TECHNICAL NOTE

Progression Factors in the HCM 2000 Queue and Delay Models for Traffic Signals

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Summary

This Technical Note recommends that the following conditions be used in implementing the HCM 2000 progression factors for delay and back of queue. The symbols used below are as follows: PF is the delay progression factor, PF₂ is the queue progression factor, Rₚ is the platoon ratio, P is the proportion of arrivals on green, u is the g/C for the lane group, and yₗ is the lane group flow ratio per lane (arrival flow / saturation flow).

(i) PF \geq 1.0 and PF₂ \geq 1.0 for Arrival Types 1 and 2,
(ii) PF \leq 1.0 and PF₂ \leq 1.0 for Arrival Types 4 to 6,
(iii) P \leq 0.95 (Rₚ \leq 0.95 / u) for both PF and PF₂,
(iv) Rₚ \leq 0.95 / yₗ for both PF and PF₂,
(v) PF₂ = 1.0 for yₗ \geq u (X₁ \geq 1),
(vi) Rₚ \geq (1 - 0.95 (1 - u) / yₗ) / u for both PF and PF₂, and
(vii) Rₚ = 1.0 (P = u), therefore PF = 1.0 and PF₂ = 1.0 for yₗ \geq 0.95.
(viii) If Conditions (iii), (iv) and (vi) create inconsistent constraints on Rₚ and P, set Rₚ = 1.0 and P = u, therefore PF = 1.0 and PF₂ = 1.0.

The numbering of conditions listed above reflects the discussion given in Section 3 of this Technical Note. An order and further suggestions for their implementation are given in Section 6. The progression factor equations are given in Section 2. The issue of large values of progression factors that may result under certain circumstances is discussed in Section 4. A list of Notations and Basic Relationships is given at the end of the document.

The reasons for each condition are discussed in Section 3, including numerical examples. In summary:
Condition (i) applies a logical constraint on PF and PF₂ calculations for Arrival Types (AT) 1 and 2 (Very Poor or Unfavourable progression), simply stating that these arrival types must not produce a PF or PF₂ less than 1.0, given that they represent performance worse than AT 3 (Random arrivals) which will always produce PF or PF₂ = 1.0.
Condition (ii) is similar to Condition (i). It applies a logical constraint on PF and PF₂ calculations for AT 4, 5 and 6 (Favourable, Highly Favourable, or Exceptional progression), simply stating that these arrival types must not produce a PF greater than 1.0, given that they represent performance better than AT 3.
Condition (iii) is a logical condition based on the definition of P and Rₚ, and negative values of PF and PF₂ can result if it is not applied.
Condition (iv) is needed to prevent the denominator of PF₂ going to zero or below, in which case the PF₂ value would be very large, undefined or negative. It places an upper bound on Rₚ and P. This has a logical equivalent that limits how many vehicles the defined arrival type can expect to process during the green period in relation to the saturation flow rate. Although this denominator does not explicitly appear in the PF formula for the delay model, the same logic applies for the maximum flow during green, so the constraint must be applied for PF as well.
Condition (v) essentially states that the PF₂ value is equal to 1.0 at saturation, and it cannot change from this value for oversaturated conditions. This is because the coordination effects virtually disappear under these conditions and the oversaturation effects are handled by the second term of the queue model.
Condition (vi) is needed to limit the demand during the red period dictated by the Rₚ, placing a lower bound on Rₚ and P, in a similar fashion to Condition (iv).
Condition (vii) is used when the average flow rate is so high that it may not be possible to meet Conditions (iv) and (vi) simultaneously. This condition requires that Rₚ = 1.0 (P = u), and therefore PF = 1.0 and PF₂ = 1.0, are used, i.e. the case is treated as Arrival Type 3, when the flow ratio (yₗ) exceeds a practical limit.
Condition (viii) is applicable when Conditions (iii), (iv) and (vi) create inconsistent constraints that cannot all be met simultaneously. In this case, Rₚ = 1.0 (P = u), therefore PF = 1.0 and PF₂ = 1.0, are set, and therefore, the case is treated as Arrival Type 3.

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1. Introduction

The signalised intersection chapter of the latest "HCM 2000" edition of the US Highway Capacity Manual (TRB 2000, Chapter 16) introduced a back of queue model, which was developed by the author. A paper by Viloria, Courage and Avery (2000) presented a detailed discussion on the HCM 2000 queue model and comparisons with the aaSIDRA and various other queue models for signalised intersections.

This technical note discusses the progression factors that are used in the HCM 2000 queue and delay models in order to allow for the effects of platooned arrivals generated by coordinated signals. The purpose is to propose refinements to the application of various conditions on the parameters used in the progression factor equations, and discuss the cases of large progression factors and large back of queue values.

The two progression factors for the delay and queue models (PF and PF₂, respectively) are derived using the same method, and therefore use the same concept, basic simplifying assumptions about the platooned arrival patterns, and the same traffic parameters. These progression factors have been used in the aaSIDRA software package (Akcelik & Associates 2000) since 1995. For detailed background information on this subject, refer to previous publications by the author (Akçelik 1995, 1996).

The discussion given here will use the HCM 2000 notation, and needs to be read in conjunction with Chapter 16 (Signalized Intersections), Appendix G of HCM 2000. A full list of Notations and Basic Relationships is given before the reference list at the end of this document.

2. Progression Factors

The progression factors PF and PF₂ are derived using a simple platooned arrivals model that assumes different arrival flow rates (veh/h) during the green and red periods (vₐ, vᵣ) as shown in Figure 1 where r = effective red time, g = effective green time, c = cycle time, gₛ = saturated part of the green period (queue clearance time), gᵤ = unsaturated part of the green period, and vₐ = (vᵣ r + vₛ g) / c is the average arrival flow rate (veh/h) during the signal cycle.

In HCM 2000, the queue progression factor PF₂ is given by Equation (G16-8) in Chapter 16, Appendix G. It is used in the first term of the back of queue model (Equation G16-7) to adjust the "uniform" back of queue value for progression effects. The delay progression factor is given by Equation (16-10) in Chapter 16 for use in the first term of the delay model (Equation 16-9) to adjust the "uniform" delay value for progression effects.

For simplicity, HCM 2000 Equations (G16-7) and (G16-8) will be expressed here using u = g/c (lane group green time ratio), y = vᵣ/sₐ (lane group flow ratio per lane) and qₐ = vᵣ/3600 (lane group flow rate per lane in vehicles per second). The delay equation will not be given since the focus of this paper is on the new HCM 2000 queue model. However, the HCM expression for the delay progression factor (PF) will be given since the two progression factors are interrelated and most of the discussion presented here applies to both factors. Similar notations will be used in both expressions.

First-Term Back of Queue

\[ Q₁ = PF₂ Qₐ = PF₂ \frac{qₐ C (1-u)}{1-[\min(1.0,Xₐ)] u} \]  \hspace{1cm} (1)

where

\[ Q₁ = \text{first-term back of queue (veh)}, \]
\[ Qₐ = \text{first-term back of queue for uniform (non-platooned) arrival flows (veh)}, \]
\[ PF₂ = \text{queue progression factor (Equation 2)}, \]
\[ qₐ = \text{lane group flow rate per lane (veh/s), } qₐ = vᵣ/3600, \]
\( v_L \) = lane group flow rate per lane (veh/h),
\( C \) = cycle time (s),
\( u \) = green time ratio (g/C), and
\( X_L \) = lane group degree of saturation, i.e. \( v_l/c_L \) ratio where \( c_L \) is the lane group capacity per lane (veh/h).

From Equation (1), the value of the first-term back of queue at capacity \( (X_L = 1.0) \) is:

\[
Q_{1m} = PF_2(X_L=1) q_L C \tag{1a}
\]

where \( PF_2(X_L=1) \) is the value of \( PF_2 \) at capacity. It is shown below that \( PF_2(X_L=1) = 1.0 \), and therefore:

\[
Q_{1m} = q_L C = (v_l/3600) C \tag{1b}
\]

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**Figure 1 - Simple platooned arrivals model as the basis of HCM progression factors**
Queue Progression Factor

\[ PF_2 = \frac{(1-P)(1-y_L)}{(1-u)(1-R_p u)(1-y_L)} \frac{(1-R_p)(1-y_L)}{(1-u)(1-R_p y_L)} \]  

subject to conditions

(i) \( PF_2 \geq 1.0 \) for Arrival Types 1 and 2,
(ii) \( PF_2 \leq 1.0 \) for Arrival Types 4 to 6,
(iii) \( P \leq 1.0 \) (\( R_p \leq 1 / u \)),
(iv) \( R_p < 1 / y_L \), and
(v) \( PF_2 = 1.0 \) for \( y_L \geq u \) (\( X_L \geq 1 \)).

where

\( PF_2 = \) queue progression factor,
\( u = \) green time ratio (g/C),
\( y_L = \) lane group flow ratio, i.e. \( v_L/s_L \) ratio where \( v_L \) is the lane group flow rate per lane (veh/h) and \( s_L \) is the lane group saturation flow rate per lane (veh/h), and
\( R_p = \) platoon ratio:
\[ R_p = \frac{v_{Lg}}{v_L} = \frac{P}{u} \]  

where \( v_{Lg} \) is the arrival flow rate (veh/h) during the green period (\( v_{Lg} = R_p v_L \)), \( v_L \) is the average arrival flow rate (veh/h) during the signal cycle, and \( P \) is the proportion of traffic arriving during the green period (see the notation list for further explanation of \( R_p \) and \( P \)).

Delay Progression Factor

\[ PF = \frac{(1-P) f_{pA}}{(1-u)} = \frac{(1-R_p u) f_{pA}}{(1-u)} \]  

subject to conditions

(i) \( PF \geq 1.0 \) for Arrival Types 1 and 2,
(ii) \( PF \leq 1.0 \) for Arrival Types 4 to 6, and
(iii) \( P \leq 1.0 \) (\( R_p \leq 1 / u \)).

where \( R_p \) and \( u \) are as in Equation (2), and
\( f_{pA} = \) delay progression factor, and
\( PF = \) delay progression factor, and
\( f_{pA} = \) an additional adjustment factor.

Equation (3) is derived from the following full expression for delay progression factor (Akçelik 1995), ignoring the additional parameter \( f_{pA} \):

\[ PF = \frac{(1-R_p u) [1-u + y_L (1-R_p)](1-y_L)}{(1-u)^2 (1-R_p y_L)} \]  

This expression contains \( (1-y_L) \) in the numerator and \( (1-R_p y_L) \) in the denominator, as in Equation (2) for queue progression factor. Therefore Equation (3a) for PF would require Conditions (iv) and (v) as in the case of Equation (2) for PF_2. Note that the value of PF from Equation (3a) at \( X_L = 1.0 \) (\( y_L = u \)) is equivalent to the constant values of PF given by Equation (3), ignoring the effect of \( f_{pA} \). Therefore, there is no need for a constraint on PF from Equation (3) for \( X_L > 1.0 \) similar to Condition (v) for PF_2. Conditions applicable to PF and PF_2 equations are discussed in detail in Section 3.
Table 1

Arrival types, platoon ratios (R_p), and the proportion arriving during green (P) for various green time ratios

<table>
<thead>
<tr>
<th>Arrival Type</th>
<th>Progression Quality</th>
<th>Platoon Ratio (R_p)</th>
<th>Proportion arriving during green (P = R_p u)</th>
<th>f_PA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Range</td>
<td>Default u = 0.20</td>
<td>u = 0.40</td>
</tr>
<tr>
<td>1</td>
<td>Very poor</td>
<td>≤ 0.50</td>
<td>0.333</td>
<td>0.067</td>
</tr>
<tr>
<td>2</td>
<td>Unfavourable</td>
<td>&gt; 0.50 - 0.85</td>
<td>0.667</td>
<td>0.133</td>
</tr>
<tr>
<td>3</td>
<td>Random arrivals</td>
<td>&gt; 0.85 - 1.15</td>
<td>1.000</td>
<td>0.200</td>
</tr>
<tr>
<td>4</td>
<td>Favourable</td>
<td>&gt; 1.15 - 1.50</td>
<td>1.333</td>
<td>0.267</td>
</tr>
<tr>
<td>5</td>
<td>Highly favourable</td>
<td>&gt; 1.50 - 2.00</td>
<td>1.667</td>
<td>0.333</td>
</tr>
<tr>
<td>6</td>
<td>Exceptional</td>
<td>&gt; 2.00</td>
<td>2.000</td>
<td>0.400</td>
</tr>
</tbody>
</table>

Table 1 shows the six Arrival Types (ATs) given by HCM 2000 for determining platoon ratios (default values of R_p) when the green time ratio (u) and the proportion of traffic arriving during the green period (P) are not known. The values of P corresponding to the default values of R_p for various green time ratios (u) are given in Table 1.

If the proportion of traffic arriving during the green period (P) and the green time ratio (u) are known, the platoon ratio (R_p) is calculated from Equation (2a). In this case, an Arrival Type can be assigned according to the ranges of R_p given in Table 1.

3. Conditions Applicable to Progression Factor Equations

Conditions (i) to (v) included in Equations (2) and (3) are based on those used in aaSIDRA version 1.0 (Akcelik & Associates 2000). These are discussed below, their practical meanings are explained and various refinements are recommended. Three additional conditions are introduced (Conditions vi to viii). An order and further suggestions for the implementation of the full set of conditions are given in Section 6.

Conditions (i) and (ii)

These are used to ensure that the progression factors are consistent with the definitions of progression quality relative to random arrivals (see Table 1). HCM 2000 recommends using a maximum PF value of 1.0 for Arrival Type 4 "as a practical matter". Condition (i) applies a minimum PF value of 1.0 for Arrival Types 1 and 2, and Condition (ii) applies a maximum PF value of 1.0 for Arrival Types 4, 5 and 6.

Numerical conditions that result in the violation of these conditions for Arrival Types other than Arrival Type 4 may occur, for example, R_p = 0.667 for AT = 2 with a green time ratio of u = 0.10 gives PF = 0.964 < 1.0. Although occurrence of such numerical conditions are limited, it is advisable that full set of Conditions (i) and (ii) are used in implementing the progression factor equations.
Condition (iii)

Condition $R_p \leq \frac{1}{u}$ is needed to avoid negative values of PF and PF$_2$. It is equivalent to $P \leq 1.0$, i.e. the proportion of traffic arriving during the green period should not exceed 100 per cent. If $R_p = \frac{1}{u}$, this will result in $P = R_p u = 1.0$ and therefore PF = 0 and PF$_2$ = 0. This is not a desirable condition since, in reality, it is very likely that there will be some vehicles that arrive during the red period! It is therefore recommended that a more strict condition such as $R_p \leq 0.95 / u$ is adopted. This is equivalent to the use of a $P_{\text{max}} = 0.95$, or the condition $P \leq 0.95$ (95 per cent of traffic arriving during the green period and 5 per cent during the red period).

Example 1: Consider the case of a movement with arrival flow rate, $v_L = 1083$ veh/h and saturation flow rate, $s_L = 1900$ veh/h (therefore $y_L = 0.570$), receiving exceptional progression quality (AT = 6) with a green time ratio of $u = 0.60$ ($g = 60$ s, $C = 100$ s). From Table 1, default $R_p = 2.000$. Therefore, $P = R_p u = 1.200$ is found. However, the condition $R_p \leq \frac{1}{u}$ means that $R_p = \frac{1}{0.60} = 1.667$ should be used as the maximum value, resulting in $P = R_p u = 1.000$ (no vehicles arriving during red), therefore PF = 0 and PF$_2$ = 0. Similarly, the recommended condition $R_p \leq 0.95 / u$ would result in $R_p = 1.583$ (AT = 5), and $P = R_p u = 0.950$ (5% arriving during red). This means that AT = 6 is not a realistic assumption in this case. AT = 5 is more appropriate, and PF = 0.125 and PF$_2$ = 0.551 are found as a result.

Condition (iv)

Condition $R_p < \frac{1}{y_L}$ is needed for $(1 - R_p y_L) > 0$ in the denominator of Equation (2). The condition is equivalent to $v_{Lg} < s_L$, i.e. the arrival flow rate during the green period should be less than the saturation flow rate. This equivalence can be seen by putting $R_p = \frac{v_{Lg}}{v_L}$ (by definition) and $y_L = \frac{v_L}{s_L}$.

Although $(1 - R_p y_L)$ does not appear in Equation (3), this condition is relevant to the delay progression factor as seen from the implied requirement $v_{Lg} < s_L$, otherwise an unrealistic platoon ratio will be used in calculating PF.

It is recommended that a maximum acceptable value of $v_{Lg} / s_L$ is used, e.g. $(v_{Lg} / s_L)_{\text{max}} = 0.95$ may be appropriate for a more strict condition. Condition (iv) would then be expressed as $v_{Lg} \leq 0.95 s_L$, or $R_p \leq 0.95 / y_L$.

Example 2: Consider the same movement as in Example 1, i.e. $v_L = 1083$ veh/h, $s_L = 1900$ veh/h ($y_L = 0.570$), $u = 0.60$, but receiving highly favourable signal coordination (AT = 5). From Table 1, default $R_p = 1.667$. This means $P = R_p u = 1.000$. Applying Condition (iii) as $P \leq 0.95$, we need to set $P = 0.950$ and $R_p = 0.950 / 0.600 = 1.583$ as in Example 1. As a result, Condition $R_p < 0.95 / y_L$ is also satisfied since $R_p = 1.583 < 0.95 / y_L = 1.667$. The platoon ratio of $R_p = 1.583$ implies an arrival flow rate during the green period, $v_{Lg} = 1.583 \times 1083 = 1714$ veh/h, which is less than $0.95 s_L = 1805$ veh/h. PF = 0.125 and PF$_2$ = 0.551 are found as in Example 1.

Condition (v)

Condition $P F_2 = 1.0$ for $y_L \geq u$ is related to the value of queue progression factor, PF$_2$ at capacity, $P F_2(X_L=1)$. This is obtained using $y_L = u$ (i.e. $v_L / s_L = g/C$ for $X_L = 1.0$) in Equation (2), which results in:

$$P F_2(X_L=1) = 1.0 \quad (4)$$

This condition requires that, for arrival flow rates above capacity ($X_L > 1.0$), the values of PF$_2$ must be restricted to $P F_2(X_L=1) = 1.0$. This is consistent with the use of a constant value of $Q_u$ (the first-term back of queue for uniform arrivals) for $X_L > 1.0$ as seen from in Equations (1), (1a) and (1b).
While relevant to the delay progression factor PF from the Equation (3a), there is no need for a similar condition on Equation (3) since this simplified form of the equation used in the HCM gives constant values.

**Condition (vi)**

Similar to Condition (iv), a constraint on the flow rate during the red period is also considered. This is \( v_{Lr} < s_L \), i.e. the arrival flow rate during the red period should be less than the saturation flow rate (both in veh/h), or more strictly \( v_{Lr} \leq 0.95 s_L \).

The arrival flow rate during the red period, \( v_{Lr} \), can be calculated from \( v_{Lr} = v_L(1 - R_p u) / (1 - u) \). From this, the required condition can be expressed as \( R_p \geq (1 - 0.95 (1 - u)) / y_L \), i.e. proportion of traffic arriving during the red period must not exceed a maximum value, or \( P \geq 1 - 0.95 (1 - u) / y_L \), i.e. proportion of traffic arriving during the green period must not be less than a minimum value.

As in the case of Condition (iv), this condition should be applied to both delay and queue progression factors.

**Example 3:** Consider the same movement as in Example 1, i.e. \( v_L = 1083 \) veh/h, \( s_L = 1900 \) veh/h (\( y_L = 0.570 \)), \( u = 0.60 \), but with very poor signal coordination (AT = 1). From Table 1, default \( R_p = 0.333 \). Proportion arriving during green is calculated as \( P = R_p u = 0.200 \), and therefore Condition (iii) is satisfied (\( P \leq 0.95 \)). Similarly, Condition (iv) is satisfied since \( R_p (= 0.333) < 0.95 / y_L (= 1.667) \). However, this platoon ratio implies an excessive arrival flow rate during the red period: \( v_{Lr} = 1083 x (1 - 0.200) / (1 - 0.60) = 2166 \) veh/h > \( s_L (= 1900 \) veh/h). For \( v_{Lr} \leq 0.95 s_L \), the minimum value of proportion arriving during green is \( P = 1 - 0.95 (1 - u) / y_L = 0.333 \). Setting \( P = 0.333 \), we find \( R_p = P / u = 0.556 \) (AT = 2). This implies \( v_{Lr} = 0.95 s_L = 1805 \) veh/h, and means that AT = 1 is not a realistic assumption in this case. AT = 2 is more appropriate, and PF = 1.667 and PF2 = 1.049 are found as a result.

**Condition (vii)**

This is related to circumstances when the average flow rate is very high. Violation of both Conditions (iv) and (vi), i.e. \( v_{Lg} > 0.95 s_L \) and \( v_{Lr} > 0.95 s_L \), implies that the average flow rate \( v_L > 0.95 s_L \). Applying both conditions (\( v_{Lg} = v_{Lr} = 0.95 s_L \)), \( v_L = 0.95 s_L \) is implied, which means that \( R_p = 1.0 \), therefore AT = 3. The condition \( v_L > 0.95 s_L \) means a very high flow ratio of \( y_L > 0.95 \). This would correspond to highly oversaturated conditions at signalised conditions. The use of AT = 3 is acceptable under such oversaturated circumstances.

Note that the principle of the conservation of arrival flows is satisfied when Condition (iv) or (vi) is applied individually. When applying these saturation flow related conditions, we do not lose any vehicles since we reduce the number of vehicles arriving in the red or green period and make them arrive in the other period without changing the average flow rate. In other words, we do not change the number of vehicles arriving per cycle, but we change the numbers arriving during green and red periods. The two conditions are applied simultaneously only for the purpose of progression factor calculations, and the average flow rate should not be reduced below \( 0.95 s_L \) for other purposes because of the principle of conservation of flows.

Thus, when \( y_L > 0.95 \) (or \( v_L > 0.95 s_L \), Conditions (iv) and (vi) cannot be met simultaneously, and this case should be treated as Arrival Type = 3, i.e. the platoon ratio and proportion arriving green should be set to \( R_p = 1.0, P = u \), therefore \( PF = 1.0 \) and \( PF_2 = 1.0 \).
Condition (viii)

A further condition needs to be considered in order to make allowance for the situation when Conditions (iii), (iv) and (vi) create inconsistent constraints on $R_p$ and $P$ that cannot all be met simultaneously. As in the case of Condition (vii), this case should be treated as Arrival Type 3, i.e. the platoon ratio and the proportion arriving during the green period should be set to $R_p = 1.0$ and $P = u$, therefore $PF = 1.0$ and $PF_2 = 1.0$.

4. Large Values of Progression Factors

Under very poor and unfavourable progression conditions (AT = 1 and 2), manifested by low values of proportion of traffic arriving during green (P), and with large green time ratios ($u$), large values of progression factors $PF$ and $PF_2$ may result from Equations (2) and (3). Figures 2 and 3 show graphs of such progression factors that can be obtained under AT = 1.

For example, $P = 0.100$ and $u = 0.800$ (therefore $R_p = 0.125$, AT = 1) will give $PF = 4.500$. For the same example, a low flow ratio of $y_L = 0.040$ gives $PF_2 = 4.342$, and a high flow ratio of $y_L = 0.600$ gives $PF_2 = 1.263$. Note that $PF_2$ always reduces with increasing $y$, reaching $PF_2 = 1.0$ when $y_L = u$ ($X_L = 1.0$) as seen from Equation (4) and Figure 3.

Such large $PF$ and $PF_2$ values represent the effect of poor progression quality realistically. An easy test can be applied to the model in this respect. For this purpose, consider the worst-case scenario, which is when all vehicles arrive during the red period ($P = 0$, $R_p = 0$, AT = 1). Using $R_p = 0$ and $f_{pa} = 1.0$ (AT = 1) in Equations (2) and (3), the progression factors for this case are found as:

$$PF_{2m} = \frac{(1-y_L)}{(1-u)} \quad (5)$$

$$PF_m = \frac{1}{(1-u)} \quad (6)$$

These equations show that the largest values of $PF_m$ and $PF_{2m}$ will be obtained with largest green time ratios ($u$) as stated above. For example, $u = 0.8$ will result in $PF_m = 5.0$. Condition $PF_{2m} = 1.0$ applies for $y_L = u$ in Equation (5) as in Equation (2).

The queue length at the start of the green period is $(v_L r /3600)$ for random arrivals ($P = u$, $R_p = 1.0$) and $(v_L C /3600)$ when all vehicles arrive during the red period ($P = 0$, $R_p = 0$). This is shown for a fully-saturated signal cycle in Figure 4 (P = 0, AT = 1). The case of random arrivals (distributed uniformly throughout the signal cycle ($P = u$, AT = 3) is also shown in the bottom part of Figure 4. Since $v_L C = s_L g (y_L = u$, $X_L = 1.0$) in this fully-saturated case, $PF_2 = 1.0$ and the first-term back of queue, $Q_1 = v_L C / 3600$ are found.

For random arrivals, the first-term delay (uniform delay) from HCM Equation (16-11) is $d_{1u} = 0.5 r$. This can be calculated from the lower triangular area in Figure 4 as $d_{1u} = (0.5 \times v_L r) C / (v_L C) = 0.5 r$. Similarly, the first-term delay in the worst-case platooned arrivals scenario can be calculated from the upper triangular area in Figure 4 as $d_1 = (0.5 \times v_L C) C / (v_L C) = 0.5 C$. The delay progression by definition is the ratio of these two delay values, $PF = d_1 / d_{1u} = 0.5 C / (0.5 r) = C / r = C / (C - g) = 1 / (1 - u)$ as in Equation (6).
Figure 2 - Delay progression factor for Arrival Type = 1 (very poor progression) as a function of flow ratio ($y_L$) for various green time ratios ($u$)

Figure 3 - Queue progression factor for Arrival Type = 1 (very poor progression) as a function of flow ratio ($y_L$) for various green time ratios ($u$)
Figure 4 - Worst-case scenario for platooned arrivals (all vehicles arriving during the red period) and the non-platooned arrivals case

5. Conclusion

It is recommended all the conditions discussed in Section 3 are used in implementation of the HCM 2000 progression factors. These are summarised as follows:

(i) \( PF \geq 1.0 \) and \( PF_2 \geq 1.0 \) for Arrival Types 1 and 2,
(ii) \( PF \leq 1.0 \) and \( PF_2 \leq 1.0 \) for Arrival Types 4 to 6,
(iii) \( P \leq 0.95 \) (\( R_p \leq 0.95 / u \)) for both \( PF \) and \( PF_2 \),
(iv) \( R_p \leq 0.95 / y_L \) for both \( PF \) and \( PF_2 \),
(v) \( PF_2 = 1.0 \) for \( y_L \geq u \) (\( X_L \geq 1 \)),
(vi) \( R_p \geq (1 - 0.95 (1 - u) / y_L) / u \) for both \( PF \) and \( PF_2 \), and
(vii) \( R_p = 1.0 \) (\( P = u \)), therefore \( PF = 1.0 \) and \( PF_2 = 1.0 \) for \( y_L \geq 0.95 \).
(viii) If Conditions (iii), (iv) and (vi) create inconsistent constraints on \( R_p \) and \( P \), set \( R_p = 1.0 \) and \( P = u \), therefore \( PF = 1.0 \) and \( PF_2 = 1.0 \).

The numbering of conditions listed above reflects the discussion presented in this paper rather than an order for their implementation. It is recommended that the above conditions be implemented in the following order.

1. Apply Condition (vii) for both \( PF \) and \( PF_2 \) to satisfy the maximum value of the average arrival flow rate. If \( R_p = 1.0 \) and \( P = u \) (therefore \( PF = 1.0 \) and \( PF_2 = 1.0 \)) have been set as a result of this
condition, and the case is being treated as Arrival Type 3, finish the process as there is no need to check any other conditions.

2. Apply Condition (iii) for both PF and PF2 to satisfy the maximum value of the proportion of traffic arriving during the green period.

3. Apply Condition (iv) for both PF and PF2 to satisfy the maximum arrival flow rate during the green period.

4. Apply Condition (vi) for both PF and PF2 to satisfy the maximum arrival flow rate during the red period.

5. Apply Condition (viii) when Conditions (iii), (iv) and (vi) create inconsistent constraints that cannot all be met simultaneously. If \( R_p = 1.0 \) and \( P = u \) (therefore PF = 1.0 and PF2 = 1.0) have been set as a result of this condition, and the case is being treated as Arrival Type 3, finish the process as there is no need to check further conditions.

6. Apply Condition (v) for queue progression factor PF2 only (use the value at capacity for arrival flow rates above capacity). This is satisfied for the delay progression factor PF, which is a constant.

7. Apply Condition (i) if the Arrival Type is 1 or 2, or apply Condition (ii) if the Arrival Type is 4, 5 or 6. If one of these conditions is violated for the delay or queue progression factor, set PF = 1.0 or PF2 = 1.0 as applicable.

In software packages implementing these conditions, it would be useful to give a warning message when these conditions come into effect, thus modifying user input. This would help the user to understand that some input specifications have been changed by software, possibly implying a different Arrival Type.

It is emphasised that, although some conditions do not appear to be relevant to the progression factor for delay calculations (PF) on the basis of numerical requirements, they should be applied to both queue and delay progression factors due to their practical meanings as constraints on arrival flow rates that can be achieved during the red and green periods.

The simple platooned arrivals model used in the HCM is realistic in predicting large values of progression factors that may result under very poor and unfavourable progression conditions (low values of proportion of traffic arriving during green) and with large green time ratios. Unrealistic limits should not be placed on progression factors predicted by the HCM equations since they provide valuable information regarding the combination of flow and signal coordination conditions that result in large back of queue values.
## Notations and Basic Relationships

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</table>

*Continued >>>*
HCM 2000 and this document | aaSIDRA User Guide | Description and relationships
--- | --- | ---
$s_L$ | $s$ | Lane group saturation flow rate per lane (veh/h)
$u, \ g/C$ | $u$ | Green time ratio
$u = g/C$
$v_L$ |  | Lane group arrival (demand) flow rate per lane (veh/h)
$v_L = (v_{Lg} g + v_{Lr} r) / C$
$v_{Lg}$ | $q_{Lg}$ | Arrival flow rate (veh/h) during the green period
(for random arrivals: $v_{Lg} = v_L$)
$v_{Lg} = R_p v_L$
$v_{Lr}$ | $q_{Lr}$ | Arrival flow rate (veh/h) during the red period
(for random arrivals: $v_{Lr} = v_L$)
$v_{Lr} = v_L (C - R_p g) / r = v_L (1 - R_p u) / (1 - u)$
$X_L$ | $x$ | Lane group degree of saturation, i.e. the ratio of arrival (demand) flow rate to capacity
$X_L = v_L / c_L = v_L C / (s_L g) = y_L / u$
$y_L$ | $y$ | Lane group flow ratio per lane, i.e. the ratio of arrival (demand) flow rate to the saturation flow rate
$y_L = v_L / s_L$

REFERENCES


