

## REPRINT

# Acceleration profile models for vehicles in road traffic

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### NOTE:

This paper is related to the intersection analysis methodology used in the SIDRA INTERSECTION software. Since the publication of this paper, many related aspects of the traffic model have been further developed in later versions of SIDRA INTERSECTION. Though some aspects of this paper may be outdated, this reprint is provided as a record of important aspects of the SIDRA INTERSECTION software, and in order to promote software assessment and further research.

# Acceleration Profile Models for Vehicles in Road Traffic

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*Three new models of acceleration profile (a two-term sinusoidal, a three-term sinusoidal, and a polynomial model) are described. These models yield the S-shaped speed-time trace indicated by data from driving in real-life traffic conditions, satisfy the realistic conditions of zero jerk (except the two-term sinusoidal model) and zero acceleration at the start and end of the acceleration, and allow for the position and the value of the maximum acceleration to vary for a given average acceleration rate. A comparative evaluation of these three models and the previously known constant and linear-decreasing acceleration models is reported. The evaluation criteria are distance traveled and fuel consumed during acceleration. The performances of the five models are compared under three sets of conditions: acceleration time and distance known, time known but distance unknown, and both time and distance unknown. The comparisons are made separately for central business district (CBD), other urban and nonurban traffic conditions. The polynomial model has been found to be the best overall for predicting acceleration distance and fuel consumption. Similar results have been found for deceleration profiles. Dependence of fuel consumption on acceleration rate and profile is also discussed. Suggestions for further work using the results of this report are included.*

## INTRODUCTION

The modeling of the acceleration and speed profiles of vehicles in a traffic stream is a fundamental question in traffic and fuel consumption modeling. The term *acceleration profile* will be used to refer to the acceleration-time trace of a vehicle during an acceleration from an initial speed of  $v_i$  to a final speed of  $v_f$ . Similarly, the corresponding speed-time trace will be called the *speed profile* of the vehicle. Models of acceleration/speed profiles of vehicles in road traffic can be used in microscopic traffic simulation models in conjunction with an instantaneous fuel consumption model to estimate fuel consumption during accelerations and decelerations. They are useful in investigating the effect of the acceleration profile and the acceleration rate on fuel consumption.

The need for a mathematical description of the acceleration/speed profile for the purpose of fuel consumption modeling was discussed and constant and linear-decreasing models of acceleration considered in early papers by BAYLEY,<sup>[1]</sup> AKCELIK<sup>[2]</sup> and AKCELIK AND BAYLEY.<sup>[3]</sup> While the constant acceleration model is a convenient simplification, some empirical evidence for the linear-decreasing acceleration model was given by SAMUELS<sup>[4]</sup> and LEE et al.<sup>[5]</sup> However, both of these models assume a high initial acceleration

value which is not realistic. Our observation of data from driving in real-life traffic indicated that the speed-time curve typically had an S shape as shown in Figure 1. This was based on the examination of extensive second-by-second speed-time data collected by the Sydney University, Mechanical Engineering Department, by means of an instrumented car in urban, suburban and rural road conditions (TOMLIN et al.<sup>[6]</sup>). These data were collected using the chase-car method, and included all types of acceleration in traffic. There is large variation in data, and the acceleration and speed profiles are not often as smooth as indicated by data points in Figure 1 which represents a fairly typical case. Data collected by JARVIS<sup>[7]</sup> also indicate S-shaped speed profiles for vehicles accelerating from a rural intersection. An interesting aspect of Figure 1 is that it emphasizes the physical requirements for the model of zero acceleration,  $a = 0$ , and zero jerk,  $da/dt = 0$ , at the start and end of acceleration (at times  $t = 0$  and  $t = t_a$ ). Complete notation for this study is given in Table I.

Acceleration profiles vary widely from driver to driver and are also dependent on the type of vehicle and the traffic and weather conditions. A robust model is required which will provide good estimates of acceleration and speed profiles in a wide variety of situations. Three new models of acceleration profile,

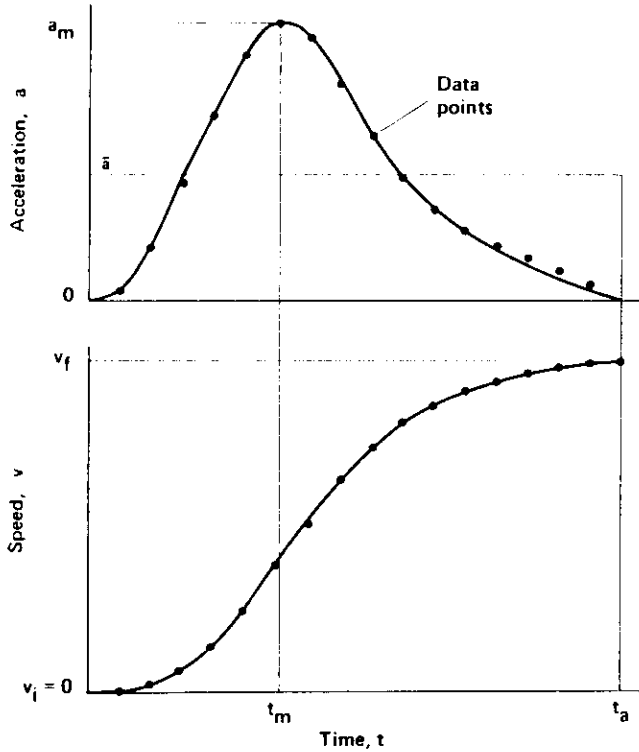


Fig. 1. Typical acceleration and speed profiles for accelerations during on-road driving when initial speed is zero.

TABLE I

Notation

$t$	= time since the start of acceleration
$a(t)$	= acceleration rate at time $t$ ( $dv/dt$ )
$v(t)$	= speed at time $t$ ( $dx/dt$ )
$x(t)$	= distance traveled at time $t$
$t_a$	= acceleration time, i.e. the total time to reach the final speed
$t_m$	= time of maximum acceleration
$a_m$	= maximum acceleration, $a(t_m)$
$\bar{a}$	= average acceleration, $(v_f - v_i)/t_a$
$x_a$	= distance traveled during acceleration, $x(t_a)$
$v_i$	= initial speed, $v(0)$
$v_f$	= final speed, $v(t_a)$
$v_a$	= average speed during acceleration, $x_a/t_a$
$F_a$	= total fuel consumed during acceleration
$F_v$	= excess fuel consumed during acceleration, $F_a - f_c x_a$ where $f_c$ is the steady-speed cruise fuel consumption per unit distance
$\rho$	= a ratio which relates to the shape of the speed-time curve of acceleration, $(v_a - v_i)/(v_f - v_i)$
$\sigma$	= acceleration ratio, i.e. the ratio of acceleration rate at time $t$ to the average acceleration rate, $a/\bar{a}$
$\sigma_m$	= maximum acceleration ratio, $a_m/\bar{a}$
$\theta$	= time ratio, i.e. the ratio of time since the start of acceleration to the total acceleration time, $t/t_a$
$\theta_m$	= time ratio for maximum acceleration, $t_m/t_a$

namely a two-term sinusoidal, a three-term sinusoidal and a polynomial model were considered for this purpose. The sinusoidal models, as well as the constant and linear-decreasing acceleration models

are described in Appendix A. The polynomial model, which was found to be the best model in a comparative evaluation of these models, is described in detail in the following section. The method of evaluation is described in Appendix B. All of the three new models yield the S-shaped profiles indicated by real-life data, satisfy the realistic conditions of zero acceleration at the start and end of the acceleration, and allow for the position and the value of maximum acceleration to vary for a given average acceleration rate. Only the three-term sinusoidal and polynomial models satisfy the conditions of zero jerk at the start and end of the acceleration.

Following the description of the polynomial model, the method of estimating model parameters is explained. In the subsequent sections, the results of the comparison of the constant, linear-decreasing, two and three-term sinusoidal and polynomial acceleration models are summarized for both acceleration and deceleration cases. Finally, a discussion of the dependence of fuel consumption on acceleration rate and profile is presented. In the concluding section, the findings are summarized and recommendations for further work are given. For more detail on the findings reported in this paper, see AKCELIK et al.<sup>[8]</sup> and BIGGS AND AKCELIK.<sup>[9]</sup>

#### THE POLYNOMIAL MODEL OF ACCELERATION

THE GENERAL form of the polynomial function to estimate the acceleration rate at time  $t$  is given by:

$$a(t) = r a_m \theta^n (1 - \theta^m)^2 \quad (n > 0, m > -0.5 n) \quad (1)$$

where

- $a(t)$  = acceleration rate at time  $t$ ,
- $a_m$  = maximum acceleration,
- $\theta$  = time ratio,  $t/t_a$ ,
- $t_a$  = acceleration time,
- $m, n$  = parameters to be determined, and
- $r$  = a parameter which depends on the values of  $m, n$ .

This function satisfies the following conditions:

- (a) zero acceleration at the start and end of the acceleration:

$$a = 0 \quad \text{at} \quad t = 0 \quad \text{and} \quad t = t_a \quad (2)$$

(i.e. at  $\theta = 0$  and 1)

- (b) zero jerk at the start and end of the acceleration:

$$da/dt = 0 \quad \text{at} \quad t = 0 \quad \text{and} \quad t = t_a \quad (3)$$

(i.e. at  $\theta = 0$  and 1).

It should be noted that there is a discontinuity in the jerk function at the point where  $n = 1$ . The jerk is

zero for values of  $n$  which approach one from above and below one. However, the fact that the function gives nonzero jerk for  $n = 1$  has little practical importance as shown by the small difference overall, and the negligible difference near the start of acceleration, between the acceleration profiles derived using  $n = 1$  and  $n = 1.1$  (Biggs and Akcelik<sup>[9]</sup>). Therefore, the condition of  $n > 0$  is used in Equation 1.

Given the conditions  $n > 0$  and  $m > -0.5n$ , the parameters  $m$  and  $n$  can be adjusted so that the function represents the range of acceleration profiles which occur on the road. The values of parameters  $n$  and  $m$  could be chosen according to one, or a combination, of the following profile characteristics.

- The ratio of average speed in acceleration to the final speed,  $\rho = v_a/v_f = x_a/v_f t_a$  for zero initial speed. This is equivalent to choosing acceleration distance as a profile characteristic for determining  $n$  and  $m$ .
- The position of maximum acceleration,  $\theta_m = t_m/t_a$ , where  $t_m$  is the time when maximum acceleration is achieved,  $a(t_m) = a_m$ . Unlike the sinusoidal models described in Appendix A, the polynomial model has practically no restriction on the value of  $t_m/t_a$ , i.e. the maximum acceleration can occur anytime in the range 0 to  $t_a$ .
- The sharpness of the acceleration-time curve as characterized by the maximum acceleration ratio  $\sigma_m = a_m/\bar{a}$ .

The values of  $n$  and  $m$  should ideally be chosen to satisfy conditions on both  $\theta_m$  and  $\rho$  (or both  $\sigma_m$  and  $\rho$ ), simultaneously. However, it is not possible to give analytical expressions for the simultaneous calculation of  $m$  and  $n$ . As a simple procedure, a value of  $n$  can be chosen first, and then the value of  $m$  can be chosen using the given value of  $n$ . Tests using variable values of  $n$ , compared with  $n$  set to 1.0 and 1.1 have shown that the increase in fuel consumption accuracy when  $n$  is allowed to vary is very small. See Biggs and Akcelik<sup>[9]</sup> for details. Comparison of errors in acceleration distance and fuel consumption for the cases where  $n$  is constant indicate that the model with  $n = 1.0$  is at least as good as, if not better than, the model with  $n = 1.1$ . Therefore, the following simpler model with  $n = 1.0$  is recommended for use in practice:

$$a(t) = ra_m\theta(1 - \theta^m)^2 \quad (m > -0.5) \quad (4)$$

where  $a$  is in the same units as  $a_m$  (m/s<sup>2</sup> or km/h/s).

Other important relationships for the polynomial model given by Equation 4 are as follows:

$$v(t) = v_i + t_a r a_m \theta^2 [0.5 - 2\theta^m/(m+2) + \theta^{2m}/(2m+2)] \quad (5)$$

where  $v$  is km/h and  $a_m$  is km/h/s (or  $v$  is in m/s and  $a$  is in m/s<sup>2</sup>).

$$r = [(1 + 2m)^{2+1/m}]/4m^2 \quad (6)$$

$$a_m = \bar{a}/rq = (v_f - v_i)/rqt_a \quad (7)$$

$$q = m^2/[(2m+2)(m+2)] \quad (8)$$

$$x_a = v_i t_a + r s a_m t_a^2 \quad (9)$$

$$s = 1/6 - 2/[(m+2)(m+3)] + 1/[(2m+2)(2m+3)] \quad (10)$$

$$\theta_m = t_m/t_a = (1 + 2m)^{-1/m} \quad (11)$$

$$\sigma_m = a_m/\bar{a} = 1/rq = 8(m+1)(m+2)(1+2m)^{-(2+1/m)} \quad (12)$$

$$\rho = \frac{v_a - v_i}{v_f - v_i} = \frac{s}{q} = \frac{2m^2 + 15m + 19}{3[(m+3)(2m+3)]} \quad (13)$$

where  $v_a = x_a/t_a$ .

The values of  $\theta_m$ ,  $\sigma_m$  and  $\rho$  as a function of  $m$  from Equations 11 to 13 are given in Table II and are shown in Figure 2 for positive values of  $m$ . The observed ranges which include approximately 98% of accelerations in the Sydney data are also given for these parameters in Table II. The following characteristics of the observed accelerations should also be noted:

- all accelerations with  $\theta_m$  larger than 0.70 are for speeds below 60 km/h,
- all accelerations with  $\sigma_m$  larger than 3.0 are for average acceleration rates below 3.5 km/h/s, and
- all accelerations with  $\rho$  less than 0.44 are for speeds below 60 km/h.

Considering these characteristics of data, the values of  $\theta_m$ ,  $\sigma_m$  and  $\rho$  predicted by the polynomial model correspond to the ranges observed in practice reason-

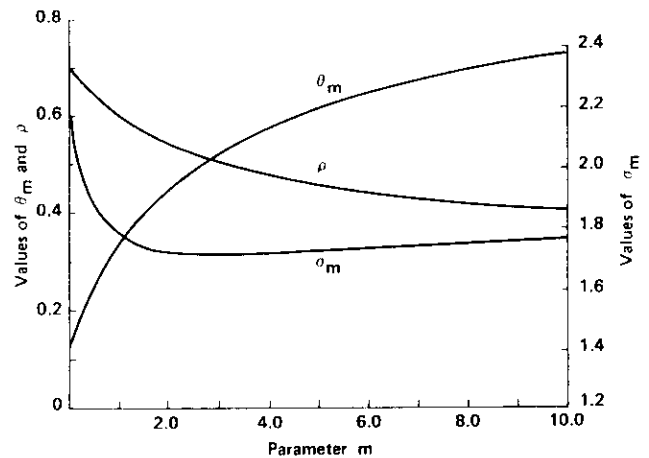


Fig. 2. Values of  $\theta_m$ ,  $\sigma_m$  and  $\rho$  as a function of parameter  $m$  for the polynomial acceleration model ( $n = 1.0$ ).

TABLE II  
Values of  $\theta_m$ ,  $\sigma_m$  and  $\rho$  as a Function of Parameter  $m$  for the Polynomial Acceleration Model ( $n = 1.0$ )

$m$	$\theta_m = t_m/t_a$	$\sigma_m = a_m/\bar{a}$	$\rho = v_a/v_f$ (*)
-0.2	0.08	2.49	0.74
-0.1	0.11	2.30	0.72
0.01	0.14	2.16	0.70
0.06	0.15	2.11	0.70
0.1	0.16	2.07	0.69
0.5	0.25	1.88	0.64
1.0	0.33	1.78	0.60
2.0	0.45	1.72	0.54
4.0	0.58	1.71	0.48
6.0	0.65	1.73	0.45
8.1	0.70	1.75	0.43
10.0	0.74	1.77	0.41
Observed Range	0.08 - 0.78	1.45 - 4.20	0.40 - 0.75

(\*) For acceleration from rest, i.e.  $v_i = 0$  in Equation (13)

ably well. Among these three variables, prediction of  $\rho$  is the most important and prediction of  $\sigma_m$  is the least important in terms of the estimation of acceleration distance and fuel consumption. For this reason, the polynomial model of acceleration performs well as it will be seen in the following sections.

In Figure 3, acceleration profiles predicted by the polynomial model are shown for three values of  $m$ , which represent the range of profiles observed in practice. The corresponding speed profiles are shown in Figure 4. Smaller values of  $m$  (curve A) correspond to accelerations to higher final speeds (curve A) where relatively high maximum accelerations occur closer to the start of acceleration as seen in Figure 3. The area under curve A is the largest in Figure 4, which indicates the largest acceleration distance for a given final speed. The shape of curve A is closest to the shape of the speed profile from a linear-decreasing acceleration model. If the first few seconds of acceleration is ignored, profile A in Figure 3 could, in fact, be approximated by a linear-decreasing acceleration profile (see Appendix A).

The instantaneous fuel consumption values which correspond to the acceleration and speed profiles given in Figures 3 and 4 are shown in Figure 5. These are calculated for an acceleration from rest to  $v_f = 60$  km/h at an average rate of  $\bar{a} = 3.0$  km/h/s using the fuel consumption model described in Appendix C. Figure 5 is further discussed in the section on dependence of fuel consumption on acceleration rate and profile.

ESTIMATION OF MODEL PARAMETERS

AN INDIVIDUAL acceleration is commonly specified by the values of four variables: initial and final

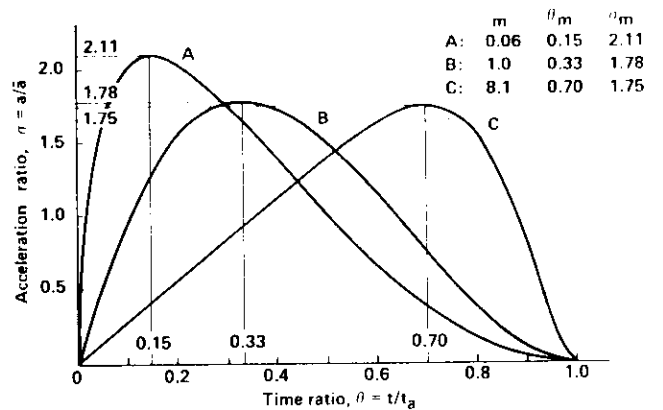


Fig. 3. Acceleration profiles predicted by the polynomial model for varying values of parameter  $m$ .

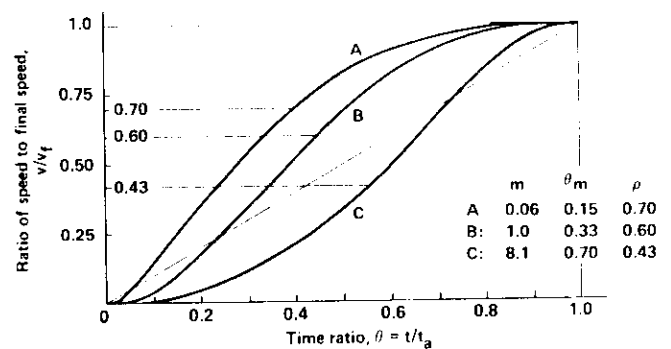


Fig. 4. Speed profiles predicted by the polynomial model for varying values of parameter  $m$  (these profiles correspond to the acceleration profiles in Fig. 3).

speeds,  $v_i$  and  $v_f$ , acceleration time,  $t_a$ , and acceleration distance,  $x_a$ . If the values of these four variables are known, the following steps can be taken to calibrate the model given by Equation 4 for a given

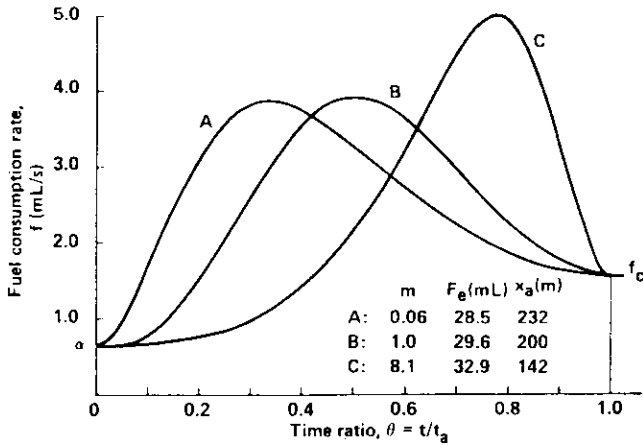


Fig. 5. Fuel consumption rate,  $f$  (mL/s), during acceleration from rest to a final speed at a given average acceleration rate ( $v_f = 60$  km/h,  $\bar{a} = 3$  km/h/s) for various acceleration profiles ( $m = 0.06$ ; 1.0; 8.1 as in Figs. 3 and 4). Fuel consumption is predicted by the energy model described in Appendix C ( $\alpha$  is the idle fuel rate and  $f_c$  is the constant-speed fuel consumption rate at speed  $v_f$ ).

acceleration profile:

(a) First, calculate  $\rho$  as follows:

$$\rho = (3.6 x_a/t_a - v_i)/(v_f - v_i) \quad (14)$$

where  $x_a$  is in meters,  $t_a$  is in seconds,  $v_i$  and  $v_f$  are in km/h.

(b) Then choose a value of  $m$  so that the modeled acceleration distance corresponds to the observed distance. For this purpose, calculate  $m$  from the following equation using the value of  $\rho$  calculated in the first step:

$$m = (-A_1 + (A_1^2 - 4A_0A_2)^{1/2})/2A_2 \quad (15)$$

where

$$A_0 = 27\rho - 19 \quad (15a)$$

$$A_1 = A_0 + 4 \quad (15b)$$

$$A_2 = 6\rho - 2. \quad (15c)$$

It should be noted that it is necessary to truncate the value of  $\rho$  to a value slightly larger than  $1/3$  since the value of  $m$  approaches infinity as  $\rho$  approaches  $1/3$  and  $A_2$  approaches zero.

(c) Finally, calculate the value of  $ra_m$  from:

$$ra_m = [2(m + 1)(m + 2)/m^2]\bar{a} \quad (16)$$

where  $\bar{a} = (v_f - v_i)/t_a$ .

**Example**

Given  $v_i = 0$ ,  $v_f = 81$  km/h,  $t_a = 27$  s,  $x_a = 340$  m.

(a) From Equation 14,  $\rho = 45.3/81 = 0.560$ .

(b) From Equation 15,

$$A_0 = -3.88, \quad A_1 = 0.12, \quad A_2 = 1.36 \quad \text{and} \quad m = 1.65.$$

(c) From Equation 16,

$$\bar{a} = 81/27 = 3.0 \text{ km/h/s}$$

$$\text{and} \quad ra_m = 7.106 \times 3.0 = 21.317.$$

Therefore, from Equation 4, acceleration in km/h/s is given by

$$a(t) = 21.317(t/27)(1 - (t/27)^{1.65})^2.$$

From Equation 5, speed in km/h is given by

$$v(t) = 575.56(t/27)^2 \cdot (0.5 - 0.548(t/27)^{1.65} + 0.189(t/27)^{3.3}).$$

From Equation 11, the maximum acceleration is predicted to occur when  $\theta_m = t_m/t_a = 0.413$  ( $t_m = 11.2$  s). Therefore, the value of maximum acceleration is

$$a_m = a(t_m) = 21.317 \times 0.413 \times 0.589 = 5.2 \text{ km/h/s},$$

$$\text{and} \quad \sigma_m = a_m/\bar{a} = 5.2/3.0 = 1.73.$$

The speed at the time when maximum acceleration is reached is

$$v(t_m) = 575.56 \times (0.413)^2 \times 0.383 = 37.6 \text{ km/h}.$$

**Estimation when Acceleration Time and/or Distance Are Unknown**

If the acceleration time,  $t_a$ , and/or the acceleration distance,  $x_a$ , are not known, the following regression equations, which are based on Sydney data can be used to estimate  $t_a$  and/or  $x_a$  from known initial and final speeds,  $v_i$  and  $v_f$ :

$$t_a = \frac{v_f - v_i}{2.08 + 0.127(v_f - v_i)^{1/2} - 0.0182v_i} \quad (17)$$

$$x_a = (0.467 + 0.0020v_f - 0.0021v_i)(v_i + v_f)t_a/3.6 \quad (18)$$

where  $v_i$ ,  $v_f$  are in km/h,  $t_a$  is in seconds, and  $x_a$  is in meters.

For the example given above,  $v_i = 0$ ,  $v_f = 81$  km/h, and hence,  $t_a = 81/3.223 = 25.1$  s and  $x_a = 0.629 \times 81 \times 25.1/3.6 = 355$  m are estimated. Using these values,  $\rho = 0.629$  is found from Equation 13 (compare with  $\rho = 0.560$  for the known values of  $t_a = 27$  s and  $x_a = 340$  m).

**COMPARISON OF ALTERNATIVE MODELS**

THE BASIS of evaluating alternative models of acceleration profile was the comparison of relative errors

in the prediction of acceleration distance and acceleration fuel consumption. The method of evaluation is described in detail in Appendix B. The following five models of acceleration were compared in terms of their ability to predict the acceleration distance and the acceleration fuel consumption:

- (a) Constant acceleration model (Equation A.1 in Appendix A).
- (b) Linear-decreasing acceleration model (Equation A.5 in Appendix A).
- (c) Two-term sinusoidal acceleration model (Equation A.9 in Appendix A).
- (d) Three-term sinusoidal acceleration model (Equation A.18 in Appendix A).
- (e) Polynomial acceleration model (Equation 4 in the previous section).

The observed acceleration profiles were extracted from extensive on-road data collected by the Sydney University, Mechanical Engineering Department, using an instrumented 3.3 L automatic GMH Commodore sedan (Tomlin et al.<sup>[6]</sup>). The chase-car technique was used while driving in traffic. Tests were conducted over 956 km of urban roads and 1361 km of nonurban roads. A weighted three-point moving average method of smoothing was applied to the raw speed-time data provided by the Sydney University. This method of smoothing was applied in order to decrease the amount of random variation in the second-by-second speed values without losing real changes in speed which occur in on-road driving. The second-by-second acceleration values were calculated using smoothed speed values. A detailed description of the method used is given in Akcelik et al.<sup>[8]</sup>

In the study reported here, only accelerations from the "stop" position were analyzed. A "stop" was taken as a speed of less than 1 km/h. At the same time, accelerations to less than 20 km/h were not included. Therefore, the data were limited to conditions represented by  $v_i = 0$  and  $v_f \geq 20$  km/h. The end of the acceleration was taken as the time at which the speed stopped increasing and did not increase greatly during the next 5 seconds. On this basis, a total of 1037 accelerations were identified for analysis.

The comparisons of different acceleration models were made separately for the three driving environments: central business district (CBD), urban (excluding CBD) and nonurban. The comparisons were made under three sets of conditions:

- (a) acceleration time and distance ( $t_a, x_a$ ) known,
- (b) acceleration time ( $t_a$ ) known but distance ( $x_a$ ) unknown, and
- (c) acceleration time and distance ( $t_a, x_a$ ) unknown.

In case (a), the estimated model parameters  $B$ ,  $P$  and  $m$  for the sinusoidal and polynomial models were based on known acceleration distances. In cases (b) and (c), where the acceleration distance was treated as unknown, the model parameters were estimated as a function of the final speed,  $v_f$ , using the following equations which are regression equations derived for the Sydney data:

2-term sinusoidal:

$$B = 0.13 + 0.0054v_f, \quad (19)$$

or 0.5 whichever is smaller.

3-term sinusoidal:

$$P = -0.097 - 0.0018v_f, \quad (20)$$

or -0.25 whichever is larger.

Polynomial:

$$\rho = 0.53 + 0.0013v_f. \quad (21)$$

For the polynomial model, parameter  $\rho$  is used in Equation 15 to calculate the value of parameter  $m$ .

In case (c) where the acceleration time,  $t_a$ , was treated as unknown, it was estimated from Equation 17 by putting  $v_i = 0$ .

## Results

The percentage errors in distance and fuel consumption (Equations B.4 and B.5 in Appendix B) were calculated for all accelerations for each of the five models under the three driving locations. Fuel consumption values were calculated for the Melbourne University test car whose characteristics are described in Appendix C. The mean and standard deviation (SD) of the percentage errors were found for each of the three driving locations and are given in Tables III and IV. The SD of the percentage errors for any one subset of the data varies among the models by only a small amount indicating that the variation is due, to a large extent, to driver behavior. The mean percentage error, or the bias, is a measure of the error in estimation when accelerations are repeated many times. This is important if the models are to be included in traffic network simulations where the average distance and total fuel consumption over many accelerations are of interest. The SD of the percentage error is of less importance since the SD of the average values of acceleration distance and fuel consumption decrease as the number of accelerations made by vehicles in the network increases. The average of the standard percentage errors, S%E (Equation B.6 in Appendix B) over the three driving locations for each model and each set of conditions regarding  $t_a, x_a$  are given near

the bottom of Tables III and IV. The results are discussed below.

(a) *The results for  $t_a$  and  $x_a$  known:* The constant and linear-decreasing models of acceleration do not make use of the fact that  $x_a$  is known. The errors are therefore greater than for the other three models where a parameter is dependent on  $x_a$  (calculated through Equations 19 to 21). Errors in estimated distance still occur for the sinusoidal and polynomial models due to restrictions on parameter values. These errors are largest for the three-term sinusoidal model. The polynomial model is superior to the other models in terms of mean and SD of the percentage errors in both distance and fuel consumption.

(b) *The results for  $t_a$  known,  $x_a$  unknown:* Although SD values of percentage errors are similar for all models, the polynomial model is the best model in predicting acceleration distance and fuel consumption in terms of the mean percentage errors. As summarised by the average S%E values, the two-term sinusoidal model is the second best model in predicting acceleration distance and fuel consumption as is the case where both  $t_a$  and  $x_a$  are known.

(c) *The results for  $t_a$  and  $x_a$  unknown:* In terms of S%E values, the predictions from different models are seen to be comparable. However, the mean values of percentage errors in estimated distances are high for the constant, linear-decreasing and two-term sinusoidal models, while the polynomial

TABLE III  
Percentage Errors in Predicted Distance for Five Acceleration Profile Models for Sydney On-Road Acceleration Data

Condition	Location	Mean and SD* of % Error in Estimated Distance				
		Constant	Lin-dec	2-Sin	3-Sin	Polyn
$t_a, x_a$ known	CBD	-12.8 (11.4)	16.3 (15.2)	-1.2 (3.3)	-3.1 (5.2)	0.0 (0.2)
	Urban	-15.1 (10.7)	13.2 (14.3)	-2.2 (3.8)	-4.6 (5.7)	0.0 (0.1)
	Non-urban	-15.2 (10.6)	13.1 (14.2)	-1.8 (3.2)	-4.5 (5.3)	0.0 (0.1)
$t_a$ known $x_a$ unknown	CBD			1.1 (13.2)	-2.7 (12.7)	0.8 (13.2)
	Urban	As for $t_a, x_a$ known		0.9 (12.0)	-4.2 (11.7)	0.2 (12.2)
	Non-urban			4.2 (11.2)	-2.7 (11.2)	3.6 (11.2)
$t_a, x_a$ unknown	CBD	-14.4 (26.6)	14.1 (35.4)	-13.6 (26.8)	-4.5 (29.8)	-0.7 (31.2)
	Urban	-17.7 (26.8)	9.8 (35.7)	-16.9 (27.0)	-7.1 (30.1)	-2.2 (31.7)
	Non-urban	-17.5 (24.5)	10.0 (32.6)	-16.7 (24.7)	-5.4 (27.6)	1.3 (29.2)
Average S%E for:						
$t_a, x_a$ known		18.1	20.4	3.9	6.8	0.1
$t_a$ known, $x_a$ unknown		18.1	20.4	12.4	12.5	12.4
$t_a, x_a$ unknown		30.8	36.4	30.6	29.8	30.8
Average SE (m) for $t_a, x_a$ unknown		76.0	57.3	75.1	64.2	59.4

\* Standard deviation of percentage error is given in brackets below mean percentage error.



TABLE IV  
*Percentage Errors in Predicted Acceleration Fuel Consumption for Five Acceleration Profile Models for Sydney On-Road Acceleration Data*

Condition	Location	Mean and SD* of % Error in Estimated Fuel Consumption				
		Constant	Lin-dec	2-Sin	3-Sin	Polyn
$t_a, x_a$ known	CBD	-0.4 (4.8)	1.9 (7.6)	1.9 (3.3)	6.6 (4.6)	1.2 (2.7)
	Urban	-0.5 (4.7)	2.3 (5.6)	2.4 (3.4)	7.2 (4.6)	1.2 (2.8)
	Non-urban	10.7 (4.6)	2.5 (5.5)	4.0 (3.6)	9.2 (5.1)	2.6 (2.3)
$t_a$ known $x_a$ unknown	CBD			0.8 (4.9)	5.5 (5.5)	0.9 (4.8)
	Urban	As for $t_a, x_a$ known		1.9 (4.5)	6.5 (5.1)	1.2 (4.2)
	Non-urban			3.6 (4.9)	8.6 (5.8)	1.6 (4.1)
$t_a, x_a$ unknown	CBD	-2.2 (7.2)	0.1 (8.3)	1.2 (7.5)	3.4 (7.8)	-1.1 (7.3)
	Urban	-1.0 (7.3)	2.5 (7.8)	2.9 (7.6)	6.1 (8.0)	0.8 (7.3)
	Non-urban	-0.5 (9.1)	3.2 (9.9)	4.2 (9.3)	9.1 (10.0)	2.1 (9.2)
Average S%E for:						
$t_a, x_a$ known		7.8	6.7	4.5	9.1	3.2
$t_a$ known, $x_a$ unknown		7.8	6.7	5.3	8.9	4.6
$t_a, x_a$ unknown		8.0	9.0	8.7	10.9	8.1
Average SE (mL) for $t_a, x_a$ unknown		3.29	3.76	3.78	5.31	3.46

\* Standard deviation of percentage error is given in brackets below mean percentage error.

model is best is this respect. It should also be noted that the constant and two-term sinusoidal acceleration models consistently underestimate the acceleration distances for large final speed values. This is indicated by high values of standard error (SE) for these models given in the bottom line of Table II (when the errors are proportioned to the value being predicted, the SE values place more weight than the S%E values on the errors for high distance and fuel consumption values). Although to a lesser extent, the three-term sinusoidal model also underestimates acceleration distances for high final speeds. In terms of the estimation of fuel consumption, the polynomial and constant acceleration models have similar performances when estimated values of  $t_a$  as well as  $x_a$  are used.

It is concluded that the polynomial model is the best overall in predicting the acceleration distance and fuel consumption. This is clearly so when at least the acceleration time,  $t_a$ , is known. The predictive power of the polynomial model is displayed in the plots of the fuel consumption (estimated by energy model and adjusted to observed acceleration distance (Equation B.2) in Appendix B), and distance of the observed profile versus those for the profile predicted by the polynomial model in Figures 6 and 7. It is seen in Figure 6 that the accuracy of fuel consumption estimates is very high when the acceleration time and distance are known. On the other hand, the accuracy of distance estimates is not as high when both the acceleration time and distance are unknown as seen in Figure 7. The scatter is mainly because of variation in the acceleration rate for a given final speed which

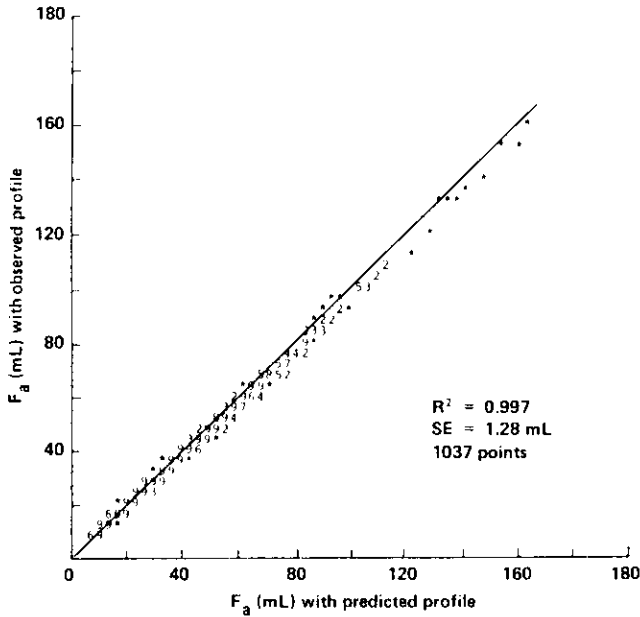


Fig. 6. Observed versus predicted acceleration fuel consumption where the acceleration profile is predicted by the polynomial model using the observed values of acceleration time and distance,  $t_a$  and  $x_a$ .

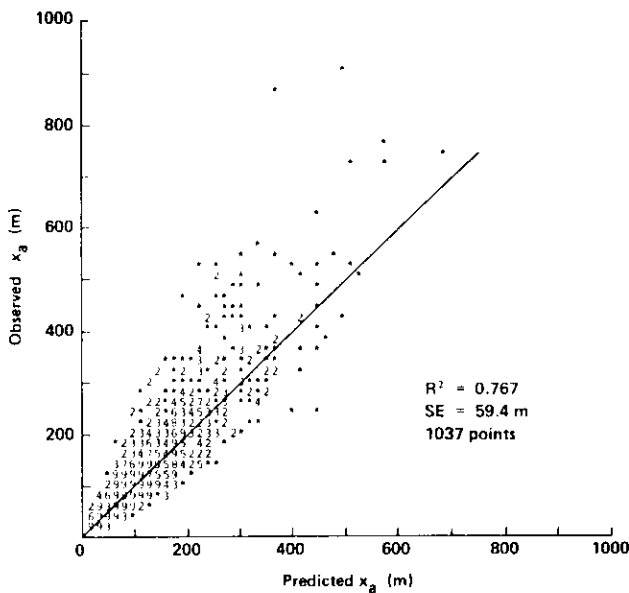


Fig. 7. Observed versus predicted acceleration distance where the acceleration profile is predicted by the polynomial model. The acceleration distance is calculated from  $x_a = \rho v_i t_a$  where  $\rho$  is estimated from Equation 21 and  $t_a$  is estimated from Equation 17.

is not allowed for when the estimated mean values of acceleration time are used.

To determine the general applicability of these acceleration profile models it is necessary to test the models on independent data sets. A small set of on-

road speed data collected in Melbourne was available for this task and the results of this analysis showed that the polynomial model (with parameters derived from Sydney data) was found to be the best of the five models at predicting acceleration distance and fuel consumption under the three sets of testing conditions on  $t_a$  and  $x_a$ . However, the Melbourne data included only 11 accelerations and further testing using more extensive data is necessary.

APPLICATION TO DECELERATION PROFILES

DECELERATIONS from an initial speed,  $v_i$ , to a final speed,  $v_f < v_i$ , were identified in the Sydney data using a procedure which is similar to, but the reverse of, the acceleration case ( $v_f > v_i$ ). As with the case of accelerations, only decelerations from cruise to rest ( $v_f < 1$  km/h) were identified. Only decelerations from  $v_i \geq 20$  km/h were considered, which gave 1058 decelerations for analysis. See Biggs and Akcelik<sup>[9]</sup> for details.

The acceleration-time and speed-time traces during a deceleration have the typical shapes shown in Figure 8. Comparing with Figure 1, it is seen that the acceleration and speed profiles for decelerations are almost mirror images of the profiles for accelerations but with the time of absolute maximum acceleration occurring later.

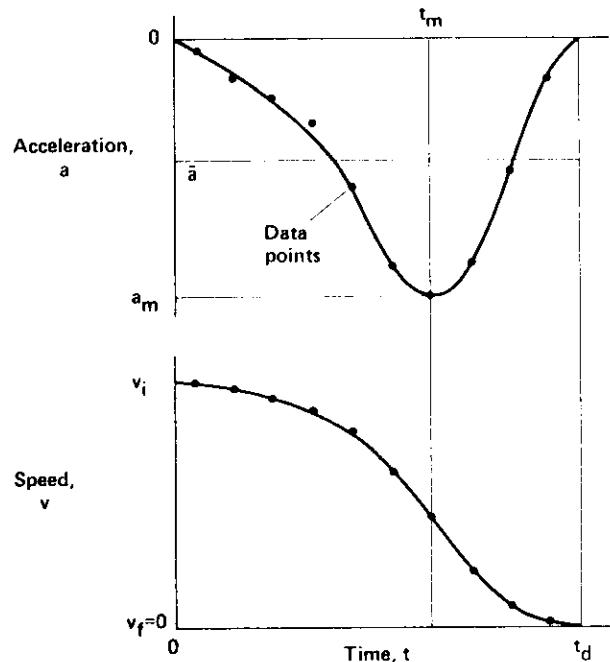


Fig. 8. Typical acceleration and speed profiles for decelerations during on-road driving when final speed is zero.

deceleration (slowing down) profiles. Similar to Equations 17 and 18, the following regression equations derived from Sydney data can be used to estimate deceleration time,  $t_d$ , and distance,  $x_d$ , when these are not known.

$$t_d = \frac{v_i - v_f}{1.71 + 0.238(v_i - v_f)^{1/2} - 0.0090v_f} \quad (22)$$

$$x_d = (0.473 + 0.00155v_i - 0.00137v_f)(v_i + v_f)t_d/3.6 \quad (23)$$

where  $v_i$ ,  $v_f$  are in km/h,  $t_d$  is in seconds, and  $x_d$  is in meters.

For example, for a deceleration from 81 km/h to rest,  $t_d = 81/3.852 = 21.0$  s and  $x_d = 0.599 \times 81 \times 21.0/3.6 = 283$  m are estimated. Average speed during the deceleration is  $v_a = 3.6 \times 283/21.0 = 48.5$  km/h, hence  $\rho = 48.5/81 = 0.599$  is found.

The five acceleration models were tested with profiles for decelerations from a final speed to rest in a similar way to comparisons for acceleration profiles. The results are summarized in Tables V and VI. In the cases where the deceleration distance,  $x_d$ , was treated as unknown, the parameters of the sinusoidal and polynomial models were calculated from the following regression equations derived from the Sydney

TABLE V  
Percentage Errors in Predicted Distance for Five Deceleration Profile Models for Sydney On-Road Deceleration Data

Condition	Location	Mean and SD* of % Error in Estimated Distance				
		Constant	Lin-dec	2-Sin	3-Sin	Polyn
$t_d, x_d$ known	CBD	10.9 (13.5)	14.7 (17.5)	0.0 (0.0)	-2.8 (5.4)	-0.4 (1.4)
	Urban	13.3 (12.5)	12.5 (16.1)	-0.0 (0.0)	-4.2 (6.2)	-1.0 (2.7)
	Non-urban	-11.7 (11.9)	14.9 (15.2)	0.0 (0.0)	-3.4 (5.4)	-0.7 (2.2)
$t_d$ known $x_d$ unknown	CBD			1.5 (15.2)	5.1 (15.9)	0.9 (15.1)
	Urban	As for $t_d, x_d$ known		1.0 (14.1)	2.3 (14.7)	0.0 (14.1)
	Non-urban			5.7 (14.1)	4.2 (14.0)	4.2 (13.8)
$t_d, x_d$ unknown	CBD	-0.5 (32.9)	28.6 (42.6)	13.4 (37.6)	17.4 (38.8)	13.6 (37.7)
	Urban	-7.7 (31.6)	19.9 (40.9)	7.4 (36.4)	8.9 (37.2)	7.7 (36.5)
	Non-urban	3.6 (39.6)	35.2 (51.7)	24.0 (47.4)	22.2 (46.7)	24.3 (47.6)
Average S%E for:						
$t_d, x_d$ known		17.4	21.5	0.0	6.7	2.3
$t_d$ known, $x_d$ unknown		17.4	21.5	14.9	15.4	14.6
$t_d, x_d$ unknown		33.7	49.4	40.4	41.3	40.5
Average SE (m) for						
$t_d, x_d$ unknown		65.7	60.1	59.1	59.2	59.1

\* Standard deviation of percentage error is given in brackets below mean percentage error.

TABLE VI  
Percentage Errors in Predicted Fuel Consumption for Five Deceleration Profile Models for Sydney On-Road Deceleration Data

Condition	Location	Mean and SD* of % Error in Estimated Fuel Consumption				
		Constant	Lin-dec.	2-Sin	3-Sin	Polyn
$t_d, x_d$ known	CBD	2.9 (11.0)	-10.2 (9.5)	1.8 (6.4)	5.3 (6.8)	-1.7 (3.3)
	Urban	1.3 (13.5)	-11.2 (11.3)	5.8 (33.8)	8.7 (9.4)	-2.5 (4.9)
	Non-urban	-2.9 (18.5)	-16.7 (14.3)	3.0 (11.0)	9.9 (11.5)	-4.3 (7.4)
$t_d$ known $x_d$ unknown	CBD			-0.8 (9.1)	2.3 (11.0)	-1.5 (9.0)
	Urban	As for $t_d, x_d$ known		-0.1 (10.9)	6.4 (13.3)	-2.7 (10.3)
	Non-urban			-0.3 (14.0)	(8.4) (15.8)	-8.2 (14.0)
$t_d, x_d$ unknown	CBD	5.4 (14.8)	-9.0 (14.8)	1.6 (14.0)	5.0 (16.5)	0.7 (13.8)
	Urban	1.61 (15.5)	-12.0 (16.3)	0.0 (14.5)	6.7 (16.6)	-2.7 (14.0)
	Non-urban	-8.6 (26.3)	-23.4 (31.7)	-4.7 (26.5)	5.6 (25.9)	-13.2 (27.9)
Average 5%E for:						
$t_d, x_d$ known		14.9	17.6	21.2	12.5	6.3
$t_d$ known, $x_d$ unknown		14.9	17.6	11.5	14.9	12.4
$t_d, x_d$ unknown		20.5	27.5	19.4	21.0	21.2
Average SE (mL) for $t_d, x_d$ unknown		3.7	3.5	3.0	3.8	3.1

\* Standard deviation of percentage error is given in brackets below mean percentage error.

data (for  $v_f = 0$ ):

2-term sinusoidal:

$$B = -0.14 - 0.0039v_i, \tag{24}$$

or  $-0.5$  whichever is larger

3-term sinusoidal:

$$P = 0.097 + 0.0013v_i, \tag{25}$$

or  $0.25$  whichever is smaller

Polynomial:

$$\rho = 0.46 + 0.0008v_i, \tag{26}$$

When the deceleration time,  $t_d$ , was treated as unknown, it was estimated from Equation 22 by putting  $v_f = 0$ .

**The main conclusions from Tables V and VI are:**

- (a) If deceleration time is known, the errors using the polynomial model are smaller for fuel consumption than with the other models and at least as small as the other models for distance, when distance is unknown.
- (b) If deceleration distance is unknown (and time is known or unknown), the two-term sinusoidal and polynomial models have similar errors in distance

and fuel consumption and these errors are generally smaller than for the other three models tested. The percentage errors in distance are smallest for the constant model but the standard error is greatest for that model.

Overall, the polynomial model is the best of the five models tested in representing deceleration profiles.

**DEPENDENCE OF FUEL CONSUMPTION ON ACCELERATION RATE AND PROFILE**

THE DEPENDENCE of fuel consumed during acceleration on the acceleration rate and profile can be investigated using "excess" acceleration fuel consumption as this avoids the problem of different acceleration distances with different acceleration rates and profiles. Excess acceleration fuel consumption,  $F_e$ , is the actual acceleration fuel consumption less fuel consumed while cruising along the acceleration distance at a constant speed equal to the final speed in acceleration.

The excess fuel consumption values for accelerations with average rates of  $\bar{a} = 2, 3$  and  $5$  km/h/s from rest to final speeds of  $v_f = 30, 60$  and  $90$  km/h were calculated for a wide range of profiles and the results are given in Table VII. Excess acceleration fuel consumption as a function of the average acceleration rate and final speed are shown in Figure 9 for fixed acceleration profiles of  $m = 0.06$  and  $m = 8.1$  in the polynomial model. The existence of an optimum acceleration rate for minimum fuel consumption can be observed from Figure 9. This subject has been dis-

cussed in detail in Biggs and Akcelik,<sup>[9]</sup> and observed optimum acceleration rates of 4.7, 1.6 and 1.6 km/h/s have been reported for accelerations from rest to final speeds of 30, 60 and 90 km/h, respectively. Using Equations 18 and 15,  $m = 2.4, 1.2$  and  $0.5$  have been predicted for these speeds and the optimum acceleration rates for these profiles are 3.4, 2.2 and 1.0 km/h/s, respectively, as shown in Figure 9. The corresponding average acceleration rates used in practice are predicted to be 2.8, 3.1 and 3.3 km/h/s from Equation 17. Although these values do not allow for variability among individual drivers, a very low

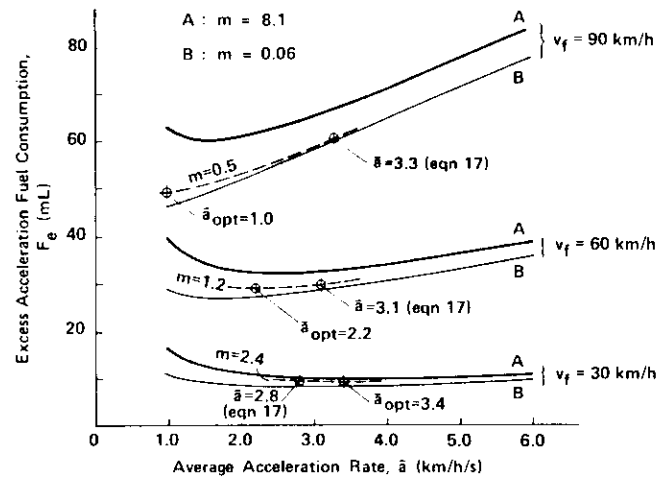


Fig. 9. Excess acceleration fuel consumption as a function of final speed,  $v_f$ , average acceleration rate,  $\bar{a}$ , and acceleration profile (represented by parameter  $m$  of the polynomial model).

TABLE VII  
Excess Acceleration Fuel Consumption for a Range of Acceleration Profiles, Rates and Final Speeds\*

m	$\theta_m$	$\rho$	$v_f = 30$ km/h			$v_f = 60$ km/h			$v_f = 90$ km/h		
			$\bar{a} = 2$	$\bar{a} = 3$	$\bar{a} = 5$	$\bar{a} = 2$	$\bar{a} = 3$	$\bar{a} = 5$	$\bar{a} = 2$	$\bar{a} = 3$	$\bar{a} = 5$
0.06	0.15	0.70	8.6	8.4	9.0	27.1	28.5	33.2	51.8	58.1	71.1
0.5	0.25	0.64	9.1	8.6	9.2	28.0	29.0	33.3	53.0	58.6	70.9
1.0	0.33	0.60	9.5	8.9	9.3	28.8	29.6	33.6	54.2	59.5	71.4
1.2	0.36	0.56	9.7	9.0	9.4	29.1	29.8	33.8	54.6	59.8	71.7
2.4	0.48	0.48	10.3	9.5	9.7	30.4	30.8	34.6	56.8	61.4	73.2
8.1	0.70	0.43	11.4	10.3	10.4	32.9	32.9	36.6	60.8	65.1	77.1
Maximum Percentage Difference between above Profiles.			33%	23%	16%	21%	15%	10%	17%	12%	9%

\* Acceleration rates,  $\bar{a}$ , are in km/h/s. The ratio  $\theta_m = t_m/t_a$  is the position of maximum acceleration. The ratio  $\rho = x_a/v_f t_a$  is for acceleration from rest to  $v_f$  (acceleration time,  $t_a = v_f/\bar{a}$ , and acceleration distance,  $x_a = \rho v_f t_a$  in meters if  $v_f$  is in m/s).

average acceleration rate such as 1.0 km/h/s would be rarely used to accelerate to a high final speed (90 km/h) in practice. Low fuel consumption for  $\bar{a} = 1.0$  km/h/s occurs because less time is spent at speeds near 90 km/h where fuel consumption is high due to the high aerodynamic drag component. However, this is only achieved with a high time penalty, and is likely to have adverse effects in general traffic terms. The optimal rates for accelerations to 30 and 60 km/h are close to observed values indicating that most drivers' acceleration behaviors are not far from optimal.

The effect of *acceleration profile* can be gauged by comparing the values of  $F_c$  for the same  $v_f$  and  $\bar{a}$  in Table VII (i.e. down the columns). The percentages at the bottom of the table indicate that varying profiles can result in differences in excess fuel consumption of 10–30%. It is seen that the profiles with the maximum acceleration at the start use less fuel but this effect is not as strong for hard accelerations ( $\bar{a} = 5$  km/h/s in Table VII). This can be explained with the help of Figures 3 to 5. For small values of  $m$  (e.g.  $m = 0.06$  for curve A), relatively high speeds (about 50 km/h) are reached earlier in the profile and the vehicle will travel at close to its optimal cruise speed for a greater part of the acceleration (optimal cruise speed is the speed which gives minimum fuel consumption per unit distance). For that part of the acceleration above the optimal cruise speed, the acceleration rates are small and more time is spent with speed close to the optimal speed, and hence less fuel is used. The energy-acceleration term of the fuel consumption model ( $a^2v$ ) also indicates that more fuel will be used with high acceleration rates at high speeds (toward the end of acceleration) as seen for  $m = 8.1$  (curve C) in Figure 5. It therefore appears that the best general strategy during acceleration, given an acceleration time or average acceleration rate, is to accelerate hard at the start and slowly after the optimal cruise speed is reached. However, the constraints of a traffic stream may not allow an individual driver to implement such a strategy.

### CONCLUSIONS

THE FOLLOWING conclusions are drawn from the analyses of on-road acceleration profiles and alternative acceleration models for both speeding up and slowing down cases:

- (a) There are large variations between acceleration profiles in real driving conditions.
- (b) The speed-time curve of a typical acceleration has an S shape with the position and value of the maximum acceleration depending on the cruise speed.
- (c) Sinusoidal and polynomial acceleration models can be specified which yield an S-shaped speed profile, satisfy the realistic conditions of zero acceleration and zero jerk (rate of change of acceleration) at the start and end of the acceleration, and allow the position and value of maximum acceleration to vary for a given average acceleration rate.
- (d) The polynomial and sinusoidal acceleration models predict acceleration distance and fuel consumption better than the constant and linear-decreasing acceleration models when acceleration time is known.
- (e) The polynomial acceleration model predicts acceleration fuel consumption better than the other models in the three driving locations, CBD, other urban and nonurban, and for the three sets of testing conditions: acceleration time and distance,  $t_a$  and  $x_a$  known;  $t_a$  known,  $x_a$  unknown; and  $t_a, x_a$  unknown.
- (f) Of the five models tested, the polynomial acceleration model is the best overall for predicting acceleration distance and fuel consumption. Satisfactory results will be obtained from this model only if the measured values of acceleration time are known. However, the best results will be achieved if both the acceleration time and distance are measured.
- (g) When acceleration time,  $t_a$ , and distance,  $x_a$ , are not known, regression equations are available to predict these values as functions of the initial and final speeds,  $v_i$  and  $v_f$ .
- (h) Acceleration fuel consumption is found to depend on both the acceleration rate and the acceleration profile.
- (i) For a given acceleration profile (position of the maximum acceleration fixed), there is an optimum average acceleration rate which depends on the final speed. The optimum acceleration rate predicted by the polynomial model for the car described in Appendix C for accelerating from rest to 60 km/h is 2.2 km/h/s ( $t_m/t_a = 0.36$  predicted).
- (j) For a given acceleration rate, varying acceleration profiles can result in differences in excess fuel consumption of about 10 to 30%. These differences are larger for lower average acceleration rates. Profiles with the maximum acceleration occurring closer to the start of acceleration use less fuel.

### The following subjects are recommended for further study:

- (a) The results reported in this paper are for accelerations from rest ( $v_i = 0$ ), or for decelerations to

rest ( $v_i = 0$ ). Testing of alternative models could be carried out for accelerations with non-zero initial speed and decelerations with nonzero final speed.

- (b) The results given in this paper are for accelerations in all traffic situations. It would be useful to collect and analyze real-life acceleration and deceleration data in specific traffic control situations such as traffic signals, roundabouts, give-way and stop signs.
- (c) A study of the acceleration profiles and rates of the first vehicles to arrive at, and depart from, the stop line of a signalized intersection approach in comparison to the subsequent vehicles in queue would be useful.
- (d) Acceleration rates and profiles of different vehicle types such as manual and automatic transmission, light and heavy vehicles, etc., need to be studied.
- (e) In microscopic (vehicle-by-vehicle) models of traffic, car-following models have commonly been used to generate speed-time traces of vehicles, e.g. NETSIM<sup>[10]</sup> and MULTSIM.<sup>[11]</sup> In this type of model, the first vehicle in a queue is assigned a maximum acceleration value and the other vehicles react to those in front according to a car-following equation. It would be interesting to compare the speed and acceleration profiles resulting from various car-following models used in microscopic traffic simulation models with real-life profiles and with those given by the polynomial model.

#### APPENDIX A: CONSTANT, LINEAR-DECREASING AND SINUSOIDAL ACCELERATION MODELS

THE VARIABLES  $\bar{a}$  (average acceleration rate),  $a_m$  (maximum acceleration),  $\theta$  (time ratio),  $\theta_m$  (time ratio for maximum acceleration) and  $v_a$  (average speed during acceleration) used for the models given in this appendix are defined in Table I.

##### Constant Acceleration Model

The constant acceleration model is the simplest model and assumes that the average acceleration is maintained throughout the acceleration (Fig. 10):

$$a(t) = \bar{a} = (v_f - v_i)/t_a. \quad (\text{A.1})$$

The speed at time  $t$  is given by:

$$v(t) = v_i + (v_f - v_i)\theta. \quad (\text{A.2})$$

The acceleration distance and the average speed are therefore:

$$x_a = 0.5(v_i + v_f)t_a \quad (\text{A.3})$$

$$v_a = x_a/t_a = 0.5(v_i + v_f). \quad (\text{A.4})$$

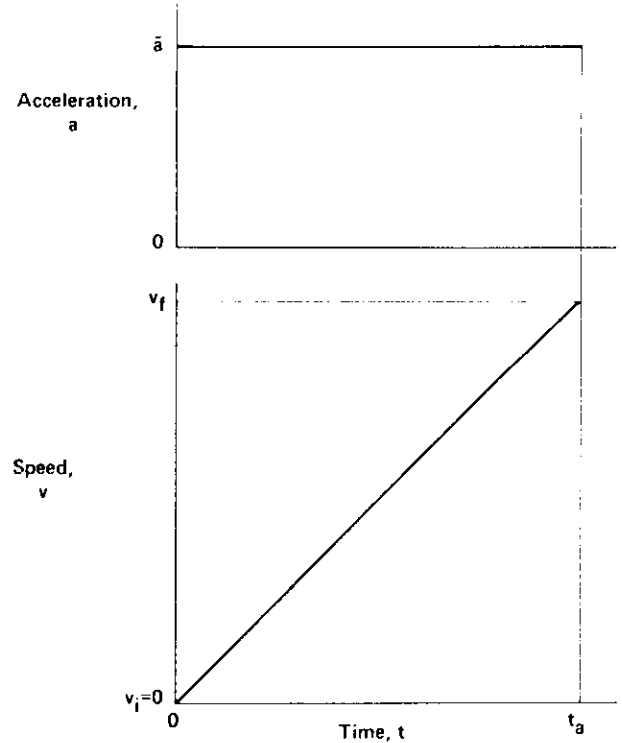


Fig. 10. Acceleration and speed profiles for the constant acceleration model when initial speed is zero.

##### Linear-Decreasing Acceleration Model

The linear-decreasing model (Fig. 11) assumes that the maximum acceleration occurs at  $t = 0$  and that the acceleration decreases steadily to zero as the speed approaches the final speed,  $v_f$ . The acceleration and speed at time  $t$  are given by:

$$a(t) = 2(1 - \theta)(v_f - v_i)/t_a \quad (\text{A.5})$$

$$v(t) = v_i + (2 - \theta)\theta(v_f - v_i). \quad (\text{A.6})$$

For this model, the maximum acceleration is  $2\bar{a}$  (at  $t = 0$ ). The acceleration distance and the average speed are:

$$x_a = (v_i + 2v_f)(t_a/3) \quad (\text{A.7})$$

$$v_a = (v_i + 2v_f)/3. \quad (\text{A.8})$$

From Equation A.4 and A.8, it can be shown that the constant and linear-decreasing acceleration models have constant values of  $\rho = 1/2$  and  $2/3$ , respectively, i.e. the shape of the acceleration and speed profiles do not change for different initial and final speeds.

##### Two-Term Sinusoidal Acceleration Model

The two-term sinusoidal model of acceleration which satisfies the condition of zero acceleration at

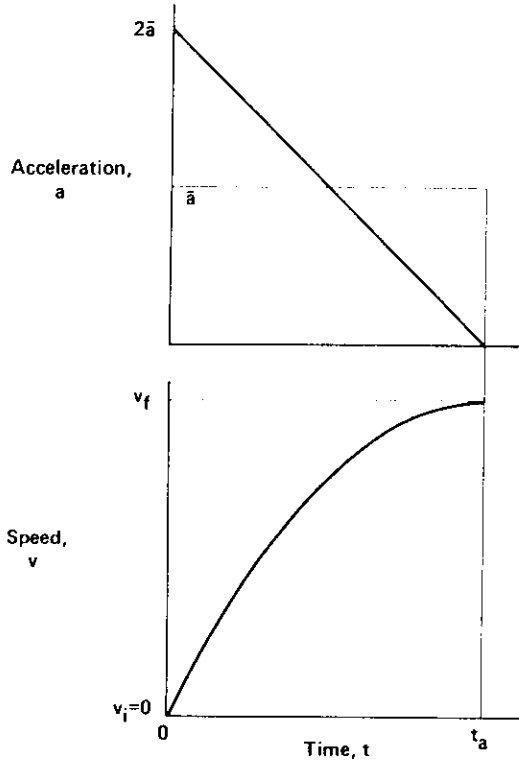


Fig. 11. Acceleration and speed profiles for the linear-decreasing acceleration model when initial speed is zero.

the start and end of acceleration is given by:

$$a(t) = Ca_m(\sin \pi\theta + B \sin 2\pi\theta) \quad (A.9)$$

where  $B$  is a parameter to be determined and  $C$  is a constant which depends on  $B$ .

To ensure that the acceleration predicted by this model is greater than zero for all  $t$  between 0 and  $t_a$ , it is necessary to make the following restriction:

$$-1/2 \leq B \leq 1/2. \quad (A.10)$$

The acceleration distance and the position of the maximum acceleration is related to the parameter  $B$  as follows:

$$B = [4(v_a - v_i)/(v_f - v_i)] - 2 \quad (A.11)$$

$$\theta_m = t_m/t_a = (1/\pi)\cos^{-1}[(-1 + (1 + 32B^2)^{1/2})/8B]. \quad (A.12)$$

The restriction on  $B$  is therefore equivalent to the restriction:

$$1/3 \leq \theta_m \leq 2/3. \quad (A.13)$$

The constant  $C$  is related to  $B$  as follows:

$$1/C = \sin \pi\theta_m + B \sin 2\pi\theta_m. \quad (A.14)$$

Expressions for the maximum acceleration, average speed and the speed at time  $t$  are given below:

$$a_m = 0.5\pi\bar{a}/C \quad (A.15)$$

$$v_a = v_i + 0.5(1 + 0.5B)(v_f - v_i) \quad (A.16)$$

$$v(t) = v_i + 0.5(v_f - v_i)[(1 + 0.5B) - (\cos \pi\theta + 0.5B \cos 2\pi\theta)]. \quad (A.17)$$

From Equation A.16, it is seen that the model gives a variable value of  $\rho = 0.5 + 0.25B$ . From Equation A.10, this means that the allowed range of  $\rho$  for this model is 0.375 to 0.625.

Figures 12 and 13 show that the two-term sinusoidal model produces a result which resembles the typical acceleration profile observed on the road (Fig. 1) more closely than the constant or linear-decreasing models. However, it has several limitations:

- (a) The position of the maximum acceleration ( $t_m/t_a$ ) is restricted to between  $1/3$  and  $2/3$ .
- (b) As  $t_m/t_a$  approaches  $1/3$  or  $2/3$ , the trajectory of the acceleration curve is too flat for times near 0 or  $t_a$ .
- (c) Except with the extreme values of  $t_m/t_a$ , there is jerk at the start and end of the acceleration.

To apply the two-term sinusoidal model given by Equation A.9, it is necessary to determine values of parameters  $B$ ,  $C$  and  $a_m$ . Using the measured or estimated values of  $x_a$  and  $t_a$ ,  $B$  can be calculated from Equation A.11. And from Equation A.15, the value of  $Ca_m = 1.571\bar{a} = 1.571(v_f - v_i)/t_a$ . This method allows the shape of the acceleration profile to vary for different initial and final speeds.

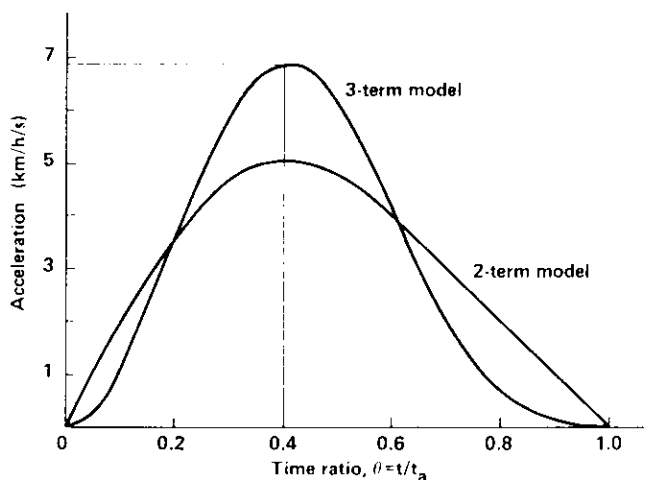


Fig. 12. Comparison of the two-term and three-term sinusoidal models where  $t_m/t_a = 0.4$ ,  $v_f = 81$  km/h and  $t_a = 27$  s.



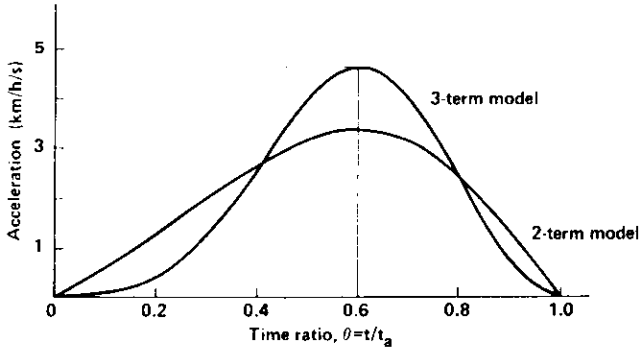


Fig. 13. Comparison of the two-term and three-term sinusoidal models where  $t_m/t_a = 0.6$ ,  $v_f = 40$  km/h and  $t_a = 20$  s.

### Three-Term Sinusoidal Acceleration Model

The two-term sinusoidal model has the important property that the acceleration starts off at zero and tapers off to zero as the speed approaches  $v_f$ . The model does not, however, satisfy the practical constraint that there is no jerk at the start and end of the acceleration (i.e.  $da/dt = 0$  at  $t = 0$  and  $t = t_a$ ). The following three-term sinusoidal model satisfies this constraint:

$$a(t) = Ra_m(0.5 - P \cos \pi\theta - 0.5 \cos 2\pi\theta + P \cos 3\pi\theta) \quad (\text{A.18})$$

$$v(t) = v_i + Rt_a(a_m/\pi)(0.5\pi\theta - P \sin \pi\theta - 0.25 \sin 2\pi\theta + (P/3) \sin 3\pi\theta) \quad (\text{A.19})$$

where  $P$  is a parameter to be determined and  $R$  is a constant which depends on  $P$ . Graphically this function has a similar form to the two-term model but for the same position of maximum acceleration, the value of the maximum is greater and the acceleration approaches zero more smoothly with the three-term model (Figs. 12 and 13).

It is necessary to apply the following constraint so that the acceleration,  $a(t)$ , is greater than 0 for all  $t$  between 0 and  $t_a$ :

$$-0.25 \leq P \leq 0.25. \quad (\text{A.20})$$

Other important relationships for the three-term sinusoidal model are given below:

$$\gamma = \pi\theta_m \quad (\text{A.21})$$

$$P = \cos \gamma / 2(3 \cos^2 \gamma - 1) \quad (\text{A.22})$$

$$R = (1 - 3 \cos^2 \gamma) / \sin^4 \gamma \quad (\text{A.23})$$

$$a_m = 2\bar{a}/R \quad (\text{A.24})$$

$$v_a = v_f - (v_f - v_i)(0.5 + 32P/9\pi^2). \quad (\text{A.25})$$

From Equation A.25, parameter  $P$  is also given as:

$$P = (9\pi^2/32)(-0.5 + (v_f - v_a)/(v_f - v_i)). \quad (\text{A.26})$$

From Equations A.21 and A.22, the position of the maximum acceleration is:

$$\theta_m = (1/\pi) \cos^{-1}(1 - (1 + 48P^2)^{1/2}/12P) \text{ for } P \neq 0 \\ = 0.5 \text{ for } P = 0. \quad (\text{A.27})$$

The restriction on  $P$  (Equation A.20) is therefore equivalent to the following restriction:

$$0.392 \leq \theta_m \leq 0.608. \quad (\text{A.28})$$

The range of possible values of  $\theta_m$  is even more restrictive than the range for the two-term sinusoidal model (Equation A.13) and is a severe limitation to the three-term model. From Equation A.25, it can be shown that the three-term sinusoidal model has a variable value of  $\rho = 0.5 - 0.3603P$ . From Equation A.20, this means that the allowed range of  $\rho$  for this model is 0.41 to 0.59. To use Equation A.18, it is necessary to determine the values of parameters  $P$  and  $Ra_m$ . Using the measured or estimated values of  $x_a$  and  $t_a$ ,  $P$  can be calculated from Equation A.26 and the value of  $Ra_m = 2\bar{a} = 2(v_f - v_i)/t_a$ .

### APPENDIX B: METHODS OF EVALUATING ACCELERATION MODELS

VARIOUS METHODS could be used to compare the predicted acceleration profiles with those observed on the road. For example, the maximum error between data and model in predicted speeds could be used as a goodness of fit measure. However, for the goodness of fit measure to be used for evaluating models of acceleration profile to have a practical meaning and be directly relevant to the problem at hand, the following two measures were used in the analysis presented in this paper:

- distance traveled during acceleration ( $x_a$ ), and
- fuel consumed during acceleration ( $F_a$ ).

Distance traveled during acceleration is the area under the speed-time curve. The error in distance is therefore equal to the sum (integral) of the error in speeds over the acceleration period ( $t_a$  seconds):

$$\Delta x_a = x_{ap} - x_{ao} \\ = \int_0^{t_a} (v_p(t) - v_o(t)) dt \quad (\text{B.1})$$

where

$\Delta x_a$  is the error in predicted acceleration distance,

$x_{ap}$  and  $x_{ao}$  are the predicted and observed acceleration distances, respectively, and  $v_p$  and  $v_o$  are the predicted and observed speeds, respectively.

In the analysis of data, the error in distance was approximated by:

$$\Delta x_a = \sum_{j=1}^{t_a} (v_p(t_j) - v_o(t_j))/3.6 \quad (\text{B.1a})$$

where the units of distance, time and speed are meters, seconds and km/h, respectively, and the summation is over intervals of 1 s.

Fuel consumed during acceleration can be predicted from an instantaneous fuel consumption model which gives fuel consumed at time  $t$ ,  $f(t)$ , as a function of the speed,  $v(t)$ , and acceleration,  $a(t)$ . An energy-related model of instantaneous fuel consumption described by Biggs and Akcelik<sup>[9]</sup> and BOWYER et al.<sup>[12]</sup> has been used for this purpose. The model is summarized in Appendix C. The error in fuel consumption is calculated as the difference between total fuel consumption values predicted using the observed and modeled acceleration and speed profiles (as explained below, a procedure equivalent to the use of excess acceleration fuel consumption is employed to avoid the influence of errors in distance prediction). Therefore, for the particular fuel consumption model used in the evaluation, the error in fuel consumption can be thought of roughly as a weighted sum of errors in speed and acceleration ( $v$ ,  $v^3$ ,  $av$ ,  $a^2v$ ) over the acceleration period.

There are several reasons for not using the measured fuel consumption to evaluate the models of acceleration profile. First, fuel consumption is difficult to measure accurately on-road over the small time intervals in which accelerations occur (10–30 s). Even if the fuel consumption values were known for the observed acceleration, they would not be known for the modeled acceleration profile. The energy-related model has been found to predict fuel consumption during accelerations adequately (Biggs and Akcelik<sup>[9]</sup>). Fuel consumption estimated by the energy model for the observed and modeled accelerations are directly comparable. In addition, since they approximate the actual fuel consumption without much of the random variation associated with observed values, they would provide a valuable guide to the real error in fuel consumption associated with the acceleration model.

Another difficulty with using fuel consumption to compare acceleration profiles is that the distance traveled under various profiles will be different. This problem can be overcome by using the observed distance as a base and subtracting or adding the fuel that would have been consumed traveling the extra dis-

tance at final cruise speed,  $v_f$ . More specifically, the fuel consumption adjusted for distance is defined:

$$F'_{ap} = F_{ap} - f_c \Delta x_a \quad (\text{B.2})$$

where

- $F_{ap}$  is the predicted fuel consumption for the acceleration,
- $F'_{ap}$  is the predicted fuel consumption adjusted to observed acceleration distance,
- $f_c$  is the fuel consumption per unit distance traveling at speed  $v_f$ , and
- $\Delta x_a$  is the error in distance as given by Equation B.1.

Thus, the error in acceleration fuel consumption,  $\Delta F_a$ , is:

$$\Delta F_a = F'_{ap} - F_{ao} = F_{ap} - F_{ao} - f_c \Delta x_a \quad (\text{B.3})$$

where  $F'_{ap}$ ,  $F_{ap}$  and  $\Delta x_a$  are as in Equations B.1 and B.2, and  $F_{ao}$  is the predicted fuel consumption under the *observed* acceleration profile.

Rather than using the absolute errors,  $\Delta x_a$ ,  $\Delta F_a$ , which increase with distance and fuel consumption, the percentage errors,  $\% \Delta x_a$ ,  $\% \Delta F_a$  were used in the evaluation of the alternative acceleration models. These are given as:

$$\% \Delta x_a = 100 \Delta x_a / x_{ao} \quad (\text{B.4})$$

$$\% \Delta F_a = 100 \Delta F_a / F_{ao} \quad (\text{B.5})$$

where  $\Delta x_a$ ,  $x_{ao}$ ,  $\Delta F_a$ ,  $F_{ao}$  are as in Equations B.1 and B.3.

As specific measures, the mean percentage error over many accelerations is a measure of the bias in the model, and the standard deviation of percentage errors gives an indication of the variation in model performance over many accelerations. When modeling a single acceleration, a better measure of the error in estimated distance and fuel consumption is the standard percentage error (square root of the mean square percentage error). The standard percentage error (S%E) of the estimated acceleration distance is given by:

$$S\%E(x_a) = \sqrt{\sum_{k=1}^N (\% \Delta x_{ak})^2 / N} \quad (\text{B.6})$$

$$= \sqrt{[\text{mean} (\% \Delta x_{ak})]^2 + [\text{SD} (\% \Delta x_{ak})]^2} \quad (\text{B.7})$$

where  $\% \Delta x_{ak}$  is the value given by Equation B.4 for the  $k$ th acceleration ( $k = 1$  to  $N$ ) and  $N$  is the total number of accelerations.

The S%E is analogous to the SD of the percentage error but the mean percentage error is replaced by

zero. The relationship between the S%E and the mean and SD of the percent error is given by Equation B.7.

#### APPENDIX C: THE ENERGY MODEL FOR ESTIMATING CAR FUEL CONSUMPTION

THE ENERGY-RELATED fuel consumption model described below estimates instantaneous values of fuel consumption from second-by-second speed and grade information. The model is an extended and modified version of the power model described by POST et al.,<sup>[13]</sup> and is described in detail in Biggs and Akcelik<sup>[9]</sup> and Bowyer et al.<sup>[12]</sup> Basically, the model relates fuel consumption during a small time increment,  $dt$ , to:

- the fuel to maintain engine operation,
- the energy consumed (work done) by the vehicle engine while traveling an increment of distance,  $dx$ , during this time period, and
- the product of energy and acceleration during periods of positive acceleration.

Part (c) allows for the inefficient use of fuel during periods of high acceleration. Since energy is  $dE = R_T dx$  where  $R_T$  is the total tractive force required to drive the vehicle along distance  $dx$ , the fuel consumed in the time increment,  $dt$ , is expressed as:

$$\begin{aligned} dF &= \alpha dt + \beta_1 R_T dx + (\beta_2 a R_T dx)_{a>0} \quad \text{for } R_T > 0 \\ &= \alpha dt \quad \text{for } R_T \leq 0 \end{aligned} \quad (C.1)$$

where

- $dF$  = increment of fuel consumed (ml) during travel along distance  $dx$  (m) and in time  $dt$  (s),
- $\alpha$  = constant idle fuel rate (ml/s), which applies during all modes of driving (as an estimate of fuel used to maintain engine operation),
- $\beta_1$  = an efficiency parameter which relates fuel consumed to the energy provided by the engine, i.e. fuel consumption per unit of energy (ml/kJ),
- $\beta_2$  = an efficiency parameter which relates fuel consumed during positive acceleration to the product of inertia energy and acceleration, i.e. fuel consumption per unit of energy-acceleration (ml/(kJ · m/s<sup>2</sup>)),
- $a$  = instantaneous acceleration ( $dv/dt$ ) in m/s<sup>2</sup>, which has a negative value for slowing down, and
- $R_T$  = total "tractive" force required to drive the vehicle, which is the sum of drag force ( $R_D$ ), inertia force ( $R_I$ ) and grade force ( $R_G$ ) in kN (kilonewtons):

$$R_T = R_D + R_I + R_G. \quad (C.2)$$

The resistive forces can be expressed as:

$$R_D = b_1 + b_2 v^2 \quad (C.3)$$

$$R_I = Ma/1000 \quad (C.4)$$

$$R_G = 9.81M(G/100)/1000 \quad (C.5)$$

where

$v$  = speed ( $dx/dt$ ) in m/s,

$G$  = percent grade which has a negative value for downhill grade,

$M$  = vehicle mass in kg, including occupants and any other load, and

$b_1, b_2$  = the vehicle parameters related mainly to rolling resistance and aerodynamic drag, but each containing a component due to drag associated with the engine. The parameters of the drag function (Equation C.3) are derived using steady-speed fuel consumption data. However, if data collected during coast-down in neutral are also available, a three-term function  $R_D = b_1 + b_2 v + b_3 v^2$  can be derived where  $b_1, b_2$  and  $b_3$  are related to rolling, engine and aerodynamic drag, respectively.

The following parameter values derived for the Melbourne University test car (4.1-L Ford Cortina station-wagon with automatic transmission) were used for the analyses reported in this paper:

$$M = 1680 \text{ kg}$$

$$\alpha = 0.666 \text{ ml/s}$$

$$\beta_1 = 0.0717 \text{ ml/kJ}$$

$$\beta_2 = 0.0344 \text{ ml/(kJ} \cdot \text{m/s}^2\text{)}$$

$$b_1 = 0.527 \text{ kN}$$

$$b_2 = 0.000948 \text{ kN(m/s)}^{-2}.$$

When the engine drag was allowed for separately, the three drag parameters were found to be  $b_1 = 0.269$ ,  $b_2 = 0.0171$  and  $b_3 = 0.000672$ . However, the two-term drag function was used for the analyses reported in this paper.

Fuel consumption per unit time (ml/s) can be expressed as:

$$\begin{aligned} f_t = dF/dt &= \alpha + \beta_1 R_T v + \left( \frac{\beta_2 M a^2 v}{1000} \right)_{a>0} \quad \text{for } R_T > 0 \\ &= \alpha \quad \text{for } R_T \leq 0 \end{aligned} \quad (C.6)$$

where the total tractive force required is:

$$R_T = b_1 + b_2 v^2 + \frac{Ma}{1000} + 9.81 \times 10^{-5} MG. \quad (C.7)$$

Note that in this form, the energy model becomes an extended form of the power model since  $R_T v = P_T$  is the total tractive power (kW) and  $Ma^2 v = a P_I$ , where  $P_I$  is the inertia power.

Fuel consumption per unit distance (ml/m) can similarly be expressed as:

$$f_x = \frac{dF}{dx} = \frac{f_t}{v} = \frac{\alpha}{v} + \beta_1 R_T + \left( \frac{\beta_2 M a^2}{1000} \right)_{a>0} \quad \text{for } R_T > 0$$

$$= \frac{\alpha}{v} \quad \text{for } R_T \leq 0.$$

(C.8)

Fuel consumption per unit time for constant speed travel along a level road ( $a = 0$ ,  $G = 0$ ) is obtained from the above equations as:

$$f_{c,t} = \alpha + \beta_1 (b_1 + b_2 v^2) v \quad \text{(C.9)}$$

where  $f_{c,t}$  is the constant speed fuel consumption rate (ml/s).

For more detail and examples for the use of the energy model of fuel consumption, see Bowyer et al.<sup>[12]</sup> and Biggs and Akcelik.<sup>[9]</sup>

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