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An energy-related model of instantaneous fuel consumption

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REFERENCE:

D. C. BIGGS and R. AKÇELIK (1986). An energy-related model of instantaneous fuel consumption. *Traffic Engineering and Control, June 1986, Vol. 2, No. 6, pp 320-325.*

An energy-related model of instantaneous fuel consumption

by **D. C. Biggs**, *Experimental Officer*, and **R. Akçelik**, *Principal Research Scientist, Australian Road Research Board**

A model for estimating fuel consumption from instantaneous speed, acceleration and grade information is described. The model is suitable for use in the design of detailed traffic management schemes and, in particular, for determining the incremental effects on fuel consumption resulting from changes in traffic management. Basically, the model relates fuel consumption to the fuel to maintain engine operation and to the energy consumed (work done) in providing tractive force to the vehicle. The tractive force is separated into drag, inertia and gradient components. A further term dependent on the acceleration and the inertial and grade forces allows for the increased engine drag and inefficient use of fuel during periods of hard acceleration. The accuracy of estimates of fuel consumption during accelerations and constant-speed driving and over short cycles which included acceleration, cruise, deceleration and idle periods and grades of up to 5 per cent was found to be good. Procedures are described for calibrating the vehicle parameters used in the model. These procedures require separate idle, constant-speed and acceleration data. A method is also given for calculating the rolling, engine and aerodynamic components of drag separately. This requires additional data on the vehicle coasting down in neutral gear.

1. Introduction. In recent years, attention has focused on traffic management as a means of reducing car fuel consumption. It has therefore been necessary to establish rigorous models for estimating car fuel consumption that are applicable to the range of traffic management applications. A hierarchy of vehicle fuel consumption models has been established by Akçelik *et al*[1] which includes models ranging from basic, instantaneous type models to the more aggregate, average travel speed models. The choice of model depends on its intended application, the available data and the accuracy required. This paper describes a model which could be classified as a basic model in the hierarchy of fuel consumption models.

Post *et al*[2] showed that instantaneous fuel consumption can be related to the instantaneous power demand experienced by the vehicle using a simple linear relation. This power-based model was not claimed to be accurate at an instantaneous level, but was shown to give errors in estimated fuel consumption of less than 3 per cent over trips of about 10 km. The model is attractive as it relates fuel consumption to the fundamentals of vehicle motion. An advantage of this model is that it can be calibrated using fuel consumption data collected on a dynamometer and only speed-time data during coast-downs need be collected on-road.

The validation of the power model by Post *et al* was not sufficient for the use of the model in the detailed assessment of the impacts of proposed traffic management schemes. For this purpose, it is necessary to show that the model is accurate during specific driving modes such as acceleration, deceleration and steady-speed driving. The model could then be used in microscopic traffic simulation to determine *incremental* effects on fuel consumption resulting from *changes* in traffic management. Alternatively, it could be used to derive higher-level models of more direct use to the traffic engineer.

Using data collected from carefully-controlled on-road acceleration, deceleration and steady-speed fuel consumption tests, Biggs and Akçelik[3] showed that the power model in the form proposed by Post *et al* gave adequate estimates of fuel consumption over trip segments of at least 60 seconds duration, as well as during cruise and slow-to-medium accelerations (mean errors generally less than 5 per cent). However, during hard accelerations fuel consumption was significantly underestimated, with mean errors of up to 20 per cent depending on the acceleration rate and final speed.

An extension of the power model which greatly improved the accuracy of estimated fuel consumption during hard accelerations was developed by Biggs and Akçelik[4]. An important feature of the extended model is the method used to calibrate parameters. This model is expressed as an 'energy'-related model of fuel consumption in a guide to fuel consumption analyses by Bowyer, Akçelik and Biggs[5]. The model given in this paper is a refinement of the energy-related model. The treatment of grade in the new model has been improved, but with zero grade the two models are equivalent. The new model is discussed in detail and the accuracy in each mode of driving and the calibration procedures are described. A method for identifying the components of drag and the contribution of all the components to fuel consumption are also given.

2. Description of the model

The energy-related fuel consumption model described below estimates fuel consumption from second-by-second speed-time and grade information. Basically, the model relates fuel consumption during a small time increment, dt, to:

(*a*) the fuel to maintain engine operation;

(*b*) the energy consumed (work done) by the vehicle engine while travelling an increment of distance, dx, during this time period; and

(*c*) the product of energy and effective acceleration (i.e. including acceleration due to gravity) during periods of positive acceleration.

Part (*c*) allows for the increased fuel consumption due to the effect of a lower gear being used during acceleration, especially on uphill grades. This is discussed further in Section 4. It also allows for the inefficient use of fuel during periods of very high power-demand. Since energy is $dE = R_T dx$ where R_T is the total tractive force required to drive the vehicle along distance dx, the fuel consumed in the time increment, dt, is expressed as:

$$\begin{aligned} dF &= \alpha dt + \beta_1 R_T\, dx + \\ &\quad [\beta_2 a_e R_{IG} dx]_{a_e>0} && \text{for } R_T>0 \\ &= \alpha dt && \text{for } R_T \leqslant 0 \quad \ldots (1) \end{aligned}$$

where dF = increment of fuel consumed (mL) during travel along distance dx (metres) and in time dt (seconds)

α = idle fuel rate (mL/sec.), which applies as a constant value during all modes of driving (as an estimate of fuel used to maintain engine operation)

β_1 = an efficiency parameter which relates fuel consumed to the energy provided by the engine, i.e. fuel consumption per unit of energy (mL/kJ)

β_2 = an efficiency parameter which relates fuel consumed during positive acceleration to the product of inertia energy and acceleration, i.e. fuel consumption per unit of energy-acceleration (mL/(kJ.m/sec.^{2}))

a = instantaneous acceleration (dv/dt) in m/sec.^{2}, which has a negative value for slowing down

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G = per cent grade which has a negative value for downhill grades, e.g. $G = -3$ for 3 per cent downhill grade

a_e = effective acceleration in m/sec.^{2} which includes both the acceleration experienced by the vehicle and the acceleration due to gravity:

$$a_e = a + 0.0981\,G \quad \ldots (2)$$

R_T = total 'tractive' force required to drive the vehicle, which is the sum of drag force (R_D), inertia force (R_I) and grade force (R_G) in kN (i.e. $R_T = R_D + R_I + R_G$)

R_{IG} = sum of inertial and grade forces (i.e. $R_{IG} = R_I + R_G$)

The resistive forces can be expressed as:

$$R_D = b_1 + b_2 v^2 \quad \ldots (3)$$

$$R_I = Ma/1\,000 \quad \ldots (4)$$

$$R_G = 9.81\,M\,(G/100)/1\,000 \quad \ldots (5)$$

where v = speed (dx/dt) in m/sec.

M = vehicle mass in kg, including occupants and any other load

b_1, b_2 = the vehicle parameters related mainly to rolling resistance and aerodynamic drag, respectively

The parameter b_1 is roughly proportional to vehicle mass and parameter b_2 is approximately proportional to frontal area. Both parameters are also related to the component of drag associated with the engine. In general, the two-term function given by Equation (3) is adequate for calculation of the drag force. However, if suitable calibration data are available, a three-term function could be used as described in Section 5. The three-term function allows the contribution of the rolling, engine and aerodynamic drag components to be estimated separately.

Fuel consumption per unit time (mL/sec.) can be expressed as:

$$f_t = dF/dt = \alpha + \beta_1 R_T v + [\beta_2 M v a_e^2/1\,000]_{a_e>0} \quad \text{for } R_T>0$$
$$= \alpha \quad \text{for } R_T \leqslant 0 \ldots (6)$$

where the total tractive force required is

$$R_T = b_1 + b_2 v^2 + M a_e/1\,000 \quad \ldots (7)$$

Note that in this form, the energy model becomes an extended form of the power model since $R_T v = P_T$ is the total tractive power (kW).

Fuel consumption per unit distance (mL/m) can similarly be expressed as:

$$f_t = dF/dx = f_t/v$$
$$= \alpha/v + \beta_1 R_T + [\beta_2 M a_e^2/1\,000]_{a_e>0} \quad \text{for } R_T>0$$
$$= \alpha/v \quad \text{for } R_T \leqslant 0 \quad \ldots (8)$$

Fuel consumption per unit time for constant speed travel along a level road ($a = 0$, $G = 0$) is obtained from the above equations as:

$$f_{c,t} = \alpha + \beta_1 (b_1 + b_2 v^2) v = \alpha + c_1 v + c_2 v^3 \quad \ldots (9)$$

where $f_{c,t}$ is the constant speed fuel consumption rate (mL/sec.).

In Table I, vehicle parameters required for the energy-related model of fuel consumption are summarised. Parameter values are given for two test cars, a 4.1-litre Cortina and a 3.3-litre Commodore (both station-wagons with automatic transmission). The two test cars were used to collect on-road data for determining the model accuracy. Also given in Table I is a set of 'default' parameters which are applicable for a fairly typical car in Australia in 1984.

In Fig 1, the speed-time trace of a vehicle during a 'cruise-decelaration-idle-acceleration-cruise' cycle is shown. Note that the initial cruise speed is 60 km/h and the final cruise speed is 90 km/h, and there are fluctuations around the average cruise speeds. The corresponding fuel consumption-time trace, computed from Equations (6) and (7) for a level road (using the

Table I. Vehicle parameters required by the energy-related model of car fuel consumption

Parameter	Cortina test car	Commodore test car	Default Value	Description
α	0.666	0.57	0.444	Idle fuel rate in mL/sec.
M	1 680	1 433	1 200	Mass in kg
β_1	0.0717	0.076	0.090	Energy efficiency parameter in mL/kJ
β_2	0.0344	0.066	0.030	Energy-acceleration efficiency parameter in mL/kJ.m/sec^2)
b_1	0.527	0.30	0.333	Drag force parameter in kN, mainly related to rolling resistance*
b_2	0.000948	0.000127	0.0008	Drag force parameter in kN/(m/sec.^{2}), mainly related to aerodynamic resistance*
$c_1 = b_1\beta_1$	0.0378	0.0241	0.030	Drag fuel consumption component in mL/m, mainly due to rolling resistance*
$c_2 = b_2\beta_1$	6.80×10^{-5}	10.2×10^{-5}	7.2×10^{-5}	Drag fuel consumption component in (mL/m)/(m/sec.^{2}), mainly due to aerodynamic resistance*

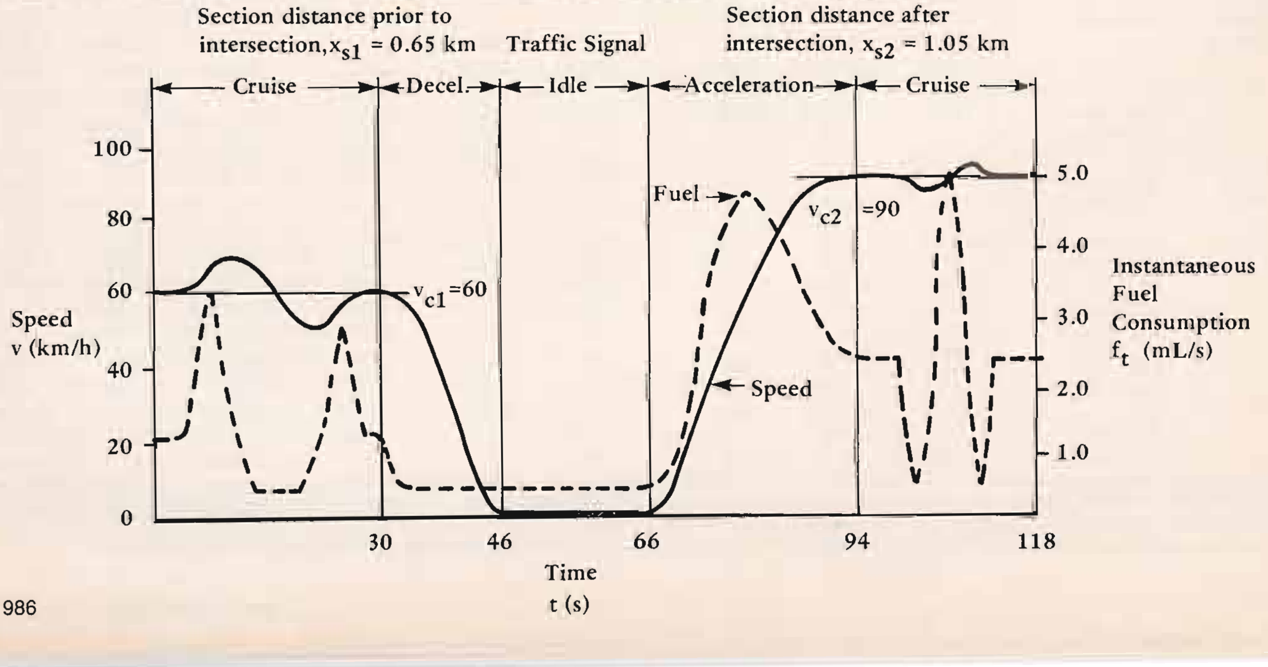
*b_1 and b_2 (and hence, c_1 and c_2) are also related to the component of drag associated with the engine

Table II. Estimated fuel consumption summarised by driving mode for a vehicle* following the speed-time trace shown in Fig 1

	Fuel consumed (mL)					
Grade	Initial cruise ($v_c = 60$)	Deceleration ($v_i = 60$, $v_f = 0$)	Idle ($v = 0$)	Acceleration ($v_i = 0$, $v_f = 90$)	Final cruise ($v_c = 90$)	Total
0	43.1	8.1	8.9	87.4	57.4	205
+5	68.9	11.1	8.9	116.8	90.0	296
−5	23.2	7.1	8.9	58.0	28.4	126

*Default vehicle parameters (Table I) used in energy-related model of fuel consumption

Fig 1. Speed-time trace and estimated instantaneous fuel consumption for the default vehicle driven over a cruise-deceleration-idle-acceleration-cruise cycle.



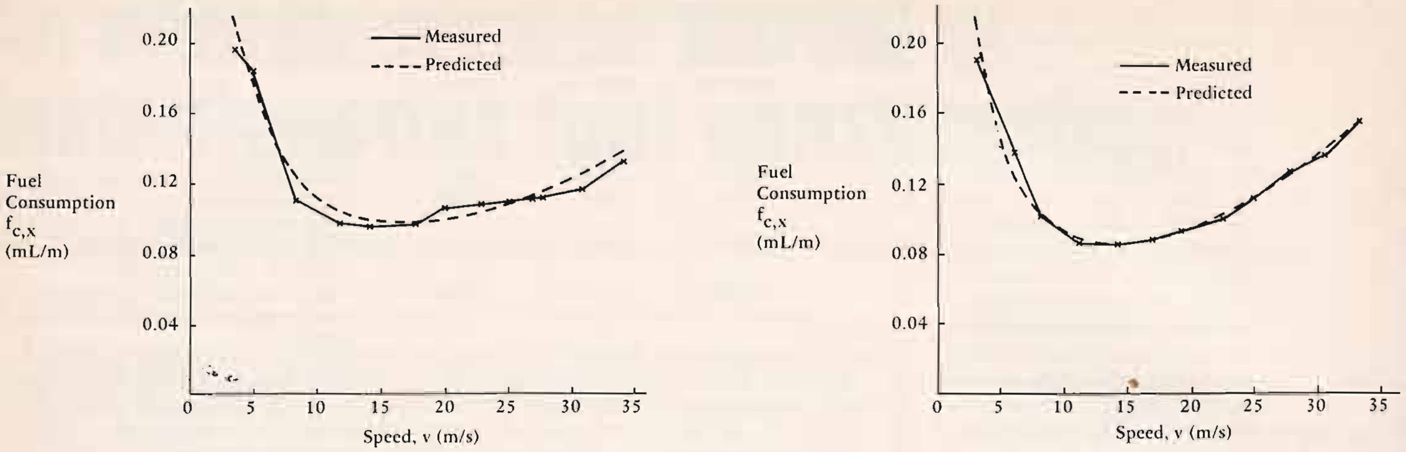


Fig 2. Speed versus fuel consumption for (left) a 4.1-litre Cortina and (right) a 3.3-litre Commodore.

'default' car parameters given in Table I) is also shown in Fig 1. The results for this example are summarised by driving mode in Table II, including the cases of 5 per cent uphill and downhill grades.

3. Accuracy of model

The accuracy of the energy model was investigated using two sets of data. The first was collected with the Cortina test car during carefully controlled on-road acceleration, deceleration and steady-speed tests[3,6]. Full details of this investigation are given in Biggs and Akçelik[4]. Note that the road gradient for these data was zero and under these conditions the models given here and by Biggs and Akçelik[4] and by Bowyer *et al*[5] are equivalent. However, with zero grade it is impossible to determine the effectiveness of the grade terms in the energy-related model. The accuracy was further evaluated using data from fuel consumption tests recently undertaken by the Australian Road Research Board using the Commodore test car[7]. This set of data includes accelerations, declerations and steady-speed driving on roads with grades varying from −6 per cent to 6 per cent. The vehicle parameters for both test cars calculated by the method outlined in Section 5 are included in Table I.

Figure 2 shows the measured and predicted steady-speed fuel consumption for speeds of between 15 and 120 km/h (4 and 34 m/sec.) for the two test cars. The errors in predicted steady-speed fuel consumption are generally less than 5 per cent except at low speeds (less than 50 km/h (14 m/sec.)) where fuel consumption is influenced by the speed at which the gear-change occurs.

The errors in predicted fuel consumption using the Cortina test car for a range of accelerations and decelerations and over 'acceleration-short cruise-deceleration' cycles are summarised in Table III. The mean errors in acceleration fuel consumption are less than 10 per cent for all accelerations and less than 6 per cent for accelerations in the range most commonly experienced on the road. Fuel consumption was also measured during six full throttle accelerations to a final speed of 110 km/h (31 m/sec.) and the model underestimated fuel consumption by 9 per cent on average. The magnitude of the errors in deceleration fuel consumption are small, but as a percentage of deceleration fuel consumption they can be quite high, especially for hard decelerations from low speeds. However, for all decelerations from initial speeds of 60 km/h (17m/sec.) or more, mean errors are less than 10 per cent of deceleration fuel consumption. The estimation accuracy of the model over 'acceleration-short cruise-deceleration' cycles was found to be very good, with mean errors less than 5 per cent of total fuel consumption. The variation in the errors was also found to be relatively small, the standard deviation of the errors ranging from 1 to 8 per cent of total fuel consumption.

Similar results were found using the more comprehensive data collected by the Commodore test car. The model was tested over 1 112 'cruise-deceleration-idle-acceleration-cruise' (CDIAC) cycles collected over 36 routes through three intersections. Cruise speeds varied between 50 and 85 km/h (14 and 24 m/sec.) and average total section distance was 600 m. During the tests the weather was generally dry, with little wind (less than 1 m/sec.,) although on two of the 19 test days light drizzle fell part of the time and wind speeds of 2.1, 2.8 and 5.3 m/sec. were recorded on three days. Wind speeds of this magnitude are estimated by the model

Table III. Errors in predicted fuel consumption during accelerations, decelerations and over acceleration-cruise-deceleration cycles for the Cortina test car

Maximum speed (m/sec.)	Error Measure	Errors in fuel consumption (mL) for acceleration/deceleration rates*								
		Slow			Medium			Hard		
		Accel.	Decel.	Cycle	Accel.	Decel.	Cycle	Accel.	Decel.	Cycle
8.3	Obs. fuel	24.0	6.6	33.5	15.8	4.4	25.1	15.4	3.0	23.6
	Mean error	−2.3	0.4	−0.5	−0.7	0.8	0.9	−0.9	1.0	−0.1
	(%)	(−10%)	(6%)	(−1%)	(−4%)	(18%)	(4%)	(−6%)	(33%)	(0%)
	SD error	1.0	1.1	1.1	0.9	1.0	1.0	1.0	1.4	1.8
	No. of tests	13	26	13	18	19	18	19	2	11
16.7	Obs. fuel	70.8	18.0	91.8	54.5	11.8	69.3	50.0	8.5	71.5
	Mean error	3.3	−1.5	2.3	−0.8	−0.3	−0.8	0.2	−0.5	−0.2
	(%)	(5%)	(−8%)	(3%)	(−1%)	(−4%)	(−1%)	(0%)	(−6%)	(0%)
	SD error	1.2	1.3	4.3	1.2	1.1	1.0	0.7	1.2	1.1
	No. of tests	8	4	8	8	9	8	8	11	8
25	Obs. fuel	170	27.7	192.0	122	19.1	141.0	108	13.6	137.7
	Mean error	1.1	−0.7	0.7	2.9	0.9	1.6	−0.3	−0.4	−0.9
	(%)	(1%)	(−3%)	(0%)	(2%)	(5%)	(1%)	(0%)	(−3%)	(−1%)
	SD error	2.0	2.8	1.6	1.8	1.3	2.3	2.1	1.4	3.0
	No. of tests	8	6	7	8	7	6	8	10	8

*Acceleration rates for slow, medium and hard accelerations were 0.4 m/sec.^{2}, 0.8 m/sec.^{2} and 1.4 m/sec.^{2}, respectively. Similarly, deceleration rates for slow, medium and hard decelerations were in the ranges 0 to 0.8 m/sec^2, 0.8 to 1.1 m/sec.^{2} and greater than 1.1 m/sec^2, respectively
For the cycles, acceleration rates were the same as above acceleration rates and deceleration rates were a mixture of the above rates

Table IV Errors in predicted fuel consumption over Cruise-Deceleration-Idle-Acceleration-Cruise (CDIAC) cycles for Commodore test car

Mean F_s (mL)	Mean error in F_s (mL)	(%)	SD of error (mL)	Number of runs	Main features of 'CDIAC' cycle*
27.8	1.1	(4)	1.5	58	Cruise at 60 downhill, $G = -3.0$
60.6	1.2	(2)	2.1	61	Cruise at 60 uphill, $G = 3.0$
117.6	2.3	(2)	4.5	101	Accel. to 60 uphill, $G = 4.5$
74.6	1.0	(1)	2.4	97	Accel. to 60 downhill, $G = -1.2$
65.6	1.6	(2)	2.9	99	Accel. to 60 uphill, $G = 1.5$
81.0	1.3	(2)	4.3	265	Accel. to 60 on small grades**
91.6	2.0	(2)	4.7	108	Accel. to 60 uphill, $G = 2.2$
176.5	1.4	(1)	7.6	89	Accel. to 80 uphill, $G = 5.4$
90.3	−1.7	(2)	4.7	104	Accel. to 60 uphill, $G = 2.5$
128.8	−3.3	(−3)	6.8	91	Accel. to 80 plus cruise downhill, $G = -2.1$
142.6	4.5	(3)	6.2	19	Cruise at 80 uphill, $G = 3.8$
63.9	4.8	(8)	3.1	20	Cruise at 80 downhill, $G = -3.8$
92.7	0.91	(1.0)	4.6	1112	All above cycles

* Speed given in km/h

** Winds of 2.1, 2.8 and 5.3 m/sec. were recorded on 3 of the 4 days on which these data were collected and average grades during acceleration were between ±2 per cent

to affect fuel consumption by less than 3 mL over a cycle and would therefore have little effect on the overall result. The fuel consumption errors over the CDIAC cycles were divided into groups according to the grade and final acceleration speed and are summarised in Table IV. For almost all cycles the mean errors are less than 4 per cent of cycle fuel consumption and the overall mean error of 1.0 per cent is very small. The greatest errors occur over cycles where the vehicle cruises, with some fluctations in speed, on roads with fairly steep uphill grades (greater than 3 per cent).

It should be noted that fuel consumption was measured using a fuel flow meter and that there is a small variable delay between the time the meter measures fuel and the time the engine uses the fuel. Thus the predicted and measured fuel consumption values may not correspond exactly. This could result in errors of up to about 2 mL and will have the greatest effect on percentage errors over short intervals.

In summary, the energy-related model generally predicts fuel consumption well for steady-speed driving, during accelerations and over CDIAC cycles. However, for some types of driving mean errors of 10 per cent occur. These are discussed further in Section 4.

4. Discussion of model

The energy model described in this paper is designed for use in the area of urban traffic management. It therefore uses such variables as speed, acceleration and grade which are the most detailed type of information likely to be available in this area. The model attempts to estimate the various components of fuel consumption separately so that the incremental effects of changes in traffic management can be estimated. Several approximations and assumptions are made in the development of this model and the implications of these approximations are discussed below.

One of the model approximations is that fuel required to maintain engine operation is the same when idling as when the engine is providing tractive force to the vehicle. The engine drag will be greater when the vehicle is moving, but this extra drag is included in the drag function, Equation (3), rather than in the fuel to maintain engine operation. This assumption allows the α parameter to be estimated independently of the drag and efficiency parameters and has been found to have almost no effect on the prediction accuracy of the model.

Fuel consumption is dependent not only on speed and acceleration, but also on the engine revolution rate (RPM) of the test vehicle. This is clearly shown by comparing the fuel consumption rates for steady-speed driving in different gears shown in Fig 3. The rolling resistance and aerodynamic drag will be the same in each gear for a given speed; the difference between the curves represents the effect of engine drag on fuel consumption. However, RPM is a vehicle rather then a traffic variable and has not been included specifically in the energy-related model. Instead, the model estimates the drag component of fuel consumption at steady-speeds where the vehicle is in a relatively high gear and the β_2 term allows for the increased fuel consumption caused by the transmission being in a low gear (therefore high RPM) during acceleration, especially accelerations uphill. This term also allows for the decrease in engine efficiency at very high levels of power.

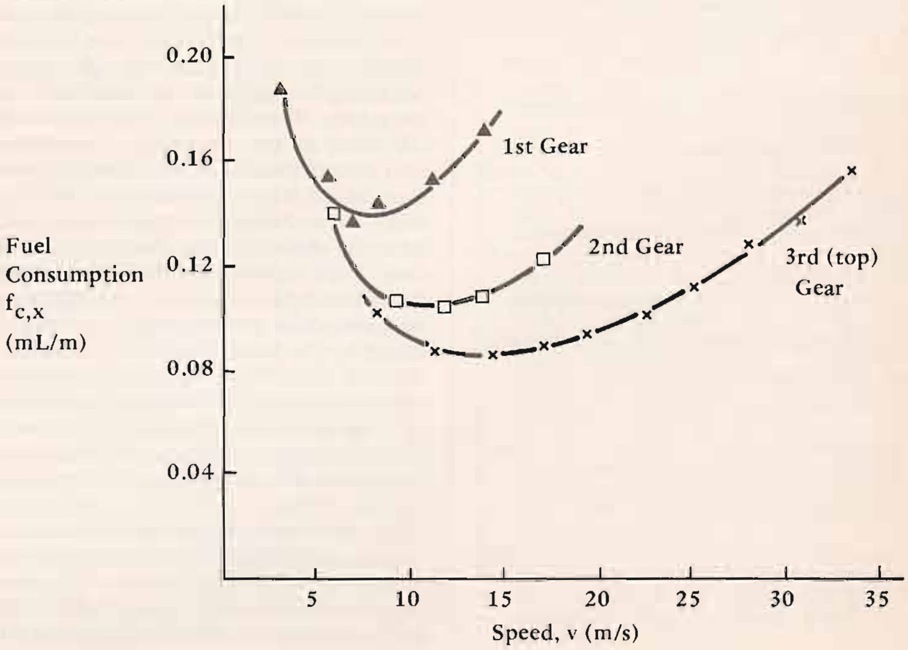
The use of the β_2 term to reflect the increase in fuel consumption at high RPM has been found to work well when the car is driven in a typical manner through a range of gears. However, two types of driving where this approximation gives relatively larger errors are discussed below:

(*a*) At low steady-speeds, the fuel consumption rate is very dependent on gear, and errors in predicted fuel consumption will occur since RPM is not included. The model predicts steady-speed fuel consumption for the vehicle in a relatively high gear so that the RPM is low. This is typical of the gear used in traffic situations where the vehicle must maintain a low steady-speed. For example, vehicles with automatic transmission select a high gear at low steady-speeds.

(*b*) When accelerating slowly in top gear (e.g. from 60 to 70 km/h) the β_2 term will contribute to predicted fuel consumption, but since the car remains in top gear this contribution is not required. These types of accelerations in top gear occur when cruising (with speed fluctuations) at about 60 km/h or faster and result in overestimation of cruise fuel consumption.

The errors in predicted fuel consumption for these types of driving are not great (about 10 per cent) and for most types of driving the errors are less than 5 per cent. The model is therefore suitable for

Fig 3. Speed versus fuel consumption for a 3.3-litre Commodore in first, second and third gears.



application in the area of traffic management. Several other important properties of this model are discussed below.

It is generally recognised that the mass of the vehicle is effectively increased by between about 2 and 10 per cent because of the inertia of the engine, wheels, tyres and drive-line. The effect of the increased mass has not been included specifically in the model as the effective mass is difficult to measure and is dependent on speed and gear. However, the model allows for this effect through the efficiency parameters, β_1 and β_2, which relate to overall vehicle efficiency, not just engine efficiency.

The relationship between acceleration rate and fuel consumption can be investigated using the energy-related model. Biggs and Akçelik[4] calculated the optimal acceleration rates for the Cortina test car to be 3.5 and 2.0 km/h/sec. (1.0 and 0.6 m/sec.^{2}) for accelerations to 30 and 60 km/h (16.7 and 25 m/sec.), respectively. Typical acceleration profiles were assumed. These compare with the values calculated from measured on-road data of 4.7 and 1.6 km/h/sec. (1.3 and 1.45 m/sec.^{2}). The power model proposed by Post *et al*[2] cannot predict optimal acceleration rates and the extension of the model outlined in this paper has therefore greatly enhanced the model.

5. Calibration of the model

The idle, drag and efficiency parameters are calibrated using different data-sets so that the model will accurately esimate the contribution of each energy *component* to fuel consumption. It is usually overlooked in the literature that this may not be the case if free regression is used to estimate all parameters based on one data-set containing all modes of driving (eg. drive cycle data). All data should be collected with the vehicle fully warmed-up.

Two methods have commonly been used for estimating the drag parameters. Post *et al*[2] suggest calculating the drag force on a vehicle from the speed-time trace of a vehicle coasting-down in neutral gear. Another method involves measuring fuel consumption rates during steady-speed driving and relating these, through the fuel consumption model, to the drag force on the vehicle. The drag force calculated using the latter method is roughly 30 to 40 per cent greater than when based on the coast-down data. When the vehicle is coasting-down in neutral gear any drag associated with the engine will not be present. The drag force calculated from steady-speed fuel consumption data has been found to be quite close to the drag force experienced by the vehicle while it is coasting-down in gear. From the above findings it can be concluded that engine drag makes up a significant proportion of total drag. The steady-speed data are therefore used to estimate the drag parameters to ensure that all components of drag are included.

To calibrate the model, firstly parameter α is set to the idle fuel consumption rate measured over at least 200 sec. Using on-road steady-speed fuel consumption data, the parameters c_1 and c_2 of Equation (9) are estimated by regression with α fixed to the previously determined value. Note that when collecting steady-speed data, the gear used in a manual car should be similar to that used in the same car with automatic transmission (i.e. generally a higher gear than would have been used had the vehicle been accelerating), the road gradient should be approximately zero and tests should be conducted in both directions. The efficiency parameters are found using instantaneous (usually second-by-second) speed, grade and fuel consumption data collected on the road or on a dynamometer. These data should include at least 1 000 sec. of driving and cover the full range of speeds and accelerations which occur on the road. If the data are collected on a dynamometer, it will be necessary to estimate another set of c_1 and c_2 parameters applicable to steady-speed fuel consumption on a dynamometer.

As mentioned in Section 3, time-lags between measurement and use of fuel by the engine make it necessary to aggregate data into time intervals. An aggregation interval of between 10 and 20 sec. is recommended for fuel measurements using a fuel flow meter and between 1 and 5 sec. by the gas analysis method on a dynamometer. To determine β_1 and β_2 with data divided into time intervals, each consisting of a certain number of data-points, firstly calculate for each data-point the inertial-grade terms:

$$P_{IG} = M a_e v / 1\,000 \qquad \ldots (10)$$

$$a_e P_{IG} = M a_e^2 v / 1\,000 \qquad \ldots (11)$$

Then, aggregate each of these terms as well as the inertial-grade component of fuel consumption (difference between total measured and steady-speed fuel consumptions) into the time intervals as follows:

$$F_{IG}^{(k)} = \Sigma\,(f_t - f_{c,t}) \qquad \ldots (12)$$

$$P_{IG}^{(k)} = \Sigma [P_{IG}]_{P_T>0} \qquad \ldots (13)$$

$$E^{(k)} = \Sigma [a_e P_{IG}]_{P_T>0,\, a_e>0} \qquad \ldots (14)$$

Where the summations are over the points in time interval k subject to additional restrictions on total power, P_T, and effective acceleration, a_e

$f_{c,t}$ is the steady-speed fuel consumption estimated using Equation (9)

the total power for a data point in interval k is:

$$P_T = P_{IG} + c_1 v / \beta_1^{(0)} + c_2 v^3 / \beta_1^{(0)} \qquad \ldots (15)$$

where $\beta_1^{(0)}$ is a first guess of β_1 (e.g. use default value of β_1)

Estimate the values of β_1 and β_2 jointly by regression of $P_{IG}^{(k)}$ and $E^{(k)}$ on $F_{IG}^{(k)}$ through the origin (i.e. with no constant). Substitute the new estimate of β_1 for $\beta_1^{(0)}$ in Equation (15) and re-estimate β_1 and β_2. Repeat this procedure until the estimated value of β_1 does not change. Note that negative values of β_2 should not be obtained and a likely cause of this error is that the time interval for aggregation is too small. The drag parameters, b_1 and b_2 can then be found by: $b_1 = c_1/\beta_1$ *and* $b_2 = c_2/\beta_1$.

Table V. The contribution of components to fuel consumption for the Cortina and default cars based on a three-term drag function

Location	Car*	Components of fuel consumption (%) Engine Operation	Roll-drag	Engine drag	Air-drag	Inertia	aR_I	Grade
CBD	Default	65.1	9.9	4.0	2.8	11.0	5.7	1.5
	Cortina	70.1	7.9	3.6	1.6	8.8	6.6	1.4
Other urban	Default	44.2	14.9	12.7	11.9	9.4	5.4	1.5
	Cortina	51.2	12.9	12.4	7.4	8.2	6.7	1.2
Non-urban	Default	21.9	16.9	19.1	33.6	3.1	2.3	3.1
	Cortina	28.8	16.6	20.2	23.6	3.1	3.2	4.5

*The drag function for the Cortina test car is given by Equation (18) and for the default car this drag function is adjusted for the smaller mass and engine capacity to be:
$R_D = 0.269 \times 1\,200/1\,680 + 0.0171 \times 2.6/4.1v + 0.000672v^2$

6. Components of drag

The three components of drag (i.e. rolling, aerodynamic and engine drag) can be estimated separately using both coast-down in neutral data and steady-speed fuel consumption data. The coast-down (in neutral) data include drag associated with rolling resistance (all frictional forces from the rear end of the gearbox to the tyres) and aerodynamic drag. With b_1 and b_2 of Equation (3) found using coast-down (in neutral) data, b_1 will be a measure of the rolling resistance on the vehicle and b_2v^2 will reflect the aerodynamic force on the vehicle. For the Cortina test car the function for estimating the rolling and aerodynamic drag was found to be:

$$R_{RA} = 0.269 + 0.000672\,v^2 \quad \ldots (16)$$

where R_{RA} is the total rolling and aerodynamic components of drag.

The total drag force on the vehicle can be calculated from steady-speed data by:

$$R_D = (f_{c,t} - \alpha)/(\beta_1 v) \quad \ldots (17)$$

The engine drag can then be found by subtraction of the drag, R_{RA}, found from coast-down in neutral data, from R_D given by Equation (17).

The engine drag is related to the engine revolution rate (RPM) and to the engine size. As discussed in Section 4, RPM is not included in the energy-related model, but the engine drag can be related roughly to vehicle speed. For the Cortina test car, the engine drag was found to be linearly related to speed and the following equation was found for estimating total drag:

$$R_D = 0.269 + 0.0171v + 0.000672\,v^2 \quad \ldots (18)$$

Thus, the three terms of Equation (18) explicitly represent rolling, engine and aerodynamic drag, respectively. The constant term in the three-term drag function is proportional to vehicle mass since rolling resistance is proportional to mass. The final term is related to the shape (therefore the drag coefficient) and frontal area of the vehicle. Note that if the coefficients of a three-term function for drag are estimated simultaneously by regression using only the steady-speed data, multicollinearity (caused by the high correlation between v and v^2) will make the interpretation of parameters unreliable and often a negative parameter value is found.

One use of the instantaneous fuel consumption model is determining the contribution of different components to fuel consumption. Using the 1 929 km of on-road instantaneous speed, acceleration and grade data collected by Post *et al*[2] in Sydney, the contributions of various components for the Cortina test car and the default car were calculated (the procedure is documented in Biggs and Akçelik[4]). The results are given in Table V. The importance of fuel to maintain engine operation is clearly illustrated in this table, although it should be noted that the Cortina had a high idle fuel consumption rate. From such a table, it is possible to calculate the effect on fuel consumption of changes in vehicle parameters. For example, since mass affects the rolling drag component, both inertia components and the grade component, a 10 per cent decrease in the mass of the Cortina test car will decrease fuel consumption in the CBD by ([7.9 + 8.8 + 6.6 + 1.4] × 10/100) = 2.5 per cent.

7. Conclusions

An instantaneous fuel consumption model has been specified which relates fuel consumed to the tractive forces provided by the engine. The model separates the fuel required to maintain engine operation and to provide tractive force to the vehicle and further separates the tractive force components into drag, inertia and gradient forces.

Mean errors in steady-speed and acceleration fuel consumption and fuel consumed during cycles which include acceleration, cruise, deceleration and idle periods, were less than 12 per cent. These cycles included grades of up to 5 per cent. For cruise speeds, accelerations and cycles most commonly experienced on the roads mean errors were less than 6 per cent.

The idle, drag and efficiency parameters must be calibrated using separate data-sets so that the model will accurately estimate the contribution of each component of fuel consumption. Idle, steady-speed and acceleration fuel consumption data are therefore required and the procedure outlined in Section 5 should be used. If speed-time data for the vehicle coasting-down in neutral gear are also available, the three components of drag (rolling, aerodynamic and engine) can be estimated separately.

Careful calibration of the parameters of the energy-related model allow the components of fuel consumption to be identified. The incremental effects on fuel consumption of changes in traffic management can therefore be estimated. The model is therefore suitable for the design of detailed traffic management schemes and can be used when speed-time data are available from a field survey or from a microscopic traffic simulation model which generates speed-time traces of individual vehicles. More aggregate models have been derived by Biggs and Akçelik[4] which are suitable for the design of traffic management schemes when less detailed data are available (e.g. cruise speeds, number of stops, stopped time, etc.). These models are described in a guide to fuel consumption analyses for urban traffic management[5] and detailed examples on the use of all models are given. Note that these models were derived from the energy-related model described by Biggs and Akçelik[4] which is less sensitive to grade than the model described here, but the two models are equivalent when grade is zero.

ACKNOWLEDGEMENTS

The authors thank the Executive Director of the Australian Road Research Board (ARRB), Dr M. G. Lay, for permission to publish this paper. The work reported in this paper was undertaken through a project supported by the National Energy Research, Development and Demonstration Council (NERDDC) of Australia. The views expressed are those of the authors, and not necessarily those of ARRB or NERDDC.

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