 0.883 d + 38.5 h \]

E2. Elemental model with data from Robertson, Lucas and Baker (1980) for a medium family saloon (6 cyl., 2.2 L) at \( v_c = 52 \text{ km/h} \):

\[ f_e = 94 + 0.417 d + 14.1 h \]


\[ f_e = 89 + 0.700 d + 13.4 h \]
TABLE II

<table>
<thead>
<tr>
<th>TRAFFIC PERFORMANCE DATA</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Cruise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average delay, $d$ (s/km)</td>
<td>87.0</td>
<td>22.6</td>
<td>17.3</td>
<td>0</td>
</tr>
<tr>
<td>Average number of stops, $\bar{n}$ (stops/km)</td>
<td>1.55</td>
<td>0.97</td>
<td>0.86</td>
<td>0</td>
</tr>
<tr>
<td>Average speed, $v_s$ (km/h)</td>
<td>23.4</td>
<td>40.3</td>
<td>42.9</td>
<td>54 (= $v_c$)</td>
</tr>
<tr>
<td>PKE (m/s²)</td>
<td>0.349</td>
<td>0.216</td>
<td>0.194</td>
<td>0</td>
</tr>
</tbody>
</table>

TABLE III

<table>
<thead>
<tr>
<th>CHANGES IN TRAFFIC PERFORMANCE</th>
<th>Case A-Case B</th>
<th>Case B-Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average delay, $d$ (s/km)</td>
<td>64.4 (74%)</td>
<td>5.3 (23%)</td>
</tr>
<tr>
<td>Average number of stops, $\bar{n}$ (stops/km)</td>
<td>0.58 (37%)</td>
<td>0.11 (11%)</td>
</tr>
<tr>
<td>Average speed, $v_s$ (km/h)</td>
<td>-16.9 (72%)</td>
<td>-2.6 (6%)</td>
</tr>
</tbody>
</table>

P1. PKE model by Watson et al. (1980) for the same car as in E3:

\[ f_s = -11.2 + 2597/v_s + 0.811v_s + 121.1 \text{ PKE} \]

P2. PKE model by Watson (1980) for the same car as in P1:

\[ f_s = -30.7 + 2903/v_s + 1.216v_s + 94.21 \text{ PKE} \]

P3. PKE model by Poynton and Dawson (1980) for the same car as in P1:

\[ f_s = -46.9 + 3093/v_s + 1.342v_s + 90.66 \text{ PKE} \]

The following should be noted about these formulae:

(a) The elemental model coefficients were derived individually by direct measurement: on the road for E1 and E2, and in the laboratory (chassis dynamometer tests) for E3.

(b) The PKE model coefficients were derived for the same car by regression analysis using data from real-life traffic conditions for P1, and data from various driving cycle tests in the laboratory for P2 and P3 (ADR 27A cycle for P2; ADR 27A as well as U.S. Highway, Sydney and Melbourne Initial cycles for P3).

(c) All cars had automatic transmissions.

(d) Different assumptions regarding deceleration and acceleration rates and profiles during a stop are built into the above models. For E3, constant deceleration and acceleration rates were used (average $a_n = 5.33 \text{ km/h/s}$). For E1, the average deceleration-acceleration rate is $a_n = 4.9 \text{ km/h/s}$ but the deceleration/acceleration pattern is not clear. The PKE model (P1 to P3) assumes independence from the acceleration rates and profiles.

(e) Variables in the above formulae are:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_s$</td>
<td>Fuel consumption per unit distance</td>
<td>mL/km</td>
</tr>
<tr>
<td>$\bar{d}$</td>
<td>Average delay per unit distance</td>
<td>s/km</td>
</tr>
<tr>
<td>$\bar{n}$</td>
<td>Average number of stops per unit distance</td>
<td>stops/km</td>
</tr>
<tr>
<td>$v_s$</td>
<td>Average interrupted speed</td>
<td>km/h</td>
</tr>
<tr>
<td>PKE</td>
<td>Sum of positive kinetic energy changes</td>
<td>m/s²</td>
</tr>
</tbody>
</table>

(f) The first term of the elemental model is the steady-speed cruise fuel consumption (at $v_c = 54 \text{ km/h}$). The corresponding figure from the PKE model is obtained by putting $v_s = v_c = 54$ and $\text{PKE} = 0$ in the formula.

The fuel consumption results are given in Table IV together with normalised values based on cruise fuel consumption = 1.0. Absolute and percentage changes in fuel consumption from Case A to Case B, and from Case B to Case C are given in Table V. The predictions of the incremental fuel consumption due to stops as a percentage of the total are given in Table VI (the stops component is the last term of each formula). The results are summarised in Fig. 1.

2.4 DISCUSSION OF RESULTS

The following observations can be made from the above results.

(a) The differences in the basic fuel consumption data for different cars (as reflected by the coefficients of the elemental model in E1 to E3) result in different sensitivities to changing traffic control conditions (Tables IV and V). Analyses for different vehicle types in the traffic stream is therefore necessary. For simple applications, data for different vehicles can be combined to produce a 'composite vehicle' model to represent a standard composition of traffic.
Fig. 1 — Normalised fuel consumption results from the elemental model and the PKE model for different data

- **Trip A**: $v_s = 45$ km/h
  - PKE = 0.278 m/s²
  - $d_s = 10$ s

- **Trip B**: $v_s = 30$ km/h
  - PKE = 0.278 m/s²
  - $d_s = 50$ s

- **Trip C**: $v_s = 15$ km/h
  - PKE = 0.278 m/s²
  - $d_s = 170$ s

Fig. 2 — Three trips with identical acceleration-cruise-deceleration patterns but different idling times
TABLE IV

FUEL CONSUMPTION PER UNIT DISTANCE, \(f_s\) (mL/km)*

<table>
<thead>
<tr>
<th>Model</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
<th>Cruise</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>253(2.18)</td>
<td>173(1.49)</td>
<td>164(1.41)</td>
<td>116(1.0)</td>
</tr>
<tr>
<td>E2</td>
<td>152(1.62)</td>
<td>117(1.24)</td>
<td>113(1.20)</td>
<td>94(1.0)</td>
</tr>
<tr>
<td>E3</td>
<td>171(1.92)</td>
<td>118(1.33)</td>
<td>113(1.27)</td>
<td>89(1.0)</td>
</tr>
<tr>
<td>P1</td>
<td>161(1.99)</td>
<td>112(1.38)</td>
<td>108(1.33)</td>
<td>81(1.0)</td>
</tr>
<tr>
<td>P2</td>
<td>155(1.74)</td>
<td>111(1.25)</td>
<td>107(1.20)</td>
<td>89(1.0)</td>
</tr>
<tr>
<td>P3</td>
<td>148(1.78)</td>
<td>104(1.25)</td>
<td>100(1.20)</td>
<td>83(1.0)</td>
</tr>
</tbody>
</table>

* Normalised values based on cruise fuel consumption = 1.0 are shown in brackets.

TABLE V

CHANGES IN FUEL CONSUMPTION

<table>
<thead>
<tr>
<th>Model</th>
<th>Case A-Case B</th>
<th>Case B-Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>80(32%)</td>
<td>9(5.3%)</td>
</tr>
<tr>
<td>E2</td>
<td>35(23%)</td>
<td>4(3.4%)</td>
</tr>
<tr>
<td>E3</td>
<td>53(31%)</td>
<td>5(4.2%)</td>
</tr>
<tr>
<td>P1</td>
<td>49(30%)</td>
<td>4(3.6%)</td>
</tr>
<tr>
<td>P2</td>
<td>44(28%)</td>
<td>4(3.6%)</td>
</tr>
<tr>
<td>P3</td>
<td>44(30%)</td>
<td>4(3.9%)</td>
</tr>
</tbody>
</table>

TABLE VI

INCREMENTAL FUEL CONSUMPTION DUE TO ‘STOPS’ AS A PERCENTAGE OF THE TOTAL

<table>
<thead>
<tr>
<th>Model</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>24</td>
<td>22</td>
<td>20</td>
</tr>
<tr>
<td>E2</td>
<td>14</td>
<td>12</td>
<td>11</td>
</tr>
<tr>
<td>E3</td>
<td>12</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>P1</td>
<td>26</td>
<td>24</td>
<td>22</td>
</tr>
<tr>
<td>P2</td>
<td>21</td>
<td>18</td>
<td>17</td>
</tr>
<tr>
<td>P3</td>
<td>21</td>
<td>19</td>
<td>18</td>
</tr>
</tbody>
</table>

(b) The sensitivities of the elemental model (E3) and the PKE model (P1 to P3) for the same car are reasonably similar (Tables IV and V; Fig. 1). Considering the approximate nature of the formulae to predict the traffic performance variables (i.e. the simplified nature of the traffic model), the differences between models may not be considered to be significant in terms of overall model sensitivity to changing traffic conditions.

(c) However, as seen from Table VI, the predicted fuel consumption only due to ‘stops’ differ markedly between the elemental model (E3) and the PKE model (P1 to P3) for the same car. A difference between the PKE model calibrated using on-road data (P1) and those calibrated using laboratory data (P2 and P3) is also observed. It is seen that the PKE model implies much higher stop penalties compared with the elemental model. This could be due to the assumption in the PKE model that fuel consumption due to stops are independent of the acceleration rate and profile, and/or because the coefficient of the PKE term is derived by regression and there may be other factors affecting it. For example, it is possible that the PKE model under-estimates delays but compensates by attributing higher contributions to stops. This particular point is explored below by means of an example.

3. INCREMENTAL FUEL CONSUMPTION DUE TO EXTRA IDLING TIME

In the example shown in Fig. 2, three trips along the same road section (\(x_s = 1\) km) are considered, which have identical acceleration-cruise-deceleration patterns but different idling times, \(d_i\) (hence different average speeds, \(v_i\)). Figure 1 indicates constant acceleration and deceleration profiles, but the discussion below applies to any profile as long as they are identical for each trip. In this example acceleration and deceleration times are equal (\(t_a = t_d = 10\) s) and the cruise time, \(t_c = 50\) s for all trips. Thus, the...
running time, \( t = t_s + t_c + t_d = 70 \) s is constant. However, the stopped (idling) times, \( d_s \), are 10, 50 and 170 s for Trips A, B and C, respectively. The corresponding interrupted travel times \( (t_c = t + d_s) \) are 80, 120 and 240 s, and the average 'interrupted' speeds \( (v_s = 3600 \times k_m/t_c) \) are 45, 30 and 15 km/h, respectively. The value of PKE is constant: \( \text{PKE} = (60^2 - 0)/(12960 \times 1.0) = 0.278 \text{ m/s}^2 \).

The following fuel consumption values are predicted by the PKE model (P2 given above) using these data:

- Trip A: \( f_A = 114.7 \text{ mL/km} \)
- Trip B: \( f_B = 128.7 \text{ mL/km} \)
- Trip C: \( f_C = 207.3 \text{ mL/km} \)

The difference between fuel consumptions for Trips B and A is \( \Delta f_{BA} = 14.0 \text{ mL/km} \), and for Trips C and A is \( \Delta f_{CA} = 92.6 \text{ mL/km} \). The only differences between these trips are due to the idling times: \( \Delta d_{BA} = 40 \) s and \( \Delta d_{CA} = 160 \) s. If the idling fuel consumption rate is known, the expected values of \( \Delta f_{BA} \) and \( \Delta f_{CA} \) can be calculated directly (in the elemental model fashion). If the coefficient of the second term of the PKE model is the idling fuel consumption rate \( \text{PKE} \text{ fuel consumption rate} = 29.03 \text{ mL/h} = 0.8064 \text{ mL/s} \), then the expected values are 32.3 and 129.0 mL. Alternatively, using the directly measured value of \( 2640 \text{ mL/h} = 0.7333 \text{ mL/s} \) (Watson 1980), 29.3 and 117.3 mL are found. Thus, the PKE model shows a tendency to underestimate the incremental fuel consumption due to extra idling time. This supports the suggestion above that high stop penalties implied by the PKE model may be a result of the underestimation of idling fuel consumption which is compensated by an incremental fuel consumption associated with the PKE term. This suggestion is further supported by the discussion of the equivalence between the PKE and elemental models in Part 3 of this report.

### 4. CONCLUSION

In the simple examples considered, the elemental and the PKE models gave similar performances in terms of the overall fuel consumption prediction, but they differed significantly in terms of the incremental fuel consumption predictions (i.e. in terms of the contributions of delays and stops). The PKE model implies higher stop penalties for the same car, and as indicated by the second example, it is possible that the PKE model underestimates the incremental fuel consumption due to delays, and overestimates that due to stops (compensation by the PKE term whose coefficient is determined by regression). In the model choice for a particular purpose, the differences between the overall and incremental fuel consumption prediction abilities of alternative models should therefore be considered carefully. The elemental model, with its incremental prediction ability is particularly useful for traffic management applications which involve design and optimisation by small changes to the components affecting operating conditions in specific situations.

### REFERENCES


Part 3

ON THE ELEMENTAL MODEL
OF FUEL CONSUMPTION

by

R. AKCELIK
Principal Research Scientist
Australian Road Research Board

(First written in October 1981)
1. INTRODUCTION

Introductory discussions and references to the literature on the elemental model of fuel consumption can be found in Akcelik (1981a) and in Part 1 of this report. Different analytical formulations of the elemental model are presented in Section 2 of this paper in order to facilitate its use with different traffic variables, namely:

(a) idling time (delay in stopped position);
(b) delay time including both stopped (idling) delay and deceleration-acceleration delays;
(c) interrupted travel time (uninterrupted travel time plus delay time); and
(d) average speed (interrupted travel time per unit distance).

The fundamental relationships among these variables are employed to show how the appropriate coefficients of the elemental model can be calculated for the chosen variable.

In Section 3, an attempt is made to relate the elemental model to the 'PKE' model reported by Watson, Milkins and Marshall (1980) and Watson (1980). The PKE variable is considered as two separate variables: one to represent speed fluctuations while cruising without interruptions from traffic controls and the other to represent stops imposed by traffic controls. A formula is given to relate the latter to the number of stops. A problem of correspondence between the PKE model and the elemental model is pointed out. Questions are also raised about whether the coefficients of the two separate PKE variables are the same and constant, and whether they depend on the amount of speed change and/or deceleration and acceleration rates and patterns.

The effects of deceleration and acceleration rates are investigated in Section 4 using data from Claflery (1976). The analyses show that the elemental model coefficient for excess fuel consumption per stop is dependent on deceleration and acceleration rates. This raises a question about the validity of the corresponding coefficient in the PKE model. In Section 5, Bayley's (1980) method is shown to give a different form of the function which relates the excess fuel consumption coefficient to the cruise speed. An analysis of the Claflery (1976) data to establish a form of the function which is consistent with the basic relationships and which describes data satisfactorily leads to inconclusive results.

In Section 6, the calculation of the elemental model coefficients for a 'composite vehicle' model is discussed. Putting aside the theoretical problems discussed in earlier sections, a simple-to-use practical method is described in Section 7 to obtain fuel consumption data for the elemental model. The same method is recommended for use in research to investigate the problems raised in this paper. Several other questions for further analysis and research are given in Section 8.

2. DIFFERENT FORMS OF THE ELEMENTAL MODEL

The basic assumption of the elemental model is the independence of the amounts of fuel consumed during three fundamental driving manoeuvres, namely of cruise, idling and the deceleration-acceleration manoeuvre. It is therefore assumed that consumptions associated with these three manoeuvres can be added together irrespective of the order in which they occur. This basic principle was used in a pollutant emission model by Watson (1973) who treated deceleration and acceleration manoeuvres separately (i.e. four 'modes' of traffic were employed). For each mode, Watson's model predicted pollutant emissions by integrating a function which expresses emissions in terms of instantaneous accelerations and velocities. The calculations for this model were carried out by means of a computer program.

The elemental model discussed in Akcelik (1981a) and in Part 1 of this report employs a simplifying approach in that it combines together the deceleration and acceleration manoeuvres imposed by traffic controls (e.g. Give way/Stop signs, roundabouts, traffic signals, etc.) and treats them as 'effective stops'. This involves finding equivalents of slow-down and speed-up manoeuvres caused by traffic controls in terms of their fuel consumption values relative to the consumption associated with a 'complete' stop/start manoeuvre from the cruise speed. In Part 1 of this report, it is argued that the consumption associated with the slow-down and speed-up manoeuvres while cruising ('mid-block' component not associated with traffic controls) should be accounted for as part of the cruise component of the model. However, there may be a need to model this consumption component explicitly if it is expected to be affected by traffic controls, e.g. for testing the effectiveness of a clearway system.

The elemental model employs travel distance, delay time and the number of stops as the traffic performance variables which represent the three basic driving manoeuvres. These performance variables need to be predicted by a traffic model for all vehicles in each traffic movement (stream) as average values. For this purpose, each separate movement in a traffic system needs to be identified by its unique set of control, physical and flow characteristics, e.g. see Akcelik (1981b) for detailed rules to identify movements at signalised intersections.

According to the basic assumption of the elemental model, fuel consumption for an average vehicle can be expressed as follows:

\[ F = F_c + F_s + F_h = \varphi_1 x_c + \varphi_2 d_s + \varphi_3 h \]  

where

- \( F \) = total fuel consumption (mL),
- \( F_c \) = fuel consumed while cruising (\( \varphi_1 x_c \)),
- \( F_s \) = fuel consumed while idling (\( \varphi_2 d_s \)),
- \( F_h \) = fuel consumed during stop-start manoeuvres (\( \varphi_3 h \)),
- \( x_c \) = distance travelled while cruising.

The elemental model allows for the calculation of various fuel consumption components that are affected by traffic control measures and their associated traffic patterns. Further research and analysis are needed to refine the model and improve its accuracy in predicting fuel consumption changes in response to traffic control strategies.
\[ d_s = \text{time spent while idling, i.e. in stopped position (s)}, \]
\[ h = \text{number of (effective) stops}, \]
\[ \varphi_1 = \text{fuel consumption per unit distance while cruising (mL/km)}, \]
\[ \varphi_2 = \text{fuel consumption per unit time while idling (mL/s)}, \]
\[ \varphi_3 = \text{actual fuel consumption per 'stop', i.e. a complete stop-start manoeuvre which involves a deceleration from an initial (cruise) speed, \( v_c \), to zero speed and an acceleration back to a final (cruise) speed, \( v_c \).} \]

The coefficient \( \varphi_1 \) is constant while the coefficients \( \varphi_2 \) and \( \varphi_3 \) depend primarily on the cruise speed. This is discussed in detail in Sections 3 to 5. It is important to note that the average values of the traffic performance variables \( d_s \) and \( h \) are for the stopped and unstopped vehicles alike, and \( h \) is the number of 'effective stops', and hence these variables must be predicted by the traffic model accordingly.

It may be more convenient to express the first term of the elemental model in terms of the total section distance \( x \), rather than the cruise distance, \( x_c \). If the average deceleration-acceleration distance per stop is \( x_h \), then

\[ x_s = x_c + h x_h, \]

Putting \( x_c = x_s - h x_h \) in eqn (1),

\[ F = \varphi_1 x_s + \varphi_2 d_s + (\varphi_3 - \varphi_1) h \] (2)

is obtained. Putting \( \varphi_1 = f_1, \varphi_2 = f_2 \) and \( \varphi_3 - \varphi_1, x_h = f_3 \) in eqn (2),

\[ F = f_1 x_s + f_2 d_s + f_3 h \] (3)

is obtained. The coefficient \( f_3 \) is the 'excess' fuel consumption rate per stop, i.e. absolute consumption per stop less the consumption if the deceleration-acceleration distance is travelled at cruise speed.

Traditionally, traffic models predict an average delay, \( d \), which includes the deceleration-acceleration delays. The relationship between this delay and the stopped delay (idling time), \( d_s \), used in above equations is

\[ d = d_s + h d_h \]

where \( d_h \) is the average deceleration-acceleration delay per stop. Putting \( d_s = d - h d_h \) in eqn (3),

\[ F = f_1 x_s + f_2 d + (f_3 - f_2) d_h \] (4)

is obtained. Defining

\[ f_1' = f_1 - f_2 d_h \] (5)

as the adjusted excess fuel consumption rate per stop, i.e. normal excess fuel consumption rate less the idling fuel consumption during time \( d_h \),

\[ F = f_1' x_s + f_2 d + f_3' h \] (6)

is derived.

The deceleration-acceleration delay, \( d_s \), is the time spent during a deceleration-acceleration manoeuvre, \( t_s \), less the time taken to travel the deceleration-acceleration distance, \( x_h \), at cruise speed, \( v_c \):

\[ d_s = t_s - 3600 x_h/v_c \] (7)

where \( d_s, t_s \) are in seconds, \( x_h \) is in km and \( v_c \) is in km/h. Assuming constant deceleration and acceleration rates (\( a_1 \) and \( a_2 \), respectively), the deceleration-acceleration time and distance are given by

\[ t_n = 2 v_c/a_n \] (8)

\[ x_n = v_c^2/(3600 a_n) \] (9)

where

\[ a_h = \frac{2}{(1/a_1) + (1/a_2)} \] (10)

is the average deceleration-acceleration rate per stop (harmonic mean of \( a_1 \) and \( a_2 \)). From eqns (7) to (10), the deceleration-acceleration delay is

\[ d_h = v_c/a_h \] (11)

In eqns (8) to (11), \( a_1, a_2, a_h \) are in km/h/s.

Further, the elemental model can be expressed in terms of the average travel time, \( t_s \), rather than delay, \( d \). Since

\[ t_s = d + 3600 x_h/v_c \]

(where \( t_s \) and \( d \) are in seconds, the section distance \( x_h \) is in km and \( v_c \) is in km/h), eqn (6) can be re-written as

\[ F = f_1' x_s + f_2 t_s + f_3' h \] (12)

where

\[ f_1' = f_1 - (3600 t_s/v_c) \]

(12a)

is the adjusted cruise fuel consumption rate.

Fuel consumption rate per unit distance, \( f_s \) (mL/km), and per unit time, \( f_t \) (mL/s), for the above equations can be calculated from \( f_s = F/x_s \) and \( f_t = F/t_s \), where \( x_s \) and \( t_s \) are total section distance and travel time, respectively. The following formulae are useful:

\[ f_s = f_1' + f_2 x_s + f_3' \] (13a)

\[ f_t = f_1' + f_2 t_s + f_3' \] (13b)

where \( f_1' = 3600 f_2/v_s + f_3' \)

(13a)

(13b)

(14)

where \( d_s, t_s \) are stopped delay time and number of stops per unit distance, and \( v_c \) is the average section speed (km/h).
Example:

Basic fuel consumption rates for $v_c = 52$ km/h, $a_i = 2.2$ km/h/s and $a_s = 8.6$ km/h/s (constant rates) are known as $v_c = 140$ mL/km, $v_s = 610$ mL/s and $v_s = 60$ mL per stop (coefficients for eqn (1)). Determine the coefficients for eqns (3), (6) and (12). For all equations, the idling fuel consumption rate is the same:

$$f_i = v_c = 610$$ mL/s

For eqns (3) and (6), the cruise fuel consumption rate is $f_c = v_c = 140$ mL/km, but the adjusted rate for eqn (12) is

$$f_i = 140 - (3600 \times 0.610/52) = 98$$ mL/km.

To determine $f_i$ and $f_i'$, firstly from eqns (9) to (11), $a_i = 3.5$ km/h/s, $x_i = 0.215$ km and $d_i = 15$ s are found. Hence,

$$f_i = v_i - v_s, x_i = 60 - (140 \times 0.215) = 30$$ mL and

$$f_i' = f_i - f_c, d_i = 30 - (0.610 \times 15) = 21$$ mL

are found for eqns (3) and (6), respectively. For $x_c = 0.650$ km, $d_c = 24$ s and $h = 1.4$ stops (hence $f_c = 90$ s, $v_c = 26$ km/h), the above formulae give $PKE = 148$ mL, $f_c = 228$ mL/km and $f_c = 1.644$ mL/s.

Various forms of the elemental model given above help to relate it to the other models given in the literature. However, a direct comparison of the model coefficients must not be made with the corresponding coefficients of any model based on regression analysis for reasons discussed in Part 1 of this report. It should be remembered that, for the elemental model, each coefficient is determined individually by controlled experimentation. With this point in mind, an attempt is made below to relate the elemental model to the PKE model by Watson, et al. (1980).

3. RELATION TO THE PKE MODEL

Watson et al. (1980) and Watson (1980) gave the following expression for predicting fuel consumption per unit distance:

$$f_c = k_i + k_i/v_s + k_i v_s + k_i PKE$$ (15)

where $k_i$ (i = 1 to 4) are constants, and

$k_i$ represents the fuel used in overcoming vehicle resistance losses,

$k_i$ represents idling fuel consumption,

$k_i$ represents consumption due to aerodynamic forces, and

$k_i$ represents fuel consumption due to work to accelerate the vehicle per unit distance,

$v_s$ is the average interrupted travel speed, and

PKE is the sum of the positive kinetic energy changes (during acceleration), and is given by:

$$PKE = \frac{\Sigma(v_f^2 - v_i^2)}{12960 x_s}$$ (16)

where $v_f$ and $v_i$ (km/h) are the final and initial velocities in a positive acceleration, $x_s$ is the total section distance (km) and PKE is in (mL/s). For example, for a speed change manoeuvre 70-40-60 (slow-down from 70 km/h to 40 km/h and speed-up to 60 km/h) over a distance of 500 m,

$$PKE = (60^2 - 40^2)/(12960 \times 0.5) = 0.31$$ m/s²

is found.

Watson et al. (1980) derive the coefficients $k_i$ to $k_i$ of the PKE model (eqn (15)) by regression analysis. Watson (1980) refers to this as the 'lumped coefficient model', and emphasises that the coefficients derived from the regression analysis should not be used as the elemental model coefficients 'because regression allows inter-coefficient transfer'. As an example, Watson contrasts the measured value of $k_i = 2640$ mL/h with the value of $k_i = 2903$ mL/h obtained from regression analysis. The following discussion assumes that each coefficient of eqn (15) can be measured directly by controlled experimentation, and as such it can be related to the elemental model described above.

Firstly, consider the prediction of steady-speed fuel consumption by eqn (15). Putting $v_s = v_c$ (no delay) and PKE = 0 (no speed changes), then

$$f_c = k_i + k_i/v_c + k_i v_c$$

where

$f_c = \text{steady-speed fuel consumption rate (mL/km)}$ at cruising speed, $v_c$ (km/h); $k_i$ (mL/km), $k_i$ (mL/h) and $k_i$ (mL-h/km²).

The following features of eqn (17) should be noted:

(a) The minimum fuel consumption is obtained at the optimum steady speed given by

$$v_o = \sqrt{k_i/k_i}$$ (18)

and the corresponding minimum fuel consumption is given by

$$f_o = k_i + 2 \sqrt{k_i k_i} - k_i + 2 k_i v_o$$ (19)

For example using Watson's (1980) formula $f_c = -30.7 + (2903/v_c) + 1.216 v_c, v_o = 49$ km/h and $f_o = 88$ mL/km are found. However, it should be noted that $k_i, k_i, k_i$ in this example were obtained from regression analysis. Bayley (1980) suggests a method to calculate coefficients $k_i$ and $k_i$ from eqns (18) and (19) using known values of $k_i$, $v_o$ and $f_o$. This is discussed in Section 5.

(b) As $v_s$ approaches zero, $f_c$ approaches infinity. This is due to the contribution of the second term of eqn (17), and results from travel time per unit distance approaching infinity. This contrasts with the steady-speed fuel consumption formula used by Vincent, Mitchell and Robertson (1980), which is of the form

$$f_c = a + b v_c + c v_c^2$$

Vincent et al.'s (1980) formula

$$f_c = 170 - 4.55 v_c + 0.049 v_c^2$$

gives a finite fuel consumption figure for zero speed ($f_c = 170$ mL/km).

Now find the difference between the consumption predictions given by eqns (15) and (17) which must be due to delays and speed change manoeuvres. This is found as

$$f_x - f_c = k_2 \left( \frac{1}{v_s} - \frac{1}{v_c} \right) + k_3 (v_s - v_c) + k_4 PKE$$
Putting $1/v_c = \bar{t}_s - \bar{t}_c + \bar{d} = (1/v_c) + \bar{d}$,

$$f_x - f_c = k_2 \bar{d} \left( 1 - \frac{(v_c/v_o)^2}{1 + dv_c} \right) + k_4 PKE$$

is found. From eqn (18), $(k_1/k_2) = 1/v_o^2$, and hence

$$f_x - f_c = k_2 \bar{d} \left( 1 - \frac{(v_c/v_o)^2}{1 + dv_c} \right) + k_4 PKE (20)$$

is obtained.

Furthermore, treat the PKE model in a deterministic fashion, and define two separate PKE’s: $PKE_1$ to represent speed fluctuations while cruising uninterrupted by traffic controls; and $PKE_2$ to represent stops and slow-downs imposed by traffic controls.

Now write

$$k_1 PKE = k_1 PKE_1 + k_6 PKE_2 \quad \text{(20a)}$$

Therefore the PKE model given by eqn (15) is re-written as

$$f_x = (k_1 + k_2/v_c + k_3 v_c + k_5 PKE_1)$$

$$+ k_2 \bar{d} \left( 1 - \frac{(v_c/v_o)^2}{1 + dv_c} \right) + k_6 PKE_2 \quad \text{(21)}$$

A comparison of eqn (21) with eqn (13a) indicates that, for the elemental model,

$$f_1 = k_1 + k_2/v_c + k_3 v_c + k_5 PKE_1 \quad \text{(22a)}$$

$$f_2 = k_2 \left( 1 - \frac{(v_c/v_o)^2}{1 + dv_c} \right) = k_2 \frac{(v_c - v_o)}{d} \quad \text{(22b)}$$

$$f_3' = k_6 \frac{PKE_2}{h} \quad \text{(22c)}$$

It is seen that the expected equivalency of the idling fuel consumption rates, $f_3' = f_2$, is not obtained. Eqn (22b) indicates that this can be obtained only if the third term of the PKE model (eqn (15)) is expressed as $k_3 v_c$ rather than $k_5 v_c$. It appears that this problem is a result of interpreting 'steady speed' as the 'average speed', $v_c$, rather than the cruise speed, $v_c$, in the original derivation of the PKE model as indicated by Watson et al. (1980) eqns (12) to (18), and Watson’s (1980) eqns (2) to (4). To compensate for the underestimation of the effects of stopped delays, the regression method would produce a higher value of the PKE coefficient $k_6$. This would result in overestimation of the effects of stops relative to the effects of stopped delays, i.e. high stop penalties would be implied. In addition, this would cause the PKE model (eqn (15)) to fail to explain fuel consumption differences due to extra idling time for a given stop pattern (see Part 2).

Now have a close look at the PKE, and PKE2 variables in eqns (22a) and (22c), irrespective of the problem with eqn (22b). The speed fluctuations during cruise, which contribute to PKE, in eqn (22a), are due to the internal friction of the traffic stream as well as various road-side (environmental) friction factors as discussed in Part 1 of this report. These fluctuations can be considered to be around the average cruise speed, $v_c$, and

$$PKE_1 = \left\{ (v_c^2 - v_o^2) \right\} / 12960 x_s \quad \text{(23)}$$

where the speeds $v_o, v_p, v_c$ are in km/h, the distance $x_s$ is in km, and PKE is in m/s$^2$. The speed $v_o$ represents the local peak speed during a speed-up manoeuvre ($v_e - v_o - v_p$), e.g. passing a slower vehicle, and the speed $v_p$ represents the local minimum speed during a slow-down manoeuvre ($v_c - v_p - v_o$). An example is given in Fig. 1 which illustrates travel in Section A on a road of length $x_s = 1$ km. The speeds are $v_o = 48$ km/h, $v_c = 64$ km/h and $v_p = 32$ km/h, and hence

$$PKE_1 = \left\{ (64^2 - 48^2) + (48^2 - 32^2) \right\} / 12960 = 0.237 \text{ m/s}^2$$

is found. Using Claffey’s (1971) data, $f_c = 104 \text{ mL/km}$ (at 48 km/h), $\Delta f = 14 \text{ mL}$ for (48-64-48) cycle assuming this is equivalent to (64-48-64) cycle, and 13 mL for (48-32-48) cycle. Fuel consumed per unit distance while cruising in Section A is

$$f_x = f_c + \Delta f/x_s = 104 + 14 + 13 = 131 \text{ mL/km}.$$ 

Since $k_{16} PKE_1 = \Delta f/x_s$,

$$k_{16} = 27/0.237 = 114 \text{ mL-h}^2/\text{km}^2$$

is found. The ratio of the consumption allowing for speed fluctuations to the steady-speed consumption for this example is $131/104 = 1.26$.

The speed change cycle $(v_c - 0 - v_c)$ represents a complete stop from cruise speed. Putting $v_i = v_c, v_o = 0$ in eqn (16),

$$PKE_2 = \frac{\Sigma v_c^2}{12960 x_s} \quad \text{(24)}$$

is found. Assuming $h$ identical 'effective' stops per vehicle, $\Sigma v_c^2 = h v_c^2$, and hence $PKE_2 = h v_c^2/12960$ is found. From eqn (22c),

$$f_3' = k_6 \frac{v_c^2}{12960} \quad \text{(25)}$$

is obtained. For example, during travel in Section B ($x_b = 1$ km) of Fig. 7, there is a full stop from 48 km/h. Using Claffey’s (1971) data, $f_s = 37 \text{ mL/stop}$, $f_o = 0.610 \text{ mL/s}$, and $\Delta f = 10 \text{ s}$ from Fig. 1 (at $a_s = 4.8 \text{ km/h/s}$). $f_s = 37 - 0.610 \times 10 = 31 \text{ mL/stop}$ is found. The PKE value from eqn (24) is $PKE_s = 0.178 \text{ m/s}^2$, and from eqn (25),

$$k_o = 31 \times 12960/48^2 = 174 \text{ mL-h}^2/\text{km}^2$$

is found. From eqn (13a), fuel consumption per unit distance in Section B is

$$f_s = 104 + 0.61 \times 30 + 37 \times 1 = 159 \text{ mL/km}$$

(note $d = 40 \text{ s}, d_s = 30 \text{ s}, h = 1$).

In this example, the fuel consumption rate for
travel in Sections A and B together is
\[ f' = \frac{(131 + 159)}{2} = 145 \text{ mL/km}. \]

If an attempt is made to combine PKE's for cruise and for stops,
\[ \text{PKE} = \text{PKE}_1 + \text{PKE}_2 = \frac{0.237 + 0.178}{2} = 0.208 \text{ m/s}^2 \]
is found. The corresponding fuel consumption is
\[ \frac{14 + 13 + 31}{2} = 29 \text{ mL/km}. \]

Writing \( k_i \) as in eqn (15),
\[ k_i = 29/0.208 = 140 \text{ mL-h}^2/\text{km}^2 \]
is found. However, the combined PKE and the corresponding \( k_i \) do not distinguish between speed fluctuations during cruise and stops due to traffic controls.

### 4. EFFECT OF ACCELERATION AND DECELERATION RATES

An important question arises from the above analysis about whether \( k_i \) and \( k_n \) are the same and constant, or whether they depend on the amount of speed change and/or deceleration and acceleration rates and patterns. There is insufficient data for a complete analysis of this question. However, an attempt is made below to investigate the coefficient \( k_n \) and its relation to the elemental model coefficient \( f' \), as expressed in eqn (25) using data given by Claffey (1976).

The excess fuel consumption data for stops, \( f_{st} \), have been taken from Claffey's (1976) Tables 41 and 43. The data are for a 1972 Chevrolet sedan (V-8, 6.5 L, automatic), for a level road, and are limited to a range of cruise speeds from 16 km/h to 64 km/h. Eight sets of excess fuel consumption data, \( f_{st} \), are given by Claffey for the different combinations of deceleration and acceleration rates (km/h/s) given in Table I (\( a_n \), calculated from eqn (10)). The adjusted excess fuel consumption values, \( f_{st}' \), have been calculated from eqn (5) using \( f_t = 0.883 \text{ mL/s} \) (Claffey 1976, p. 210) and \( d_i \) ('excess time consumption') data given in Claffey's Tables 44 and 45.

Linear regressions of \( f'_{st} \) on \( v_c^2 / 12960 \) (all forced through the origin) have been carried out in order to estimate the value of coefficient \( k_n \) in eqn (25). The results are given in Table I, and are shown in Figs 2 and 3. In terms of \( R^2 \) values, eqn (25) appears to provide a good basis for predicting excess fuel consumption. At the same time, the results indicate that coefficient \( k_n \) is dependent on the deceleration and acceleration rates. The tendency is for \( k_n \) to increase as \( a_1 \), \( a_2 \), or \( a_n \) increase. It has been found that the relation between \( k_n \) and \( a_n \) could be described by

\[ k_n = 222 - \left( \frac{248}{a_n} \right) \quad (R^2 = 0.97). \]

This is illustrated in Fig. 4. This relation gives \( k_n = 109 \) and 194 for \( a_n = 2.2 \) and 8.8 km/h/s, respectively. The differences between these values and the constant value of \( k_n = 157 \) (neglecting acceleration and deceleration effects) are -34 per cent and +24 per cent, respectively. These results have important implications on the use of the PKE model if the equivalence between \( f'_{st} \) and \( \text{PKE} \) (eqn (22c)) is valid. Furthermore, lumping together of \( k_i \) and \( k_n \) may result in similar problems if \( k_i \) is different from \( k_n \), and \( k_i \) varies with speed and acceleration values.

The relation between \( k_n \) and \( a_n \) implies a function for \( f'_{st} \), which is of the form

\[ f'_{st} = \alpha v_c^2 - \beta \frac{v_c^2}{a_n} \quad (26) \]

where \( \alpha = 0.0171 \) and \( \beta = 0.0191 \) for the Claffey (1976) data analysed. Eqn (26) means that excess fuel consumption is zero for an average deceleration-acceleration rate of \( a_n = \beta/\alpha \), irrespective of the cruise speed. Such implications of the dependence of excess fuel consumption per stop on deceleration and acceleration rates necessitates further investigations on this subject. Further discussion with reference to Bayley's (1980) work is given below.

**TABLE I**

<table>
<thead>
<tr>
<th>Deceleration Rate</th>
<th>Acceleration Rate</th>
<th>Average Rate</th>
<th>( k_n )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>( a_2 )</td>
<td>( a_n )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>6.8</td>
<td>2.2</td>
<td>3.52</td>
<td>153</td>
</tr>
<tr>
<td>II</td>
<td>6.8</td>
<td>4.4</td>
<td>5.87</td>
<td>171</td>
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<tr>
<td>III</td>
<td>6.8</td>
<td>6.6</td>
<td>7.54</td>
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</tr>
<tr>
<td>IV</td>
<td>6.8</td>
<td>8.8</td>
<td>8.60</td>
<td>198</td>
</tr>
<tr>
<td>V</td>
<td>2.2</td>
<td>2.2</td>
<td>2.20</td>
<td>113</td>
</tr>
<tr>
<td>VI</td>
<td>2.2</td>
<td>4.4</td>
<td>2.93</td>
<td>137</td>
</tr>
<tr>
<td>VII</td>
<td>2.2</td>
<td>6.6</td>
<td>3.30</td>
<td>142</td>
</tr>
<tr>
<td>VIII</td>
<td>2.2</td>
<td>8.8</td>
<td>3.52</td>
<td>148</td>
</tr>
<tr>
<td>I-IV</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V-VIII</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I-VIII</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f'_{st} - k_n v_c^2 / 12960 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 1 — Cruise speed fluctuations and intersection stops (an example)
Fig. 2 — Adjusted excess fuel consumption rate per stop, $f'_s$, as a function of the cruise speed, $v_c$, for individual data sets.

Table of coefficients:

<table>
<thead>
<tr>
<th>Set</th>
<th>$a_1$ (km/h/s)</th>
<th>$a_2$ (km/h/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>8.8</td>
<td>2.2</td>
</tr>
<tr>
<td>IV</td>
<td>8.8</td>
<td>8.8</td>
</tr>
<tr>
<td>V</td>
<td>2.2</td>
<td>2.2</td>
</tr>
<tr>
<td>VIII</td>
<td>2.2</td>
<td>8.8</td>
</tr>
</tbody>
</table>

IV: $f'_s = 0.0153 v_c^2$

I: $f'_s = 0.0118 v_c^2$

VIII: $f'_s = 0.0114 v_c^2$

V: $f'_s = 0.0087 v_c^2$

Fig. 3 — Adjusted excess fuel consumption rate per stop, $f'_s$, as a function of the cruise speed, $v_c$, for all data.
5. AN ANALYSIS OF BAYLEY'S EXCESS FUEL CONSUMPTION FORMULA

Bayley (1980) used the following instantaneous fuel consumption function to derive an excess fuel consumption function.

\[ f = \frac{dF}{dt} = c_i + c_2 v + c_3 v^2 + c_4 a \]  

(27)

where

- \( f \) = instantaneous fuel consumption per unit time (mL/s),
- \( v \) = instantaneous speed (km/h),
- \( a \) = \( \frac{dv}{dt} \) = instantaneous acceleration rate (km/h/s), and
- \( c_i \) to \( c_4 \) = constants dependent on vehicle characteristics (\( c_i \) = idling fuel consumption rate).

For constant-speed cruise (\( a = 0, v = v_i \)), eqn (27) gives a function which is equivalent to eqn (17) with \( k_1 = 3600 c_2, k_2 = 3600 c_3, k_3 = 3600 c_4 \) for correct units. However, Bayley suggests that the coefficients \( k_1, k_2 \), and \( k_3 \) for the steady-speed fuel consumption relationship (eqn (17)) can be found deterministically. If the minimum-fuel speed, \( v_o \), the corresponding fuel consumption, \( f_o \), and the idling fuel consumption \( k_i \) are known, coefficients \( k_1 \) and \( k_2 \) can be found from eqns (18) and (19) (see eqn (34) in Section 9). For example, Watson's (1980) regression equation gives \( v_o = 49 \) km/h and \( f_o = 88 \) mL/km, and the measured value of \( k_i \) is 2640 mL/h as discussed in Section 3. Using these values, \( k_1 = -19.8 \) and \( k_2 = 1.10 \) are found. The difference between the predictions of \( f_o \) from eqn (17) with these coefficients and those from regression has been found to be negligible. The advantage of the deterministic method over the regression approach is to use the measured idling fuel consumption rate unchanged (inter-coefficient transfer problem is avoided).

Bayley (1980) eqn (7) for excess fuel consumption per stop can be shown to be equivalent to

\[ f'_1 = e_1 v^3 - e_2 \frac{v^3}{a_h} \]  

(28)

where \( e_1 \) and \( e_2 \) are constants (\( e_2 \), however, is dependent on the deceleration and acceleration profiles as discussed in detail by Bayley).

For Claffey's data, regression analysis has given \( e_1 = 0.65 \) and \( e_2 = 0.662 \times 10^{-5} \) with \( R^2 = 0.93 \) but the second term of eqn (34) has not been found statistically significant. Furthermore, the prediction of \( f'_1 \) by this formula has not been found satisfactory due to consistent overestimation for low speeds and underestimation for high speeds. Similar to eqn (26), eqn (28) means that excess fuel consumption may be zero, but it is dependent on the speed: \( \frac{a_h}{e_1 a_2} \) in eqn (28) will give \( f'_1 = 0 \). In view of the inconclusive results of this investigation, further work is recommended since the findings would be useful for both the PKE and the elemental models.

![Figure 4 - Coefficient \( k_i \) in \( f'_1 = k_i v^3/12960 \) as a function of the average deceleration-acceleration rate, \( a_h \).](image-url)
6. ON COMPOSITE VEHICLE MODELS

The above discussion applies to the basic modelling question for a single car. Whatever model is chosen for use, it is necessary to apply it to a number of different vehicle types and add the results according to the proportion of different vehicles in the traffic stream. The amount of calculations can be decreased by developing a single fuel consumption formula for a ‘composite vehicle’ representing the traffic stream. For the elemental model, e.g. eqn (3), this can be done by calculating the model coefficients $f_j$ ($i = 1$ to 3) from

$$f_i = \sum_{j=1}^{K} p_j f_{ij}$$

where

- $f_i =$ fuel consumption rate for an average or ‘composite’ vehicle during a manoeuvre of type $i$ ($i = 1$ for cruise, $i = 2$ for idling, $i = 3$ for stops),
- $f_{ij} =$ fuel consumption rate for vehicle type $j$ during a manoeuvre of type $i$ (total $K$ vehicle types),
- $p_j = q_j/q =$ proportion of vehicle type $j$ in the traffic stream,
- $q_j =$ flow rate for type $j$, and
- $q =$ total flow rate.

Thus, the fuel consumption by all vehicles in the traffic stream can be calculated as $(Fq)$ where $F$ is the fuel consumption per vehicle and $q$ is total flow rate (or volume).

7. SIMPLE MEASUREMENTS TO PRODUCE ELEMENTAL MODEL DATA

The discussion in Sections 3 to 5 in search of a general fuel consumption relationship is rather theoretical. In practice, coefficients for the elemental model can be determined easily by controlled experimentation. Measurement of the idling fuel consumption rate $(mL)$ is straightforward. The cruise fuel consumption rate $(mL/km)$ is

$$F_0/n X$$

where $F_0/n X$ is the sum of idling times during Trip A (hence $d_s$ during each stop needs to be recorded separately). Experiments can be repeated at various cruise speeds.

Experimentation similar to the above can be carried out to obtain fuel consumption rates for partial stops and multiple stops. The same method can be used for theoretical work using well-defined deceleration/acceleration rates and patterns. In the simple experimentation described above, deceleration and acceleration patterns and rates observed at traffic control devices should be duplicated as far as possible.

8. OTHER ISSUES

There are various questions which need to be investigated for improved use of the elemental model. These are briefly as follows.

(a) As discussed in Section 2, the elemental model relies on the prediction of the number of stops by a traffic model in terms of ‘effective’ stops imposed by traffic controls. Consider for example a partial stop (slow-down) cycle, $v_i - v_c - v_i$, or a multiple stop (speed-up) cycle $0 - v_i - 0$, where $v_i$ is less than $v_c$. If the excess fuel consumption rate for such a manoeuvre is $f_i$, and the excess fuel consumption rate for a complete stop cycle, $v_i - 0 - v_i$, is $f_i$, then the effective stop value of the manoeuvre is $(f_i + f_i)$ which is less than $f_i$. Formulation of an effective stop relationship should be examined in the light of the discussion in Sections 3 to 5.

(b) In the TRANSYT program (Vincent et al. 1980) for co-ordinated signals, the elemental model coefficients are applied according to the average platoon speed. However, delays and stops are calculated allowing for different speeds in the platoon (through the ‘platoon dispersion’ process). Errors which result from using the average platoon speed as the cruise speed need to be investigated in this type of application.
(c) Again in a program like TRANSYT, exit speed, \( v_e \), may be different from the approach speed, \( v_a \), in which case a stop cycle is \( v_a - v_e \), where both \( v_a > v_e \) and \( v_a < v_e \) are possible. The errors due to calculating fuel consumption according to the stop cycle \( v_a - v_e \) should be investigated considering that a subsequent speed-change manoeuvre \( v_e - v_e' \) (speed-up or slow-down) would normally be counted as part of the cruise component of the fuel consumption on the exit link (because this change is to occur irrespective of traffic controls).

9. CONCLUSION

The work reported in earlier papers by Watson, Milkins and Marshall (1980) and Bayley (1980) has been useful in the analyses towards deriving explicit functions for the elemental model parameters. There is an agreement between Bayley's method and Watson et al.'s method about the form of the steady-speed fuel consumption function. This is

\[ f_s = k_1 + k_2 / v_c + k_3 v_c \]  

(33)

The parameters \( k_1 \) to \( k_3 \) can be determined using the deterministic method suggested by Bayley. To implement the method in practice,

(a) measure idling fuel consumption rate \( k_1 \) in \( \text{mL/h} \) (\( \equiv f_i \), using elemental model notation) directly,

(b) measure various values of \( f_s \) for cruise speeds in the range 40 to 70 km/h using the method described in Section 7, and determine the optimum speed, \( v_o \) (km/h), and minimum fuel consumption \( f_o \) (mL/km), and

(c) calculate \( k_1 \) and \( k_2 \) from

\[ k_1 = f_o - 2k_2 / v_o \]  
\[ k_2 = k_2 / v_o^2 \]  

(34a)

(34b)

(see example in Section 5). The prediction ability of eqn (33) using this method should be compared with that when it is used as a regression equation, using actual (ideally on-road) data for a wide range of speeds (e.g. 10 to 120 km/h).

The discussion presented in this paper is inconclusive regarding the equivalence of the PKE and elemental models. Further analysis and research is required to resolve some fundamental problems. In particular, the following questions need to be answered.

(i) Are the coefficients of the separate PKE variables for speed fluctuations while cruising and for stops due to traffic controls the same and constant, or do they depend on the amount of speed change and/or deceleration and acceleration rates and patterns?

(ii) What is the form of the function which expresses the elemental model coefficient for excess fuel consumption, \( f_x \), in relation to the cruise speed and the average deceleration-acceleration rate (see eqns (26) and (28) )?

A thorough analysis of the subject is necessary using extensive fuel consumption data representing different deceleration/acceleration rates and patterns, and a wide range of speeds including low and high cruise speeds.

In conclusion, the coefficients of the elemental model need to be determined separately and by controlled experimentation in view of various problems raised in this paper. The use of regression equations whose coefficients are determined simultaneously may result in mis-calculation of the effects of delays and stops due to traffic controls relative to the effects of factors unrelated to traffic controls.

Depending on the answers to the basic questions raised in this paper, other issues such as fine-tuning of traffic models for better prediction of partial-stop and multiple-stop effects can be resolved.

REFERENCES


Part 4

CALIBRATION AND APPLICATION OF TWO FUEL CONSUMPTION MODELS

by

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University of Melbourne

(First written in October 1981)
1. INTRODUCTION

There exists a continuing need for fuel consumption models which can accurately forecast fuel usage from variables readily available to traffic engineers. It is desirable that the model coefficients are easily obtained from existing or slightly modified vehicle test procedures. Models should also be amenable to extension for estimating exhaust emissions.

Herein, further thoughts are developed on two of the currently available simple models and some performance comparisons are made. It is hoped that the paper will stimulate discussion rather than provide complete answers.

2. MODELS CONSIDERED

Almost the entire range of model types applicable to fuel consumption and emissions modelling can be found by careful investigation of the papers in SAE/ARRB (1980). It is appropriate to list these in approximate order of decreasing requirements placed on data input (references cited are illustrative only of model types):

1. engine mapping models (Milkins and Watson
2. vehicle mapping models (Kent 1981);
3. regression surfaces (Kunselman et al. 1974;
4. elemental models (Watson 1973, Akcelik 1981);
5. travel speed (or time) models (Evans and Herman

In the present examination, consideration of models (1), (2) and (3) is outlined only since they require a representative and continuous velocity-time record. For models (2) and (3), a joint velocity-acceleration probability density function of the type illustrated in Fig. 1 will suffice. This should not be interpreted as a statement that they cannot be employed in evaluating the effects of traffic management changes. If the required driving pattern data are available for the before-and-after change, it is possible that they may adequately forecast results of the change. Those models (1) and (2) which employ measurements directly, without recourse to regression, are likely to be superior in forecasting ability.

2.1 OBJECT

The present examination of the calibration of the remaining models (4) and (5) is directed towards the prediction of fuel consumption in:

(a) traffic simulation studies, and
(b) the results of before-and-after studies.

It is not expected that predictions can be made at the microscopic scale (second-by-second) as can be achieved by models (1), (2) and (3). Accurate prediction at the macroscopic scale is desirable. This scale can be considered to be on a link-by-link basis or a distance of about 1 km and containing an intersection or intersections with a roundabout, traffic signal or other control. In Part 1 of this report, Richardson and Akcelik (1982) argue that it is more important to predict the correct sensitivity of fuel consumption response to traffic engineering initiatives rather than absolute values of fuel consumption. This argument is only valid when the absolute values of fuel usage in a comparison are of similar magnitude. Clearly, an initiative bringing about a 10 per cent reduction from 200 mL/km is preferred to a 10 per cent reduction at 100 mL/km, provided that all other factors in the example comparison are equal. Thus it may be important to achieve accuracy in both fuel consumption level and response.

2.2 THE ELEMENTAL MODEL

Watson (1973) employed a 'modal' model in which driving was subdivided into acceleration, cruise, deceleration and idle modes, the so called ACDI cycle seen in Fig. 2. It was found that exhaust emissions rates could be expressed with some precision as functions of velocity and acceleration. In an illustrative example predictions were made for driving conditions in Edinburgh on the basis of derived average acceleration and deceleration rates, stopped delay time, total travel time and number of stops. Cruise speed and cruise time could then be directly calculated.
Bulach (1977) was unable to overcome the major defects of this method: generation of negative cruise time on some links; sensitivity of results to acceleration rates; and the problems of weighting factors required in deriving acceleration rates (Watson, Milkins and Bulach 1974). Bulach (1977) demonstrated the superiority of the instantaneous emissions rate model as a polynomial function of instantaneous velocity and acceleration.

A model of the above family has been renamed ‘elemental’ by Akcelik (1981) and also used by Robertson, Lucas and Baker (1980) for estimating fuel consumed as follows:

$$ F = f_1 x_s + f_2 d_s + f_3 h $$  \hspace{1cm} (1)

where

- $F$: average fuel consumption per vehicle (mL),
- $x_s$: total section distance (km),
- $d_s$: average stopped delay, i.e. idling time (s),
- $h$: average number of (effective) stops per vehicle (stop rate),
- $f_1$: fuel consumption rate while cruising (mL/km),
- $f_2$: fuel consumption rate while idling (mL/s), and
- $f_3$: excess fuel consumption per vehicle stop (mL).

It is important to recognise that the terms in this equation are associated with easily discernable (though often difficult to mathematically quantify) features of the driving pattern, namely cruising, idling and stopping.

### 2.3 TRAVEL SPEED MODELS

The travel speed model presently under consideration was developed by Watson, Milkins and Marshall (1980) and has been referred to as a ‘lumped’ coefficient model (Watson 1980). Fuel consumption is expressed as:

$$ f_x = k_1 + k_2 \frac{v_f}{v_s} + k_3 v_s + k_4 PKE $$  \hspace{1cm} (2)

where $k_1$ to $k_4$ are constants,

- $PKE = \Delta v_f^2 / x_s$,
- $\Delta v_f^2 = \Sigma (v_{f_i}^2 - v_{f_i}^2)$, final and initial velocities in an acceleration manoeuvre respectively, and
- $x_s$: total section distance.

The coefficients result from regression analysis, but can be associated with, but do not directly quantify (hence lumping), fuel use to overcome the following combined effects:

- $k_1$: rolling resistance, transmission losses, engine efficiency,
- $k_2$: idle consumption (no load, or base engine friction),
- $k_3$: aerodynamic drag, transmission losses, engine efficiency,
- $k_4$: vehicle mass, transmission losses, engine efficiency.

### 2.4 COMPARISON BETWEEN MODELS

Eqn (2) can be employed to calculate the fuel used over the distance $x_s$ and time $t_s$. As

$$ v_s = x_s / t_s $$  \hspace{1cm} (3)

then eqn (2) can be rearranged as

$$ F = k_1 x_s + k_2 t_s + k_3 \frac{x_s^2}{t_s} + k_4 \Delta v_f^2 $$  \hspace{1cm} (4)

### TABLE I

Values of Coefficients $k$ in Fuel Consumption Eqn (2) for the Melbourne University Test Car* at Steady Speed

<table>
<thead>
<tr>
<th>$k_1$</th>
<th>$k_2$</th>
<th>$k_3$</th>
<th>$k_4$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.7</td>
<td>2903</td>
<td>1.216</td>
<td>0.994</td>
<td></td>
</tr>
<tr>
<td>But $k_4$ = idle</td>
<td>9.17</td>
<td>2640</td>
<td>0.922</td>
<td>0.927</td>
</tr>
</tbody>
</table>

* 4.1 L, 6-cylinder automatic transmission Ford Cortina Wagon

The term $k_1 t_s$ may be expanded to

$$ k_1 t_s = (k_1 + k_1') t_s $$  \hspace{1cm} (5)

where $k_1'$ is the idle fuel flow rate and $k_1'$ represents the increment in the time-dependent fuel flow which is the result of the average increase in engine friction when operating at above idle speeds and load (or torque). For the University of Melbourne test car the value of $k_1'$ is about 264 mL/h. Table I demonstrates that forcing $k_1 = k_1'$ reduces the correlation coefficient by about 4 per cent.

Now the total travel time $t_s$ is comprised of the idle time (or stopped time) $d_s$ and the running (or moving) time $t_r$. Thus eqn (4) may be written as

$$ F = k_1 x_s + k_2 d_s + k_3 t_r + k_3 \frac{x_s^2}{t_s} + k_4 \Delta v_f^2 $$  \hspace{1cm} (6)

The positive acceleration kinetic energy change term $\Delta v_f^2$ may be associated with two components that arise from acceleration after a stop, $\Delta v_f^2$ per stop and cruise speed perturbations $\Delta v_c^2$. Hence eqn (6) may be expanded to

$$ F = k_1 x_s + k_2 d_s + k_3 t_r + k_3 \frac{x_s^2}{t_s} + k_4 \Delta v_f^2 $$  \hspace{1cm} (7)

where $h$ is the number of stops.

Comparison of coefficients with those of the elemental model (eqn (1)) yields:

$$ f_1 = k_1 $$  \hspace{1cm} (8)

$$ f_2 = k_2 $$  \hspace{1cm} (9)

$$ f_3 = k_3 \Delta v_f^2 $$  \hspace{1cm} (10)

If $k_2 t_r + k_3 \frac{x_s^2}{t_s} + k_4 \Delta v_f^2$ is non-zero, then the effect of these corrective terms must be incorporated into the coefficients $f_1$ and $f_3$ and eqns (8) and (10) no longer hold.
3. MEASUREMENT OF FUEL CONSUMPTION

The calibration of the coefficients for a model requires measurement of fuel consumption. Errors in measurement can have three main sources:
(a) meter deficiency,
(b) meter installation, and
(c) vehicle variability.

Johnston and Rogers (1979) have reviewed the performance of some commonly available fuel flow meters. No existing fuel flow meter is fault free. Even those having good (± 1 per cent full scale) accuracy during steady-state calibration may lose precision under dynamic operating conditions. These problems may range from major deficiencies, such as leaking seals to ‘overshoot’ when fuel flow rate is suddenly diminished and ‘undershoot’ when the fuel flow suddenly increases.

3.1 INSTALLATION PROBLEMS

These arise principally because of the presence of fuel vapour in fuel lines or in the meter itself. Under-bonnet conditions are severe enough to frequently cause the lighter fractions of the fuel to vapourise. Ordinarily, a modern carburettor will feed both vapour and petrol into the engine should vapourisation occur in the fuel line. Further, significant changes in fuel temperature lead to change in fuel density and incorrect mass flow when the usual volumetric means of measurement is employed.

Recommended practice includes location of the flow meter away from the under-bonnet environment (usually ahead or to one side of the engine compartment) and fuel lines must be continually upward travelling from flow meter to carburettor to avoid errors arising in the variable volume of vapour that may be trapped in the line downstream of the flowmeter if ‘humps’ or vertical loops in the fuel line occur, e.g. sudden fuel vapourisation may temporarily cause the flow of fuel through the meter to cease, conversely sudden condensation of vapour will speed up fuel flow.

3.2 VEHICLE VARIABILITY

There are three major sources of vehicle variability when making fuel flow measurements.
(a) In a typical carburetted engine the float bowl or chamber of the carburettor acts as a ‘buffer’ and fuel inflow is usually intermittent at low flows and fluctuating at high flow rates as demonstrated in Fig. 3. When undertaking on-road measurements, ‘g’ forces due to acceleration, cornering or gradient considerably influence fuel flow. Thus micro- (second-by-second) measurement of fuel flow is often not meaningful. However, integrated results for macro-scale analysis are often repeatable with coefficients of variability as little as 0.3 per cent.
(b) The second source of variability is the change which occurs throughout the life of the vehicle. This is illustrated in Fig. 4, where it can be seen that at about 16 000 km a minimum was reached and from then on the fuel consumption rate increased.
(c) Variability as the result of engine tune up or the replacement of worn or defective parts. The influence of a major retune and muffler replacement on the Hot start ADR27A test cycle fuel use is demonstrated in Fig. 4.

3.3 EXHAUST ANALYSIS

It will be recognised that, when the measurement of carbon dioxide CO₂ is added to the measurement of HC, CO and NOₓ (the usual pollutants measured in a vehicle emissions test facility), instantaneous fuel flow rates can be deduced by the carbon balance method. Unfortunately, fluctuating exhaust flow rates, with varying engine operation, give rise to variable exhaust gas residence times in the exhaust system, in addition to the problem that some mixing of sequential ‘slugs’ of exhaust also occurs.

Both the fuel delivery float bowl and the exhaust system serve to frustrate accurate, dynamic fuel consumption measurement.

3.4 ROUTINE CALIBRATION

The problems with fuel consumption measurement necessitate routine calibration of the entire fuel flow-meter-vehicle system and the use of correction factors to eliminate any long-time scale dependent variability in a long series of tests. If correction factors greater than about 5 per cent are needed then the measurement procedure/test system warrant careful scrutiny for sources of error.

Steady-speed tests over the range idle to 100 km/h at say 10 km/h speed increments can form the basis for routine checking of vehicle/equipment. For on-road checking care should be exercised in selecting roads which are level, straight and smooth.
(to avoid effects of bumps on fuel float chamber levels) and wind speeds should be less than about 10 km/h. A two-way (ideally square law weighted in accordance with vehicle air speed) average of fuel consumption should be used.

At present it can only be speculated that some difference might be observed in fuel consumption on the road and that achieved on the chassis dynamometer vehicle test facility, even when the dynamometer correctly reproduces the road load. (Few of the presently employed dynamometers are capable of replicating an aerodynamic drag force, square law, curve passing through zero load at zero speed.) This difference is likely to arise through float chamber level differences, component temperature variation (e.g. gearbox oil), etc.

3.5 CALIBRATING THE ELEMENTAL MODEL

There are considerable physical difficulties in instrumenting and operating a vehicle to follow prescribed velocity changes with time on the road: it is dangerous for the driver to follow 'head down' the 'drivers aid' chart whilst attempting to simultaneously steer the vehicle. Even with a 'head up' display, on the windshield, the steering task will detract from the driver's ability to keep to the prescribed schedule.

3.6 SURVEILLANCE DRIVING SCHEDULE

A test cycle, suitable for collecting 'mode' data for the elemental model for tests on chassis dynamometer is the Surveillance Driving Schedule (SDS). The SDS was developed in 1974 by the U.S. Environmental Protection Agency to measure vehicle emissions over a variety of steady-state and transient driving conditions (Kunselman et al. 1974b). The acceleration and deceleration modes represented in SDS consist of all possible combinations of the following five speeds: 0, 24, 48, 72 and 96 km/h.

The average acceleration or deceleration rate observed for each mode in the Los Angeles basin is used during the operation of 20 of the 26 transient modes. The remaining six transients are repeated using average acceleration rates higher (1.07 m/s²) or lower (0.58 m/s²) and similarly average deceleration rates higher (−1.34 m/s²) and lower (−0.56 m/s²) to determine the effect of acceleration/deceleration rates on emissions. These accelerations and decelerations were chosen to represent the full range of accelerations and decelerations observed in the CAPE-10 project (Scott Research Laboratories 1971).
4. APPLICATION OF THE RESULTS TO ELEMENTAL MODEL

4.1 CRUISE

Richardson and Akcelik (1982) in Part 1 of this report rightly point out the difficulty in assigning steady-speed fuel consumption values to cruise fuel usage rate, since even under low traffic flow conditions steady-speed cruising does not occur. Fig. 7 illustrates typical velocity time traces for a range of driving conditions. Further discussion of the task and what the concept of cruising speed represents is described in Appendix A.

The question may be asked, at what non zero acceleration does acceleration/deceleration become cruise (0.05, 0.1, 0.2, 0.5 m/s²)? The decision has a significant effect on the proportion of cruise to acceleration/deceleration time.

Fig. 8 illustrates three of several simple elemental approximations that may be employed. Method 1 complies with the concepts developed in eqn (7):

\[ v_c = \text{average running speed} \]

\[ a_c = \text{constant acceleration} \]

\[ \Delta t = \text{constant time} \]

\[ \theta = \text{constant angle} \]

The range of deviation from the steady-speed fuel consumption can be seen as about 50 per cent to 70 per cent. (Negative values are possible if the speed leaving the cruise time is less than the entering speed.) The maximum possible deviation is illustrated at 32 km/h to be +350 per cent in synthetic driving.

4.2 ACCELERATION/DECELERATION

A comparison has been made between the fuel used for the constant acceleration approximation, when acceleration times are known, and actual fuel usage in the same time during the prescribed accelerations of about 30 segments or micro-trips (stop to stop) of the driving cycles shown in Fig. 11. Fuel usage during constant accelerations of values for each micro-trip was obtained from specially conducted tests, the results from which are documented in Appendix B. The results are described graphically in Fig. 12. The constant acceleration approximation describes only 50 per cent of the observed variance (R² = 0.50) in fuel consumption.
Fig. 7 — Typical speed-time traces on arterial roads for peak and off-peak periods.
Corresponding analyses for decelerations are described in Fig. 13. It should be recalled that the smaller fuel usage during deceleration increases the error in measurement. However, under closed throttle deceleration the fuel used is independent of deceleration rate, and dependent only upon deceleration time $t_d$. It is therefore not surprising to find that more of the variance is explained ($R^2 = 0.67$).

An alternate approach to Method 2 in Fig. 8 would be to assume constant acceleration to $v_f$. However, this would lead to variation in the distance travelled during acceleration. Therefore Method 2 is likely to be the most realistic of the linearisation techniques, yet it is seen to perform poorly in practice for elemental (or modal) analysis.

5. EXTENSION OF THE PKE METHOD TOWARDS THE MICROSCALE

As reported by Watson et al. (1980) and Poynton and Dawson (1980), the coefficients in eqn (2) may be determined from fuel consumption per unit distance for steady-speed driving and micro-trip analysis of the ADR27A driving cycle. The regression for the coefficients $k_1$, $k_2$, and $k_3$ is performed on the steady-speed results and $k_1$ is found by further regression for the 18 micro-trip fuel consumptions in the equation

$$ \Delta f_a = k_1 \text{PKE} $$

(11)

where

$$ \Delta f_a = f_a - f_c $$

(12)

and $f_c$ is the steady-speed fuel consumption per unit distance.

---

![Fig. 8 — Three 'elemental' approximations to actual driving patterns](image)

![Fig. 9 — Cruise fuel consumption v. speed for driving cycles and 'synthetic' driving](image)
Fig. 10 — Emission rates during cruise elements of ADR27A drive cycle compared to steady speed
### Cycle Data

<table>
<thead>
<tr>
<th>Cycle</th>
<th>Average Speed</th>
<th>Max Speed</th>
<th>Max/Idle Speed</th>
<th>Max Mean Acceleration</th>
<th>Max Mean Deceleration</th>
<th>Idle Mean</th>
<th>Idle Time</th>
<th>PKE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECE (ADR27)</td>
<td>18.8 km/h</td>
<td>50.0 km/h</td>
<td>1.06 m/s²</td>
<td>0.64 m/s²</td>
<td>-1.05 m/s²</td>
<td>28.7</td>
<td>0.284</td>
<td></td>
</tr>
<tr>
<td>Melbourne Initial Cycle</td>
<td>26.7 km/h</td>
<td>85.8 km/h</td>
<td>2.84 m/s²</td>
<td>-3.04 m/s²</td>
<td>0.74 m/s²</td>
<td>32.0</td>
<td>0.357</td>
<td></td>
</tr>
<tr>
<td>U.S. 1972 FTP</td>
<td>31.5 km/h</td>
<td>91.2 km/h</td>
<td>1.63 m/s²</td>
<td>-1.62 m/s²</td>
<td>0.58 m/s²</td>
<td>17.8</td>
<td>0.406</td>
<td></td>
</tr>
<tr>
<td>Sydney</td>
<td>33.5 km/h</td>
<td>80.9 km/h</td>
<td>2.73 m/s²</td>
<td>-3.31 m/s²</td>
<td>0.78 m/s²</td>
<td>18.4</td>
<td>0.505</td>
<td></td>
</tr>
<tr>
<td>U.S. Highway</td>
<td>77.6 km/h</td>
<td>96.3 km/h</td>
<td>1.53 m/s²</td>
<td>-1.60 m/s²</td>
<td>0.26 m/s²</td>
<td>0.8</td>
<td>0.138</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 11 — Driving cycles employed in the simulation studies

Fig. 12 — Constant acceleration fuel consumption v. driving cycle fuel consumption

Fig. 13 — Constant deceleration fuel consumption v. driving cycle fuel consumption
TABLE II
MEASURED v. PREDICTED FUEL CONSUMPTION DURING STOP-STARTS

<table>
<thead>
<tr>
<th>End of Micro-Trip</th>
<th>Δt (s)</th>
<th>Δx (km)</th>
<th>PKE (m/s²)</th>
<th>Measured FC (mL/km)</th>
<th>Predicted FC (mL/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sydney Cycle</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>69</td>
<td>0.256</td>
<td>0.005</td>
<td>172</td>
<td>171</td>
</tr>
<tr>
<td>2</td>
<td>23</td>
<td>0.163</td>
<td>1.09</td>
<td>184</td>
<td>188</td>
</tr>
<tr>
<td>3</td>
<td>39</td>
<td>0.156</td>
<td>1.00</td>
<td>215</td>
<td>221</td>
</tr>
<tr>
<td>4</td>
<td>48</td>
<td>0.052</td>
<td>1.00</td>
<td>317</td>
<td>331</td>
</tr>
<tr>
<td>5</td>
<td>31</td>
<td>0.028</td>
<td>1.10</td>
<td>953</td>
<td>939</td>
</tr>
<tr>
<td>6</td>
<td>48</td>
<td>0.430</td>
<td>0.75</td>
<td>157</td>
<td>150</td>
</tr>
<tr>
<td>7</td>
<td>34</td>
<td>0.273</td>
<td>0.70</td>
<td>149</td>
<td>152</td>
</tr>
<tr>
<td>Melbourne Initial Cycle</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>60</td>
<td>0.469</td>
<td>0.69</td>
<td>158</td>
<td>176</td>
</tr>
<tr>
<td>2</td>
<td>68</td>
<td>0.231</td>
<td>0.63</td>
<td>173</td>
<td>173</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>0.114</td>
<td>0.82</td>
<td>204</td>
<td>208</td>
</tr>
<tr>
<td>4</td>
<td>70</td>
<td>0.165</td>
<td>1.16</td>
<td>213</td>
<td>218</td>
</tr>
<tr>
<td>5</td>
<td>79</td>
<td>0.170</td>
<td>0.56</td>
<td>177</td>
<td>185</td>
</tr>
</tbody>
</table>

Fig. 14 — Generalised fuel consumption or emission surface on average speed and positive kinetic energy per unit distance axes \( \left( \nu_s, PKE \right) \)

\[ \Delta f_s = 72.07 \text{ PKE} \quad \left( R^2 = 0.83 \right) \]  

where \( \Delta f_s \) is in mL/km and \( PKE \) is in m/s². Therefore, eqn (2) can be written as

\[ f_s = -30.7 + 2903/\nu_s + 1.216\nu_s + 72.07 \text{ PKE} \quad (14) \]

where \( \nu_s \) is in km/h. Application of eqn (14) to predicting the fuel consumed in the deceleration-accelerations of the Sydney and Melbourne initial cycles is given in Table II. These results indicate that in excess of 98 per cent of the observed variance in fuel consumption is explained and the mean prediction error is 1.9 per cent.

6. COMPARISON BETWEEN METHODS

6.1 COEFFICIENTS FOR THE ELEMENTAL MODEL

When the elemental model is expressed as in eqn (1) in terms of distance related fuel \( f_x \), stop time fuel \( f_d \), and stopping fuel \( f_h \), the coefficients may be derived from fuel usage in various driving cycles when the idle and cruise or distance related fuel consumption is known. Use of data in Table I and Appendix B, together with the number of halts per driving cycle, allows calculation of the fuel per stop as quantified in Table III for the test car.

TABLE III
FUEL USAGE PER STOP

<table>
<thead>
<tr>
<th>Test Cycle</th>
<th>( \nu_s ) (km/h)</th>
<th>Number of Stops</th>
<th>Fuel/Stop (mL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADR27A</td>
<td>31.5</td>
<td>16</td>
<td>23</td>
</tr>
<tr>
<td>Sydney</td>
<td>33.5</td>
<td>8</td>
<td>42</td>
</tr>
<tr>
<td>Melb. Init.</td>
<td>26.7</td>
<td>8</td>
<td>28</td>
</tr>
<tr>
<td>ADR27</td>
<td>18.8</td>
<td>12</td>
<td>7</td>
</tr>
</tbody>
</table>

The extreme variability in fuel/stop leads to the observation that the distance related fuel used should not be the steady cruise speed fuel usage, but rather some higher value to allow for speed perturbations, but how much larger? Further, it has been
Fig. 15 — Incremental fuel consumption over average-speed fuel usage for stop-starts with constant deceleration-acceleration and for driving cycle micro trips

Fig. 16 — Comparison between predicted fuel consumption using ACDI elemental model and measured fuel consumption for ADR27A cycle assumed to consist of (a) 17 identical micro trips and (b) 17 non-identical but linearised micro trips

Fig. 17 — Comparison of predicted v. measured fuel consumption for La Trobe Street/Victoria Parade on a link-by-link basis (PKE predictive equation based on Swanston Street regression)
demonstrated that the fuel used in a stop is related to the speed from which the stop was made (or rather the speed regained after the stop) (Watson 1980).

It seems unlikely that the speed from which a stop is made can be accurately forecast, on a link-by-link basis, but more likely an estimate could be made for cruise speed for several links. In this case it is expected that the stop fuel coefficient \( f_s \) would be constant.

6.2 INCLUSION OF ACCELERATION/DECELERATION

The best one could expect for an elemental model is some estimate of the acceleration, cruise and deceleration times \( t_a \), \( t_c \) and \( t_d \). With knowledge of the stop time \( d_s \), the number of stops \( h \) and the distance travelled \( x \), the fuel used can be calculated from the data in Appendix B.

For example, if we examine the Los Angeles trip which is the basis of ADR27A and remove the second micro-trip which represents freeway driving, then \( x = 8.75 \text{ km} \), \( h = 17 \) stops, trip time = (1372 s less 2nd micro-trip 208 s) = 1164 s, idle (stop) time = 224 s, and hence running time for an average micro-trip without idle time:

\[
 t_r = (1164 - 224)/17 = 55.3 \text{ s}
\]

If we assume \( t_c = t_a = t_d \), then the cruise speed is found as 51 km/h. Fuel consumption for this speed is 88 mL/km.

Total fuel is the sum of the acceleration/cruise/deceleration/idle components. From Appendix B, acceleration/deceleration fuel = 17 x 41 = 697 mL.

<table>
<thead>
<tr>
<th>Cycle</th>
<th>ADR27A</th>
<th>Sydney</th>
<th>Melb. Initial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurement</td>
<td>142.0</td>
<td>146.3</td>
<td>157.7</td>
</tr>
<tr>
<td>Travel Time</td>
<td>152.4 (7)</td>
<td>147.9 (1)</td>
<td>166.6 (6)</td>
</tr>
<tr>
<td>PKE Method</td>
<td>140.2 (−1)</td>
<td>146.4 (0)</td>
<td>160.8 (2)</td>
</tr>
<tr>
<td>Elemental (ACDI)</td>
<td>130.8 (−8)</td>
<td>127.7 (−13)</td>
<td>141.5 (−10)</td>
</tr>
<tr>
<td>Method 2 ( t_c / t_d = 50% )</td>
<td>134.5 (−5)</td>
<td>124.3 (−15)</td>
<td>128.1 (−18)</td>
</tr>
</tbody>
</table>

Percentage errors are shown in brackets.
7. FURTHER RESEARCH

A major problem with the present predictions of PKE is that they rely on variables such as intersection frequency with no accommodation of traffic signal control variables such as cycle time etc. This is because of the 'worm's eye' nature of our present driving pattern sampling process. More data collection is needed in which driving patterns are related to signal status. This should enable the statistical determination of the relationship between PKE and with signal settings and other acceptable variables including position in platoon as well as those described above. Hopefully the statistical relations may avoid the need for subjective evaluation of road environment as proposed by Richardson and Akcelik (1982) in Part 1 of this report.

Further, the proposed experiment could provide data to determine 'cruise' speeds for the elemental model and provide measured data for back-to-back evaluation of each of the models to establish their suitability for fuel consumption estimation.

8. CONCLUSIONS

(a) The PKE-average speed model for fuel consumption, has associated with each of its coefficients a plausible physical concept of vehicle design.

(b) By elimination of some non-trivial terms it can be shown that the PKE-average speed model reduces to the elemental fuel consumption equation.

(c) The coefficients for the PKE-based equation may be derived from simple laboratory tests including steady-speed driving and a micro-trip or segmental analysis of standard ADR27A (and AS2077) driving.

(d) The elemental analysis calls for higher resolution of fuel flow measurements and is likely to suffer from errors arising in the fluctuating fuel flow to the carburettor and restricted sensitivity of the fuel flow meter.

(e) Even when the number of stops term in the elemental model is extended to include finite, but constant accelerations and decelerations, it has been shown to perform less well than the travel speed based model.

(f) In urban driving non-freeway conditions results indicate that steady speed does not often occur. This means that it is difficult to prescribe 'cruise' speed fuel consumption.

9. FURTHER INVESTIGATION

Whereas Richardson and Akcelik (1982) have proposed that fuel consumption might be related to the cruise speed and environment, and fuel usage identified for speed changes including those to rest (stops), it is proposed that uninterrupted travelling speeds (link distances/travelling time) and speed perturbations (PKE) may be statistically related to a vehicle's position in a platoon and its surrounding environment.

A new series of experiments should be conducted using instrumented vehicle(s) in which position and time of the test vehicle(s) is recorded along with a log of signal status. The results of regression analysis could be provided as look up tables or as explicit functions of the correlating variable.

Fuel usage can then be accurately forecast from coefficients for average speed and PKE terms derivable from routine tests to AS2077 and steady-speed driving for which a wide range of data already exists without recourse to special tests. The present use of average speed, \( v_r \), can be modified to running speed, \( \bar{v} \), and to explicitly include the stopped delay (idle) fuel flow rate as follows:

\[
\bar{f}_x = b_1 + b_2/v_r + b_3 v_r + b_4 d_i/x_s + b_5 PKE
\]

TABLE V

<p>| Regression Coefficients for Fuel Consumption (mL/km) FOR eqns (15) AND (1) |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|</p>
<table>
<thead>
<tr>
<th>Data Source</th>
<th>Regression Method</th>
<th>Coefficients ( b_1 )</th>
<th>Coefficients ( b_2 )</th>
<th>Coefficients ( b_3 )</th>
<th>Coefficients ( b_4 )</th>
<th>Coefficients ( b_5 )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DYNO. 2-STEP</td>
<td>30.7</td>
<td>2900</td>
<td>1.22</td>
<td>2540</td>
<td>94.8</td>
<td>0.990</td>
<td></td>
</tr>
<tr>
<td>DYNO. MULT.</td>
<td>34.2</td>
<td>2900</td>
<td>1.20</td>
<td>2700</td>
<td>93.6</td>
<td>0.990</td>
<td></td>
</tr>
<tr>
<td>ROAD MULT.</td>
<td>19.4</td>
<td>2460</td>
<td>0.975</td>
<td>2740</td>
<td>115.6</td>
<td>0.897</td>
<td></td>
</tr>
</tbody>
</table>

(b) Equation 1

<table>
<thead>
<tr>
<th>Data Source</th>
<th>Regression Method</th>
<th>( f_1 )</th>
<th>( f_2 )</th>
<th>( f_3 )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DYNO. MULT.</td>
<td>108.7</td>
<td>2400</td>
<td>15.40</td>
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<td>2320</td>
<td>8.44</td>
<td>0.764</td>
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</table>

* Falls to 0.796 when steady speed results included since \( f \) is treated as a constant.

Units \( v_r \) (km/h), \( d_i \) (h), \( x_s \) (km), PKE (m/s²), h (stops/km)
Examples of regression coefficients are given in Table V for this new expression for the Ford Cortina test vehicle. Two equations are presented, for dynamometer results. In two-step regression, coefficients $b_1$ to $b_4$ for cruise and $b_5$ for idling are determined separately in step 1, followed by stage 2 to obtain incremental fuel usage associated with PKE. The second equation is obtained from multiple regression. The data base is a combination of ADR27A, Sydney and Melbourne Initial Cycle microtrips plus 15 steady-speed results over the range 7 to 113 km/h, all data being measured in early 1979.

For comparison regression to 1463 measurements made in 1978 on links on Melbourne roads are included. The amount of variance explained ($R^2$) is about 10 per cent less, probably as the result of residual kinetic energy, wind, grade and other effects on the link-by-link analysis.

Also given are multiple regression results to eqn (1) for further comparison. It will be noted that eqn (15) explains more of the fuel consumption variance than eqn (1) (however, the use of eqn (1) for regression analysis does not represent the normal use of the 'elemental model' approach). It is concluded that eqn (15) in explaining 99 per cent of the dynamometer results, away from additional variability on the road, clearly indicates the effectiveness of this new method.

REFERENCES


ACKNOWLEDGEMENT

The author wishes to record his appreciation to his colleagues E. Milkins, M.O. Preston and P. Beardsley for their assistance in preparing this paper, and to J. Skazas for providing information from his M.Eng Sci. thesis. Funding from the Australian Road Research Board (Project 369) and NERDDC (Project 79/9239) made this paper possible.
When describing fuel consumption by means of elemental modes, i.e. acceleration, cruise, deceleration and idling, we are attempting to approximate the variable manoeuvres executed by vehicles in traffic flow to more rigid behaviour.

In the free flow situation distant from the intersection it might be expected that a normal distribution of speed would be found. This normal distribution originates in vehicles travelling at different speeds which reflect capability, viz. laden trucks, and mild acceleration/decelerations since steady-speed driving rarely occurs in practice. Cruise speed reductions from the free speed may be expected to occur as traffic flow increases (Freeman, Fox and Associates 1972). At the intersection the range of situations from complete stops, partial stops and uninterrupted flow will be found. Greatest frequency of stopping is found at the stop line and the maximum queue length can be observed.

It is the spectrum of speed variation remote from the intersection that Richardson and Akcelik (1982) suggest may approximate to a constant speed cruise, with factors applied to fuel consumption to allow for the deviations in speed from the mean. In contrast, actual deviations are accommodated in the driving cycle approach and can be shown to be statistically representative (Braunsteins 1981).

Representation of the fuel used in accelerating and decelerating by a single value, independent of driving path must only be an approximation to reality, since a large number of factors, particularly position in platoon (Herman, Lam and Rothery 1971) affect the path. Even a linearised path is a poor approximation to reality as can be seen in Figs 7 and 8 in the main text.
APPENDIX B

FUEL USAGE DURING CONSTANT ACCELERATION AND DECELERATION

The results given in Table VI were obtained from chassis dynomometer tests on the Melbourne University test car (4.1 L, 6-cylinder Ford Cortina Wagon with automatic transmission). The results are the average of at least three tests. Steady-speed fuel usage can be deduced from Fig. 9 or Table I. Idle fuel flow rate was 0.700 mL/s.

<table>
<thead>
<tr>
<th>Accel./ Decel. (km/h-s)</th>
<th>Initial Speed (km/h)</th>
<th>Final Speed (km/h)</th>
<th>Fuel Used* (mL)</th>
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<tbody>
<tr>
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<td>0</td>
<td>14.3</td>
</tr>
</tbody>
</table>

* Values less than 15 mL are estimated to be subject to errors greater than approximately 10 per cent.
Part 5
RELATION BETWEEN TWO FUEL CONSUMPTION MODELS

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(First written in May 1982)
1. INTRODUCTION

At the Australian Road Research Board Seminar on Fuel Consumption Modelling for Urban Traffic Management, October 1981, where the preceding papers were presented, the authors agreed to place on record the resolution of differences in the approaches adopted by them for developing simple fuel consumption models. This paper presents a joint statement on this question, paying particular attention to the relation between the two simple models of interest, namely the elemental and the PKE-average speed models. Establishing the relation aids the collection and analysis of data on a standard basis and allows the conversion of the results for use with the elemental model.

The differences between the approaches adopted by the traffic engineer and the vehicle design engineer stem from different modelling needs, that is:

(a) the traffic engineer needs to employ models which make an explicit allowance for the effects of traffic management/control actions on easily discernable characteristics of traffic movement, namely cruise, delay and stops; and

(b) the vehicle design engineer needs to monitor vehicle performance characteristics by employing models which can be calibrated from vehicle tests using standard driving cycles.

However, it is to be expected that the models developed from a traffic movement viewpoint and a single vehicle viewpoint agree to some extent. The relation between the elemental model representing the former approach and the PKE-average speed model representing the latter approach is discussed in Part 3 of this report. This paper presents a discussion of the relation between the elemental model and the modified version of the PKE model proposed in Part 4 of the report.

2. RELATION BETWEEN MODELS

To establish the relation between the two models, an understanding of the differences between the following speed definitions is necessary (see Figs 1 and 2):

(a) cruise speed, $v_c$, which is the average speed while travelling uninterrupted by traffic control devices;

(b) running speed, $v_r$, which is the average speed including the effects of deceleration-acceleration delays due to traffic control devices, but excluding stopped delay time; and

(c) interrupted travel speed, $v_s$, which is the average speed including the effects of both deceleration-acceleration delays and stopped delay time imposed by traffic control devices.

The relationships between these three variables and the elemental model variables of delay and number of stops are described in detail in the Appendix.

The following expression is the PKE-$v_s$ model discussed in the preceding papers:

$$ f_x = k_1 + \frac{k_2}{v_s} + k_3 v_s + k_4 PKE $$

where

- $f_x = \text{fuel consumption per unit distance (mL/km)}$
- $k_1$ to $k_4 = \text{model coefficients (constants)}$
- $v_s = \text{average interrupted travel speed (km/h)}$
- $PKE = \text{a variable related to positive kinetic energy changes, and given by}$

$$ PKE = \frac{\sum (v_i^2 - v_f^2)}{3600 x_s} $$

where $v_i$ and $v_f$ (km/h) are the final and initial velocities in an acceleration, $x_s$ (km) is the total section distance, and $PKE$ is in km/h/s.

An example given in Part 2 of this report illustrates a deficiency of the PKE-$v_s$ model in that the model fails to predict accurately the changes in fuel consumption due to changes in stopped delay time. The following formula proposed by Watson (eqn (16) in Part 4) overcomes this deficiency by treating the stopped time as an explicit variable and by replacing the average interrupted travel speed ($v_s$) by the average running speed ($v_r$):

$$ f_x = b_1 + b_2 v_r + b_3 v_r + b_4 d_s + b_5 PKE $$

where $f_x$, $v_r$, $PKE$ are as described above, $d_s$ is the stopped delay time per unit distance (s/km), and $b_1$ to $b_5$ are the model coefficients (constants).

By measuring travel time $t_s$ and stopped delay time $d_s$ along the total section distance $x_s$, the values of $v_r$ and $d_s$ can be easily calculated $(v_r = 3600 x_s / (t_s - d_s))$ and $d_s = d_s / x_s$, where $t_s$, $d_s$ are in seconds, $x_s$ is in km). Coefficients $b_1$ to $b_5$ can be derived by regression of measured values of steady-speed fuel consumption with constant cruise speed, $v_c$, as for the PKE-$v_s$ model, although care should be taken to account for correlation between independent variables. Coefficient $b_1$ is the idling fuel consumption rate (mL/s) obtained by direct measurement. Coefficient $b_5$ is found by regression of the excess fuel consumption (calculated as the actual fuel consumption less the sum of steady-speed and idling fuel consumption) on $PKE$. The units of the coefficients are as follows: $b_1$ (mL/km), $b_2$ (mL/h), $b_3$ (mL-h/km²), $b_4$ (mL/s), and $b_5$ (mL-h-s/km³).

To show the relation with the PKE-$v_s$ model (eqn (3)), consider the following form of the elemental model (see Part 3):

$$ f_x = f_1 + f_2 d_s + f_3 h $$

where $f_1$, $v_r$, $PKE$ are as described above, $d_s$ is the stopped delay time per unit distance (s/km), and $f_1$, $f_2$, $f_3$ are the model coefficients (constants).
Fig. 1 — Time-distance diagrams showing relationships among various traffic variables
(a) An uninterrupted trip

(b) An interrupted trip with no stopped delay time

(c) An interrupted trip with stopped delay time

Fig. 2 — Speed-time profiles for Fig. 1
where \( f_e \) = fuel consumption per unit distance (mL/km),
\( \bar{d}_s \) = stopped time per unit distance (s/km),
\( \bar{h} \) = average number of stops per unit distance (stops/km),
\( f_i \) = fuel consumption per unit distance while cruising (mL/km),
\( f_c \) = fuel consumption per unit time while idling (mL/s), and
\( f_x \) = excess fuel consumption per stop (mL/stop).

As shown in the Appendix, the relation between the elemental model (eqn (4)) and the PKE-\( v_i \) model (eqn (3)) is:

\[
 f_1 = b_1 + b_2 v_i + b_3 v_i^2 + \frac{\Delta v_i^2}{3600} \tag{5} \\
 f_2 = b_i \\
 f_3 = b_5 \frac{v_i^2}{3600} + b_2 \frac{d_n}{3600} \tag{7}
\]

with the following unexplained term:

\[
 \theta = -\frac{b_3 v_i}{1 + 3600/v_i \bar{h} d_n} \tag{8}
\]

where

\( \Delta v_i^2 / x_s \) = PKE term related to speed perturbations about the cruise speed, \( v_c \), while cruising unaffected by traffic controls,

\( d_n \) = deceleration-acceleration delay per stop (the time to decelerate from \( v_c \) to zero speed and to accelerate back to \( v_c \) less the time to travel the deceleration-acceleration distance at uninterrupted speed \( v_c \)), and

\( f_e \) = [eqns (5) to (7) in eqn (4)] + \theta.

Eqns (5) to (7) indicate that the elemental and PKE-\( v_i \) models are very similar. This is subject to various minor simplifications described in the Appendix. The unexplained term (eqn (8)) could be due to an omission in the PKE-\( v_i \) model. Alternatively, it could be related to a term which may need including in eqn (7) for excess fuel consumption.

3. CONCLUSIONS

The findings are encouraging in that the models developed using two different approaches are shown to be very similar. Resolution of the problem regarding the unexplained term could enable the elemental model coefficients to be derived from dynamometer tests of vehicles at steady speed and to standard driving cycles such as ADR 27A.

It is important to obtain vehicle fuel consumption and driving pattern data under conditions free from gradient effects, or this effect explicitly allowed for, under a wide range of monitored traffic control conditions. This could form an agreed data base for testing the present and alternative models. Special tests are necessary to enable the testing of the relationship between the PKE-\( v_i \) and elemental models put forward in this paper. Particular attention needs to be paid to the effect of different acceleration and deceleration rates and profiles. Future work should also concentrate on the production of data for different vehicle types. In these respects, vehicle maps may prove to be useful as a way of storing and manipulating data for comparisons between alternative models.

The reader of the preceding papers will be aware that there still exist differences of opinion amongst the authors about a preferred method for fuel consumption prediction. Continued investigation, and importantly, dialogue between traffic engineers and vehicle designers should reveal the models which are best suited to the range of problems to be tackled and to the resources available.
APPENDIX

DERIVATION OF THE RELATION BETWEEN THE ELEMENTAL AND PKE-\(v\), MODELS

In order to establish the relation between the elemental and PKE-\(v\) models (eqns (3) and (4)), it is necessary to understand the relationships among the traffic variables used in these models. These relationships are shown in Figs 1 and 2, and are summarised below (constant 3600 appears in the formulae because time and delay variables are in seconds, and speed variables are in km/h).

The running time, \(t_r\), is the sum of the uninterrupted cruise time along the total section distance \((t_u = 3600 \frac{x_s}{v_c}\), where \(v_c\) is the average cruise speed) and the delay due to stops and starts imposed by traffic controls (not including any stopped delay time):

\[
t_r = t_u + h d_h = \frac{3600 x_s}{v_c} + h d_h
\]  

where \(h\) = average number of stops per vehicle, and \(d_h\) = average deceleration-acceleration delay per stop (see Part 2 for formulae to calculate \(d_h\)).

Therefore, the relation between the running speed and the cruise speed is:

\[
\frac{3600}{v_r} = \frac{3600}{v_c} + \bar{h} d_h
\]  

where

\[
\frac{3600}{v_r} = \frac{\bar{t}_r}{x_s} - \text{average running time per unit distance (s/km)},
\]

\[
\frac{3600}{v_c} = \frac{\bar{t}_c}{x_s} - \text{average cruise speed per unit distance (s/km)},
\]

\[
\bar{h} = h /x_s - \text{average number of stops per unit distance (stops/km)}.
\]

Travel time including the stopped delay time \((d_s)\) is:

\[
t_s = t_r + d_s = \frac{3600 x_s}{v_r} + d_s
\]  

From eqns (9) and (11),

\[
t_s = t_u + d_s + h d_h = \frac{3600 x_s}{v_c} + d
\]  

where \(d = d_s + h d_h\) is the 'delay' experienced during travel along distance \(x_s\) (difference between interrupted and uninterrupted travel times, i.e. \(t_s - t_u\)).

Therefore, the average interrupted speed allowing for both stopped delay time and deceleration-acceleration delays is related to the average running and cruise speeds as follows:

\[
\frac{3600}{v_s} = \frac{3600}{v_r} + \bar{d} = \frac{3600}{v_c} + \bar{d}
\]  

where

\[
\frac{3600}{v_s} = \bar{t}_s = t_s /x_s - \text{interrupted travel time per unit distance (s/km)},
\]

\[
d_s = \text{average stopped time per unit distance (s/km)},
\]

\[
\bar{d} = \bar{d}_s + \bar{h} d_h - \text{average delay per unit distance (s/km)}.
\]

As discussed in Part 2, the elemental model requires the following modification to the PKE term of eqn (3):

\[
b_1 PKE = b'_s PKE_1 + b''_s PKE_2
\]  

where

PKE_1 is related to speed fluctuations while cruising uninterrupted by traffic controls, and
PKE_2 is related to stop-start manoeuvres imposed by traffic controls.

For the following analysis, assume \(b'_s = b'_c = b_s\), and put PKE_1 = \(\Delta v^2 /3600x_s\). Further, neglecting minor speed perturbations during acceleration and deceleration manoeuvres, PKE_2 = \(hv^2 /3600x_s = \bar{h} v_c^2 /3600\).

Thus the PKE-\(v\) model (eqn (3) ) can be written as:

\[
f_x = b_1 + b_2/v_r + b_3 v_r
\]

\[+ b_4 \bar{d}_s + b_5 \frac{\Delta v^2}{3600 x_s} + b_6 \bar{h} \frac{v_c^2}{3600}
\]  

From eqns (10) and (15),

\[
f_x = b_1 + b_2/v_c + b_3 \bar{h} \frac{d_s}{3600}
\]

\[+ b_3 \frac{v_c}{1 + v_c \bar{h} d_s /3600}
\]

\[+ b_4 \bar{d}_s + b_5 \frac{\Delta v^2}{3600 x_s} + b_6 \bar{h} \frac{v_c^2}{3600}
\]  

\[= \frac{3600 v_r}{v_c} + \bar{d} = \frac{3600 v_c}{v_r} + \bar{d}
\]
Comparing eqn (16) with eqn (4) in the main text, the relation between the elemental and PKE-v, models is

\[
\begin{align*}
&= (b_1 + \frac{b_2}{v_c} + b_3 v_c) + b_s \frac{\Delta v_i}{3600 x_t} \\
&+ b_4 \bar{d}_d + \left( b_5 \frac{v_c^2}{3600} + b_2 \frac{d_h}{3600} \right) \bar{h}
\end{align*}
\]

This term results from the \((b_3, v_c)\) term in the PKE-v, model and cannot be related to \(f_i\) (eqn (19)) because of the form of the denominator.

The adjusted excess fuel consumption per stop discounting for idling fuel consumption during deceleration-acceleration delay per stop \((f'_i = f_i - f_d)\) can be found from eqns (18) and (19) by putting \(\frac{b_2}{3600} = b_i\):

\[
f'_i = b_s \frac{v_c^2}{3600}
\]

Watson (eqn (5) of Part 4) states that \(\frac{b_2}{3600} = b_s + b'_i\), where \(b'_i\) is the incremental fuel flow rate due to the increase in engine friction when operating at above idle speeds and load. This suggests that \(b'_i\) is speed-independent, although this needs further investigation. For deriving eqn (21) \(b'_i\) is neglected. It should also be noted that, if coefficient \(b_i\) is determined by regression of measured steady-speed fuel consumption values with cruise speed, \(\frac{b_2}{3600} = b_s\) is unlikely to hold as discussed in Parts 3 and 4. The equivalence expressed by eqns (17) to (21) also neglects this point. In spite of this and several other minor simplifications described above, eqns (17) to (19) demonstrate that the elemental and PKE-v, models are substantially similar.
Part 6

SOME RESULTS ON FUEL CONSUMPTION MODELS

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(First written in July 1982)
1. INTRODUCTION

Alternative simple models for predicting fuel consumption of vehicles in urban traffic, specifically the elemental and PKE models and their relationship have been discussed in detail in previous parts of this report. A study of the derivation of the elemental model parameters from an expression of instantaneous fuel consumption (Bayley 1980) has been discussed in Part 3. The results of further work on this subject reported in detail in Akcelik and Bayley (1981) and Akcelik (1982) are summarised in this part. These results answer some of the questions raised in previous parts of the report.

The fuel consumption formulae given here apply to a level road, but they can be extended by including the road gradient as an additional term, e.g. see Bester (1981). Detailed listing of data used to derive the results presented in this paper can be found in Akcelik (1982).

2. INSTANTANEOUS FUEL CONSUMPTION

Instantaneous fuel consumption models can be used directly in association with microscopic traffic simulation models which can calculate the instantaneous speed and acceleration of individual vehicles, e.g. MULTSIM (Gibbs and Wilson 1980) and NETSIM (Lieberman et al. 1979), or when speed-time traces are available as in the cases of driving cycle data or on-road data from instrumented cars. This class of model also provides the basic relationships from which simpler fuel consumption models such as the elemental model and the PKE model can be derived.

An investigation of a comprehensive form of instantaneous fuel consumption function has shown that the following simpler form of the function is adequate:

\[ f = \frac{dF}{dt} = k_1 + k_2 v + k_3 v^2 + |k_4 a v + k_5 a^2 v^2| > 0 \]  

where  
- \( F \): fuel consumption (mL),  
- \( t \): time (s),  
- \( f \): \( dF/dt \) = instantaneous fuel consumption per unit time (mL/s),  
- \( v \): instantaneous speed (km/h),  
- \( a \): \( dv/dt \) = instantaneous acceleration rate (km/h/s),  
- \( k_1 \): constant idling fuel consumption rate (mL/s),  
- \( k_2, k_3 \): constants representing fuel consumption related to rolling resistance and air resistance, and  
- \( k_4, k_5 \): constants related to fuel consumption due to positive acceleration.

The coefficients of the instantaneous fuel consumption function can be determined as follows.

(a) Determine the idle fuel consumption parameter, \( k_1 \), by direct measurement.
(b) Determine the rolling resistance and air resistance parameters, \( k_2 \) and \( k_3 \), from constant-speed cruise fuel consumption data (see Section 3).
(c) Determine the positive acceleration coefficients, \( k_4 \) and \( k_5 \), from acceleration fuel consumption data (see Section 4). It should be noted that the \( a^2 v \) term in eqn (1) gives an overall improvement in prediction ability by providing sensitivity to high acceleration fuel consumption as found by Evans and Takasaki (1981) and Waters and Laker (1980).

When calibrated in this way, the instantaneous fuel consumption function is sensitive to different conditions of travel. This is in contrast with the method of determining all model coefficients by regression analysis, in which case the parameters describing the idle, cruise and acceleration conditions are unlikely to have individually correct values. This is because of high inter-correlations of predictor variables (multi-collinearity). Using such a regression equation would be a source of error in evaluating alternative traffic management/control strategies, e.g. evaluating minimum-delay against minimum-fuel consumption strategy (see Hurley, Radwan and Benevelli 1981). It is therefore necessary to use separate functions for idle, cruise and other travel conditions, or to use a single function with the coefficients related to idle, cruise and acceleration conditions determined as described in this paper. The same considerations apply to the aggregate fuel consumption functions such as the PKE model. The elemental model satisfies this requirement by definition.

For the Melbourne University test car (Ford Cortina Wagon, 6-cylinder, 4.1 L, automatic transmission), \( k_1 = 0.700 \), \( k_2 = 0.00442 \), \( k_3 = 0.220 \times 10^{-5} \), \( k_4 = 0.00762 \), \( k_5 = 0.886 \times 10^{-3} \) are found by separate analyses of constant-speed cruise and acceleration fuel consumption data.

3. CONSTANT-SPEED CRUISE FUEL CONSUMPTION

For steady-speed travel, the fuel consumption per unit distance can be found from eqn (1) as \( f/v \) and putting \( v = v_c \) and \( a = 0 \):

\[ f_c = b_1 + b_2 \frac{v_c^2 - b_3 v_c^2}{v_c} \]  

where  
- \( f_c \): fuel consumption per unit distance (mL/km),  
- \( v_c \): constant cruise speed (km/h), and  
- \( b_1 \) to \( b_3 \): coefficients related to the first three coefficients of the instantaneous fuel consumption function as follows: \( b_1 = 3600 k_1 \), \( b_2 = 3600 k_2 \), \( b_3 = 3600 k_3 \).
Fig. 1 — Constant-speed fuel consumption per unit distance for the Melbourne University test car

\[ v_c = 90 \text{ km/h} \]

Fig. 2 — Acceleration fuel consumption as a function of the acceleration time for the Melbourne University test car

\[ v_c = 90 \text{ km/h} \]

\[ \bar{a}_o = 5.7 \]

\[ v_c = 60 \text{ km/h} \]

\[ \bar{a}_o = 6.0 \]

\[ v_c = 30 \text{ km/h} \]

\[ \bar{a}_o = 7.7 \]
The recommended method for determining the coefficients of the cruise fuel consumption function is to measure the idling fuel consumption rate, \( b_i \), the minimum fuel consumption rate while cruising, \( f_o \), and the (optimum) speed at which this consumption is achieved, \( v_o \) (usually in the range 40 to 60 km/h), and to calculate \( b_i \) and \( b_o \) from:

\[
b_i = f_o - \frac{1.5 b_2}{v_o} \quad \text{and} \quad b_o = \frac{b_2}{2v_o^2} \tag{2a}
\]

The results obtained for the Melbourne University test car using this method are \( b_i = 2520 \), \( b_o = 15.9 \) and \( b_o = 0.00792 \) (see Fig. 1 which shows very high correlation between predicted and measured data, \( R^2 = 0.998 \)).

The users of the TRANSYT 8 computer program (Vincent, Mitchell and Robertson 1980) should note that the recommended eqn (2) differs from the cruise fuel consumption function used in that program.

### 4. ACCELERATION FUEL CONSUMPTION

The function to predict the fuel consumed during acceleration from rest to a final cruise speed of \( v_c \) can be derived by integrating eqn (1) with respect to time. The general form of the function is

\[
F_a = (a_1 + a_2 v_c + a_3 v_c^3) t_a + a_4 v_c^2 + a_5 \frac{v_c^3}{t_a} \tag{3}
\]

where

- \( F_a \) = fuel consumed (mL) during acceleration from rest to a final cruise speed of \( v_c \) (km/h),
- \( t_a \) = acceleration time (s),
- \( a_1 \) to \( a_5 \) = coefficients related to the coefficients of the instantaneous fuel consumption function as follows: \( a_1 = k_1 \), \( a_2 = m_1 k_1 \), \( a_3 = m_2 k_1 \), \( a_4 = m_3 k_1 \), \( a_5 = m_4 k_1 \),
- \( m_1 \) to \( m_5 \) (\( i = 2, 3, 5 \)) are integration constants which depend on the functional form of the speed-time profile during the manoeuvre (acceleration model).

For 'constant' acceleration model, \( m_1 = 0.5k_1 \), \( m_2 = 0.25k_1 \), \( m_3 = 0.5k_1 \), \( m_4 = 0.5k_1 \), \( m_5 = 0.5k_1 \). For 'linear-decreasing' acceleration model, \( m_1 = 0.67k_1 \), \( m_2 = 0.46k_1 \), \( m_3 = 0.53k_1 \). (See Akcelik (1982) for detailed description of the two acceleration models.)

The results for the Melbourne University test car with constant acceleration rates are \( \alpha_i = 0.70 \), \( \alpha_i = 0.00221 \), \( \alpha_i = 0.055 \times 10^{-5} \), \( \alpha_i = 0.00381 \), \( \alpha_i = 0.443 \times 10^{-3} \) (data were available for constant acceleration only). These results were obtained by using the values of \( k_1 \), \( k_2 \), \( k_3 \) as values pre-determined for the cruise fuel consumption function (using the method explained above), and then by finding the values of \( k_1 \) and \( k_2 \) by regression for constant-acceleration fuel consumption data. The correlation between predicted and measured data was found to be very high (\( R^2 = 0.999 \)). The results are illustrated in Fig. 2 for \( v_c = 30, 60 \) and 90 km/h.

5. EXCESS FUEL CONSUMPTION PER STOP

The elemental fuel consumption model which expresses fuel consumption as a function of the three principal elements of driving patterns (idle, cruise and stop-start manoeuvres) has been discussed in previous parts of the report:

\[
F = f_i x_s + f_s d_s + f_i h \tag{4}
\]

where

- \( F \) = fuel consumed (mL),
- \( x_s \) = total section distance (km),
- \( d_s \) = stopped delay time (s),
- \( h \) = number of complete stop-start manoeuvres,
- \( f_i \) = fuel consumption per unit distance while cruising (mL/km),
- \( f_s \) = fuel consumption per unit time while idling (mL/s), and
- \( f_i \) = excess fuel consumption per complete stop-start manoeuvre (mL/stop).

The idle fuel consumption rate per unit time, \( f_i \), in eqn (4), is obtained from eqn (1) by putting \( v = 0 \) and...
a = 0, i.e. \( f = k_o \) (mL/s) in eqn (1), or \( b_i \) (mL/h) in eqn (2). For fuel consumption while cruising, \( f_c \), can be estimated. This underestimates the actual cruise fuel consumption by an amount which corresponds to speed-fluctuations while cruising (see Part 5). However, the effect of this error is likely to be negligible in urban traffic management applications because the amount underestimated tends to be constant (assuming negligible effect of traffic controls on mid-block cruise conditions) and this amount is small relative to the contributions of delay and stop-starts to total fuel consumption.

A complete stop-start manoeuvre is defined for eqn (4) as a speed-change manoeuvre which involves a deceleration from the cruise speed, \( v \), to zero speed and an acceleration back to the cruise speed; and excess fuel consumption per stop is the total fuel consumed during such a stop-start manoeuvre (with no stopped time) less the consumption when the distance travelled during this manoeuvre is travelled at the cruise speed. The total fuel consumed during a stop-start manoeuvre can therefore be calculated as the sum of the deceleration and acceleration fuel consumptions. Acceleration fuel consumption is given by eqn (3). The investigation reported in Akcelik (1982) was not conclusive regarding the deceleration fuel consumption, partly due to data limitations. However, it appears that the assumption that deceleration fuel consumption, \( F_d \), is equal to idle fuel consumption is a good approximation, i.e. \( F_d = k_d t_d \) where \( k_d \) is the deceleration fuel consumption rate(s) and \( t_d \) is the deceleration time(s). However, the form of the excess fuel consumption function becomes rather complicated with this assumption. As a simplifying assumption, all first three terms of eqn (1) can be included in integration as fully effective. The resulting deceleration fuel consumption function is the same as eqn (3) except for the deletion of the last two terms (detailed information can be found in Akcelik (1982). The resulting excess fuel consumption function is

\[
f_3 = \beta_1 t_h + \beta_2 v_c^2 + \beta_3 \frac{v_c^3}{t_a} - \beta_4 v_c^2 t_h \quad (5)
\]

where

\[
f_1 \quad \text{excess fuel consumption (mL)}
\]
\[
t_h \quad \text{stop-start time (s), i.e. sum of the deceleration time, } t_d, \text{ and acceleration time, } t_a
\]
\[
\beta_1 \text{ to } \beta_4 \quad \text{coefficients related to the coefficients of the instantaneous fuel consumption function as follows:}
\]
\[
\beta_1 = \gamma k, \quad (\gamma = 1/2 \text{ for constant acceleration model and } 1/3 \text{ for linear acceleration model}), \quad \beta_2 = 0.5k, \quad \text{for constant acceleration, } 0.53k, \quad \text{for linear acceleration (note that } \beta_2 = {\alpha}_s, \quad \beta_3 = {\alpha}_s \text{, compared with eqn (3) )}, \quad \text{and}
\]
\[
\beta_4 = \text{coefficient determined by regression.}
\]

For the Melbourne University test car when constant acceleration rates are used: \( \beta_1 = 0.350, \beta_2 = 0.00381, \beta_3 = 0.443 \times 10^{-2} \text{ and } \beta_4 = 0.096 \times 10^{-5}. \) These results were found by partial regression, i.e. \( k_i, k_s \text{ and } k_t \text{ are pre-determined values which are used to calculate } \beta_1, \text{ to } \beta_4 \text{ (} k_t \text{ found by direct measurement, } k_s \text{ and } k_i \text{ found by the analysis of acceleration data as given above), and then coefficient } \beta_4 \text{ is determined by regression. The results are shown in Fig. 3 for two different average deceleration-acceleration rates, } \bar{a} = 2v/t. \text{ The correlation between the predicted and measured data was found to be good (} R^2 = 0.911 \text{, but not as good as those for the cruise and acceleration fuel consumption functions. This decrease in prediction accuracy is due to the assumption regarding deceleration fuel consumption discussed above.}

A simpler function which neglects the last two terms of eqn (5) has been used in TRANSYT 6 program (Vincent et al. 1980), and it has been shown in Part 5 of this report that the PKE model implies the same excess fuel consumption function. However, this function may result in very large errors, particularly for high \( v \) and low \( \bar{a} \) values. Based on the analyses reported in Akcelik (1982), a limited application of the formula is recommended as follows:

\[
f_3 = e_1 t_h + e_2 v_c^2 \quad \text{for } v_c < 70 \text{ km/h} \quad (6)
\]

where

\[
e_1 = \beta_1 \text{ (mL/s), related to the pre-determined idling fuel consumption rate as in eqn (5)},
\]
\[
e_2 = \text{coefficient determined by regression, and}
\]
\[
f_1, t_h, v_c = \text{as in eqn (5)}.
\]

For the Melbourne University test car with constant acceleration and deceleration rates \( e_1 = 0.350 \text{ and } e_2 = 0.00319 \text{ were found. The correlation between the predicted and measured data for the limited data range is fairly good (} R^2 = 0.855\), but the use of the function should be strictly limited to the specified data range.

6. APPLICATION TO THE PKE MODEL

Two different forms of PKE model were discussed in previous parts of the report. A small modification to the previous form of the model makes it consistent with the instantaneous fuel consumption function given in this part (eqn (1)):

\[
f_x = b_1 + \frac{b_2}{v} + b_3 v^2 + b_4 d + b_5 PKE \quad (7)
\]

or alternatively,

\[
f_x = b_1 + \frac{b_2}{v} + b_3 v^2 + b_5 PKE \quad (8)
\]
where

- \( f_s \) = fuel consumption per unit distance (mL/km),
- \( b_1 \) to \( b_6 \) = model coefficients (constants),
- \( v_s \) = average interrupted travel speed including all delays (km/h),
- \( v_t \) = average running speed (km/h) excluding any stopped delay time but including deceleration and acceleration delays,
- \( \tilde{a}_s \) = \( d_s / x_s \) = stopped delay time per unit distance (s/km),
- \( PKE \) = variable describing total "positive kinetic energy" changes (m/s^2), given by

\[
PKE = \frac{\sum (v_f^2 - v_i^2)}{12960 x_s}
\]

where

- \( v_s \) = final and initial speeds (km/h) in an acceleration, and \( x_s \) = total section distance (km).

Only the third terms of eqns (7) and (8) differ from the original formulae: \( v^2 \) instead of \( v \), and \( v^2 \) instead of \( v_s \) are used. Analyses reported in Akcelik (1982) indicate that better results are obtained with this modification. The analyses were carried out using Watson's on-road data (collected in 1978 on Melbourne roads). The results of free regressions for level road data (identified as those with a net gradient less than 0.5 per cent, leaving 160 measurements from the total of 1463) are \( b_1 = 10.2, b_2 = 2623, b_3 = 0.00741, b_4 = 111.1 (R^2 = 0.936) \) for eqn (8), and \( b_1 = 14.2, b_2 = 2178, b_3 = 0.00771, b_4 = 0.796, b_5 = 115.9 (R^2 = 0.950) \) for eqn (7).

7. CONCLUSION

A five-term instantaneous fuel consumption model (eqn 1) has been derived which can be used:

(a) for predicting fuel consumption when speed-time traces of individual vehicles are known; and

(b) as a basis for deriving functions describing the elemental model parameters as well as aggregate fuel consumption functions such as the PKE model. The instantaneous fuel consumption model coefficients can be determined using data for separate idle, constant-speed cruise and acceleration manoeuvres with minimum reliance on regression analyses. It has been shown that the excess fuel consumption per stop-start manoeuvre depends on the initial and final speeds, as well as the deceleration and acceleration rates and profiles.

Considering the limitations of the data used for the analyses whose results are presented in this paper, similar studies are recommended using good quality on-road data representing:

(a) a wide range of speeds, and acceleration and deceleration rates,
(b) realistic acceleration and deceleration profiles (speed-time traces), and
(c) different vehicle types (manual as well as automatic transmission).

For further work, it is also recommended to extend the work reported in detail in Akcelik (1982) to include the road gradient as a parameter, to develop a physical interpretation of the $a^2 v$ term in the instantaneous fuel consumption function, which provides sensitivity to high acceleration rates, to derive formulae for speed-up and slow-down manoeuvres involving non-zero initial and final speeds, and to carry out similar analyses to derive and calibrate pollutant emission models.

REFERENCES


